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Credit Booms, Debt Overhang and Secular Stagnation

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Abstract

Why do economies fall into prolonged periods of economic stagnation, particularly in the aftermath of credit booms? We study the interactions between household debt, liquidity and asset prices in a model of persistent demand shortage. We show that financially more deregulated economies are more likely to experience persistent stagnation. Credit booms or asset price booms mask this structural aggregate demand deficiency. However, the resulting debt overhang permanently depresses spending in the long run. Hence, the contractionary long run effects of relaxing lending standards are the opposite of their expansionary short run effects. These findings are in line with the macroeconomic developments in Japan during its lost decades and other advanced economies before and during the Great Recession.

Keywords: Secular Stagnation, Aggregate Demand Deficiency, Liquidity Preferences, Financial Frictions, Credit Booms, Debt Overhang

JEL Classification: E13, E21, E32, E41, E51

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1 Introduction

Many advanced economies suffer from insufficient aggregate demand in the aftermath of the global financial crisis despite unconventional monetary policy actions of unprecedented scales. In addition, the experience of Japan shows that economic stagnation and deflationary tendencies can prevail for decades without any natural recovery. Hence, worries that economies might permanently fail to operate at full employment are widespread and expressed in the “secular stagnation” hypothesis.¹ Proponents of this view emphasize the importance of asset prices, credit availability and private sector debt.

Credit booms or asset price booms, particularly those initiated by a relaxation of lending standards, are seen as a means to temporarily stimulate a stagnating economy. In particular, Summers (2014) argues that the credit boom in the United States in the early 2000s was masking the underlying lack of aggregate demand by initiating unsustainable consumption spending of households. Similar effects were at play during the stock market boom of the 1990s. Therefore, he concludes that “the difficulty that has arisen in recent years in achieving adequate growth has been present for a long time but has been masked by unsustainable finances” and “it has been close to 20 years since the American economy grew at a healthy pace supported by sustainable finance”.²

At the same time, the resulting indebtedness of the private sector is considered a major impediment to economic recovery. From a theoretical perspective, Eggertsson and Krugman (2012) and Korinek and Simsek (2016) illustrate the reduction in private demand due to debt overhang during balance sheet recessions. On the empirical side, the increase in private sector leverage in the United States in the early 2000s is recognized as a major cause of the subsequent prolonged recession (cf. Mian and Sufi, 2011; Mian et al., 2013). In addition, credit growth is a strong indicator for financial crises. These crises are associated with substantially higher output losses than normal recessions, particularly when the credit expansion is driven by real estate lending (cf. Borio and Lowe, 2002; Schularick and Taylor, 2012; Jordà et al., 2015, 2016).

More generally, an expansionary short run effect followed by a significant negative medium run impact of higher household debt on employment and income growth is identified in cross-country studies (cf. Mian and Sufi, 2018; International Monetary Fund, 2017), the analysis of long time series (cf. Schularick and Taylor, 2012; Jordà et al., 2015, 2016) as well as several case studies.³

¹The term “secular stagnation” itself goes back to Hansen (1938) and was taken up by Larry Summers (2013). Yet, Keynes (1936) in Chapter 17 of the *General Theory* already argues that permanent demand shortage can exist as a steady state phenomenon in a monetary economy.

²A related argument is made by Krugman (2013): “In other words, you can argue that our economy has been trying to get into the liquidity trap for a number of years, and that it only avoided the trap for a while thanks to successive bubbles.”

³Ogawa and Wan (2007) provide empirical evidence for Japan. Mian et al. (2017) study the effects of the credit supply expansions associated with deregulation in the 1980s in the United States.

For illustration, consider the transition of Japan from a high growth to a stagnating economy, many features of which are reminiscent of recent developments in other advanced economies. Figure 1 summarizes the macroeconomic developments in Japan.

The secular decline in real economic growth and the persistence of deflationary tendencies over several decades are apparent in panels (a) and (b): From 1980 to 1991, the Japanese economy grew at an average rate of 4.4% in real terms with an annual inflation rate of 1.9%. In contrast, real GDP grew at only 0.9% on average in the period since 1992 with inflation falling into negative territory despite substantial increases in the money supply, particularly during QE in the early 2000s and in the context of “Abenomics”.⁴ Monetary policy became ineffective at stimulating output as the formerly stable relationship between base money and nominal spending substantially changed. Increases in the money supply simply resulted in a decline in the circulation velocity of money.

A distinguishing feature of the high growth and the stagnation period is the behavior of asset prices and credit. As shown in panels (c) and (d), Japan experienced a credit and asset price boom during its high growth period. Within ten years, the outstanding amount of credit to the private non-financial sector as a fraction of GDP increased from 1.4 in 1980 to 2.2 in 1992 primarily driven by aggressive bank lending to small and medium-sized corporations and declining lending standards (see Posen, 2003). Credit to the private sector grew on average by 7.9% in real terms during the period from 1980 to 1991 while residential property prices (as a proxy for collateralizable assets) increased by 5.1% in real terms.⁵ Credit expansion and asset price inflation in terms of stock, land and housing prices were at the core of Japan’s bubble economy.

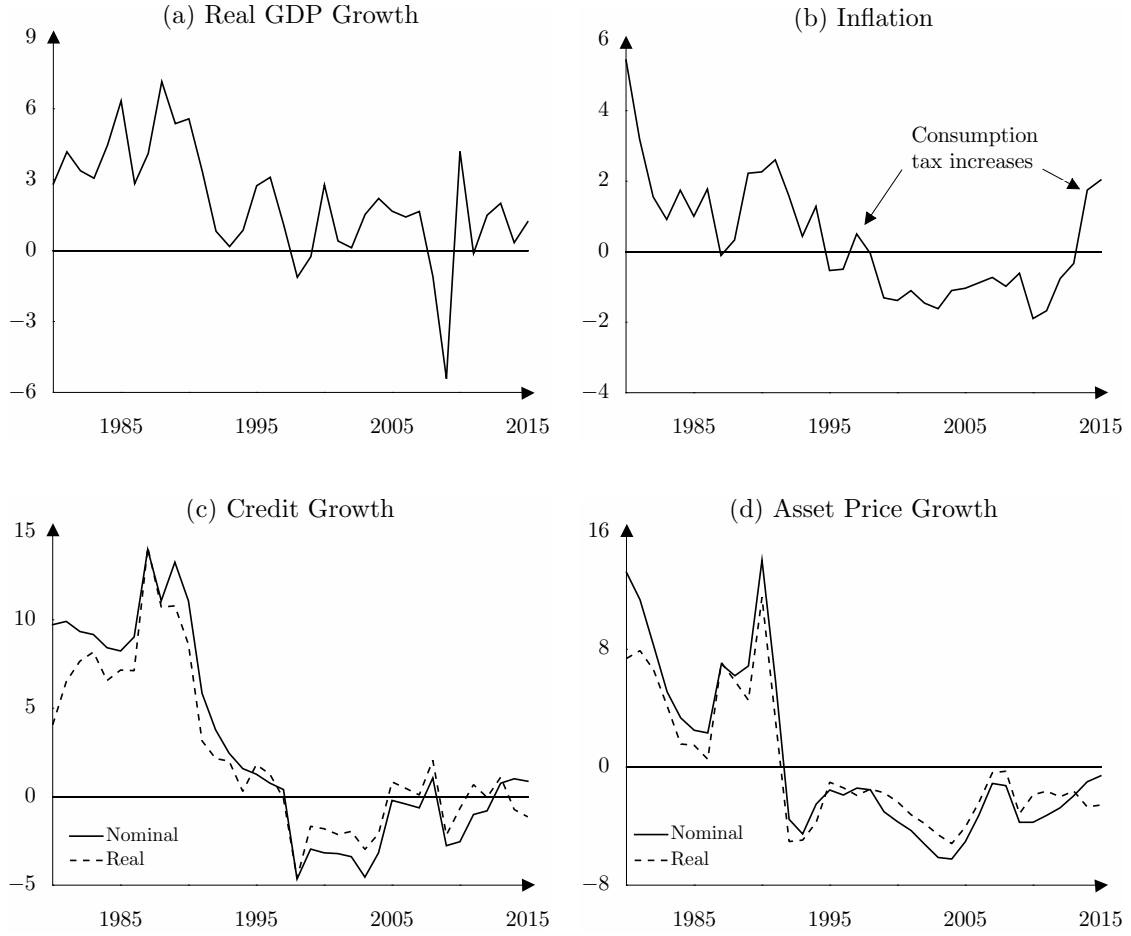
In contrast, asset prices declined and the private sector disencumbered in the stagnation period following the asset price crash of the early 1990s: Credit declined by 0.8% on average each year after 1991 while the real amount of credit stagnated. Credit as a share of GDP declined by almost 20% from 2.2 in 1992 to a level of 1.6 in 2015. At the same time, nominal property prices decreased substantially by 3.1% per year on average. The decline in asset prices continued throughout the stagnation period without any indication of a sustained recovery.

The extent of the credit expansion preceding the financial crisis in the United States and the United Kingdom is reminiscent of the experience of Japan as is the subsequent prolonged period of depressed spending because of persistent debt overhang. This is illustrated in Figure 2.

⁴We measure inflation by the GDP Deflator. The patterns is the same for CPI inflation at 2.6% (1980-1991) and 0.25% (1992-2015). Note that the recent increase in inflation in panel (b), as well as the spike in 1997, can be explained by an increase in the consumption tax in April 2014 (and 1997). Apart from the tax effect, there is no indication of a persistent increase in inflation. These tendencies are similar for other measures of economic activity (e.g. real consumption expenditure growth declines from 4.0% to 1.0%) and hold when excluding the financial crisis episode since 2008.

⁵Price increases were higher for commercial property (6.0%) and in the six major cities (12.1%).

Figure 1: Macroeconomic Developments in Japan, 1980-2015



Data sources and modifications:

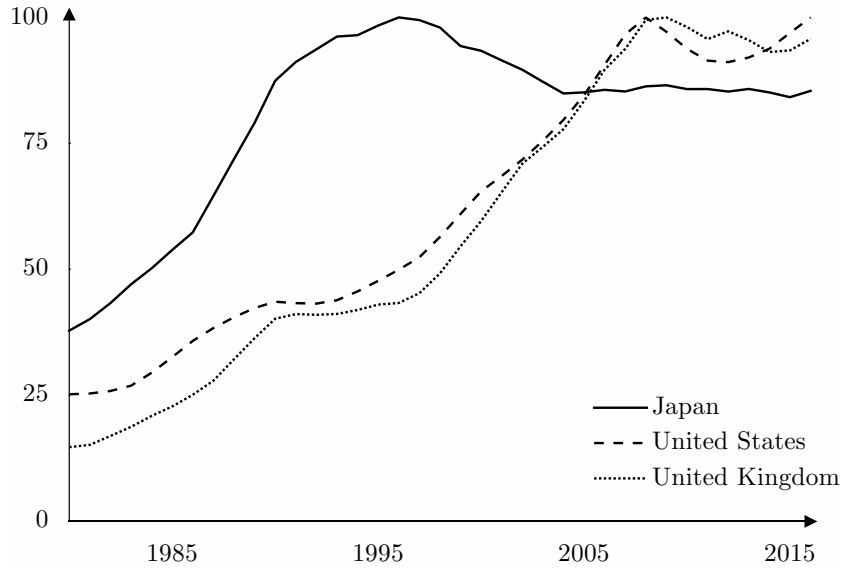
- (a) Real GDP, growth rate in percent, World Bank (WDI), *Series: NY.GDP.MKTP.CN*;
- (b) GDP Deflator, in percent, World Bank (WDI), *Series: NY.GDP.DEFL.ZS*;
- (c) Credit to private non-financial sector from all from all sectors at market value, growth rate in percent, Bank for International Settlements, *Series: Q:JP:P:A:M:XDC:A*;
- (d) Residential property price index, growth rate in percent, Bank for International Settlements, BIS Residential Property Price database, www.bis.org/statistics/pp.htm; *Series: Q:JP*

Obviously, the interactions of asset prices, credit availability and private sector debt are important factors for the emergence and the severity of economic stagnation. Episodes of persistent stagnation tend to be precluded by expansionary credit booms that result in substantial debt overhang of the private sector.

In this paper, we develop a stylized dynamic macroeconomic model that theoretically explains these observations. The model features three types of assets and two types of households: Borrowers obtain funds from savers, but their borrowing ability is limited by the value of collateral that is endogenously determined in the housing market following Iacoviello (2005). Households gain utility from consumption, housing and money. The last follows Sidrauski (1967) and reflects, among other things, the demand for liquidity.

Similarly, the subsequent decline was stronger for commercial property (-5.6%) and in cities (-4.8%).

Figure 2: Credit Booms and Debt Overhang



Data sources and modifications: Real credit to private non-financial sector, deflated by GDP Deflator and normalized to 100 in year of peak, Bank for International Settlements, *Series: Q:GB:P:A:M:XDC:A (UK), Q:US:P:A:M:XDC:A (US), Q:JP:P:A:M:XDC:A (Japan)*

We follow the research line initiated by Ono (1994, 2001) and assume insatiable liquidity preferences:⁶ The marginal utility of money stays strictly positive even for very large money holdings, which prevents consumption of the saver from increasing as potential output rises. This in turn creates stagnation if consumption of the borrower is sufficiently restricted as is the case when the economy suffers from debt overhang. Hence, economies with a higher leverage are more prone to suffering from insufficient demand.

Our setting implies that asset price or credit booms can temporarily stimulate an economy that would otherwise suffer from demand deficiency. A credit boom, which is triggered by financial liberalization, enables borrowers to temporarily increase their consumption spending, stimulating aggregate demand and inflation.⁷ Housing demand is stimulated and the house price increases, thereby reinforcing the initial credit boom as the value of collateral increases. Yet, in the new steady state, borrowers' consumption is depressed by interest payments to savers. Savers however do not increase their consumption accordingly as they prefer to hoard money because of strong liquidity preferences. In fact, the real money stock continuously expands as a result of deflation but fails to stimulate spending. As a consequence, aggregate demand falls permanently short of potential output and the economy experiences persistent deflation. The debt burden of borrowers permanently depresses spending so that the economy does not naturally recover.

⁶This preference reflects both a preference for transaction liquidity and for wealth. Our arguments also hold with wealth instead of liquidity preferences as discussed in section 5.4.

⁷When labor income is endogenous, additional amplification mechanisms may be at work, similar to the results of Bilbiie and Straub (2013). They show in a model with limited asset market participation that financial deregulation affects the elasticity of aggregate demand to interest rates.

It follows that the contractionary long run aggregate demand effects of relaxing lending standards are the opposite of their expansionary short run effects. This implies that there is a temptation for policymakers to stimulate sluggish growth by initiating lending booms that come at the cost of greater damage in the long run.

These findings are in line with the macroeconomic developments in Japan and with the situation in other advanced economies during the Great Recession as illustrated in Figure 2. In addition, our model can be interpreted as a formalization of the empirical literature cited above. In particular, it provides a theoretical foundation for the credit-driven housing demand channel described in detail by Mian and Sufi (2018).

Related Literature This paper contributes to the growing literature on secular stagnation and provides novel insights into the role of household credit for aggregate demand.

In general, stagnation occurs when the return on investing in assets, particularly the return on holding money, exceeds the natural interest rate, which is the short-term real interest rate consistent with full employment. Then the incentives to save for households are excessively high and an oversupply of savings occurs depressing aggregate demand.

The traditional liquidity trap literature views aggregate demand shortages as the consequence of temporary negative shocks in combination with a lower bound on the nominal interest rate and well-anchored inflation expectations (see Krugman, 1998; Eggertsson and Woodford, 2003; Eggertsson and Krugman, 2012). However, full employment is eventually restored even in the absence of policy measures.⁸

Our approach differs from the traditional liquidity trap literature. In our model of secular stagnation, no natural recovery occurs because this mismatch is the result of structural factors captured by strong preferences for liquidity as in Ono (1994, 2001). Other models generating secular stagnation focus on structural factors such as strong preferences for wealth (cf. Michailat and Saez, 2014; Michau, 2018), demographic developments and inequality (cf. Eggertsson et al., 2017), a shortage of safe assets (cf. Caballero and Farhi, 2018) or international considerations including the notion of a “global savings glut” (cf. Caballero et al., 2016; Eggertsson et al., 2016).⁹

We build on the literature of persistent aggregate demand shortage based on the insatiability of liquidity or wealth preferences, which was initiated by Ono (1994, 2001) and substantially extended by Ono and Ishida (2014). A key assumption of these models is the existence of a strictly positive lower bound on the marginal utility of money or wealth. As a consequence, increases in money holdings or wealth at some point cease to stimulate consumption spending as agents prefer to hoard money or wealth instead.

⁸This is in contrast to the experience of Japan where deflationary forces already prevail for more than two decades. It is difficult to make the case for the prevalence of price rigidities over such a long period. In our model, stagnation occurs in steady state despite the possibility of continuous price adjustment.

⁹In addition, some recent contributions analyze the effects of (the burst of) asset price bubbles on the natural rate and economic growth (cf. Bouillot, 2017; Hanson and Phan, 2017; Biswas et al., 2018).

The idea of a causal relationship between aggregate demand shortage and the insatiability of liquidity preferences goes back as far as Chapter 17 in Keynes (1936) as described in detail by Ono (2001). Moreover, Murota and Ono (2011) provide an explanation of this feature based on behavioral economics. Specifically, they show that this property can be linked to relative status preferences with respect to money. Ono et al. (2004) offer empirical support for the insatiability of liquidity preferences based on quarterly data in Japan using parametric and non-parametric methods. Michailat and Saez (2014) and Michau (2018) develop similar models of stagnation based on preferences for wealth.¹⁰ In fact, Michau (2018) shows that secular stagnation can occur with standard wealth preferences if the real money stock does not affect the utility from wealth in equilibrium.

However, these model consider homogeneous agents and perfect financial markets.¹¹ We introduce borrowing and lending via heterogeneity in time preference rates and borrowing frictions in the spirit of Kiyotaki and Moore (1997) and Iacoviello (2005) such that the debt capacity is endogenously determined by the value of collateral. This allows us to analyze interactions of household credit, asset prices and aggregate demand.

Our work is related to other recent contributions that analyze the effects of household credit for economic stagnation. But the results of our analysis differ substantially.

Eggertsson and Krugman (2012) and Korinek and Simsek (2016) also use a borrower-saver framework with credit constraints. In these liquidity trap models, a reduction in the borrowing limit triggers a temporary recession if the nominal interest rate cannot fall sufficiently. While the short run dynamics are similar in our model, it is not the zero lower bound but strong liquidity preferences that prevent the nominal rate from falling. In contrast to these models, our framework allows for stimulative effects of credit booms when aggregate demand is depressed. In addition, household credit affects the steady state of our model, whereas it is unaffected by the composition of household balance sheets in Eggertsson and Krugman (2012). The reason is that savers are not willing to substitute for the lack of demand of borrowers in our setting. In fact, their consumption levels are positively related in steady state due to aggregate demand spillovers.

Eggertsson et al. (2017) analyze household credit in an overlapping generations model of secular stagnation. Households are both borrowers and lenders over their life cycle and engage in inter-generational debt contracts. Specifically, young agents take out loans to finance consumption but face a borrowing constraint. Stagnation occurs as the result of a negative natural rate, the zero lower bound and nominal price rigidities. Under stagnation, the relaxation of the borrowing constraint has expansionary effects on aggregate

¹⁰Further examples for models built on insatiable liquidity preferences include Rodríguez-Arana (2007) who analyzes fiscal deficits and Murota and Ono (2012) who explain zero nominal interest rates and excess reserve holdings by commercial banks. Open economy considerations are analyzed in Ono (2006, 2014).

¹¹Ono (1994), Matsuzaki (2003) and Hashimoto (2004) introduce agents that differ in their initial wealth levels. However, financial markets are perfect and there is no credit. An exception is Kumhof et al. (2015), who analyze inequality in a model with wealth preferences and credit constraints.

demand even in steady state. Higher debt stimulates consumption when young but implies less disposable income at the later stages of the life cycle. As a consequence, the supply of savings contracts and the natural rate increases in the new steady state. As the young have a higher propensity to consume, aggregate demand expands. Households do not suffer from persistent debt overhang as they are both savers and borrowers at different stages of their life. In contrast, we model borrowing as an intra-generational contract between borrowers and savers. The higher real interest payments associated with a relaxation of the borrowing constraint permanently depress spending of borrowers while savers are not willing to expand their consumption accordingly. As a consequence, aggregate demand contracts in steady state in response to a relaxation of the borrowing constraint due to debt overhang.

We proceed as follows. First, we present a model of economic stagnation that gives a prominent role to household credit. This model is analyzed in section 3. We discuss the role of leverage for economic stagnation in section 4 and some extensions of the model as well as policy recommendations in section 5. The final section concludes.

2 The Model Economy

We use a continuous time model with money-in-the-utility that features competitive firms, two types of households and a central bank but abstracts from taxation or government expenditures. Agents have perfect foresight and there is no uncertainty in the model. We build on Ono (1994, 2001) for the idea of permanent demand shortage based on insatiable liquidity preferences and Iacoviello (2005) for modeling endogenous borrowing constraints with durable assets as collateral to introduce private sector debt.

2.1 Firms

The supply side is modeled as a Lucas tree. Firms are price takers and produce the amount \bar{y} of the consumption good without any inputs or costs. This constitutes the economy's production capacity or a measure of potential output. Yet, actual sales are determined by aggregate demand C_t so that realized income y_t falls short of potential output in case of aggregate demand shortage. Firm sales are hence given by

$$y_t = C_t \leq \bar{y} . \tag{1}$$

Nominal firm profits are given by $P_t y_t$ as production is costless. These are distributed equally across households and show up as exogenous income in the budget constraints. When falling short of potential output, aggregate demand determines household income. As a consequence, there are feedback loops between spending and income.

In addition, we abstract from the labor market and the wage-setting process and instead introduce a reduced-form Phillips curve for the inflation rate π_t . Specifically, the price level dynamics under full employment differ from those in the presence of aggregate demand shortage as follows:

$$\pi_t = \frac{\dot{P}_t}{P_t} = \begin{cases} \mu & \text{if } C_t = \bar{y} , \\ \alpha \left(\frac{C_t}{\bar{y}} - 1 \right) & \text{if } C_t < \bar{y} . \end{cases} \quad (2)$$

It should be clear that equations (1) and (2) are interrelated. Under full employment, actual firm sales and aggregate demand equal potential output and the dynamics of the price level are similar to the standard Money-in-the-Utility framework. The price level adjusts to clear the money market and the inflation rate is determined by the growth rate of the money supply μ , such that the quantity equation holds and money is neutral. In contrast, firm sales are constrained by a lack of aggregate demand and the associated output gap determines inflation in case of secular stagnation, where the parameter $\alpha > 0$ governs the speed of price adjustment. A negative output gap will result in deflation. If the output gap persists in steady state, deflation will persist and the goods market does not clear at full employment despite continuous price adjustments.

Similar relations are derived in standard macroeconomic models with a labor market based on downward nominal wage rigidity.¹² Specifically, Ono and Ishida (2014) and Ono (2015) provide the following microfoundation for equation (2) based on fairness concerns in the wage setting process:¹³ In their model, the productivity of workers depends on their perception of being treated in a fair way. In particular, workers withhold effort when they are not remunerated at least with a “fair wage”. Under full employment, competition among firms for workers determines the wage offer. Therefore, the dynamics of the price level determine the wage dynamics. The former are in turn dependent on the money supply growth. In contrast, firms have bargaining power when there is unemployment. However, the fair wage provides a lower bound on wage offers to prevent shirking. As a consequence, it is the dynamics of the fair wage that determine the wage and hence the price dynamics. These are in turn related to the level of unemployment or the output gap. Taken together, inflation is governed by an expression similar to equation (2), where α can be interpreted as the (exogenous) job separation rate faced by workers.

¹²As argued by Schmitt-Grohé and Uribe (2016): “There is abundant empirical evidence on downward nominal wage rigidity stemming mostly from developed countries.” An overview of the empirical evidence is presented in section 8 of their paper.

¹³In these models, the representative household has a fixed labor endowment. In equilibrium, competitive firms make zero profits, which is why the real wage equals labor productivity, which is constant due to the linear production function. $P_t y_t$ then is not a lump-sum transfer of profits but labor wages and the deflation gap is related to the labor market instead of the commodity market. Yet, the implications for the emergence of stagnation are only modestly affected.

This Phillips curve is formally equivalent to wage setting frictions as in Eggertsson et al. (2017), Michau (2018) or Schmitt-Grohé and Uribe (2016, 2017). These contributions introduce some form of downward nominal wage rigidity that becomes binding in case of unemployment. Eggertsson et al. (2017) assume that wages cannot fall below a “wage norm”, which is a linear combination of past wages and the marginal product of labor. In Michau (2018), wage demands of workers are guided by a reference rate of inflation, which creates an asymmetry in the wage dynamics similar to the one discussed above. Finally, Schmitt-Grohé and Uribe (2016, 2017) introduce an exogenous lower bound on the growth rate of the nominal wage that becomes binding in case of unemployment. The same mechanism is used by Hanson and Phan (2017) and Biswas et al. (2018).

It is worth pointing out that our conclusions on the role of asset prices and household debt for economic stagnation continue to hold in the presence of a richer modeling of the labor market. Specifically, the introduction of a production function and wage setting frictions in the spirit of Ono and Ishida (2014) does not alter our results qualitatively but comes at the cost of computational complexity. It is for this reason that we decided to rely on a reduced-form expression for inflation.

2.2 Households

There is a mass one of infinitely-lived households. Each household is one of two types based on his time preference rate ρ_i : A fraction n of households are savers ($i = 1$) whereas the remaining fraction $1 - n$ are borrowers ($i = 2$) in the sense that $\rho_1 < \rho_2$.¹⁴ This setting will endogenously result in differences in wealth levels and we will hence model an economy in which the “rich” (savers) lend to the “poor” (borrowers).¹⁵

Households have three means of savings: money $M_{i,t}$, credit contracts $B_{i,t}$ and real assets in the form of housing $h_{i,t}$. Money yields an interest rate of $R_M = 0$ whereas loans are contracted at the non-negative nominal interest rate R_t . Let $B_{i,t} > 0$ denote savings in the form of loans issued and $B_{i,t} < 0$ debt in the form of credit. Housing is always owner-occupied. It provides its owner with a convenience yield $w(h_{i,t})$, but does not generate rental income. Let Q_t denote the nominal house price. The return on housing depends on the resale value of the house.

Total nominal wealth $A_{i,t}$ is given by the sum of the household’s money holdings, bond holdings and the value of its housing investment: $A_{i,t} = B_{i,t} + M_{i,t} + Q_t h_{i,t}$. In real terms, wealth is given by

$$a_{i,t} = b_{i,t} + m_{i,t} + q_t h_{i,t} , \quad (3)$$

¹⁴The borrower-saver separation based on differences in time preference rates is a standard method to introduce borrowing incentives in macroeconomic models, see Sufi (2012). Since these differences are permanent, the roles of lenders and borrowers are static. Alternative ways of modeling include idiosyncratic income shocks or an uneven life-cycle income distribution.

¹⁵Alternatively, we could assume that agents differ in their initial wealth $a_{i,0}$ such that $a_{1,0} >> a_{2,0}$.

where lowercase letters denote the respective variables in real terms such that q_t denotes the real house price defined as $Q_t \equiv P_t q_t$. Households are the owners of firms and receive firm profits $P_t y_t$, where y_t is defined in (1). These profits are distributed equally across both types and considered exogenous by the households. In addition, households receive all income from seignorage in a lump-sum transfer $Z_{i,t}$. For the moment, this transfer is not important. Later, we will assume that $\mu = 0$ and hence $Z_{i,t} = 0$. Yet, it becomes relevant for the discussion of $\mu > 0$ in section 5. In real terms, the flow of funds constraint is given by¹⁶

$$\dot{a}_{i,t} = r_t a_{i,t} - R_t m_{i,t} - (r_t q_t - \dot{q}_t) h_{i,t} - c_{i,t} + y_t + z_{i,t} , \quad (4)$$

where the nominal interest R_t and the real interest rate r_t are related via the Fisher Equation as

$$R_t = r_t + \pi_t . \quad (5)$$

The household incurs opportunity costs when holding money because of the foregone interest income that would be associated with lending. Similar costs arise when investing in housing. Yet, housing investment involves the possibility of capital gains (or losses) associated with changes in the real house price as captured by \dot{q}_t .

Impatient households have a strong motive to borrow. However, lenders require sufficient collateral in the form of housing because of problems of asymmetric information in the credit market. Specifically, savers will only lend up to a fraction θ of the value of the borrowers' collateralizable assets. We refer to the parameter θ as the loan-to-value ratio. In real terms, the associated borrowing constraint takes the form

$$b_{2,t} \geq -\theta q_t h_{2,t} . \quad (6)$$

Throughout this paper, we choose parameters to ensure that the borrowing constraint is always binding. In our model, housing is the only durable asset that serves as collateral. In contrast, money is not collateralizable because it is too fungible to be effectively seized by lenders in case of missed repayment.¹⁷

¹⁶Equation (4) is based on the following expressions for the evolution of nominal and real wealth where we use the composition of household assets in (3) to substitute for B_t and the Fisher Equation (5) to relate the nominal and the real interest rate:

$$\begin{aligned} \dot{A}_t &= R_t B_t + \dot{Q}_t h_t - P_t c_t + P_t y_t + Z_{i,t} = R_t A_t - R_t M_t - R_t Q_t h_t + \dot{Q}_t h_t - P_t c_t + P_t y_t + Z_{i,t} \\ \dot{Q}_t &= P_t \dot{q}_t + q_t \dot{P}_t \\ \dot{a}_t &= \left(\frac{\dot{A}_t}{P_t} \right) = \frac{\dot{A}_t}{P_t} - \frac{A_t}{P_t} \frac{\dot{P}_t}{P_t} = (R_t - \pi_t) a_t - R_t m_t - (R_t q_t - \pi_t q_t - \dot{q}_t) h_t - c_t + y_t + z_{i,t} \end{aligned}$$

¹⁷It is easy to introduce a collateral value for money. The constraint becomes $b_{2,t} \geq -\theta_1 q_t h_{2,t} - \theta_2 m_{2,t}$, where θ_2 determines the collateralizability of money. For the special case of $\theta_1 = \theta_2 = \theta$, this formulation implies that (6) becomes a pure wealth constraint: $b_{2,t} \geq -\theta(1-\theta)^{-1} a_{2,t}$. Our main results are unchanged (and even stronger) when using this formulation.

Apart from differences in the time preference rate, households have identical preferences. Specifically, they choose consumption, real money balances and housing to maximize their lifetime utility function

$$U_i = \int_0^\infty [u(c_{i,t}) + v(m_{i,t}) + w(h_{i,t})] e^{-\rho_i t} dt , \quad (7)$$

where ρ_i denotes the subjective discount rate of the household of type i and is strictly positive with $\rho_2 > \rho_1$. Utility from consumption and housing services satisfies the standard Inada conditions. For simplicity, we make the following functional form assumptions on these instantaneous utility functions:

$$u(c_{i,t}) = \ln(c_{i,t}) ; \quad w(h_{i,t}) = \gamma \ln(h_{i,t}) ,$$

where $\gamma > 0$ is an exogenous and positive constant. In contrast, the Inada conditions do not hold for the utility from real money balances. As discussed in the previous section and following Ono (1994, 2001), we deviate from the neoclassical assumptions and introduce insatiable liquidity preferences. Formally, the marginal utility of real money holdings does not converge to zero but approaches a strictly positive constant value:

$$\lim_{m \rightarrow \infty} v'(m) = \beta > 0 .$$

We will explain the consequences of this assumption in the following sections.

Rich Households (Savers): Savers choose consumption, money holdings, housing and bond investments to maximize lifetime utility (7) subject to the wealth composition (3) and the flow budget constraint (4) for a given initial wealth level $a_{1,0}$. They take the paths of the price level, the real house price, the nominal and the real interest rate as given and do not internalize the effects of their spending on aggregate demand and firm profits. By the maximum principle, the solution of this problem satisfies

$$\frac{\dot{c}_{1,t}}{c_{1,t}} = r_t - \rho_1 , \quad (8)$$

$$R_t = v'(m_{1,t})c_{1,t} , \quad (9)$$

$$r_t q_t - \dot{q}_t = \frac{\gamma c_{1,t}}{h_{1,t}} , \quad (10)$$

together with the transversality condition for the saver's real wealth

$$\lim_{t \rightarrow \infty} e^{-\rho_1 t} \frac{a_{1,t}}{c_{1,t}} = 0 . \quad (11)$$

For the saver, the nominal interest rate governs both the intertemporal allocation of consumption in the Euler Equation (8), as it affects the real interest rate via the Fisher Equation (5), as well as the intra-temporal trade-off between money and consumption according to optimal money demand in (9). Taken together, the rich household equates the marginal rate of substitution between present and future consumption to the marginal rate of substitution between present consumption and money holdings, i.e. the liquidity premium, which also equals the nominal interest rate that constitutes the opportunity cost of holding money:

$$\frac{\dot{c}_{1,t}}{c_{1,t}} + \rho_1 + \pi_t = R_t = v'(m_{1,t})c_{1,t} . \quad (12)$$

Under neoclassical assumptions, the liquidity premium is declining in $m_{1,t}$, all else equal, thereby stimulating consumption or decreasing the nominal interest rate. In contrast, with insatiable liquidity preferences, the marginal utility of real money holdings will reach the positive lower bound if the wealth of the patient households is sufficiently high, i.e. $v'(m_1) = \beta$. Then the liquidity premium no longer declines with additional money holdings and $R_t = R_t(c_{1,t})$ in (9). As a consequence, consumption of the rich household is unaffected by changes in his money holdings for a given nominal interest rate. Put differently: The nominal interest rate no longer responds to changes in the real money stock. For that reason monetary policy becomes ineffective in single agent models such as Ono (2001): Additional money is stored as cash and does no longer stimulate consumption. The economy is trapped in a deflationary steady state despite an infinite expansion of the real money stock.

Poor Households (Borrowers): Borrowers maximize lifetime utility (7) subject to the wealth composition (3), the flow budget constraint (4) and the borrowing constraint (6) for a given initial wealth level $a_{2,0}$. Let φ_t denote the multiplier on the borrowing constraint. Like savers, they take prices and interest rates as given and do not internalize the effects of their spending on income. By the maximum principle, the solution of this problem satisfies

$$\frac{\dot{c}_{2,t}}{c_{2,t}} = r_t - \rho_2 + \varphi_t c_{2,t} , \quad (13)$$

$$R_t + \varphi_t c_{2,t} = v'(m_{2,t})c_{2,t} , \quad (14)$$

$$r_t q_t - \dot{q}_t + \varphi_t (1 - \theta) q_t c_{2,t} = \frac{\gamma c_{2,t}}{h_{2,t}} , \quad (15)$$

together with the transversality condition for the borrower's real wealth

$$\lim_{t \rightarrow \infty} e^{-\rho_2 t} \frac{a_{2,t}}{c_{2,t}} = 0 . \quad (16)$$

The borrower also equates the marginal rate of substitution between present and future consumption to the liquidity premium. This results from the borrower's Euler Equation (13) and optimal money demand (14) and gives

$$\frac{\dot{c}_{2,t}}{c_{2,t}} + \rho_2 + \pi_t = R_t + \varphi_t c_{2,t} = v'(m_{2,t}) c_{2,t} . \quad (17)$$

The borrowing friction affects optimal money demand and the evolution of consumption in (13) and (14). Impatience creates a strong motive to borrow funds for current consumption so that current funds have a higher value to the borrowers than to the savers. When these funds are used to increase liquidity instead of consumption, the household incurs an implicit cost of φ_t due to the borrowing constraint facing in fact a higher implicit interest rate than the saver. As a consequence, optimal money demand is reduced relative to the case without borrowing frictions.

Under neoclassical assumptions, the liquidity premium decreases with money holdings for the borrower, i.e. $v''(m_2) < 0$. In contrast, with insatiable liquidity preferences, $v'(m_2) = \beta > 0$ if the borrower becomes sufficiently wealthy. As a consequence, our model features different regions depending on the behavior of $v'(m_1)$ and $v'(m_2)$.

Asset Prices and Household Credit: Households incur opportunity costs when investing in housing because of the opportunity loss of real interest income that is associated with the alternative of bond savings. Yet, agents gain utility from housing which is captured by the user cost, i.e. the marginal rate of substitution between consumption and housing. For the saver, this is expressed in (10). For the borrower, housing investment comes at a higher cost, his implicit interest rate being higher than for the saver. But since housing serves as collateral, the associated borrowing costs are lower than those for money at $(1 - \theta)\varphi_t$ which can be seen in (15).

Moreover, changes in the real house price affect the costs and benefits of housing due to valuation effects. In optimum, the real house price adjusts such that agents are indifferent between investing in an additional unit of housing and alternative uses. Put differently, the absence of arbitrage requires that housing investment yields the same real return as bonds and money. Hence, the real house price appreciates if the opportunity costs from housing exceed the user costs to compensate home owners for the higher costs with capital gains. Similarly, the real house price depreciates if the benefits of housing exceed the opportunity costs resulting in capital losses for home owners. From equations (10) and (15), the dynamics of the real house price can be expressed as the difference between opportunity costs and housing benefits for both agents as

$$\dot{q}_t = r_t q_t - \frac{\gamma c_{1,t}}{h_{1,t}} = r_t q_t - \frac{\gamma c_{2,t}}{h_{2,t}} + \varphi_t (1 - \theta) q_t c_{2,t} . \quad (18)$$

The borrowing constraint is strictly binding throughout our analysis, i.e. $\varphi_t > 0$, and the borrower always takes out loans up to the maximum. It follows from (3) and (6) that his real wealth consists of money holdings and housing investment, a fraction θ of which serves as collateral:

$$a_{2,t} = m_{2,t} + (1 - \theta)q_t h_{2,t} . \quad (19)$$

Similarly, total assets of the saver include loans to the borrower. From (17) and (18), the consumption value of borrowing $\varphi_t c_{2,t}$ - or equivalently, the consumption cost of debt-financed money holdings or housing investment - equals the difference in the liquidity premia and is proportional to the difference in the user cost of housing:

$$v'(m_{2,t})c_{2,t} - v'(m_{1,t})c_{1,t} = \varphi_t c_{2,t} = \frac{1}{1 - \theta} \left(\frac{\gamma c_{2,t}}{q_t h_{2,t}} - \frac{\gamma c_{1,t}}{q_t h_{1,t}} \right) . \quad (20)$$

Hence, a binding borrowing constraint implies that it is more costly (in terms of consumption) for the borrower to hold money or invest in housing than for the saver.

2.3 Aggregation, Market Clearing and Equilibrium

A fraction n of households are savers, while the remaining fraction $1 - n$ are borrowers. Aggregate demand for goods C_t consists of the consumption demands of both types with

$$C_t = n c_{1,t} + (1 - n) c_{2,t} . \quad (21)$$

Aggregate demand determines firm profits y_t in the flow budget constraints (4). In equilibrium, aggregate demand equals realized income y_t as is clear from (1). Under full employment, potential output determines realized income, i.e. $C_t = y_t = \bar{y}$. Under stagnation, realized income falls short of potential output, i.e. $C_t = y_t < \bar{y}$. Aggregate demand relative to potential output determines the output gap and inflation via (2).

The central bank perfectly controls the nominal money supply M_t which grows at an exogenous rate μ . Hence, the real money supply m_t evolves as

$$\frac{\dot{m}_t}{m_t} = \mu - \pi_t . \quad (22)$$

In contrast, the nominal interest rate R_t is determined endogenously in the money market. Let M_0 and P_0 denote the initial nominal money supply and price level. The real money supply at time t is related to the initial money supply m_0 via the inflation rate, which determines the price dynamics for a given P_0 , and the money growth rate. Total money demand is the weighted average of the individual money demands. Money market clearing then requires

$$n m_{1,t} + (1 - n) m_{2,t} = m_t = \frac{M_0}{P_0} \cdot e^{\int_0^t (\mu - \pi_s) ds} . \quad (23)$$

Loans are financial claims among households. As we abstract from government debt and focus on household credit, bonds are in zero net supply. Equilibrium in the credit market then requires

$$nb_{1,t} + (1 - n)b_{2,t} = 0 . \quad (24)$$

In contrast, housing is a real asset. Following Iacoviello (2005), we assume a fixed supply of houses H and abstract from depreciation and construction both of which could easily be implemented in this setting.¹⁸ Market clearing in the housing market requires

$$nh_{1,t} + (1 - n)h_{2,t} = H . \quad (25)$$

This completes the description of the model. We formally define an equilibrium as:

Definition 1 *An **equilibrium** is a set of paths for prices P_0 and $\{\pi_t, r_t, R_t, q_t, \varphi_t\}$ and for quantities $\{y_t, C_t, m_t, a_{1,t}, a_{2,t}, c_{1,t}, c_{2,t}, m_{1,t}, m_{2,t}, b_{1,t}, b_{2,t}, h_{1,t}, h_{2,t}\}$ that solve the optimization problem of savers and borrowers in (8), (9), (10), (13), (14) and (15) given (3), (4) and (6), where credit constraint (6) is binding, and are consistent with goods market equilibrium in (1), (2) and (21) as well as equilibrium in the money, housing and bond markets in (23), (24) and (25) given the No-Arbitrage relation (5) and the process for the real money supply in (22).*

3 Analysis of the Model Economy

In the following analysis, we focus on the special case of a constant nominal money supply,

$$\mu = 0,$$

for simplicity. This implies a zero trend inflation rate under full employment as is evident from (2).¹⁹ Yet, the qualitative conclusions of our analysis can be generalized and hold for positive levels of μ as will be discussed later in section 5.2 in greater detail.

Our model framework features three regions depending on the behavior of the marginal utilities $v'(m_1)$ and $v'(m_2)$. This is in turn related to the production capacity \bar{y} :

1. For low levels of potential output \bar{y} , the economy behaves as in the standard *neo-classical case*. The marginal utility of money is decreasing in money holdings for both households, i.e. $v''(m_i) < 0$, and aggregate demand equals potential output. The price level is constant and changes proportionally with the money supply.

¹⁸This assumption seems reasonable for an economy like Japan that is characterized by land scarcity and a low price elasticity of the housing supply. A study by Shimizu and Watanabe (2010) concludes that the housing supply was very price inelastic during the Japanese housing boom of the late 1980s, partly due to the incentives given by the tax system as well as regulation on land utilization.

¹⁹This parameterization also allows us to derive some expressions in closed-form that do not depend on the shape of the function $v(m)$, which helps to provide a more intuitive understanding of our results.

2. For higher levels of \bar{y} , there is an *asymmetric* steady state under stagnation. In this region, the patient household's marginal utility of money is constant while the impatient household's liquidity premium still declines with additional money holdings, i.e. $v''(m_1) = 0$ and $v''(m_2) < 0$. Aggregate demand falls short of potential output and deflation occurs.
3. For very high levels of potential output, the *symmetric* steady state under stagnation might occur. In this region, the marginal utility of money has reached its lower bound for savers and borrowers, i.e. $v''(m_i) = 0$.

Among the three mentioned above, we focus on the asymmetric steady state under stagnation for several reasons. First, this steady state features economic stagnation and deflation unlike the neoclassical case. Secondly, indebtedness and asset prices play an important role in affecting the severity of stagnation.²⁰ Thirdly, it is more in conformity with what has occurred in the Japanese economy as discussed in the introduction. The asymmetric steady state under stagnation is defined as follows:

Definition 2 *In the **asymmetric steady state** under stagnation, the real and nominal interest rates are constant, the price level is declining at a constant rate, the real consumption level of each household is constant as is the real house price, and the borrower's asset level is constant while the saver's wealth expands infinitely:*

$$\dot{r} = 0, \dot{R} = 0, \pi < 0, \dot{c}_1 = 0, \dot{c}_2 = 0, \dot{q} = 0, \dot{a}_1 > 0, \dot{a}_2 = 0.$$

3.1 The Occurrence of Persistent Stagnation

Intuitively, aggregate demand shortage occurs if potential output is so high that households are no longer willing to consume the available amount of \bar{y} due to the insatiability of liquidity preferences of the saver.²¹ For lower levels of potential output, the economy attains full employment at zero inflation (in the present case as $\mu = 0$) and the price level adjusts to clear the money market for a given level of the nominal money supply in equation (23). We define this full employment steady state as follows:

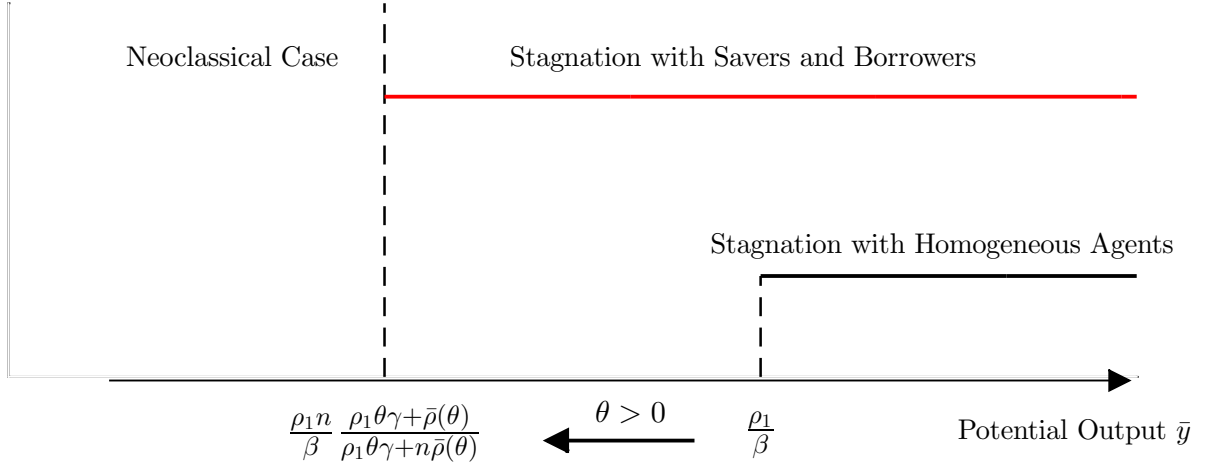
Definition 3 *In the **neoclassical steady state**, the real and nominal interest rates are constant, the price level is constant, the real house price is constant and the consumption and wealth of all households are constant:*

$$\dot{r} = 0, \dot{R} = 0, \pi = 0, \dot{c}_1 = 0, \dot{c}_2 = 0, \dot{q} = 0, \dot{a}_1 = 0, \dot{a}_2 = 0.$$

²⁰In contrast, changes in leverage cease to affect aggregate demand in the symmetric steady state, but simply affect asset prices and the distribution of the housing stock.

²¹In addition, the liquidity premium of the borrower must still decline with additional money holdings for asymmetric (instead of symmetric) stagnation to occur. This requirement is not important at the moment and will be discussed in greater detail in section 5.

Figure 3: Occurrence of Persistent Stagnation



Note: This figure shows the equilibrium of the model for different values of \bar{y} compared to homogeneous agent models. In particular, note that in the heterogeneous agent framework economic stagnation occurs for lower levels of potential output.

Put differently, stagnation occurs when it is too attractive to hold money. The natural rate in our model is determined by the time preference rate ρ_1 of the saver, which follows from (8) with $\dot{c}_{1,t} = 0$. The nominal interest rate is tied to the liquidity premium of the saver in (9). The insatiability of liquidity preferences therefore establishes a lower bound on the nominal rate for a given level of potential output. For sufficiently high levels of potential output, the return on holding money exceeds the natural rate, which results in an oversupply of savings at full employment. Then stagnation and deflation occur.

Consider first the case of homogeneous agents: Suppose there are only patient households. From (12) with $\dot{c}_1 = 0$, the economy attains full employment, i.e. $c_1 = \bar{y}$, and zero inflation as long as the marginal utility of money can adjust such that $v'(m_{1,t})\bar{y} = \rho_1$. With insatiable liquidity preferences, there is a lower bound β of the marginal utility of real money holdings. Once the production capacity \bar{y} exceeds the level of $\rho_1\beta^{-1}$, there is no longer a solution to (12) that is compatible with $\pi = 0$ and $c_1 = \bar{y}$. This is because households are no longer willing to consume the available output but prefer to accumulate money instead. As a consequence, stagnation and deflation occur in equilibrium. This condition is illustrated by the lower line in Figure 3.

Similarly, suppose there were only impatient households.²² From (17) with $\dot{c}_2 = 0$, the economy attains full employment as long as higher spending can be accommodated at zero inflation such that $v'(m_2)\bar{y} = \rho_2$. There is no solution to (17) consistent with full employment once \bar{y} is above $\rho_2\beta^{-1}$. Taken together, the relevant condition for homogeneous agent models is given by $\bar{y} > \rho_i\beta^{-1}$ where ρ_i refers to the representative household.

²²Note that in this case, the borrowing constraint would cease to be binding, i.e. $\varphi_t = 0$.

In an economy with n savers and $1 - n$ borrowers, the distribution of consumption spending under full employment determines the occurrence of stagnation. Since $\pi = 0$, we have from (5) and (12) that $R = r = \rho_1$. Then, consumption levels are derived from the flow budget constraints (4). The borrower consumes his income net of interest payments on debt. Income in turn depends on aggregate demand which equals potential output. His consumption in the neoclassical steady state is then given by

$$c_2^{NC} = \frac{\bar{\rho}_\theta}{\bar{\rho}_\theta + \theta\rho_1\gamma} \bar{y} , \quad \text{where } \bar{\rho}_\theta \equiv \theta\rho_1 + (1 - \theta)\rho_2 . \quad (26)$$

Note that $\bar{\rho}_\theta$ can be interpreted as the debt-weighted average discount rate. This follows from (4), (19) and the requirements $\pi = 0$, $\dot{q} = 0$ and $\dot{a}_2 = 0$. The rich household behaves similarly, but receives interest income on its lending. Hence, steady state consumption of the saver exceeds consumption of the borrower in the neoclassical steady state due to the redistribution associated with ownership of financial assets:

$$c_1^{NC} = \frac{n\bar{\rho}_\theta + \theta\rho_1\gamma}{n\bar{\rho}_\theta + n\theta\rho_1\gamma} \bar{y} = \frac{n\bar{\rho}_\theta + \theta\rho_1\gamma}{n\bar{\rho}_\theta} c_2^{NC} > c_2^{NC} . \quad (27)$$

It is easy to see from these expressions that aggregate demand equals potential output. Yet, it follows from (12) and (17) with $\dot{c}_i = 0$ that the consumption levels of both agents in (26) and (27) are consistent with zero inflation only if the marginal utility of money falls sufficiently. In particular, for the neoclassical case to exist it has to hold that

$$v'(m_{1,t}) = \frac{\rho_1}{c_1^{NC}} \quad \text{and} \quad v'(m_{2,t}) = \frac{\rho_2}{c_2^{NC}} . \quad (28)$$

With insatiable liquidity preferences, there exists a lower bound on the marginal utility of money such that $v'(m_{i,t}) \geq \beta$. Hence, the neoclassical case is not feasible once $\beta c_i^{NC} > \rho_i$. So with rising levels of potential output \bar{y} , at some threshold the stagnation steady state will occur. Since $\rho_1 < \rho_2$ and $c_1^{NC} > c_2^{NC}$, it is the saver's marginal utility of money that will reach its lower bound first for rising levels of \bar{y} . Then, aggregate demand falls short of the production capacity and the economy enters the stagnation steady state. We derive the following proposition from combining (27) and (28):

Proposition 1 *The neoclassical equilibrium with full employment and zero inflation cannot be attained once potential output exceeds the following threshold:*

$$\tilde{y} \equiv \frac{\rho_1 n}{\beta} \frac{\theta\rho_1\gamma + \bar{\rho}_\theta}{\theta\rho_1\gamma + n\bar{\rho}_\theta} < \frac{\rho_1}{\beta} . \quad (29)$$

The threshold \tilde{y} is affected by the model parameters as follows (see Appendix A):

$$\frac{\partial \tilde{y}}{\partial \beta} < 0 , \quad \frac{\partial \tilde{y}}{\partial \rho_1} > 0 , \quad \frac{\partial \tilde{y}}{\partial \rho_2} > 0 , \quad \frac{\partial \tilde{y}}{\partial \theta} < 0 , \quad \frac{\partial \tilde{y}}{\partial n} > 0 , \quad \frac{\partial \tilde{y}}{\partial \gamma} < 0 .$$

Once potential output exceeds \tilde{y} , the economy is in the asymmetric steady state under stagnation and suffers from insufficient demand and deflation. Additional income does no longer stimulate consumption of the saver who chooses to accumulate wealth instead. This is represented by the upper line in Figure 3.

The lower the insatiability parameter β , the higher potential output needs to be for the economy to enter stagnation. Similarly, increases in the time preference rate of the saver ρ_1 or in their fraction of the population n also increase the income threshold. The same holds for a higher time preference rate ρ_2 of the borrower.

What we add is the insight that financially more developed countries, i.e. countries with higher leverage, drift into stagnation already at a lower level of potential output. This is because the higher debt is associated with lower steady state consumption demand from the borrower. To see this, note from (26) that if financial markets are closed and no borrowing is possible, i.e. if $\theta = 0$ or $\gamma = 0$, the consumption levels of both households are equal and given by $c_i = \bar{y}$ under full employment. Once we allow for borrowing, housing investment is associated with an increase in indebtedness of the borrower. This in turn results in a higher real interest burden on poor households and reduces their affordable consumption. This gives rise to a more unequal income distribution but does not affect aggregate demand as long as the rich households expand their consumption accordingly. If they invest in liquidity holdings instead, aggregate demand falls short of the economy's production capacity and stagnation occurs.²³

Let us contrast this condition with the existence condition in models without lending and borrowing as in Ono (2001), which was discussed above. Condition (29) is reduced to the expression in single-agent models if we abstract from housing ($\gamma = 0$), if we do not allow for borrowing ($\theta = 0$) or if there are only rich households ($n = 1$). In all other cases, \tilde{y} is below the threshold of the single-agent model. Hence, the economy enters stagnation in an earlier stage, which is illustrated in Figure 3. The reason is that consumption of the saver is higher due to additional income associated with interest payments on loans.

3.2 The Asymmetric Steady State under Persistent Stagnation

The real interest rate in steady state is determined by the time preference rate of the more patient agent, which is the saver, and therefore given by

$$r^* = \rho_1 . \quad (30)$$

This follows from the Euler Equation (8) with $\dot{c}_1 = 0$. The absence of arbitrage requires that all assets yield the same real return r^* to allow for market clearing.²⁴

²³The same effect arises for a higher value of γ . Economies that invest more heavily in assets in fixed supply are hence more prone to stagnation. Also note that the effects of θ and γ are mutually reinforcing.

²⁴Steady state values of the respective variables will be characterized by “*” for notational convenience.

Money is a nominal asset, whose real return is given by the difference between the nominal interest rate and the inflation rate in (2), which follows from (5). The nominal rate is determined by the liquidity premium of the saver in (9) and is below the real rate because of deflation. Substituting expression (2) for π^* and (21) for aggregate demand C^* in (12) with $\dot{c}_1 = 0$ and $v'(m_1) = \beta$, we obtain the No-Arbitrage relationship between money holdings and bonds for the saver as

$$\rho_1 + \alpha \left(\frac{nc_1^* + (1-n)c_2^*}{\bar{y}} - 1 \right) = \beta c_1^* . \quad (31)$$

It follows that consumption spending of borrowers and savers is positively related in steady state due to aggregate demand spillovers. An increase in consumption of the borrower expands aggregate demand and mitigates deflation. For a given real rate $r^* = \rho_1$, less deflation makes bond investments less attractive such that the nominal interest rate increases. This increases the opportunity costs of holding money and hence induces the saver to expand his consumption spending since the marginal utility of money does not adjust. This is the same relation as in Ono (1994) and Matsuzaki (2003) and results from the insatiability of liquidity preferences in combination with sluggish price adjustment. Rearranging (31) gives

$$c_1^* = \frac{(\rho_1 - \alpha)\bar{y}}{\beta\bar{y} - \alpha n} + \frac{\alpha(1-n)}{\beta\bar{y} - \alpha n} c_2^* . \quad (32)$$

Spillovers from aggregate demand are stronger the higher the share of spending constrained households $(1-n)$ and the higher the speed of price adjustment α . In particular, steady state consumption c_1^* is not directly affected by the borrowing decision or asset composition of the impatient household. Yet, there are indirect effects via c_2^* .

The borrower is less patient than the saver but faces the same real return on savings r^* . The difference between these rates determines the shadow value of borrowing. From (8) and (13) with $\dot{c}_{1,t} = \dot{c}_{2,t} = 0$, the Lagrange parameter on the borrowing constraint is given by

$$\varphi^* = \frac{\rho_2 - \rho_1}{c_2^*} > 0 . \quad (33)$$

Hence, the borrowing constraint is binding in steady state. Higher consumption of the borrower reduces the value of additional funds.

For the borrower to be indifferent between money and bonds as a means of savings, the liquidity premium on money also has to match his time preference rate taking into account the rate of inflation. Substituting (2) for π^* , (21) for C^* and (33) for φ^* in (17) with $\dot{c}_2 = 0$ gives an expression similar to (31) for the borrower, i.e.

$$\rho_2 + \alpha \left(\frac{nc_1^* + (1-n)c_2^*}{\bar{y}} - 1 \right) = v'(m_2^*)c_2^* . \quad (34)$$

In contrast to the saver, the liquidity premium of the borrower is still affected by his money holdings in equilibrium. Using (32) to substitute for c_1^* and rearranging (34) implies

$$c_2^* = \frac{\chi}{v'(m_2^*)(\beta\bar{y} - \alpha n) - \beta\alpha(1 - n)} , \quad (35)$$

where $\chi \equiv \rho_2(\beta\bar{y} - \alpha n) - \alpha(\beta\bar{y} - \rho_1 n)$.

More money induces more consumption of the borrower, which is necessary to equalize the liquidity premium to the nominal interest rate. Importantly, we require parameter restrictions to guarantee positive consumption levels c_1^* and c_2^* in (32) and (35). We make the following assumptions throughout this paper:

$$(i) \ \rho_1 > \alpha \quad \text{and} \quad (ii) \ \beta\bar{y} > \alpha . \quad (36)$$

In contrast to money, housing is a real asset on which investors require the return r^* in steady state. As the real house price is constant, there are no capital gains and the user cost of housing for each agent adjusts to match the corresponding real required return. For the saver, condition (10) with $\dot{q} = 0$ implies

$$\rho_1 = \frac{\gamma c_1^*}{q^* h_1^*} . \quad (37)$$

For the borrower, condition (15) with $\dot{q} = 0$ and the expression for φ^* in (33) imply

$$\theta\rho_1 + (1 - \theta)\rho_2 \equiv \bar{\rho}_\theta = \frac{\gamma c_2^*}{q^* h_2^*} , \quad (38)$$

where $\bar{\rho}_\theta$ is a weighted average discount rate. Hence, the steady state value of housing investment is a constant fraction of each agent's consumption level. Note that $\varphi^* > 0$ in (33) and equation (38) imply that the real level of debt is constant in steady state. It also follows that real wealth and consumption spending are positively related for the borrower. An increase in consumption induces both higher money holdings via (35) and higher housing investment via (38), and hence higher real wealth.

Market clearing in the housing market requires the house price to adjust such that housing demands in (37) and (38) are consistent with the constant supply H in (25). The real house price satisfies

$$q^* = \frac{\gamma}{H} \left[\frac{n}{\rho_1} c_1^* + \frac{1 - n}{\bar{\rho}_\theta} c_2^* \right] . \quad (39)$$

As higher consumption demand implies higher housing demand, the real house price increases in response to an increase in aggregate demand. From (32), $c_1^* = c_1^*(c_2^*)$ and hence $q^* = q^*(c_2^*)$. An increase in consumption of the borrower increases the real house price in steady state as does an increase in γ , all else equal.

The borrower's real assets are constant in the asymmetric steady state. From the budget constraint (4) with $\dot{a}_2 = 0$ and (3), (6), (21) and (38), we get

$$nc_1^* + (1 - n)c_2^* + (\rho_1 - \beta c_1^*)m_2^* = \left(\frac{\theta \rho_1 \gamma}{\bar{\rho}_\theta} + 1 \right) c_2^* . \quad (40)$$

The borrower obtains real income from two sources: firm profits, which are determined by aggregate demand, and capital gains on his money holdings, which are determined by the rate of *deflation*, i.e. $-\pi^* = \rho_1 - \beta c_1^*$. This income is used to finance consumption expenditures c_2^* and to make interest payments on debt, which depend on the household's borrowing capacity. This capacity in turn is related to the value of housing collateral via (6) and hence to the borrower's consumption demand as is clear from (38). In steady state, real interest payments are a fraction $\theta \rho_1 \gamma \bar{\rho}_\theta^{-1}$ of consumption and increase with θ , γ and ρ_1 but decrease with ρ_2 , because $\bar{\rho}_\theta = \theta \rho_1 + (1 - \theta) \rho_2$.

Finally, it follows from (23) that the real money stock becomes infinitely high because of deflation. The increase in the real money supply exclusively benefits the saver in the asymmetric steady state. His real wealth expands with the rate of deflation. However, this does not violate the transversality condition (11) since $\rho_1 > \alpha > -\pi^*$.

Equations (32), (35) and (40) jointly determine c_1^* , c_2^* and m_2^* . All other variables are derived from these values: q^* follows from (39), h_1^* and h_2^* from (37) and (38), R^* and π^* from (9) and (2), y^* and C^* from (1) and (21), b_2^* and b_1^* from (6) and (24), a_2^* from (3) and r^* and φ^* from (30) and (33).

Model Dynamics under Asymmetric Stagnation The model dynamics are represented by a system of six differential equations for $c_{1,t}$, $c_{2,t}$, q_t , $a_{1,t}$, $a_{2,t}$ and m_t . All other variables are derived from this system: $\pi_t = \pi(c_{1,t}, c_{2,t})$ from (2) and (21), $R_t = R(c_{1,t})$ from (12) with $v'(m_{1,t}) = \beta$ and $r_t = r(c_{1,t}, c_{2,t})$ from (5). Given $c_{1,t}$, $c_{2,t}$, q_t and $a_{2,t}$, $m_{2,t}$, $h_{1,t}$, $h_{2,t}$ and φ_t are jointly determined by (19), (20) with $v'(m_{1,t}) = \beta$ and (25).

The consumption dynamics are determined by (12) with $v'(m_{1,t}) = \beta$ and (17) with $v'(m_{2,t}) > \beta$. The dynamics of the real house price are given by (18) and real wealth levels follow (4) where we use (10) and (15) to substitute for $\dot{q}_t - r_t q_t$, (17) for φ_t and (21) for y_t . Finally, the real money supply decreases with the inflation rate as is clear from (23). Taken together, the dynamic system is characterized by

$$\frac{\dot{c}_{1,t}}{c_{1,t}} = \beta c_{1,t} - \pi_t - \rho_1 , \quad (41)$$

$$\frac{\dot{c}_{2,t}}{c_{2,t}} = v'(m_{2,t}) c_{2,t} - \pi_t - \rho_2 , \quad (42)$$

$$\frac{\dot{q}_t}{q_t} = r_t - \frac{\gamma c_{1,t}}{q_t h_{1,t}} , \quad (43)$$

$$\frac{\dot{a}_{1,t}}{a_{1,t}} = -\pi_t + \frac{r_t q_t (h_{1,t} + (1/n - 1)\theta h_{2,t}) - (1 - n + \gamma)c_{1,t} + (1 - n)c_{2,t}}{a_{1,t}}, \quad (44)$$

$$\frac{\dot{a}_{2,t}}{a_{2,t}} = -\pi_t + \frac{v'(m_{2,t})c_{2,t}(1 - \theta)q_t h_{2,t} - (n + \gamma)c_{2,t} + nc_{1,t}}{a_{2,t}}, \quad (45)$$

$$\frac{\dot{m}_t}{m_1} = -\pi_t. \quad (46)$$

Therefore, it holds that $\dot{c}_{1,t} = \dot{c}_1(c_{1,t}, c_{2,t})$, $\dot{c}_{2,t} = \dot{c}_2(c_{1,t}, c_{2,t}, q_t, a_{2,t})$, $\dot{q}_t = \dot{q}(c_{1,t}, c_{2,t}, q_t, a_{2,t})$, $\dot{a}_{1,t} = \dot{a}_1(c_{1,t}, c_{2,t}, q_t, a_{1,t}, a_{2,t})$, $\dot{a}_{2,t} = \dot{a}_2(c_{1,t}, c_{2,t}, q_t, a_{2,t})$ and $\dot{m}_t = \dot{m}(c_{1,t}, c_{2,t}, m_t)$. Note from (44) that wealth of the saver will eventually growth with rate $-\pi^* > 0$ due to the expansion of real money balances.

Equations (41) to (46) fully describe the dynamics of the economy together with the initial asset levels $a_{1,0}$, $a_{2,0}$ and m_0 . This system satisfies saddle-point stability around the asymmetric steady state as shown in Appendix B. In the following section, we analyze the dynamic and static properties of this steady state.

4 Asset Prices and Leverage Under Stagnation

As shown above, private sector debt affects the occurrence of persistent stagnation. In addition, household credit and asset prices also affect aggregate demand under secular stagnation. Specifically, we show that credit booms can temporarily mask aggregate demand insufficiency. However, this comes at the cost of more severe stagnation in the new steady state.

4.1 Credit and Asset Price Booms under Stagnation

We have argued that an economy can enter an equilibrium of persistent stagnation as a consequence of the debt burden of some households. However, an expansion of debt via financial liberalization can in the short run mask aggregate demand shortage by creating a temporary credit and asset price boom. Specifically, consider an economy that is suffering from insufficient aggregate demand. Suppose that lending standards loosen such that borrowers can take on more loans per unit of housing net worth. This setup is in line with the claims of Larry Summers about the U.S. economy during the early 2000s and also mirrors several features of the situation of Japan in the late 1980s that we described in the introduction.²⁵

²⁵Note that we proxy the credit boom by variations in θ but do not make explicit claims about the origin of this variation. The sources of the Japanese credit boom are still up to debate. Yet, Posen (2003) argues that both partial deregulation in corporate finance and a relaxation of lending standards in the mortgage market with mortgage limits rising from 65% of the home value on average to 100% played a major role for the Japanese credit boom. According to Posen (2003), “there is a consensus view among economists on how partial financial deregulation in Japan in the 1980s led to a lending boom”. The effects of deregulation and financial liberalization are also well-documented in Tsuruta (1999).

Figure 4 shows the associated model dynamics as deviations from the initial steady state, denoted by \hat{x}_t , for two values of the housing preference parameter γ . The increase in the loan-to-value ratio triggers a substantial credit boom as indicated by the behavior of the credit to income ratio in the third row, which is reminiscent of Figure 2. Borrowers can acquire new funds for a given collateral value some of which they consume and some of which they hold as money or invest in new housing. These funds are provided by savers and financed by current income, money holdings and the sale of houses. What follows is a temporary boom in both the real economy and the housing market.

The credit boom stimulates aggregate demand driven by an increase in spending of borrowers. As a consequence, the output gap is reduced as illustrated in the first row of Figure 4 since the increase in spending of borrowers overcompensates the decline in savers' consumption. This creates inflation which lowers the real interest rate. If the credit boom is sufficiently strong, the economy can temporarily return to full employment with aggregate spending being constrained by potential output.

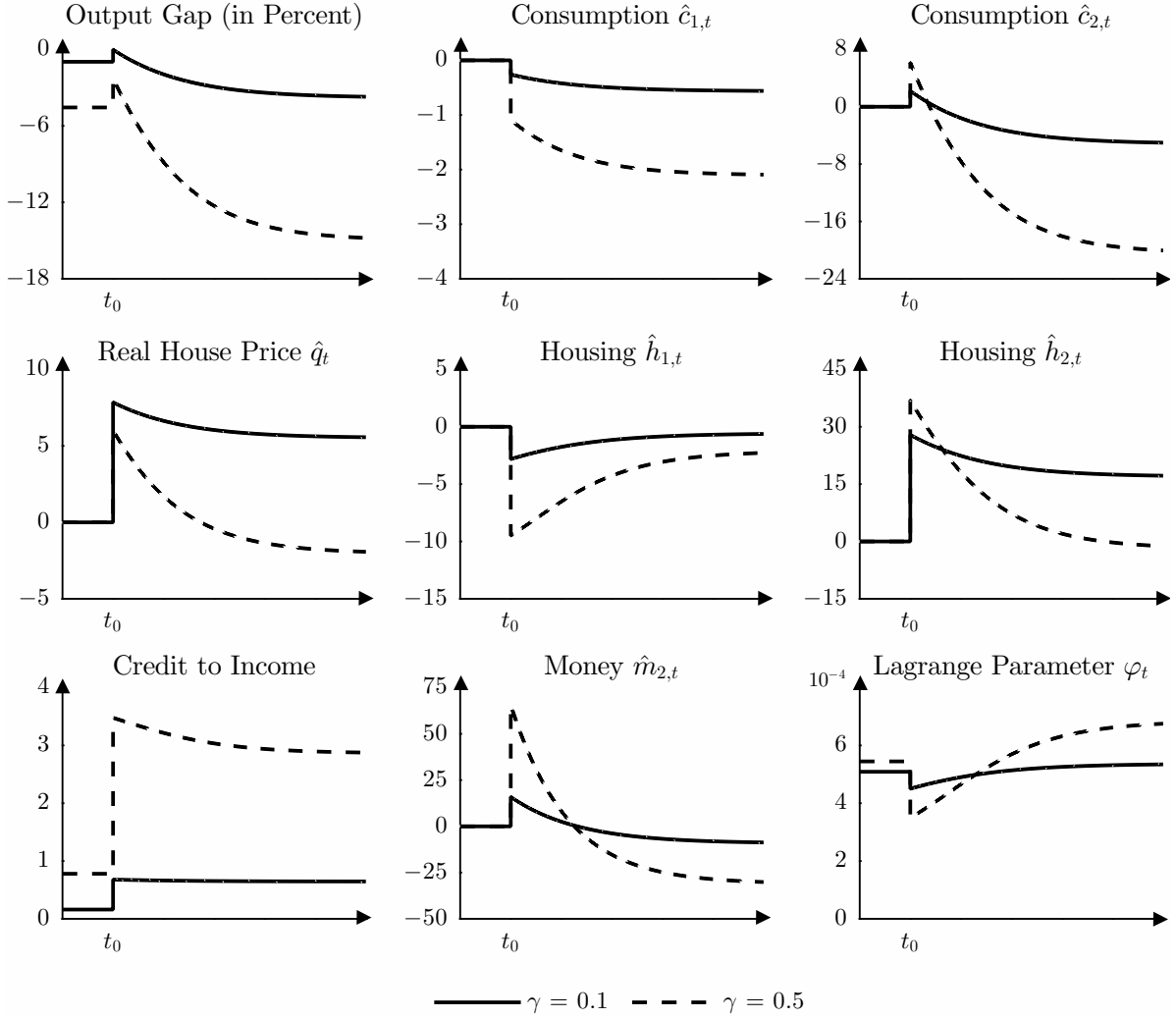
In addition, an asset price boom ensues since the real house price surges (second row) as housing demand of impatient households increases. The initial jump in the house price has a positive valuation effect on the housing holdings of both agents, which increases the real value of their assets. A feedback loop sets in with higher house prices increasing the collateral value of borrowers which in turn enhances their borrowing ability thereby reinforcing the initial credit boom. The housing allocation shifts in favor of the impatient households, which further strengthens the value of their collateral.²⁶

The allocation of new funds among consumption, money and housing investment is guided by the parameters in the utility function. Higher impatience implies a stronger increase in consumption and hence aggregate demand and inflation. In contrast, higher preferences for housing imply that more of the newly available funds are spent on purchasing fixed supply assets. As a consequence, higher preferences for housing imply a stronger amplification of the dynamics because of more pronounced collateral effects. In fact, aggregate consumption might actually fall during the credit boom for very high values of γ . Figure 4 illustrates the dependency of the dynamic responses on γ .

Over time, some of the newly acquired assets are sold by the borrower to smooth its consumption and for interest payments. Therefore, the allocation of the housing stock reverts in favor of the saver and aggregate demand remains above its new equilibrium for a prolonged period, thereby masking the underlying demand deficiency. Yet, eventually the resulting debt overhang pushes the economy into persistent stagnation, which is worse than before the credit boom, as we will show in the next subsection.

²⁶This is the same propagation mechanism as described in Kiyotaki and Moore (1997) and Iacoviello (2005) among others, which creates amplification and persistence of shocks. The borrowing constraint is binding throughout the adjustment process as illustrated by the behavior of φ_t in Figure 4, which is strictly positive. For a treatment of occasionally binding constraints, see Guerrieri and Iacoviello (2017).

Figure 4: Dynamic Effects of Credit Booms under Stagnation



Note: This figure shows the dynamics associated with a permanent increase in the loan-to-value ratio from $\theta = 0.15$ to $\theta = 0.5$. The output gap is given in percentage points. The credit to income ratio and the Lagrange parameter φ_t are given in absolute values. All other variables are depicted as deviations from the *initial* steady state in percent, denoted by \hat{x}_t . We assume the following utility from money for the borrower: $v(m_{2,t}) = \beta m_{2,t} + \delta \ln(m_{2,t})$. The figure is based on the following calibration: $\beta = 0.0005$, $\bar{y} = 100$, $\rho_1 = 0.05$, $\rho_2 = 0.1$, $\alpha = 0.01$, $n = 0.5$, $H = 1$ and $\delta = 0.1$. Simulations are based on a modification of the relaxation algorithm of Trimborn et al. (2008).

Also note that a house price boom, which is typically modeled by an increase in γ (cf. Iacoviello, 2005), can temporarily stimulate the stagnating economy though at the cost of more severe stagnation in the long run. The argument is similar: Higher housing demand creates an immediate increase in the real house price resulting in valuation gains for both households. In addition, the value of collateral that borrowers can pledge for funds increases, which initiates a credit boom. These funds are used to increase consumption, money holdings and housing investment, the last of which feeds back into the value of borrowers' collateral. The dynamics are similar to those in Figure 4.

4.2 Debt Overhang and Stagnation

While increases in θ and γ can temporarily stimulate aggregate demand by initiating a credit boom, they also affect the properties of the stagnation steady state. The former represents financial liberalization - or the degree of sustainable finance - whereas the latter is a proxy for the level of asset prices. Higher leverage θ and a higher house price reduce aggregate demand in the asymmetric steady state and hence worsen economic stagnation. This is summarized in the following proposition (see Appendix C for the proof):

Proposition 2 *In the asymmetric steady state under stagnation, an increase in the loan-to-value ratio reduces aggregate demand and worsens deflation. It holds that*

$$\frac{dC^*}{d\theta} < 0, \quad \frac{dc_1^*}{d\theta} < 0, \quad \frac{dc_2^*}{d\theta} < 0, \quad \frac{dm_2^*}{d\theta} < 0, \quad \frac{da_2^*}{d\theta} < 0, \quad \frac{d\pi^*}{d\theta} < 0.$$

The same effects arise from an increase in the housing preference γ . It holds that

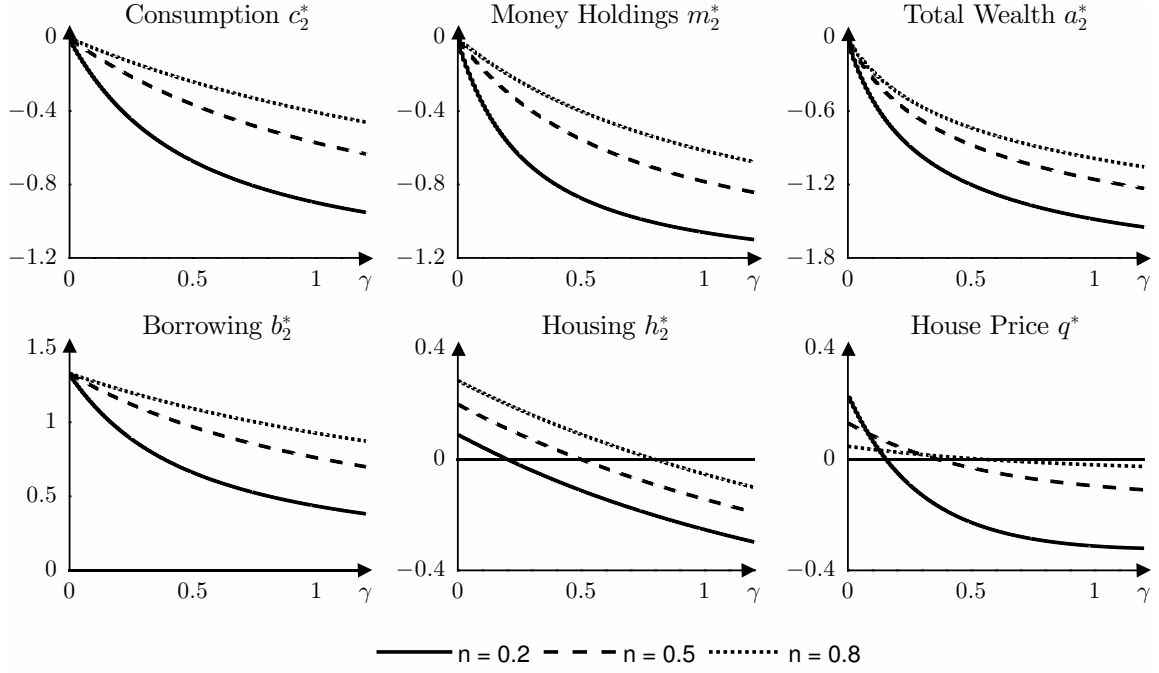
$$\frac{dC^*}{d\gamma} < 0, \quad \frac{dc_1^*}{d\gamma} < 0, \quad \frac{dc_2^*}{d\gamma} < 0, \quad \frac{dm_2^*}{d\gamma} < 0, \quad \frac{da_2^*}{d\gamma} < 0, \quad \frac{d\pi^*}{d\gamma} < 0.$$

Consider intuitively the effects of an increase in the loan-to-value ratio θ . Initially, the borrowing constraint (6) is relaxed allowing the borrower to acquire new funds. However, the new steady state is associated with higher debt and hence higher real interest payments as the steady state real interest rate is not affected. These payments are a fraction $\theta\rho_1\gamma\bar{\rho}_\theta^{-1}$ of the borrower's consumption spending where $\bar{\rho}_\theta$, given in 26, is decreasing in θ . This can be seen from (40). Therefore, higher leverage is associated with higher interest costs per unit of consumption which implies that the borrower's income is not sufficient to cover expenditures for a given c_2^* once θ increases. As this would violate the lifetime budget constraint of the borrower, his consumption spending has to decline.

This implies that the expenditures of the borrower are reduced (“spending effect”), raising disposable income. However, the lower spending negatively affects the borrower's income since aggregate demand declines (“demand effect”). This partially offsets the first effect. In addition, the real return on money holdings is affected (“capital gains effect”): Higher deflation increases the return on money. Yet, lower consumption discourages money holdings. The first effect is stronger, the higher money holdings, but the net effect is always negative in the asymmetric stagnation case (see Appendix B). This implies that a decrease in consumption reduces expenditures. Hence, consumption must decline in response to an increase in θ . These effects can be seen from the total differential of (40):

$$\underbrace{\left(n \frac{dc_1^*}{dc_2^*} + 1 - n\right) \frac{dc_2^*}{d\theta}}_{\text{Demand Effect}} + \underbrace{\left(-\pi^* \frac{dm_2^*}{dc_2^*} - m_2^* \frac{d\pi^*}{dc_2^*}\right) \frac{dc_2^*}{d\theta}}_{\text{Capital Gains Effect}} - \underbrace{\left(1 + \frac{\theta\rho_1\gamma}{\bar{\rho}_\theta}\right) \frac{dc_2^*}{d\theta}}_{\text{Spending Effect}} = \underbrace{\frac{\rho_1\rho_2\gamma}{\bar{\rho}_\theta^2} c_2^*}_{\text{Interest Cost}}.$$

Figure 5: Elasticities with Respect to θ



Note: This figure shows the elasticities of the model variables with respect to θ as a function of γ and for different values of n . The y-axis shows the %—change in each variable in response to a 1% increase in θ . We assume the following utility from money: $v(m_{2,t}) = \beta m_{2,t} + \delta \ln(m_{2,t})$. The calibration is $\beta = 0.0005$, $\bar{y} = 100$, $\rho_1 = 0.05$, $\rho_2 = 0.1$, $\alpha = 0.01$, $\delta = 0.1$, $\theta = 0.5$ and $H = 1$.

The decrease in consumption of the borrower feeds back into the other variables of the model. Aggregate demand decreases, which aggravates deflation via (2). Deflation in turn reduces the nominal interest rate via (5) since the real rate is determined by ρ_1 . This reduces consumption of the saver, which can be seen in (32). In addition, money demand of the borrower declines, as is clear from (35), as does the borrower's real wealth.

The effects of a higher θ on the real house price and the distribution of the housing stock are ambiguous because of two opposing effects: Investment in housing becomes more attractive for a given level of consumption c_2^* since housing becomes more collateralizeable. Higher housing demand bids up the house price. However, there is an indirect effect on the house price because lower consumption spending decreases housing demand of both agents which in turn lowers the real house price. This can be seen from (39).

The preference for housing γ determines the relative strength of these effects. The higher γ , the weaker the effects associated with the higher collateral value relative to the negative effect on consumption. If γ is sufficiently high, the indebtedness of the borrower might actually *decline* in response to financial liberalization since housing is reallocated to the saver. The reason is that higher levels of γ are associated with a higher collateral value and hence higher household debt. A given change in θ needs to be balanced by a stronger decline in consumption and hence a stronger reduction in housing demand.

Figure 5 illustrates the effects of a rise in the loan-to-value ratio θ on the steady state. Each subplot shows the elasticity of the respective variable to a rise in θ as a function of the housing preference parameter γ for three different values of n . We set parameters such that the economy is at full employment for $\theta = 0$. In particular, note the negative effect on the borrower's housing investment and the real house price for large values of γ . This in turn implies that financial liberalization is associated with a substantial decrease in the real wealth of the poor household. Also, the responses of consumption and asset prices are stronger the higher the share of poor households.

In the literature, γ is typically calibrated to match empirical observations on the housing market. In a similar framework, Iacoviello (2005) chooses a value of $\gamma = 0.1$ to match the value of residential housing to output in the United States. Guerrieri and Iacoviello (2017) follow a similar approach in a model with endogenous housing supply and select a value of $\gamma = 0.04$. When we apply the same criterion, the implied value of γ ranges between 0.08 and 0.1 and is substantially below unity. This implies the dominance of the collateral channel and hence financial liberalization raises asset prices and credit-financed housing investment.

These effects are in stark contrast to the standard neoclassical case with $v''(m_i) < 0$ for both types of households. From (26), it is clear that an increase in indebtedness reduces the consumption demand of the borrower in the neoclassical steady state since this agent faces higher real interest payments. Yet, aggregate demand is unaffected by variations in θ or γ because the saver increases his consumption level accordingly as long as his liquidity premium is decreasing in real money holdings, which can be seen from (27). As a consequence, changes in these parameters do result in a redistribution of available income and hence of consumption spending and housing investment. However, they do not trigger deviations from full employment because aggregate demand is not affected by these changes. In addition, the price level will adjust to clear the money market, which can be inferred from (23).

Similarly, aggregate demand is no longer affected by variations in these parameters once the model economy is in the symmetric steady state under stagnation. Then, variations in θ or γ cease to affect the consumption spending of both agents and simply lead to a redistribution of the housing stock and changes in the real house price. This case as well as other extensions of the model will be discussed in the next section.

5 Model Extensions and Discussion

In this section, we analyze two extensions of the model that have been turned off so far in order to focus on the core mechanism. In addition, we discuss policy options and their welfare implications.

5.1 Asymmetric and Symmetric Stagnation

From Proposition 1, it follows that stagnation does not occur for sufficiently low levels of potential output. In addition, it is clear from (28) and the discussion in the previous section that the borrower will also eventually choose to accumulate money holdings if his consumption level is sufficiently high. More specifically, it follows from (35) that symmetric stagnation will occur once the borrower's consumption level has reached the critical threshold of $\chi/\beta(\beta\bar{y} - \alpha)$. We first derive a sufficient condition for asymmetric stagnation to prevail and then give an intuition for the occurrence of the symmetric stagnation case in which both savers and borrowers accumulate money indefinitely.²⁷

Under symmetric stagnation, $v'(m_{1,t}) = v'(m_{2,t}) = \beta$ and both households accumulate wealth infinitely. Consumption of neither type is stimulated by additional money. Formally, the symmetric steady state is defined as follows:

Definition 4 *In the **symmetric steady state** under stagnation, the real and nominal interest rates are constant, the price level is declining at a constant rate, the real house price is constant, the real consumption levels of both agents are constant but the wealth of each household expands infinitely:*

$$\dot{r} = 0, \dot{R} = 0, \pi < 0, \dot{c}_1 = 0, \dot{c}_2 = 0, \dot{q} = 0, \dot{a}_1 > 0, \dot{a}_2 > 0.$$

The economy experiences secular stagnation once potential output exceeds the threshold \tilde{y} defined in condition (29). Then, $v'(m_{1,t}) = \beta$ and there is deflation and demand shortage, i.e. $\pi < 0$ and $C < \bar{y}$ from (2) and (21). Consider the population-weighted average of (8) and (13) with $\dot{c}_1 = \dot{c}_2 = 0$:

$$n\rho_1 + (1-n)\rho_2 = \beta nc_1 + v'(m_2)(1-n)c_2 - \pi. \quad (47)$$

Symmetric stagnation cannot occur if $\beta\bar{y} < n\rho_1 + (1-n)\rho_2$. To see this, suppose we have $v'(m_{2,t}) = \beta$ and $\beta\bar{y} < n\rho_1 + (1-n)\rho_2$. Then from (2), (21) and (47), we get

$$\begin{aligned} \beta\bar{y} - \alpha &< n\rho_1 + (1-n)\rho_2 - \alpha = \beta[nc_1 + (1-n)c_2] - \pi - \alpha \\ &= \beta C - \alpha \left(\frac{C}{\bar{y}} - 1 \right) - \alpha = (\beta\bar{y} - \alpha) \frac{C}{\bar{y}}. \end{aligned}$$

This only holds for $C > \bar{y}$ which is not the case. Hence, we always have $v'(m_{2,t}) > \beta$ for $\beta\bar{y} < n\rho_1 + (1-n)\rho_2$. Together with Proposition 1 and Condition (36), this yields the following proposition:

²⁷Alternatively, we can rule out the case of symmetric stagnation if we simply assume that only the saver's demand for liquidity is insatiable with $\beta_1 > 0$ while the borrower has standard liquidity preferences with $\beta_2 = 0$. However, as shown in this section, sufficient conditions for the existence of an asymmetric steady state can be characterized for our more general case.

Proposition 3 *Given the parameter restrictions $\rho_1 > \alpha$ and $\beta\bar{y} > \alpha$, the following condition is sufficient for the asymmetric steady state under stagnation to occur:*

$$n\rho_1 \left(\frac{\theta\rho_1\gamma + \bar{\rho}_\theta}{\theta\rho_1\gamma + n\bar{\rho}_\theta} \right) < \beta\bar{y} < n\rho_1 + (1-n)\rho_2 . \quad (48)$$

The first inequality in (48) follows from (29) and ensures that aggregate demand falls short of potential output and the second inequality ensures asymmetry. Intuitively, the second condition requires that the time preference rate ρ_2 is sufficiently high so that borrowers still strive for higher consumption. Yet, note that an increase in ρ_2 also tightens the first inequality, which is clear from Proposition 1.

Importantly, (48) is a sufficient condition for the existence of the asymmetric steady state but not a necessary condition. Under certain conditions, the asymmetric stagnation case will prevail for higher values of potential output. This is the case when further increases in potential output do not stimulate the borrower's consumption to exceed the threshold discussed above. We discuss the necessary existence condition in detail in Appendix B and only provide some intuition here.

Intuitively, the borrower's consumption depends on two factors as can be seen from (40): Income from firm profits which are determined by aggregate demand and capital gains on money holdings which depend on the rate of deflation. Under stagnation, an increase in the economy's production capacity worsens deflation which has two effects on the borrower's income. On the one hand, deflation reduces the consumption incentives of the saver. This reduces the income of the borrower since aggregate demand declines ("aggregate demand effect"). On the other hand, the purchasing power of money holdings rises which stimulates the borrower's consumption ("capital gains effect"). The second effect is stronger the higher his money holdings. If the capital gains effect dominates, the borrower's consumption increases with a higher production capacity as do his money holdings. Then, the marginal utility of money eventually reaches the lower bound and symmetric stagnation occurs.²⁸ But the asymmetric case may persist even for high levels of potential output \bar{y} as long as the capital gains effect is weak or negative.

To summarize, our model features three regions depending on \bar{y} : If potential output is below the threshold \tilde{y} given by (29), the neoclassical case applies and there is no demand shortage. In contrast, stagnation occurs for $\bar{y} > \tilde{y}$ because of the insatiability of liquidity preferences. The asymmetric case always occurs if condition (48) holds and might prevail for even higher values of potential output. Finally, the symmetric case occurs if consumption of the borrower under stagnation becomes sufficiently high.

²⁸Thus, there will be an implicit threshold \hat{y} such that there is symmetric stagnation for $\bar{y} > \hat{y}$. This threshold depends on the model parameters, particularly on those affecting equilibrium money holdings of the borrower. These in turn depend on the shape of the utility function $v(m)$. Therefore, we cannot give a closed-form expression for this threshold.

5.2 Stagnation with Positive Money Growth

So far, we have focused on the case of zero trend inflation, i.e. $\mu = 0$, under full employment. Two considerations need to be taken into account for $\mu > 0$ that affect the occurrence of stagnation as well as the existence of the stagnation steady state. For the general conditions and proofs, we refer to Ono and Ishida (2014) for the case of homogeneous households. Here, we will provide an intuitive discussion of the effects of $\mu > 0$ for the case of heterogeneous agents in the borrower-saver framework.

First, as argued above, stagnation occurs once one of the households is no longer willing to consume the amount consistent with full employment because of his insatiable desire for holding liquidity, i.e. once the following threshold is reached for any household:

$$\tilde{c}_i^{NC} > \frac{\rho_i + \mu}{\beta} . \quad (49)$$

This is a generalization of condition (28) for the case of positive money growth. Two effects emerge relative to the case of $\mu = 0$ that has been discussed so far.

Positive nominal money growth raises the nominal interest rate under full employment, due to the Fisher equation (5). This increases the opportunity cost of holding money for both agents, which stimulates their consumption, thereby increasing the liquidity premium. As a consequence, full employment can be sustained for higher levels of potential output and stagnation occurs at a later stage. In fact, for every level of potential output \bar{y} there exists a nominal money growth rate μ such that full employment prevails. However, this comes at the cost of higher inflation.

In addition, there is a more subtle effect as positive money growth might affect both households' consumption levels \tilde{c}_i^{NC} under full employment. This crucially depends on the assumption about the distribution of seignorage profits $z_t = \mu m_t$ that show up in the flow of funds constraints of both agents in (4). If these are distributed in proportion to each agents money holdings, there is no effect on the full employment levels of consumption, given by (26) and (27).²⁹ However, if seignorage income is distributed equally across households, the household with lower money holdings benefits at the expense of the household with higher money holdings. For reasonable parameter specifications, it will be the saver whose consumption will be lowered by this effect, while the borrower benefits. This further increases the income threshold for stagnation.³⁰

²⁹The intuition is simple: Each household incurs implicit costs of money holdings due to inflation. In turn, the household benefits from inflation via the seignorage profits. If profits are distributed in proportion to money holdings, these effects exactly offset each other.

³⁰It could even be the case that the saver's consumption is actually *lower* under full employment than the borrower's consumption level because of the redistributive effect of inflation. Yet, note that this effect only occurs for a very restrictive parameterization. Specifically, both the difference in discount rates and the money growth rate need to be sufficiently high. In addition, the loan-to-value ratio or the housing preference parameter need to be sufficiently low.

Secondly, the existence condition of the asymmetric steady state is affected. Because of persistent deflation, the money supply expands indefinitely and so does the wealth level of the saver. With $\mu = 0$, the rate of expansion is given by the rate of deflation as is clear from (22). Since the deflation rate is below the real interest rate, as we assume $\rho_1 > \alpha$, the transversality condition (11) holds despite this expansion. With positive nominal money growth, however, the expansion of the real money supply increases to $\mu - \pi$ as does the growth rate of household wealth. For the transversality condition to hold, we need to require that this rate of expansion is below the time preference rate of the saver that determines the real interest rate. Specifically, for an asymmetric steady state to exist, it has to hold that

$$0 > \frac{\dot{m}_t}{m_t} - \rho_1 = \mu - \beta \tilde{c}_1^*, \quad (50)$$

where \tilde{c}_1^* denotes the saver's consumption in the asymmetric steady state with $\mu > 0$.

On top of that, the occurrence condition of the symmetric stagnation steady state is affected by introducing positive money growth. The effects depend again on the assumption on the distribution of seignorage income. If this income is distributed in proportion to each household's money holdings, then there are no effects as the borrower's consumption under asymmetric stagnation is not affected. In contrast, if this income is distributed equally across households, the borrower's consumption will be stimulated under asymmetric stagnation. As the money supply expands, so does his exogenous income, which allows for higher consumption. Then, the symmetric stagnation case will eventually occur if condition (50) holds.

In conclusion, the equilibrium of the economy is conditional on the money growth rate. A sufficiently high rate of money growth may help to restore full employment, similar to other models of secular stagnation. Since this comes at the cost of high inflation, policymakers are likely to be inclined to prefer a scenario of persistent stagnation and take measures to improve aggregate demand within that equilibrium. Even worse, the interplay of conditions (49) and (50) also implies that multiple equilibria can emerge with both stagnation and full employment as steady state equilibria for the same parameterization. It might also be the case that no equilibrium exists at all. So once the economy has reached stagnation, it will be very hard and costly in terms of high inflation to move towards the full employment equilibrium.

For that reason, our analysis has focused primarily on the stagnation case with $\mu = 0$. Note, however, that the conclusions also hold for a low inflation scenario which requires sufficiently low levels of monetary growth.³¹

³¹This is similar to the assumptions of Michaillat and Saez (2014) and Michau (2018) that the central bank follows a sufficiently low inflation target.

5.3 Policy Discussion and Welfare Considerations

Two features in our model prevent the economy from reaching full employment - insatiable liquidity preferences and debt overhang. Insatiable liquidity preferences imply that stagnation occurs for sufficiently high levels of potential output, even in the absence of financial frictions. The reason is that agents prefer to hold money instead of consumption, which prevents the nominal interest rate from falling sufficiently to clear the goods market at full employment. This implies that expansionary monetary policy is ineffective in the stagnation steady state of our model.³² In fact, the deflationary steady state is characterized by an infinite expansion of the real money stock.

Several policies can increase aggregate demand in the stagnation steady state. The case for fiscal policy is straightforward: The government is not constrained by the same liquidity motives as the private sector and can expand its spending.³³ Redistributive policies work by transferring resources from rich agents to poor ones. The latter expand their consumption while spending of the former is not directly affected (unless at the margin). Therefore, targeted redistributive interventions can stimulate the economy. In reality, targeted transfers might not be feasible though. Yet, Matsuzaki (2003) shows that lump-sum transfers financed by a consumption tax can increase aggregate demand if the fraction of poor households is relatively small. Finally, policies that limit household indebtedness and help to repair balance sheets of spending-constrained households reduce private debt overhang and increase spending in steady state. However, although these policies stimulate aggregate demand, their welfare implications are less straightforward.

Consider a policy that reduces the loan-to-value ratio by $\Delta\theta$ to a permanently lower level, supplemented by debt relief for borrowers. Specifically, assume borrowers only need to repay a fraction $1 - \epsilon$ of the reduction in the debt limit while the remainder constitutes a one-time transfer from savers to borrowers given by

$$DR = \epsilon q^* h_2^* \Delta\theta , \quad (51)$$

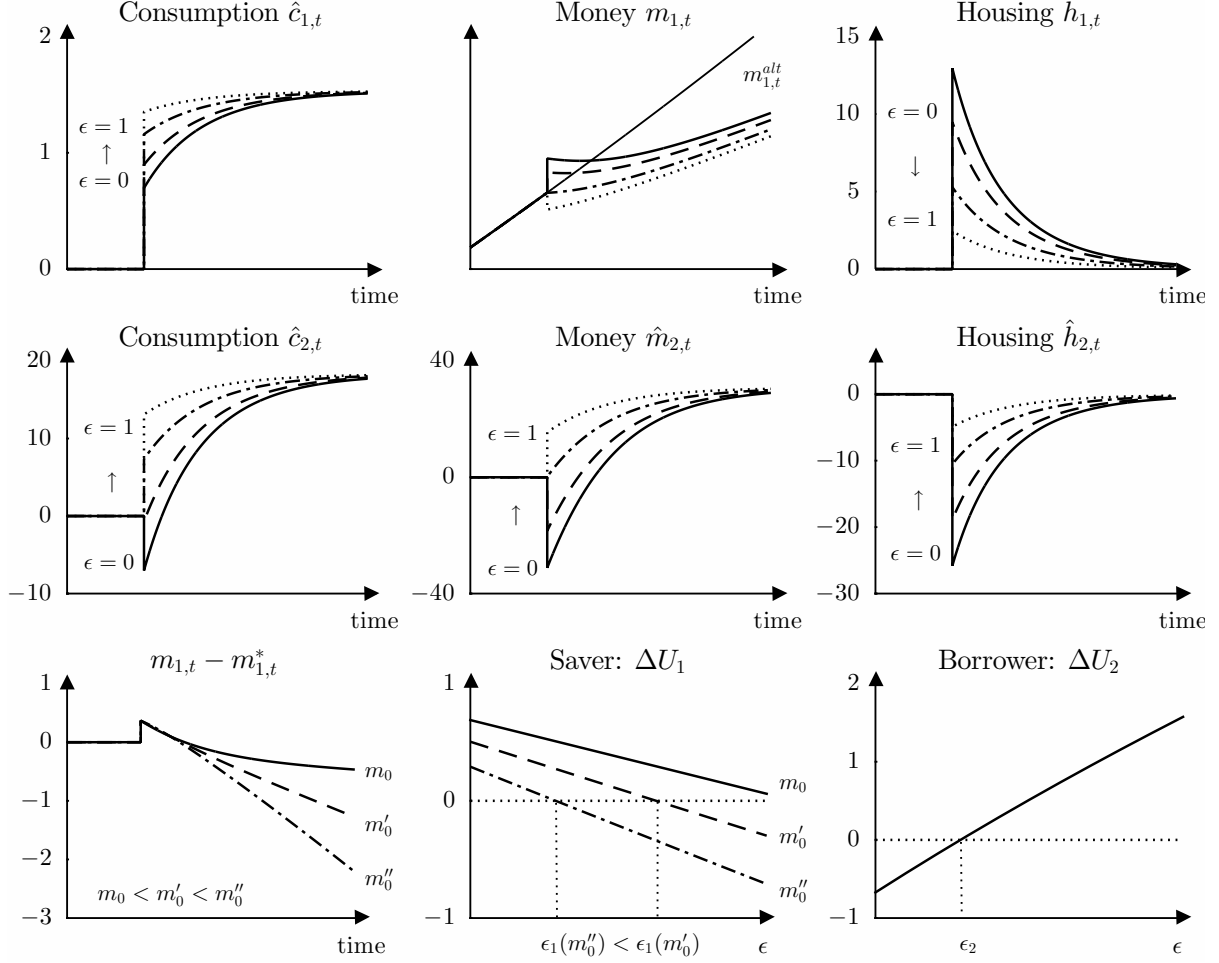
where ϵ is the haircut on the repayment and q^* and h_2^* refer to the original steady state.

The effects of this policy are illustrated in Figure 6 for various degrees of debt relief. Without debt relief, i.e. for $\epsilon = 0$ (solid lines), the effects are exactly opposite to those illustrated in Figure 4: The borrower's consumption, money holdings and housing investment decline at the implementation date, but are higher in the new steady state (potentially expect for housing), which is illustrated in the second row. In contrast, the

³²Even though a sufficiently large expansion of the money supply might restore the full employment case, this comes at the cost of inflation as discussed before.

³³The expansionary effect of government spending has nothing to do with deficit-budget financing or balanced-budget financing. It works through a direct creation of demand. We refer to Ono (1994, 2001) for an explicit modeling of government spending.

Figure 6: Welfare Analysis of Debt Relief



Note: This figure shows the welfare effects associated with a permanent decline in the loan-to-value ratio from $\theta = 0.5$ to $\theta = 0.25$ for various levels of debt relief ϵ . The solid lines in panels 1 to 6 show the baseline case of $\epsilon = 0$, while the dashed lines represent increasing levels of ϵ towards $\epsilon = 1$ (dotted lines). The dynamics are given as percentage deviations from the original steady state except for panel 2 which is in levels. The linear line in panel 2 represents the alternative path of the saver's money holdings in the absence of policy changes. The difference, $m_{1,t} - m_{1,t}^*$, is illustrated in panel 7 for $\epsilon = 0$ and different levels of the money supply m_0 (scaled relative to potential income \bar{y}) with $m_0 < m'_0 < m''_0$. Panels 8 and 9 show the associated welfare effects for savers and borrowers as a function of ϵ and for different levels of m_0 . These are calculated as the integrals over the respective short run dynamics plus a long run capitalized value that reflects permanent differences in the steady states. We assume the following utility from money for the borrower: $v(m_{2,t}) = \beta m_{2,t} + \delta \ln(m_{2,t})$. The figure is based on the following calibration: $\beta = 0.0005, \bar{y} = 100, \rho_1 = 0.05, \rho_2 = 0.1, \alpha = 0.01, n = 0.5, \gamma = 0.5, H = 1$ and $\delta = 0.1$. Simulations are based on a modification of the relaxation algorithm of Trimborn et al. (2008).

saver's consumption, money holdings and housing increase at initiation as shown in the first row. However, less deflation implies that the saver accumulates monetary balances at a lower rate relative to the original steady state. The difference between the new and the original path of money accumulation increases in the initial level of the real money stock m_0 as illustrated in the first graph of the third row.

More comprehensive debt relief implies that borrower's can afford higher consumption and money holdings in the short run, which limits the extent of the short run contraction and might even result in a positive short run demand effect if ϵ is sufficiently high. In addition, more debt relief increases the speed of convergence to the new steady state. At the same time, higher transfers to borrowers come at the expense of the saver's money holdings: As fewer funds are repaid, the saver's money holdings increase less at initiation or might even decline. In addition, the positive aggregate demand effect reduces deflation, which lowers the rate at which the saver's money stock expands. These effects are illustrated by the dashed lines in Figure 6 for different levels of ϵ .

The welfare effects for both agents relative to the original steady state can be inferred from their lifetime utility functions as shown in Appendix D. Specifically, it holds that

$$\Delta U_1 = \int_0^\infty \left[\hat{c}_{1,t} + \gamma \hat{h}_{1,t} - \frac{\beta m_2^*(1-n)}{n} \hat{m}_{2,t} + \frac{\beta}{n} \left(e^{-\int_0^t (\pi_s - \pi^*) ds} - 1 \right) m_0 e^{-\pi^* t} \right] e^{-\rho_1 t} dt ,$$

$$\Delta U_2 = \int_0^\infty \left[\hat{c}_{2,t} + \gamma \hat{h}_{2,t} + v(m_{2,t}) - v(m_2^*) \right] e^{-\rho_2 t} dt ,$$

where \hat{x}_t denotes the deviation from the original steady state in percent, x^* the original steady state and m_0 is the level of the real money stock at implementation of the policy.

Welfare effects for the borrower are positively related to the degree of debt relief as more comprehensive debt relief allows for more consumption, money holdings and housing investment. There exists a critical value $\epsilon_2 > 0$ above which the borrower is better off despite the tightening of the borrowing constraint.

The opposite holds for the saver as more debt relief reduces his lifetime utility. Hence, there exists a critical value $\epsilon_1 > 0$ above which the saver is made worse off as a consequence of this policy. In addition, the initial real money stock m_0 determines the welfare effects for the saver. A policy that stimulates aggregate demand reduces the rate of deflation below the original steady state, i.e. $\pi_s - \pi^* > 0$. As a consequence, the saver's money holdings expand at a lower rate, which is detrimental for his welfare. The higher the initial money supply, the more severe the effect of the lower inflation rate as can be seen in the first graph in the third row of Figure 6. Note that these two effects reinforce each other. More debt relief implies less deflation, which is more detrimental for higher values of m_0 . Therefore, $\epsilon_1 = \epsilon_1(m_0)$ with $\epsilon_1' < 0$.

It follows that two conditions need to hold for both agents to be better off by a policy of tighter debt limits in combination with debt relief: First, the degree of debt relief ϵ^* has to be sufficiently high to make the borrower better off, without however making the saver worse off. This requires $\epsilon_2 \leq \epsilon^* \leq \epsilon_1$. Second, we require the real money stock at implementation date m_0 to be sufficiently small for this condition to hold. The threshold money stock is implicitly defined by $m_0 < \tilde{m}_0$ where $\epsilon_1(\tilde{m}_0) = \epsilon_2$.

5.4 Further Extensions

Our conclusions hold when we impose the borrowing constraint on the supply side. The following thought experiment clarifies this point: Suppose the collateralizable asset is a factor of production and producers are constrained in their borrowing ability. As above, financial liberalization is associated with higher equilibrium collateral holdings by the borrower. These in turn imply a higher production capacity. Therefore, financial liberalization may improve equilibrium output under neoclassical assumptions. However, the economy is demand-constrained in our model so that the implied improvements in the supply side actually worsen the output gap and deflation. An increase in indebtedness hence deteriorates equilibrium income for reasonable parameter ranges, irrespective of the modeling of the borrowing friction on the demand side or supply side.

Finally, our results continue to hold with insatiable wealth preferences instead of liquidity preferences. Unlike the latter, wealth preferences affect the equilibrium real interest rate by encouraging household savings (cf. Ono, 2015; Kumhof et al., 2015). In addition, the nominal interest rate converges to zero in the secular stagnation equilibrium as the transaction demand for money becomes satiated whereas it is positive in our model (cf. Ono, 2016). As a consequence, the natural real rate of interest can turn negative in steady state (cf. Michau, 2018). In our setting, this would imply a redistribution from savers to borrowers as the real cost of debt becomes negative. However, the very existence of housing as a durable asset without depreciation prevents the real rate from turning negative in our setup. This can be easily seen from (18), which is unaffected by the introduction of wealth preferences. Housing yields a positive “dividend” stream in the form of the user cost of housing while the cost of housing investment are given by the real opportunity cost, since there is no depreciation. The real house price adjusts to make agents indifferent between housing investment and other uses of funds. Hence, from (18) a negative real rate of interest would require a decline of the real house price in steady state:

$$r^* < 0 \Leftrightarrow \frac{\dot{q}}{q} < -\frac{\gamma c_1^*}{h_1^*} < 0 .$$

This is not consistent with a stationary steady state. Moreover, it would imply that the real house price eventually converges to zero and hence that the current asset price itself is not well-defined. We can therefore exclude the possibility of a negative real rate in our model under wealth preferences. Hence, there cannot be a redistribution from savers to borrowers via negative interest cost of debt in steady state.³⁴

³⁴When we allow for housing depreciation, the real interest rate in a model with wealth preferences may in fact become negative. However, in a secular stagnation steady state with deflation, the realized real interest rate must be strictly positive. Hence, a redistribution from savers to borrowers via negative real interest payments cannot occur in the secular stagnation steady state.

6 Conclusion

Many developed countries, e.g. Japan, EU and the USA, have been suffering from persistent stagnation of aggregate demand under which some households do not increase consumption and keep wealth while others do not increase consumption because they are severely indebted. It typically occurred after a credit and stock price boom. To analyze this phenomenon, we have introduced private indebtedness into a model with two types of agents that have different time patience and insatiable preferences for money holding.

The less patient households borrow funds from the more patient ones but face a borrowing constraint that depends on the value of their housing. Therefore their consumption is restricted by this constraint. The more patient households earn interests from the lending and hence can expand consumption, but in fact do not because of high preference for money holding. Thus, aggregate demand shortages arise and deflation occurs. The deflation makes it more advantageous for the lenders to reduce consumption and hold money. It in turn expands the real value of debt of the borrowers and decreases their consumption because they have to pay high interests to the lenders.

If the borrowers could consume more, deflation would mitigate and stimulate the lenders consumption as well, leading to an expansion of total income. Thus, a government that faces this situation may be tempted to ease the borrowing constraint. It will indeed enable the borrowers to consume more and mitigate deflation, which also stimulates the lenders consumption by lowering the advantage of holding money. Moreover, easing the borrowing constraint makes the borrowers think housing investment to be more valuable because an increase in the value of housing enables them to borrow more for consumption. Thus, it triggers a housing price boom.

However, those positive effects occur only in the short run. In the long run the borrowers are more indebted so that they have to reduce consumption, which worsens deflation and makes the lenders to decrease consumption and save more because money holding is more profitable. The decrease in total consumption stops the housing price boom. The economy eventually falls into secular stagnation of aggregate demand. Thus, direct transfers from the richer to the poorer, which does not create debt overhang, will be more promising.

Appendices

A. Proof of Proposition 1

The model parameters affect the stagnation threshold \tilde{y} in (29) as follows:

$$\begin{aligned}\frac{\partial \tilde{y}}{\partial \beta} &= -\frac{n\rho_1}{\beta^2} \frac{\gamma\theta\rho_1 + \bar{\rho}_\theta}{\gamma\theta\rho_1 + n\bar{\rho}_\theta} < 0 , \\ \frac{\partial \tilde{y}}{\partial \rho_1} &= \frac{n[(1+\gamma)(\gamma+n)\theta^2\rho_1^2 + 2(1+\gamma)n\theta(1-\theta)\rho_1\rho_2 + (1-\theta)^2n\rho_2^2]}{\beta[\gamma\theta\rho_1 + n\bar{\rho}_\theta]^2} > 0 , \\ \frac{\partial \tilde{y}}{\partial \rho_2} &= \frac{n\rho_1}{\beta} \frac{(1-n)\gamma\theta(1-\theta)\rho_1}{[\gamma\theta\rho_1 + n\bar{\rho}_\theta]^2} > 0 , \\ \frac{\partial \tilde{y}}{\partial \theta} &= -\frac{n\rho_1}{\beta} \frac{\rho_1\rho_2\gamma(1-n)}{[\gamma\theta\rho_1 + n\bar{\rho}_\theta]^2} < 0 , \\ \frac{\partial \tilde{y}}{\partial n} &= \frac{\gamma\theta\rho_1^2[\gamma\theta\rho_1 + \bar{\rho}_\theta]}{\beta[\gamma\theta\rho_1 + n\bar{\rho}_\theta]^2} > 0 , \\ \frac{\partial \tilde{y}}{\partial \gamma} &= -\frac{n\rho_1}{\beta} \frac{(1-n)\rho_1\theta\bar{\rho}_\theta}{[\gamma\theta\rho_1 + n\bar{\rho}_\theta]^2} < 0 .\end{aligned}$$

B. Existence and Stability of the Asymmetric Steady State

Existence: Using (32) to substitute for c_1^* and (35) for c_2^* , we rewrite (40) as

$$F(m_2) \equiv \alpha[(\beta\bar{y} - \rho_1 n)v'(m_2) - (1-n)\beta\rho_2]m_2 + n(\rho_1 - \alpha)\bar{y}[v'(m_2) - \beta] = A , \quad (\text{B.1})$$

$$\text{where } A \equiv \frac{\theta\rho_1\gamma}{\bar{\rho}_\theta}\chi + n(\beta\bar{y} - \alpha)(\rho_2 - \rho_1) > 0 ,$$

$$\text{and } \chi \equiv \rho_2(\beta\bar{y} - \alpha n) - \alpha(\beta\bar{y} - \rho_1 n) > 0 .$$

The asymmetric steady state exists for $\bar{y} > \tilde{y}$ if there exists a finite $m_2 > 0$ as a solution to this equation. Note that the RHS of (B.1) is a positive constant that is independent of m_2 . In contrast, the LHS is a function of m_2 . It holds that $\lim_{m_2 \rightarrow 0} F(m_2) = \infty$ and

$$\lim_{m_2 \rightarrow \infty} F(m_2) = \begin{cases} -\infty & \text{if } \beta\bar{y} < n\rho_1 + (1-n)\rho_2 , \\ \lim_{m_2 \rightarrow \infty} \alpha(\beta\bar{y} - \rho_1 n)(v'(m_2) - \beta)m_2 & \text{if } \beta\bar{y} = n\rho_1 + (1-n)\rho_2 , \\ +\infty & \text{if } \beta\bar{y} > n\rho_1 + (1-n)\rho_2 . \end{cases}$$

Hence, there has to be a solution to (B.1) if $\beta\bar{y} < n\rho_1 + (1-n)\rho_2$. This is the sufficient condition in Proposition 3. For higher values of \bar{y} , there may be two solutions, exactly one solution or no solution to (B.1). Existence then requires that the minimum (or limit

if $\beta\bar{y} = n\rho_1 + (1-n)\rho_2$ of $F(m_2)$ is smaller than the RHS:

$$\min_{m_2} F(m_2) < \frac{\theta\rho_1\gamma}{\bar{\rho}_\theta}\chi + n(\beta\bar{y} - \alpha)(\rho_2 - \rho_1) . \quad (\text{B.2})$$

This condition guarantees the existence of at least one solution to (B.1). In case of multiple solutions, we choose the solution that satisfies $F'(m_2^*) < 0$. This is for two reasons: First, it is consistent with continuous variations in \bar{y} . Second, this solution satisfies saddle-point stability, whereas the other solution is unstable. Therefore, a necessary condition for the asymmetric steady state is given by

$$\left. \frac{\partial F(m_2)}{\partial m_2} \right|_{m_2^*} < 0 . \quad (\text{B.3})$$

Finally, if there is no finite value of m_2 that solves (B.1), we must have $\dot{a}_{2,t} > 0$ which implies that the economy is in the symmetric stagnation steady state.

To summarize: *The asymmetric steady state exists for $\beta\bar{y} > n\rho_1 + (1-n)\rho_2$ if there exists a finite, positive value of m_2 that solves (B.1). Moreover, (B.2) is a sufficient condition for the existence of the asymmetric steady state given $\beta\bar{y} - n\rho_1 - (1-n)\rho_2 \geq 0$. In addition, (B.3) is a necessary condition for the asymmetric steady state to occur.*

For illustration, consider the specific utility function $v(m_2) = \beta m_2 + \delta \ln(m_2)$. Figure 7 shows the behavior of the two sides of (B.1) as a function of m_2 , which is given by

$$\alpha\beta(\beta\bar{y} - \rho_1 n - (1-n)\rho_2)m_2 + \delta\alpha(\beta\bar{y} - \rho_1 n) + \frac{\delta n(\rho_1 - \alpha)\bar{y}}{m_2} = \frac{\theta\rho_1\gamma}{\bar{\rho}_\theta}\chi + n(\beta\bar{y} - \alpha)(\rho_2 - \rho_1) .$$

For $\beta\bar{y} - n\rho_1 - (1-n)\rho_2 = 0$, existence of the steady state requires a sufficiently low value of δ :

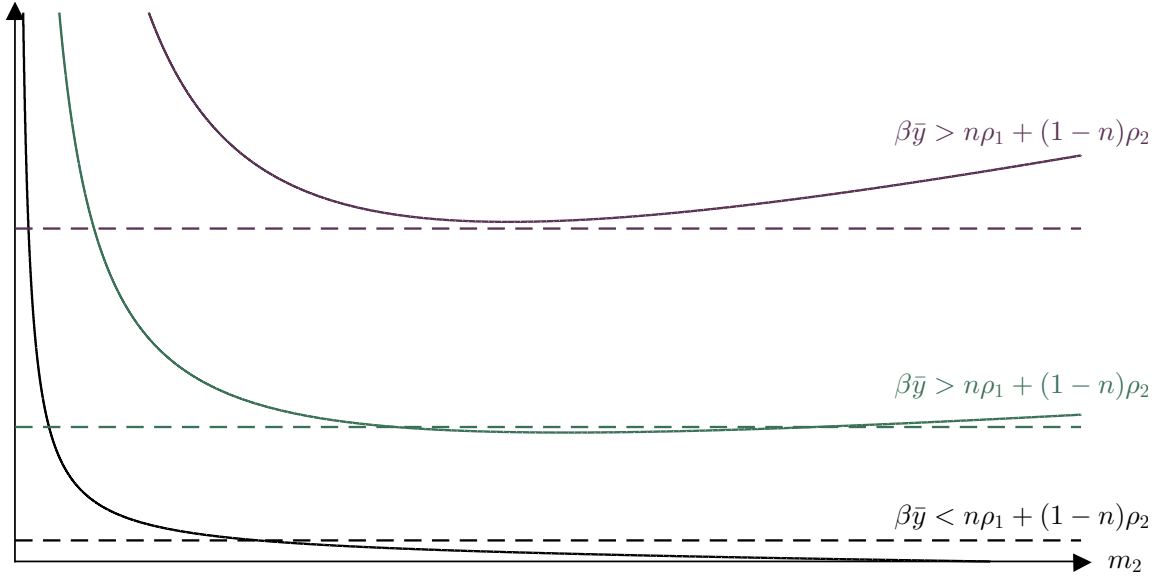
$$\delta < \bar{\delta} \equiv \frac{1}{\alpha(\beta\bar{y} - \rho_1 n)} \left[\frac{\theta\rho_1\gamma}{\bar{\rho}_\theta}\chi + n(\beta\bar{y} - \alpha)(\rho_2 - \rho_1) \right] . \quad (\text{B.4})$$

For $\beta\bar{y} - n\rho_1 - (1-n)\rho_2 > 0$, we require in addition that (B.2) holds which implies

$$\frac{\left[\frac{\theta\rho_1\gamma}{\bar{\rho}_\theta}\chi + n(\beta\bar{y} - \alpha)(\rho_2 - \rho_1) - \alpha\delta(\beta\bar{y} - \rho_1 n) \right]^2}{4\alpha\beta n(\rho_1 - \alpha)\delta\bar{y}} > \beta\bar{y} - n\rho_1 - (1-n)\rho_2 . \quad (\text{B.5})$$

Stability: The dynamic system is characterized by six differential equations for c_1 , c_2 , q , a_1 , a_2 and m given by (41), (42), (43), (45), (44) and (46) and by the static equations (19), (20) and (25) for m_2 , h_1 and h_2 . The asymmetric steady state under stagnation is characterized by a diverging real money supply and real assets of the saver. Define $z_{1,t} \equiv a_{1,t}^{-1}$ and $z_{2,t} \equiv m_t^{-1}$. Then the steady state of $\{c_{1,t}, c_{2,t}, q_t, a_{2,t}, z_{1,t}, z_{2,t}\}$ is given by

Figure 7: Existence of the Asymmetric Steady State under Stagnation



Note: This figure illustrates the LHS (solid line) and RHS (dotted line) of (B.1) for different values of potential output \bar{y} and for the specific utility function $v(m_2) = \beta m_2 + \delta \ln(m_2)$.

$\{c_1^*, c_2^*, q^*, a_2^*, 0, 0\}$. We linearize the system around this steady state using a first-order Taylor approximation:

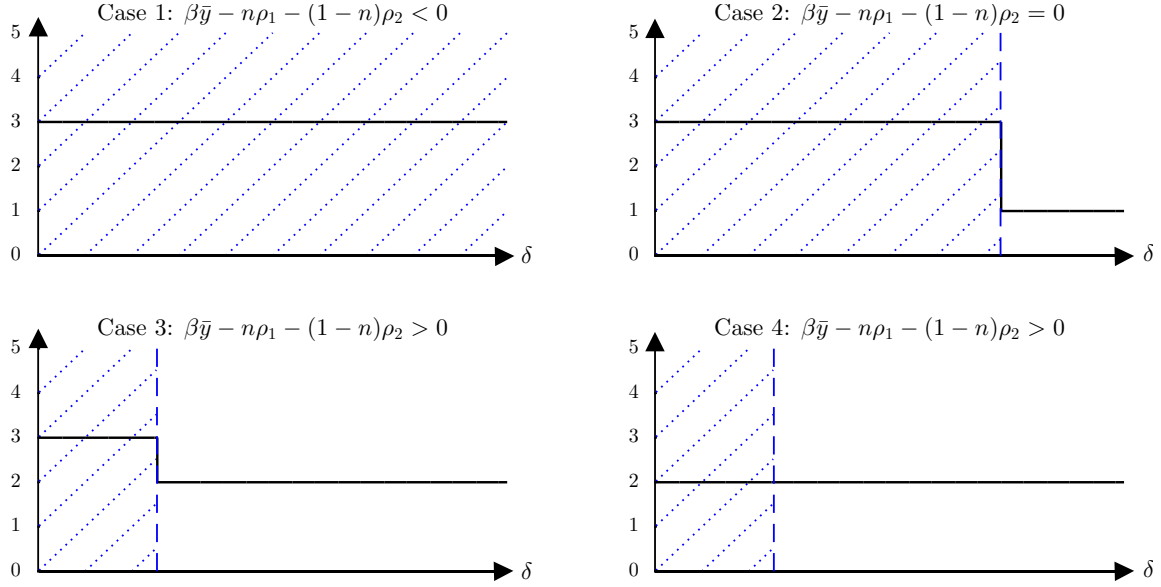
$$\begin{pmatrix} \dot{c}_{1,t} \\ \dot{c}_{2,t} \\ \dot{q}_t \\ \dot{a}_{2,t} \\ \dot{z}_{1,t} \\ \dot{z}_{2,t} \end{pmatrix} = \begin{pmatrix} v_{11} & v_{12} & 0 & 0 & 0 & 0 \\ v_{21} & v_{22} & v_{23} & v_{24} & 0 & 0 \\ v_{31} & v_{32} & v_{33} & v_{34} & 0 & 0 \\ v_{41} & v_{42} & v_{43} & v_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & v_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & v_{66} \end{pmatrix} \begin{pmatrix} c_{1,t} - c_1^* \\ c_{2,t} - c_2^* \\ q_t - q^* \\ a_{2,t} - a_2^* \\ z_{1,t} - z_1^* \\ z_{2,t} - z_2^* \end{pmatrix},$$

where the entries v_{ij} in the transition matrix V refer to the respective terms in the linearized system. The eigenvalues ξ_i of V determine the stability of this system and solve

$$\left((v_{11} - \xi) \begin{vmatrix} v_{22} - \xi & v_{23} & v_{24} \\ v_{32} & v_{33} - \xi & v_{34} \\ v_{42} & v_{43} & v_{44} - \xi \end{vmatrix} - v_{12} \begin{vmatrix} v_{21} & v_{23} & v_{24} \\ v_{31} & v_{33} - \xi & v_{34} \\ v_{41} & v_{43} & v_{44} - \xi \end{vmatrix} \right) (v_{55} - \xi)(v_{66} - \xi) = 0,$$

where $\|Q\|$ is the determinant of Q . Since only $c_{1,t}$, $c_{2,t}$ and q_t are jumpable, there must be three positive and three negative eigenvalues for the system to exhibit saddlepoint stability. $\xi_i = \beta c_1^* - \rho_1 = \pi^*$ is a solution and under stagnation $\pi^* < 0$. Thus, these two eigenvalues are negative. We use a numerical analysis for the other solutions.

Figure 8: Stability of the Asymmetric Steady State under Stagnation



Note: This figure shows the number of negative eigenvalues in V for the function $v(m_2) = \beta m_2 + \delta \ln(m_2)$. Case 1 refers to existence condition (48) and cases 2 to 4 refer to conditions (B.4) and (B.5) which are represented by vertical lines. Variations in δ are shown on the x-axis. The calibration is as follows: $\beta = 0.0005$; $\rho_1 = 0.05$; $\rho_2 = 0.1$; $\alpha = 0.01$; $n = 0.5$; $H = 1$, $\theta = 0.5$ and $\bar{y} = 120$ (case 1), $\bar{y} = 150$ (case 2) and $\bar{y} = 200$ (cases 3 and 4). In the dashed areas, the existence conditions for the asymmetric steady state are fulfilled.

Based on the functional form $v(m_2) = \beta m_2 + \delta \ln(m_2)$, we simulate V for three cases determined by $\beta \bar{y} - n \rho_1 - (1 - n) \rho_2$. For each case, we vary δ (and implicitly m_2^*), which determines the strength of the capital gains channel in (40). We then determine the number of negative eigenvalues. The results are summarized in Figure 8 which also highlights the threshold parameter $\bar{\delta}$ in (B.4) or (B.5).

For $\beta \bar{y} - n \rho_1 - (1 - n) \rho_2 < 0$ (case 1), the system is saddlepoint-stable for all $\delta > 0$. This corresponds to condition (48). For $\beta \bar{y} - n \rho_1 - (1 - n) \rho_2 = 0$ (case 2), the system is saddlepoint-stable for $0 < \delta < \bar{\delta}$. Hence, under existence condition (B.4) the steady state exhibits saddle-point stability. For $\beta \bar{y} - n \rho_1 - (1 - n) \rho_2 > 0$, there are two solutions to (B.1) shown in cases 3 and 4. Both solutions require condition (B.5) to hold. Yet, only one of these solutions shows saddle-point stability. This is the solution that fulfills condition (B.3). We therefore conclude that the model is saddlepoint-stable around the asymmetric stagnation steady state under conditions (B.1), (B.2) and (B.3).

C. Proof of Proposition 2

The effects of variations in the model parameters on the asymmetric steady state are derived from the total differential of (B.1). Define $\Omega(m_2, x) \equiv 0$ where x is any parameter in the model as

$$\Omega(m_2, x) = \alpha[(\beta\bar{y} - \rho_1 n)v'(m_2^*) - (1 - n)\beta\rho_2]m_2^* + n(\rho_1 - \alpha)\bar{y}[v'(m_2^*) - \beta] - A ,$$

where A and χ are defined in (B.1). From this expression, we recover the effect on money demand of the borrower as

$$\frac{\partial\Omega(m_2, x)}{\partial\theta}dx + \frac{\partial\Omega(m_2, x)}{\partial m_2}dm_2 = 0 \Leftrightarrow \frac{dm_2}{dx} = -\frac{\partial\Omega(m_2, x)/\partial x}{\partial\Omega(m_2, x)/\partial m_2} , \quad (\text{C.1})$$

where $\frac{\partial\Omega(m_2, x)}{\partial m_2} = F'(m_2) < 0$, which follows from (B.3) and the discussion in Appendix B. Consider the effects of variations in the loan-to-value ratio θ and the housing preference parameter γ on the asymmetric steady state under stagnation:

$$\frac{\partial\Omega(m_2, \theta)}{\partial\theta} = -\frac{\rho_1\rho_2\gamma}{\bar{\rho}_\theta^2}\chi < 0 \quad \text{and} \quad \frac{\partial\Omega(m_2, \gamma)}{\partial\gamma} = -\frac{\theta\rho_1}{\bar{\rho}_\theta}\chi < 0 . \quad (\text{C.2})$$

It hence follows from (C.1) and (C.2) that

$$\frac{dm_2^*}{d\theta} < 0 , \quad \frac{dm_2^*}{d\gamma} < 0 .$$

These results imply together with (35) that

$$\frac{dc_2^*}{d\theta} < 0 , \quad \frac{dc_2^*}{d\gamma} < 0 .$$

The effects on the steady state values of the other variables can be derived from their relation with c_2^* and m_2^* . The effects on c_1^* , a_2^* , C^* and π^* are derived from (2), (3), (21) and (32) as

$$\begin{aligned} \frac{dc_1^*}{d\theta} < 0 , \quad \frac{da_2^*}{d\theta} < 0 , \quad \frac{dC^*}{d\theta} < 0 , \quad \frac{d\pi^*}{d\theta} < 0 , \\ \frac{dc_1^*}{d\gamma} < 0 , \quad \frac{da_2^*}{d\gamma} < 0 , \quad \frac{dC^*}{d\gamma} < 0 , \quad \frac{d\pi^*}{d\gamma} < 0 . \end{aligned}$$

Also note that the cross-derivative is strictly negative which implies mutually reinforcing effects of γ and θ as illustrated in Figure 5:

$$\frac{\partial^2\Omega(m_2, \theta)}{\partial\theta \partial\gamma} = -\frac{\rho_1\rho_2}{\bar{\rho}_\theta^2}\chi < 0 .$$

D. Derivation of the Welfare Functions

Let x^* denote variable x in the original steady state. The effect of a policy on lifetime utility is determined by the difference in U_i and U_i^* as defined in (7). For the borrower, this is trivially given by:

$$\begin{aligned}\Delta U_2 &= U_2 - U_2^* \\ &= \int_0^\infty [\ln(c_{2,t}) + v(m_{2,t}) + \gamma \ln(h_{2,t})] e^{-\rho_2 t} dt - \int_0^\infty [\ln(c_2^*) + v(m_2^*) + \gamma \ln(h_2^*)] e^{-\rho_2 t} dt \\ &= \int_0^\infty [\ln(c_{2,t}) - \ln(c_2^*) + v(m_{2,t}) - v(m_2^*) + \gamma (\ln(h_{2,t}) - \ln(h_2^*))] e^{-\rho_2 t} dt \\ &= \int_0^\infty [\hat{c}_{2,t} + \gamma \hat{h}_{2,t} + v(m_{2,t}) - v(m_2^*)] e^{-\rho_2 t} dt ,\end{aligned}$$

where $\hat{x}_t \equiv \ln(x_t) - \ln(x^*)$ denotes the percent deviation from the original steady state. As the saver's money holdings are expanding in steady state, we use a first-order Taylor approximation of the utility function $v(m_{1,t})$. Let $m_{1,0}$ denote the saver's money holdings right after the implementation of a policy (including a potential jump) and $m_{1,0}^*$ the saver's money holdings at the same date in the absence of policy changes. We then have

$$v(m_{1,t}) = v(m_{1,0}) + \beta(m_{1,t} - m_{1,0}) \quad \text{and} \quad v(m_{1,t}^*) = v(m_{1,0}^*) + \beta(m_{1,t}^* - m_{1,0}^*) .$$

Using these approximations, the difference in lifetime utility of the saver is given as:

$$\begin{aligned}\Delta U_1 &= U_1 - U_1^* \\ &= \int_0^\infty [\ln(c_{1,t}) + v(m_{1,t}) + \gamma \ln(h_{1,t})] e^{-\rho_1 t} dt - \int_0^\infty [\ln(c_1^*) + v(m_{1,t}^*) + \gamma \ln(h_1^*)] e^{-\rho_1 t} dt \\ &= \int_0^\infty [\ln(c_{1,t}) - \ln(c_1^*) + \gamma (\ln(h_{1,t}) - \ln(h_1^*)) + v(m_{1,t}) - v(m_{1,t}^*)] e^{-\rho_1 t} dt \\ &= \int_0^\infty [\hat{c}_{1,t} + \gamma \hat{h}_{1,t} + \underbrace{v(m_{1,0}) - v(m_{1,0}^*) - \beta(m_{1,0} - m_{1,0}^*)}_{=0 \text{ as } v'(m)=\beta} + \beta(m_{1,t} - m_{1,t}^*)] e^{-\rho_1 t} dt .\end{aligned}$$

From money market equilibrium (23), $m_{1,t}$ is a function of the money stock m_t and the money holdings of the borrower $m_{2,t}$. Since m_0 is fixed, we have $m_0 = m_0^*$. This gives

$$\begin{aligned}\Delta U_1 &= \int_0^\infty \left[\hat{c}_{1,t} + \gamma \hat{h}_{1,t} + \beta \left(\frac{1}{n}(m_t - m_t^*) - \frac{1-n}{n}(m_{2,t} - m_{2,t}^*) \right) \right] e^{-\rho_1 t} dt \\ &= \int_0^\infty \left[\hat{c}_{1,t} + \gamma \hat{h}_{1,t} - \frac{\beta m_2^*(1-n)}{n} \frac{m_{2,t} - m_2^*}{m_2^*} + \frac{\beta}{n} \left(m_0 e^{-\int_0^t \pi_s ds} - m_0^* e^{-\pi^* t} \right) \right] e^{-\rho_1 t} dt \\ &= \int_0^\infty \left[\hat{c}_{1,t} + \gamma \hat{h}_{1,t} - \frac{\beta m_2^*(1-n)}{n} \hat{m}_{2,t} + \frac{\beta}{n} \left(e^{-\int_0^t (\pi_s - \pi^*) ds} - 1 \right) m_0 e^{-\pi^* t} \right] e^{-\rho_1 t} dt .\end{aligned}$$

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