

# Growth Theory Without the Aggregate Production Function\*

Carl-Johan Dalgaard<sup>†</sup> and Holger Strulik<sup>\*\*</sup>

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**Abstract.** The present paper provides a new theory of capital accumulation and growth. The law of motion for capital per worker is structurally identical to that of the neoclassical growth model (Solow, 1956). In contrast to the Solow model, however, the purposed theory neither requires the existence of an aggregate (Cobb- Douglas) production function, nor a constant savings rate. Instead thermodynamical principles and associations reflecting the geometric properties of energy-transporting networks underlie the derivations. The theory predicts that in the absence of technological progress growth is ultimately limited by the capacity of networks to supply sufficient energy to support further increases in the per capita stock of capital. By implication, without technological change capital accumulation will peter out even if the world's energy supply were unlimited. Empirically, we find that the purposed theory can explain empirical regularities which are difficult to reconcile with the standard neoclassical growth model.

*Keywords:* Economic Growth, Energy, Networks.

*JEL:* O11, I12, J13.

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<sup>†</sup>University of Copenhagen, Department of Economics, Studiestraede 6, 1455 Copenhagen K, Denmark; email: carl.johan.dalgaard@econ.ku.dk.

<sup>\*\*</sup>University of Hannover, Wirtschaftswissenschaftliche Fakultät, Königsworther Platz 1, 30167 Hannover, Germany; email: strulik@vwl.uni-hannover.de.

## 1. INTRODUCTION

The Solow (1956) model is an enduring contribution to economic growth theory. Half a century after its publication the model remains an important tool in academic work, and it is typically the first growth model students of economics are taught. This is scarcely surprising. It is simultaneously simple and empirically relevant in the context of explaining cross-country differences in GDP per worker.

The version of the Solow model which has been confronted with data comprises essentially three equations: (1) an aggregate Cobb-Douglas production function, (2) the national accounts identity for a closed economy, and, (3) the behavioral assumption that savings come out as a constant fraction of total income. In addition, one may allow population and technology to expand at constant exponential rates. Key predictions of the model are confirmed when confronted with data: The savings/investment share is positively associated with GDP per worker, labor force growth is negatively associated with GDP per worker, and the numerical size of these two opposing influences on labor productivity is found to be the same. The latter finding is particularly significant in that it suggest that data support the structure of the Solow model.

As is well known, however, there is also an anomaly. The size of the impact of investment and labor force growth, on long run labor productivity, is off. Given competitive markets, and the notion that aggregate marginal products of capital and labor determines factor prices, the estimates should identify capital's share in national accounts. The latter falls empirically in a 1/3 to 0.4 range. However, estimation point to a value around 0.6.

There are several ways of dealing with this anomaly. One approach, pursued by Mankiw et al. (1992) (and by many since then), is to argue the estimate is biased due to the omission of an important input in the production function: human capital. An alternative approach, pursued in the present paper, is to question the theoretical foundation for the prior about the estimate, and proceed to provide a different foundation which is consistent with the data. In this respect it is important to recognize that the finding of a capital-output elasticity of 0.6 only (necessarily) constitutes an anomaly when we assume that an aggregate production function exist. That is, the postulate of a functional relationship between GDP and inputs, on which basis marginal products can be derived and serve to determine factor prices in general equilibrium. On theoretical grounds there is good reason to be sceptical about such an aggregate

production function, as a crucial part of the model, since the assumptions under which it exists are very strong.<sup>1</sup>

Our fundamental view is that the overall success of the Solow model suggest that its *structure* does pick up “deep” relationships, which are important to the study of the wealth of nations and that are common across economic systems. However, we will argue that elements of the model do not have the interpretation stipulated by the standard theory. In particular, our derivations of the law of motion for capital dispenses with the need of appealing to an aggregate production function. They also dispense with the need to assume a constant savings rate. Instead we derive the law of motion for capital per worker on the basis of two principles which originates from physics and biology.<sup>2</sup>

The first principle is the law of energy conservation from thermodynamics; energy input and use equal each other. Specifically, we postulate that energy (human and non-human) is required to build, run and maintain capital.<sup>3</sup> This assumption delivers a law of motion for capital; the capital stock increases if total energy expenditure exceeds the energy costs associated with running and maintaining the existing stock.

The second principle, underlying the model, requires more explanation as it draws on recent advances in the field of biology. Ever since Kleiber (1932) it has been known to biologists that a strong correlation, referred to as “Kleiber’s law”, is found between energy intake of biological organisms (basal metabolism) and their energy requirements (body mass). Specifically, the two are related (scale) as follows:  $B = B_0 \cdot m^b$ , where  $B$  is basal metabolism,  $m$  is mass,  $B_0$  is a constant, and  $b = 3/4$ . Remarkably, this association holds across biological systems spanning 27 orders of magnitude in mass; from the molecular level up to whales (West and Brown, 2005).<sup>4</sup> Recently, biologists and physicists in collaboration have started to provide microfoundations for this scaling law ( West et al., 1997, 1999; Banavar et al., 1999, 2002). The common denominator of these theories is that they fundamentally seek to explain Kleiber’s law as a manifestation of how energy is diffused and absorbed in biological systems, viewed as energy transporting networks.

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<sup>1</sup>See Fisher (1969). For some recent expositions, see Felipe and Fisher (2003) and Temple (2006).

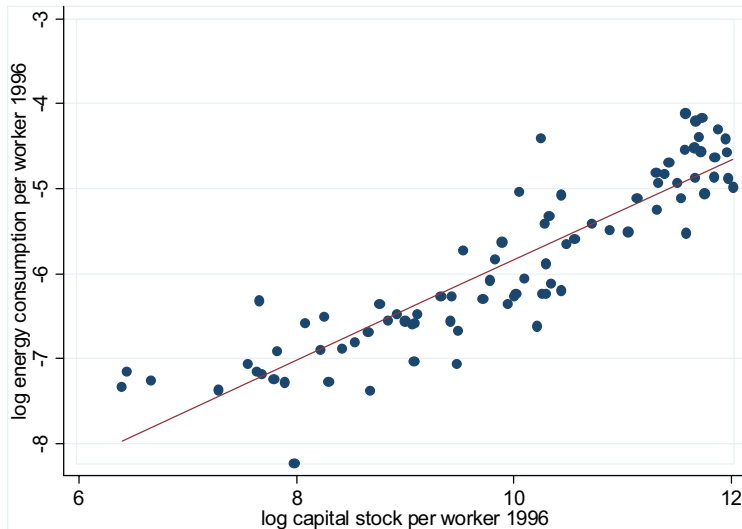
<sup>2</sup>The ideas advanced in this paper owes a great intellectual debt to a set of papers by West et al. (1997, 1999, 2001) and recent work of Banavar et al. (1999, 2002), as pointed out below.

<sup>3</sup>West et al. (2001) similarly uses this kind of an assumption to build a growth model for living tissue; energy is required for the cells to function, be maintained, and for new cells to be created.

<sup>4</sup>Admittedly, some debate persist as to the *exact* size of the exponent; some believe  $b$  closer to  $2/3$  rather than  $3/4$ . See e.g. the recent review by Agutter and Wheatley (2004).

Heavily inspired by these theories one might hypothesize that something similar holds for man-made networks. In the next section we develop a model of an economy viewed as a transport network for energy, following the work of Banavar et al. (1999). The model predicts that in economic systems energy consumption per worker ( $e$ ) can, loosely speaking, be seen as the counterpart to metabolism, and capital per worker ( $k$ ) as the counterpart to “body size”. Accordingly,  $\log(e)$  and  $\log(k)$  should be linearly related. As evidenced by Figure 1, this association is clearly visible in cross-country data.

**Figure 1:** Log energy consumption per worker vs. Log capital stock per worker , 1996. 83 countries. Data: World Development Indicators 2005 & Caselli (2006).



On the basis of these two principles we derive a law of motion for capital, which is structurally identical to the one implied by the standard Solow model. It is therefore less surprising that the Solow structure seems to do fairly well when fitted to a large and heterogenous country sample; the principles underlying the alternative derivation can be expected to hold for all countries, which provides the missing justification for fitting the same equation to all countries on the globe.

An innovation of the present theory is that it brings energy explicitly into the growth process. Indeed, the theory predicts that in the absence of technological progress growth is ultimately limited by the capacity of networks to supply sufficient energy to support an ever increasing stock of capital per capita. Hence, the inability to perpetually pump sufficient amounts of

energy into the network, in order to sustain growth, is the present model's counterpart to the classical result that "diminishing returns to capital input" limits growth.

After deriving our central equation, and examining its implications, we proceed by investigating whether the alternative theory can shed light on the anomaly mentioned above. For the purpose of comparison, we begin by re-examining the predictions of the Solow model; both with regards to the level of GDP per worker and the level of capital per worker. The regressions are conducted for a cross section of countries in 1996, thereby updating the original Mankiw et al. analysis, which focused on 1985. In both cases we confirm Mankiw et al.'s finding that the implied capital share is about 0.6; too high to match national accounts data for the same.

We then proceed to estimate the (log-)linear equation predicted by the network vision of the economy, i.e.  $e \propto k^a$ . Confining attention to the same samples of countries that underlie the estimation of the Solow model, we find that  $a$  equals roughly 0.6. This finding suggests that what is usually interpreted as a (puzzlingly high) estimate of capital's share might actually reflect the networks association derived below. This interpretation follows since  $a$  enters the law of motion for capital in the same way the Cobb-Douglas exponent would in a Solow model (i.e. " $\alpha$ " in standard notation), as demonstrated below.

The case in favor of the network approach can be strengthened. When we split the sample the estimate for  $a$  changes, which suggests the "0.6" finding actually is to be viewed as the mean value of  $a$  in the cross-country sample. For example, if we look at OECD countries in isolation  $a$  is close to  $3/4$ , whereas the coefficient is lower on e.g. the African continent. Importantly, the theory presented below can explain *why* this heterogeneity prevails. A coefficient of  $3/4$ , in the association between energy and capital, is *predicted* by the network theory, provided the network can be viewed as 3 dimensional, and efficient (in a sense to be defined below). However, the coefficient is predicted to be lower if the network is suboptimal, which may be a reasonable characterization of the current state of affairs in Africa.

The remaining part of the paper proceeds as follows. The next section lays out a model of the economy viewed as an energy transporting network. Section 3 then combines the resulting association between energy and capital with energy conservation and proceeds to derive the transition equation for capital. This section also address the empirical issue mentioned above. Section 4 discusses the determination of long-run GDP per worker, whereas Section 5 offers extensions, including a discussion of technological change. Section 6 concludes.

## 2. THE ECONOMY AS A NETWORK

While the model of Banavar et al. (1999) was developed with the purpose of explaining Kleiber’s law, the authors point out that the framework is applicable to a range of networks (aside from the cardiovascular network), including those involving flow of water, air and electrical currents (Banavar et al., 1999, p. 132). In this section we follow up on the latter, and consider the economy as an energy consuming network.<sup>5</sup>

Fundamentally, energy is assumed to originate from a source (a power plant), and is diffused across the economy to the sites at which it is used. In keeping with the terminology of Banavar et al. (1999), we shall refer to each site as “a transfer site”; this is where energy (or “nutrients” in biology) is converted into work effort. All transfer sites are locally connected, and thereby linked to the source either directly or indirectly via the distribution network (the power grid). The precise geographical location of the energy source is not restricted.

Next, we assume that the network is space-filling. In other words, the geographical size of an economy is defined by the penetration of the network; not by its “raw” area measured in  $km^2$ , say. The linear size of an economy (thus defined) is denoted by  $L$ . Hence, the total size of the network is proportional to  $L^D$ , where  $D$  is the dimension of the network.<sup>6</sup> The mean distance between transfer sites and the source is denoted by  $l$ .

The fact that different economies (or the same economy at different points in time) differ in size is reflected by the total number of transfer sites that make up the network. To drive the link between the size of the network and the *number* of transfer sites, we impose the restriction that the *size* of each transfer site is independent of the size of the entire network. In biology, transfer sites would refer to the capillaries of the body which can be regarded as scale-invariant. In an economics context, a different interpretation is needed of course. In the present context we may think of electricity outlets as the counterpart of the capillaries (West and Brown, 2004). The size of electricity outlets are independent of the size of the associated building, and the surrounding network. They also tend to have the same size across countries; electricity outlets are no bigger in rich countries than in poor countries (though they surely are more plentiful). As a result, maintaining scale invariance of the transfer site may not be unreasonable in an economic context.

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<sup>5</sup>For applications to drainage basins of rivers, see Maritan et al. (2002) and Rinaldo et al. (2006).

<sup>6</sup>Recall, the volume of a 3 dimensional sphere with radius  $(1/2) \cdot L$ , is  $(4/3) \cdot \pi \cdot ((1/2) \cdot L)^3$ , and is therefore *proportional* to  $L^3$ .

Assuming the network is space-filling, and that the transfer sites are scale-invariant, implies that the number of transfer sites,  $N$ , must rise with the “volume” of the economy (or organism in biology):  $N \propto L^D$ . Moreover, since each transfer site uses energy, one may anticipate an association between the size of the network, and energy consumed at (all) the transfer sites,  $E$ . More specifically, we assume

$$E \propto L^D \cdot P, \tag{1}$$

where  $P$  is the size of the population. Equation (1) can be restated as proportionality between energy consumption per person and the size of the network:

$$e \propto L^D. \tag{2}$$

Equation (1) says that for a given size of the network,  $L$ , total energy consumption should be (log-)linearly related to population size,  $P$ . As a result, changes in *per capita* energy consumption requires changes in the size of the network ( $L$ ), as indicated by equation (2). That is, per capita energy consumption,  $e$ , is ultimately attributable to the number of devices (e.g. television sets, washing machines, computers and so on), which a given population utilizes. The notion is that every time a new piece of equipment is connected to an electricity outlet, a new transfer site emerges, and the network expands.

Empirically, Kühnert et al. (2006) find strong support for a log-linear association between  $E$  and  $P$ . Using cross section energy delivery data and inhabitants of German cities, they estimate an elasticity of 1.0, with a 95% confidence interval of [0.96; 1.06]. In light of equation (1), an interpretation of this finding is that, at the time of data collection (year 2002), a typical city-based household (or firm) in Germany had connected an equal amount of machines and appliances with the network, irrespective of the size of city. Consequently,  $L^D$  is picked up by the constant term in the regression analysis of Kühnert et al. (2006), leading to an unbiased estimate for the association between  $E$  and  $P$ .<sup>7</sup>

A key result in Banavar et al. (1999) is a mathematical theorem, which speaks to the association between *total* flow of energy in a network,  $F$ , and the linear size of the network,  $L$ . The

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<sup>7</sup>Note, this interpretation does not require  $L$  to be *literately* the same across German cities. One could allow  $L$  to vary across cities ( $i$ ) in a random fashion, say,  $L_i = L \cdot v_i$ , where  $v_i$  is noise. In this case, we require  $\log(v_i)$  to be orthogonal to  $\log(P_i)$ .

theorem establishes that the following holds:

$$F \propto E \cdot l \propto E \cdot L^x, \tag{3}$$

where  $x$  depends on the characteristics of the network, as explained below.

The first part of the equation,  $F \propto E \cdot l$ , says that the total flow of energy in the network is proportional to energy used at the transfer sites, but with a proportionality constant that is given by the mean distance from the sites to the source. Energy is located (at a given instant in time) either at transfer sites or in the distribution network itself. Thus the total amount of energy in the network can be calculated as energy used at the sites, multiplied by the (average) distance energy has to “travel” to fill the entire network.

The second part (which stipulates that  $l \propto L^x$ ) concerns how the mean distance changes as the size of the network changes. Specifically, in the most efficient class of networks - “directed networks” -  $x = 1$ . This type of network minimizes total energy requirements needed to fuel the economy (or organism in biology), subject to the requirement that all sites are served.<sup>8</sup> At most, mean distance rises in proportion to  $L^D$ , i.e.  $x = D$ . This happens if the network can be seen as a space-filling spiral (see Banavar et al. 1999, p. 131). Accordingly,  $l$  rises as the size of the network expands, and the magnitude of the increase depends on the nature (or, efficiency) of the network in question.

Inserting equation (1) into equation (3) yields:

$$F \propto L^{D+x} \cdot P, \tag{4}$$

which implies that total energy flow per capita ( $F/P$ ) rises (scales) at least with  $L^{D+1}$ , and at most by  $L^{2D}$ .

Finally, we assume that when the capital stock,  $K$ , rises, total energy requirements, and therefore total energy flow in the system, rise proportionally

$$K \propto F. \tag{5}$$

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<sup>8</sup>A network is “directed” if, starting with the source, energy spreads throughout the network, away from the source, without “backtracking”.

This assumption is thought to capture that capital is nested at the transfer sites, *and* in the network itself. Hence capital is needed to transfer energy to the sites, where capital uses energy.<sup>9</sup>

To work out the implied association between energy use per capita, and the amount of capital per capita in the economy, we need to reduce the model, which is given by equations (2), (4) and (5). Substituting equation (5) into equation (4), isolating  $L$ , and substituting the result into equation (2) yields

$$e \propto k^{\frac{D}{D+x}}, \quad (6)$$

where  $k \equiv K/P$  is the per capita capital stock. The intuition for the concave association is the following. If  $K$  rises the size of the network ( $L$ ) increases, implying more transfer sites ( $N$ ) and a greater mean distance ( $l$ ) between the source and the transfer sites. Equation (5) implies that total energy in the network ( $F$ ) expands in direct proportion to the amount of capital accumulated. However, per capita energy availability at the transfer sites ( $e$ ) will not rise in direct proportion with the (per capita) capital stock. The reason is that more energy is required to “fuel the system”. To see this formally, notice that  $E/F \propto L^{-x}$  (cf. equations (3) and (4)). Accordingly, as the system expands, a smaller fraction of the total energy flow is available at the transfer sites, at each point in time. Therefore, increasing  $k$  by 1 percent is associated with an increase in  $e$  by  $D/(D+x) < 1$  percent.

The result is a log-linear association between energy consumption per person and capital per person, which follows from the geometric properties of energy transporting networks. Moreover, the coefficient would fall in a  $(1/2, D/(D+1))$  interval. For a more precise prior, we need to pin down  $D$ . The most natural notion is probably that of  $D = 3$ , i.e. of a three-dimensional network. In this case the coefficient should fall in an interval from  $(1/2, 3/4)$ , depending on the efficiency of the network; the more efficient the higher the coefficient.

### 3. A THEORY OF CAPITAL ACCUMULATION AND GROWTH

**3.1. The Model.** Consider a closed economy, described in continuous time. The first element of the growth model is the “network equation” derived above, which can be restated as

$$e(t) = \epsilon k(t)^a, \quad (7)$$

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<sup>9</sup>In biology  $K$  would correspond to the body mass of the animal, whereas total energy flow,  $F$ , would correspond to total nutrients in the blood stream at a given point in time; proportional to the mass of blood in the organism. Since (total) body mass and blood mass should be proportional, the association follow.

where  $a < 1$ , and  $\epsilon > 0$  is a constant in the sense that it is independent of capital per worker. In order to derive the fundamental law of motion for capital, we need to add an element. The additional element is the law of energy conservation.

To be specific, we distinguish between two forms of energy requirements: Non-human energy  $E_K(t)$  and energy supplied by humans,  $E_H(t)$ , i.e. human metabolism. The total energy supply, we assume, is used to run, maintain, and create capital. Supposing the energy costs of maintaining and running the “characteristic” machine is  $\mu$  whereas the energy requirements to create a new machine is  $\nu$ , one may express the energy balance as:

$$E_H(t) + E_K(t) = \mu K(t) + \nu \dot{K}(t). \quad (8)$$

For future reference, notice that if we were to shut off energy supply entirely,  $E_H = E_K = 0$ , the capital stock would shrink over time, due to lack of maintenance and replacement. The rate at which the stock shrinks is

$$\dot{K}(t) / K(t) = -\frac{\mu}{\nu},$$

which therefore can be viewed as the mirror image of the depreciation rate, commonly introduced in models of growth and capital accumulation. However, in contrast to the standard approach, the present theory links the rate of depreciation to underlying parameters of the model. In particular, an increase in the costs of running and maintaining machines ( $\mu$ ) should lead to accelerated depreciation. Arguably, such an increase could be motivated by increased capital utilization; running machines at greater intensity is more energy consuming, and higher capital utilization could also elevate energy requirements as a result of increased maintenance. This implication is very much in the spirit of the work of Taubman and Wilkinson (1970), according to which increasing capital utilization leads to faster capital depreciation.<sup>10</sup>

Moving on, we divide through by the size of population,  $P(t)$ , in equation (8):

$$e_h(t) + e(t) = \mu k + \nu \frac{\dot{K}(t)}{P(t)},$$

where  $e(t) \equiv E_K(t) / P(t)$  and  $e_h \equiv E_H(t) / P(t)$ . Observe that since  $k(t) \equiv K(t) / P(t)$ , it follows that  $\dot{K}(t) / P(t) = \dot{k}(t) + nk(t)$ , in so far as population expands at a constant rate  $n \equiv \dot{P}(t) / P(t)$ . Consequently

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<sup>10</sup>Epstein and Denny (1980) and Johnson (1994) find corroborating evidence for this mechanism.

$$e_h(t) + e(t) = \mu k(t) + \nu \left[ \dot{k}(t) + nk(t) \right],$$

which we can restate so as to yield

$$\dot{k}(t) = \left( 1 + \frac{e_h}{e} \right) \frac{1}{\nu} e(t) - \left( \frac{\mu}{\nu} + n \right) k(t).$$

This expression can be simplified a bit. As pointed out by Moses and Brown (2003) human metabolism only makes out for a tiny fraction of average energy consumption in modern day industrial societies, which suggest the approximation  $\frac{e_h}{e} \approx 0$ .<sup>11</sup> Needless to say, this approximation is not entirely reasonable if one thinks about the growth process in the very long run, i.e. the period preceding the industrial revolution where non-human energy supply was relatively scarce. We use the model to illustrate the mechanics of economic growth in the very long-run in Section 5. But for now we will work with  $\frac{e_h}{e} = 0$ . Accordingly, given  $\frac{e_h}{e} = 0$  and in light of equation (7) we obtain

$$\dot{k}(t) = \frac{\epsilon}{\nu} k(t)^a - \left( \frac{\mu}{\nu} + n \right) k(t). \quad (9)$$

As is visually obvious, the above equation is structurally identical to the law of motion for capital per capita, as predicted by the Solow model.<sup>12</sup> Indeed, the dynamics (including stability properties) are the same as in a standard Solow model. However, the present model holds unique predictions, related to the association between energy supply and the growth process which are not captured by the standard Solow model, as summerized in the following proposition:

PROPOSITION 1. (i) *In the absence of changes to the parameters of the model, the capital stock per capita converges to*

$$k^* = \left( \frac{\epsilon}{v(\mu/v + n)} \right)^{1/(1-a)}.$$

(ii) *Energy consumption per capita is, in the steady state, given by*

$$e^* = \epsilon \left( \frac{\epsilon}{v(\mu/v + n)} \right)^{a/(1-a)}.$$

<sup>11</sup>Moses and Brown puts it nicely into perspective (p. 296): : “The per capita energy consumption in the United States is 11.000 W ... which is approximately 100 times the rate of biological metabolism and, ... [it] is the estimated rate of energy consumption of a 30.000-kg primate”.

<sup>12</sup>In standard notation, the latter would be  $\dot{k}(t) = sAk(t)^\alpha - (\delta + n)k(t)$ .

(iii) *Increases in the supply of energy per machine,  $\epsilon$ , reductions in the energy costs of creating, running and maintaining capital ( $v$  and  $\mu$ ), and reductions in population growth, will increase capital per worker and energy consumption per worker, in the long run.*

Implicit in the proposition is the assumption that energy supply can be increased to support accumulation. In this sense equation (9) pertains to *unconstrained* growth. Now, it is common place to argue that (world) growth is ultimately limited from above, by energy availability (e.g. Daly, 1977). Interestingly, however, the present analysis demonstrate that absent technological progress growth is limited *even if* energy supply is unlimited. This result is proved by the obvious existence of the steady state  $k^*$ . As a result, the model demonstrate that while unlimited access to energy may be a necessary condition for perpetual growth, it is not sufficient. Indeed, absent technological progress, which may enable more energy to be transported through the system say, growth will ultimately peter out, regardless of supplies of energy.

If, on the other hand, energy supply is bounded from above the model does conform with the “conventional wisdom” mentioned above. To see this, suppose per capita energy supply is bounded from above at some per capita level,  $\bar{e}$ . Then we have the following result:

PROPOSITION 2. *Assume energy supply per capita is bounded from above, at  $\bar{e}$ . Absent changes to the parameters of the model, the stock of capital per worker converges to:*

$$\bar{k} = \frac{\bar{e}}{v(\mu/v + n)}.$$

Hence, equation (9) governs the dynamics of  $k$  only as long as  $k < \bar{k}$ . As long as the economy is below the threshold  $\bar{k}$ , the dynamics are similar to those of the Solow model. However, if the economy reaches the threshold,  $\bar{k}$ , further increases in the amount of energy per capita is infeasible. As a result, the capital stock stagnates at  $\bar{k}$ .

### 3.2. Evidence.

3.2.1. *Specifications.* To begin, observe that the Solow model evaluated at the steady-state would lead to the following solution for log capital per worker (see Mankiw et al., 1992):

$$\log(k) = \frac{1}{1-\alpha} \log(A) + \frac{1}{1-\alpha} \log(s) - \frac{1}{1-\alpha} \log(n + \delta).$$

The notation is standard:  $A$  is an index of technological sophistication which derives from the production function,  $s$  represents the investment rate,  $\delta$  is the rate of capital depreciation, and  $\alpha$  is the capital-output elasticity from the aggregate Cobb- Douglas production function.

The above equation can be implemented as the regression model:

$$\log(k) = \beta_{0k} + \beta_{1k} \log(s) + \beta_{2k} \log(n + \delta) + \varepsilon_k. \quad (10)$$

Accordingly, the model predicts that  $\beta_{1k} > 0, \beta_{2k} < 0$  and that  $\beta_{1k} = -\beta_{2k}$ . Moreover, an implied estimate for  $\alpha$  can be backed out as  $(\beta_{1k} - 1) / \beta_{1k}$ . It should be recognized that data on capital stocks are generated by the perpetual inventory method, i.e. as cumulated investments. As a result, a *significant* impact from  $s$  on  $k$  is all but guaranteed, i.e.  $\beta_{1k} > 0$ . However, neither the *size* of  $\beta_{1k}$  nor the sign and size of  $\beta_{2k}$  is a given.

The more familiar approach to getting at  $\alpha$  is to test the Solow model's predictions regarding long-run GDP per worker:

$$\log(y) = \beta_{0y} + \beta_{1y} \log(s) + \beta_{2y} \log(n + \delta) + \varepsilon_y \quad (11)$$

where  $\beta_{1y} = -\beta_{2y} = \alpha / (1 - \alpha)$ . This is a second specification we can employ to get an estimate for  $\alpha$ , which we can compare with the estimate obtained using equation (10).

In contrast, the model developed above evaluated at the steady-state gives rise to the following log-linear equation for long-run capital per worker

$$\log(k) = \frac{1}{1-a} \log(\epsilon) + \frac{1}{1-a} \log(1/v) - \frac{1}{1-a} \log(n + \mu/v), \quad (12)$$

Ideally, one would estimate the steady state equation, implied by the present theory so as to assess its explanatory power. Subsequently the results could be compared with those stemming from the standard theory. This direct approach is not feasible, unfortunately. The problem is that key variables in the model, i.e.  $\mu$  and  $v$ , are not presently observable (to our knowledge, at least). That is, the energy costs of running, maintaining and creating capital. Such costs undoubtedly differ from one country to the next, for which reason they should be measured and included as regressors. As a result, the empirical discussion will have a more narrow focus, in that we instead will examine whether the present theory may account for the anomaly mentioned in the Introduction.

If we compare the two steady state equations it is clear that the implied capital-output elasticity ( $\alpha$ ) enters in a way which is very similar to how  $a$  enters the alternative steady state expression. In particular, under the Solow model an increase in  $(n+\delta)$  of 1 percent should lower long-run capital per capita by  $[1/(1-\alpha)]$  percent, whereas the theory developed above would provide the answer  $[1/(1-a)]$  percent, to the same question. That is, provided  $\mu/v$  corresponds to  $\delta$ , which might be a reasonable prior since both capture the same *physical* phenomenon: capital depreciation. Consequently, one may hypothesize that  $\alpha$  and  $a$  are conveying the same information.

In order to follow up on this hypothesis, we estimate the network equation:

$$\log(e) = \beta_{0e} + \beta_{1e} \log(k) + \varepsilon_e, \quad (13)$$

where  $\beta_{1e} = a$ . As seen this linear regression enables us to obtain an estimate for  $a$  directly, without involving the unobservable variables  $\mu$  and  $v$ . This allows us to ask whether its size matches the findings of Mankiw et al. (1992).

From the perspective of identification it is worth stressing that the network theory implies that  $a$  can be identified by OLS. The constant term ( $\beta_{0e} = \log(\epsilon)$ ) is, in theory, independent of  $k$ ; there is no scope for omitted variable bias. That is, even if  $\epsilon$  varies across countries, this variation should be orthogonal to  $k$ . Nevertheless, we also provide IV estimates below. The main reason is that the data on  $k$  is likely to be very noisy (e.g. Pritchett, 2000), for which reason measurement error may bias the OLS estimates.

Finally, it is worth observing that while estimating equation (13) *is* an indirect approach to testing the theory (i.e. equation (12)), it is a rather decisive approach. Recall, the *only* other assumption we need to make, so as to derive equation (12), is the first law of thermodynamics. This element of the model is, after all, commonly accepted to be a fact of nature.

3.2.2. *Data.* Mankiw et al. (1992) tested the steady state predictions of the Solow model, with respect to GDP per worker, for the year 1985. Below we move a decade forward, and look at 1996. In addition we also examine the steady state predictions for capital per worker.

Using 1996 as our year of choice allows us to use data on capital stocks calculated by Caselli (2006); PPP GDP is also taken from Caselli (2006). The investment rate ( $s$ ), calculated as an average for the 1960-96 period, is taken from the data of Bernanke and Gurkaynak (2001), which in turn draws on Penn World Tables 6.1 (Heston, Summers and Aten, 2002). Finally, data on

the size and growth of the labor force ( $n$ ) is taken from World Development Indicators 2005, and so is the data on energy consumption ( $e$ ). Since we cannot measure the depreciation rate, we follow conventions and set it to  $\delta = 0.05$ .<sup>13</sup>

For the purpose of comparability with previous results, we limit attention to the countries contained in Mankiw et al.’s original study, corresponding to their “non- oil” and “intermediate” samples. This leaves us with 74 and 65 country observations upon which to base estimation of equations (10), (11) and (13).<sup>14</sup>

**3.2.3. Results.** Table 1 reports the results from estimating equations (10) and (11). We report both the results from OLS, and those obtained by implementing an outlier robust estimator (the least absolute deviation - LAD - estimator). The general message from the table is that the Solow model does reasonably well, especially when focusing on the robust estimates. Whether we test the steady state predictions with respect to GDP per worker or capital per worker, the following can be concluded: The model accounts for more than half the variation in the data; the predicted determinants of long-run prosperity are significant; the prediction that  $\beta_{1i} = -\beta_{2i}$ ,  $i = k, y$  is supported and finally, the implied “share of capital”,  $\alpha$ , is about 0.6. These results conform with those of Mankiw et al. (1992).

Table 2, columns 1-2, reports the results from estimating equation (13). The results are comparable to those of Table 1, in that the sample is the same. As is clear from the two first columns; the key finding is that  $\beta_{1e}$  and the implied  $\alpha$  from estimating the Solow model are basically coinciding. This result hold in both samples of countries. This is encouraging in that it suggest the network theory may in fact explain the size of what is usually interpreted as the capital-output elasticity. In the revised theory, however, the parameter should not be identified with capital’s share in national accounts.

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<sup>13</sup>Mankiw et al. also allow for technological progress (which occurs at the rate  $g$ ), but do not measure them of course. Hence, for empirical purposes one of their right hand side variables is  $\log(n + 0.05)$  implicitly because  $g = 0.02$ , and  $\delta = 0.03$ . We have ignored  $g$ ; but from an empirical perspective the two specifications are identical when we put  $\delta = 0.05$ .

<sup>14</sup>The “non-oil sample” exclude economies that more less are based entirely on natural resource extraction. The “intermediate sample” moreover excludes countries with a population smaller than 1 mio. people. Mankiw et al. also considers an OECD sample. The steady state equation of the Solow model fits very poorly in this case. An interpretation is that this selection of countries are more or less situated in the same steady state ( $\pm$  shocks). As a result, the signal to noise ratio becomes very small, which makes it hard to identify the parameters of the model by way of cross-section regression.

**Table 1. Re-estimating the Solow Model**

Dep. variable:	log(y)	log(y)	log(y)	log(y)	log(k)	log(k)	log(k)	log(k)
log(s)	1.28 <sup>a</sup> (0.17)	1.22 <sup>a</sup> (0.22)	1.55 <sup>a</sup> (0.23)	1.51 <sup>a</sup> (0.26)	2.07 <sup>a</sup> (0.18)	2.07 <sup>a</sup> (0.22)	2.46 <sup>a</sup> (0.26)	2.25 <sup>a</sup> (0.27)
log (n+0.05)	-2.30 <sup>a</sup> (0.60)	-2.29 <sup>a</sup> (0.62)	-1.69 <sup>b</sup> (0.67)	-1.65 <sup>b</sup> (0.64)	-2.71 <sup>a</sup> (0.65)	-2.57 <sup>a</sup> (0.66)	-1.92 <sup>a</sup> (0.68)	-1.98 <sup>a</sup> (0.65)
implied $\alpha$	0.56 (0.03)	0.55 (0.04)	0.61 (0.04)	0.60 (0.04)	0.52 (0.04)	0.52 (0.05)	0.59 (0.04)	0.56 (0.05)
$\beta_1 = -\beta_2$ (p-value)	0.16	0.18	0.87	0.86	0.40	0.55	0.54	0.74
$R^2$	0.61	0.57	...	...	0.77	0.74	...	...
Observations	74	65	74	65	74	65	74	65
Estimator	OLS	OLS	LAD	LAD	OLS	OLS	LAD	LAD

*Notes: All regressions include a constant. (a), (b) and (c) refer to 1, 5 and 10 percent levels of significance. Robust standard errors in parenthesis. In LAD regressions standard errors are bootstrapped with 1000 repetitions. Implied  $\alpha$  calculated using estimates for log(s).*

Countries in the non-oil (74) sample: Angola (AGO), Argentina (ARG), Australia (AUS), Austria (AUT), Belgium (BEL), Benin (BEN), Bangladesh (BGD), Bolivia (BOL), Brazil (BRA), Canada (CAN), Switzerland (CHE), Chile (CHE), Ivory Coast (CIV), Cameroon (CMR), Congo (COG), Colombia (COL), Costa Rica (CRI), Denmark (DNK), Dominican Rep. (DOM), Algeria (DZA), Ecuador (ECU), Egypt (EGY), Spain (ESP), Ethiopia (ETH), Finland (FIN), France (FRA), United Kingdom (GBR), Ghana (GHA), Greece (GRC), Guatemala (GTM), Hong Kong (HKG), Honduras (HND), Indonesia (IDN), India (IND), Ireland (IRL), Israel (ISR), Italy (ITA), Jamaica (JAM), Jordan (JOR), Japan (JPN), Kenya (KEN), Korean Republic (KOR), Sri Lanka (LKA), Morocco (MAR), Mexico (MEX), Mozambique (MOZ), Malaysia (MYS), Nigeria (NGA), Nicaragua (NIC), Netherlands (NLD), Norway (NOR), Nepal (NPL), New Zealand (NZL), Panama (PAN), Pakistan (PAK), Peru (PER), Philippines (PHL), Portugal (PRT), Paraguay (PRY), Senegal (SEN), El Salvador (SLV), Sweden (SWE), Syria (SYR), Togo (TGO), Thailand (THA), Trinidad and Tobago (TTO), Tanzania (TZA), Uruguay (URY), United States (USA), Venezuela (VEN), South Africa (ZAF), Democratic republic of Congo (ZAR), Zambia (ZMB), Zimbabwe (ZMB)

At the same time the result is seemingly not entirely consistent with the proposed theory either. The prediction would be  $a=3/4$ , if the network is three-dimensional and efficient, which seems like a reasonable prior. Still,  $a=2/3$  can be supported as well (and the results would not refute this value), if either the network is inefficient, or best conceived as 2 dimensional (and efficient). Another possibility, however, is that  $a$  varies across countries, reflecting different levels of efficiency of the local networks. If so, the point estimate of 0.6 may simply reflect the average value in the sample (e.g. Zellner, 1969). In order to look into this possibility, we re-estimated equation (13) on sub-sets of countries; these results are also reported in Table 2, columns 3-11.

The first data split consists of estimating the networks equation on a sample consisting solely of OECD countries, and non-OECD countries (column 3-4). As seen, within the OECD,  $a$  rises to a value which conform with expectations;  $a=0.75$  cannot be rejected at conventional levels of significance. Outside the OECD, however, the coefficient is smaller; here we find  $a = 0.53$ .

**Table 2. Energy Consumption and Capital**

Dep. variable:	log(e)	log(e)	log(e)	log(e)	log(e)	log(e)	log(e)	log(e)	log(e)	log(e)	log(e)
Constant	-11.6 <sup>a</sup> (0.37)	-12.1 <sup>a</sup> (0.48)	-13.7 <sup>a</sup> (1.37)	-11.3 <sup>a</sup> (0.46)	-13.4 <sup>a</sup> (0.74)	-11.1 <sup>a</sup> (1.59)	-13.0 <sup>a</sup> (0.94)	-10.2 <sup>a</sup> (0.71)	-13.0 <sup>a</sup> (0.98)	-9.8 <sup>a</sup> (0.59)	-10.8 <sup>a</sup> (0.07)
log(k)	0.59 <sup>a</sup> (0.04)	0.62 <sup>a</sup> (0.05)	0.77 <sup>a</sup> (0.12)	0.53 <sup>a</sup> (0.05)	0.74 <sup>a</sup> (0.07)	0.54 <sup>a</sup> (0.14)	0.71 <sup>a</sup> (0.10)	0.43 <sup>a</sup> (0.09)	0.71 <sup>a</sup> (0.09)	0.36 <sup>a</sup> (0.07)	0.47 <sup>a</sup> (0.15)
$R^2$	0.80	0.79	0.53	0.69	0.87	0.34	0.68	0.66	0.61	0.44	0.42
Observations	74	65	21	62	22	17	23	16	47	36	23
Sample	NOIL	INT	OECD	NOECD	ASIA	EUR	AME	AFR	RICH	POOR	PRNAFR

Notes: (a),(b) and (c) refer to 1, 5 and 10 percent levels of significance. The samples are (left to right): Non-oil, intermediate, OECD, non-OECD, Asia and Oceania, Europe, Americas, Africa, rich countries, poor countries, and poor countries without Africa. “Rich” vs. “poor” refer to above vs. below mean GDP per capita in full sample

An alternative approach consists of estimating the association on each continent: Asia/Oceania, Europe, Americas and Africa.<sup>15</sup> As seen from column 5-8, one cannot reject the 3/4 hypothesis on either the European, American or Asian continent. However, the point estimate is very low in Europe. As it turns out, a single country influences the estimate considerably: Romania. If Romania is omitted the point estimate rises to 0.66, with 0.3 for standard error (not reported). Therefore, when focusing on the first three continents,  $a$  falls in a range where the hypothesis of 3/4 cannot be rejected. In contrast, the elasticity is disturbingly low in Africa, where a 3/4 hypothesis is rejected at a 5% level of significance. Instead, the elasticity is insignificantly different from 0.5, which is the lower-end prior for its size (see Section 2). These results would suggest that the low non-OECD estimate might be ascribed mainly to the influence from the African countries in the sample.

This conclusion is supported by the third split (column 9-11). In this case we split the data according to levels of GDP per worker; above and below average in the sample. For the high income sample, the coefficient is once again near 0.75, whereas it is considerably lower for the low income sample. However, if African countries are excluded (last column), we can no longer reject (at a 5 percent level of significance) that the elasticity is 3/4, though the point estimate remains low.

As a robustness test we employ instrumental variables estimation. The main motivation is measurement error, which could render OLS misleading. As an instrument for the capital stock per worker in 1996 we use average population growth from 1960-96. Growth in the population should in theory affect capital accumulation under the theory developed above. Moreover, the

<sup>15</sup>The continent dummies are taken from Caselli (2006)

**Table 3. IV Estimates**

Dep. variable:	log(e)	log(e)	log(e)	log(e)
Constant	-12.2 <sup>a</sup> (0.40)	-12.4 <sup>a</sup> (0.47)	-12.8 <sup>a</sup> (0.54)	-13.1 <sup>a</sup> (0.66)
log(k)	0.63 <sup>a</sup> (0.04)	0.65 <sup>a</sup> (0.04)	0.69 <sup>a</sup> (0.05)	0.72 <sup>a</sup> (0.06)
F-value (First stage)	105.8	89.66	79.6	77.6
Sample	Non-oil	Inter	non Africa	75% richest
Observations	74	65	67	62

*Notes: (a), (b) and (c) refer to 1, 5 and 10. Robust standard errors in parenthesis. Instrument for log(k) is the log of population growth 1960-96*

theory predict that population growth would not affect energy consumption directly, i.e. for  $k$  given, rendering it a viable instrument for  $k$ . This is confirmed by the first stage F-values reported below, from which it is clear that the log of population growth is a strong instrument for the log of capital per worker (cf. Staiger and Stock, 1997).<sup>16</sup>

Table 3 reports the results. The first two columns report IV estimates which can be compared with the OLS results reported in Table 2, column 1 and 2; the country samples are the same. As seen, the IV estimates are consistent with our OLS results, though slightly higher. The difference could be interpreted as stemming from (classical) measurement error.

We then estimated the model (column 3) on a sample consisting of all available non-African countries, inspired by our OLS results. As seen, in this case we find a coefficient of 0.69. In contrast, if the African countries are included (not reported), which would increase the sample to 83 countries, we find  $a = 0.64$  – a lower value, as expected. Finally, we re-estimated the model excluding the 25% poorest economies in our 83 country sample. As shown in column 4 the IV estimate for  $a$  rises to 0.72, close to a 3/4 prediction.<sup>17</sup>

Overall, a higher implied elasticity in rich and middle-income countries, which is what these regressions support, mirrors the findings of Durlauf and Johnson (1995). Using regression tree techniques, they split the original Mankiw et al. sample into sub-samples and re-estimate the

<sup>16</sup>We also ran the regressions below using *both* the growth rate of the labor force, and population growth as instruments for  $\log(k)$ . Using two instruments allow us to test the exclusion restriction that neither should be correlated with  $\log(e)$  conditional on  $\log(k)$ . Our point estimates are virtually unchanged, compared with the results reported below, and data does not allow us to reject the exclusion restriction.

<sup>17</sup>We also experimented with an IV solution in the context of the continent-by-continent regressions. Unfortunately, in none of these cases did we obtain strong identification (in the sense of the “rule of thumb”:  $F > 10$ ). With weak instruments (and very small samples) the 2SLS estimator may perform worse than the OLS estimator. As a result, we have not pursued the matter further.

model. Generally speaking the implied  $\alpha$  is higher in samples consisting of countries which had more favorable initial conditions. Given the standard theory of growth, such findings could be taken to imply that the aggregate production function must differ across countries (see Durlauf and Johnson, 1995).

Our results suggest that the networks theory may account for the regularity; the interpretation would be that the efficiency of the energy transporting network differ, when comparing developed to less developed countries. This interpretation is supported by the fact that, generally speaking, the size of the point estimate for the networks equation conforms with priors. The coefficient tend to fall within the predicted interval:  $(1/2, 3/4)$ .

#### 4. THE EVOLUTION OF GDP PER WORKER

The Solow model holds predictions for capital per worker, but also for GDP per worker. In contrast, the present theory may appear to be silent as for the determination of GDP,  $Y(t)$ . But this impression is more apparent than real.

If we recognize the existence of the national accounts identity,  $Y(t) = I(t) + C(t)$ , where  $I(t)$  is gross investments and  $C(t)$  consumption, the above model will also hold implications for GDP (per worker). All capital accumulation reflect deliberate investment. As a result, in so far as the capital stock per capita is expanding investments per capita must be growing. Unless consumption per capita is declining, for some reason, capital accumulation will thereby imply increasing output per capita. The interpretation of this increase would be that expanded investment demand propels growth in total production and thus income. In a steady state, where the stock of capital per capita is constant, output per capita would be determined (in part) by gross investments, which mirror capital replacement. This, however, still leaves out consumption per capita, in the steady state. The level of GDP per capita is therefore not fully determined by the model developed in Section 3.

Accordingly, in order to derive a *closed form* solution for long-run GDP per capita, one needs to add a theory of consumption/savings choice to the model developed in Section 3.

**4.1. A Consumption-Augmented Model.** Following Becker (1965) we adopt the “home production” view of consumption.<sup>18</sup> Accordingly, the total amount of the consumed good,  $C$ ,

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<sup>18</sup>In principle any theory of consumption would do; therefore “A” consumption augmented model. We find the Becker (1965) framework appealing in the present context, however, because it can allow human energy supply to play a role in consumption, as will be clear.

is produced using inputs of human energy (by analog to “time” in the Becker framework),  $E_H = P e_h$ , and marketed goods,  $K_c$ :

$$C = \min (Z e_h P, K_c), \quad (14)$$

where  $Z > 0$  is a constant technological parameter. The variable  $K_c$  represents capital equipment used in the context of consumption, and therefore constitutes part of the aggregate capital stock available in the economy, at a given point in time. In the present model we are not distinguishing explicitly between “investment goods” and “consumption goods”. In this respect, the model mirrors the standard Solow approach where output can be either used for investments, or consumption. In the present model, energy is used to run, maintain and create capital, which can either find its use in the context of consumption or investment.

Assuming constant returns to scale in the home production function above, we find that per capita demand for marketed goods used in consumption fulfill

$$Z e_h = k_c.$$

On this basis we may determine the share of the total stock of capital,  $\sigma_c$ , which is used for consumption purposes, in the steady state, as

$$\sigma_c^* = Z e_h (k^*)^{-1} = Z e_h \cdot \left[ \frac{v \left( \frac{\mu}{v} + n \right)}{\epsilon} \right]^{1/(1-a)}.$$

Consequently, per capita consumption in the steady state can be written

$$c^* = Z e_h = \sigma_c^* \cdot k^*.$$

Further, since per capita investments in the steady state is given by the level of capital replacement,

$$i^* = \left( \frac{\mu}{v} + n \right) k^* = v^{-1} e^*,$$

we can solve for steady state GDP per worker, by invoking the national accounts identity and inserting the expressions for  $i^*$  and  $c^*$ :

$$y^* = k_c^* + i^* = Z e_h + v^{-1} e^* = \left( v^{-1} + \frac{Z e_h}{e^*} \right) e^*. \quad (15)$$

Finally, if we insert the steady state result for  $e^*$  from the last section, we obtain:

PROPOSITION 3. *In the absence of changes to the parameters of the model, and assuming consumption is described by equation (14): (i) The long run level of GDP per worker converges to*

$$y^* = \left(\frac{\epsilon}{v}\right)^{\frac{1}{1-a}} (\mu/v + n)^{-\frac{a}{1-a}} + Ze_h$$

*(ii) The long run level of GDP per worker increases if  $\epsilon, 1/v, Z$  and  $e_h$  rises, and falls if labor force growth,  $n$ , goes up.*

Comparing Proposition 1 and 3, the key difference lie with the presence of  $e_h$  and  $Z$ , which affect long run GDP per worker, but do not affect capital per worker. The reason is simply, that in generating capital, human energy represents (these days) a vanishing contribution. In contrast, human energy input is instrumental in facilitating consumption, for which reason it will nevertheless impact on GDP per worker, through this demand channel. However, absent the presence of  $Z$  (i.e. without technological progress in consumption activities), one would have cause to ignore the effect based on the approximation  $e_h/e = 0$ , discussed above.

Finally, note that if we combine the network equation, with (15) we obtain

$$y^* = \left(v^{-1} + \frac{Ze_h}{e^*}\right) \epsilon (k^*)^a \equiv A (k^*)^a.$$

Accordingly, the (consumption-augmented) model admits a reduced form log-linear association between  $y$  and  $k$ , structurally identical to a Cobb-Douglas specification. From the perspective of interpretation there are three major differences. (i) In the present case  $a$  should not be parameterized using national accounts data; (ii)  $A$ , i.e. total factor productivity in standard terminology, reflects the underlying parameters of the model (including aspects of the energy transporting network and thereby technology), and, (iii), the above association is not technological per se, but reflects a steady state association.

**4.2. A First-Pass Look at the Data.** In confronting the consumption-augmented model with data we once again face the problem of having to control for e.g.  $v^{-1}$ . Hence, a direct test is not feasible. As a result, we will have to resort to an indirect approach, for the time being.

Notice that if  $\left(v^{-1} + \frac{Ze_h}{e^*}\right)$  is roughly the same, for a group of countries, the model would predict that  $Y \propto E$ ; the association between GDP and total energy consumption, plotted in a log-log diagram, is linear with slope 1 (cf. equation (15)). On *a priori* grounds, such a condition might be satisfied within the OECD. As is well known, if initial income is plotted against

subsequent growth one obtains a highly significant negative association between the two (e.g. Baumol, 1986). The finding is consistent with the neoclassical growth model, and the model developed above, provided the countries in question are converging towards the same steady state. That is, if they have similar structural characteristics; the parameters of the model.

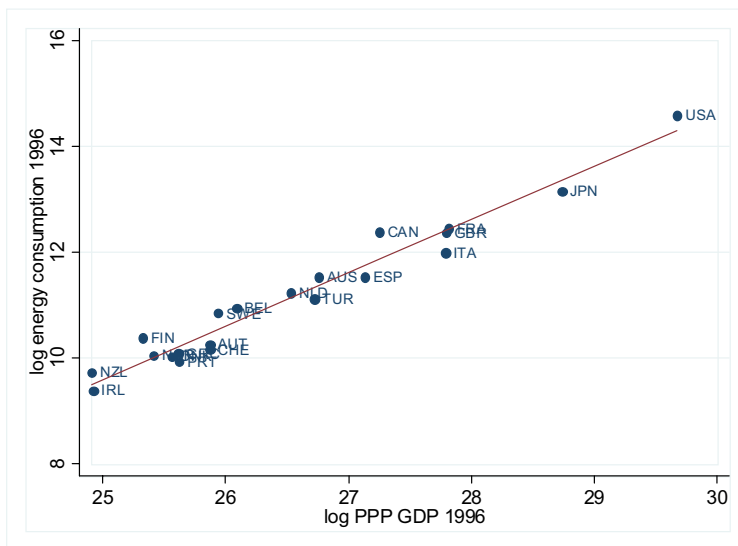
Based on this logic we estimated (by OLS) the association between  $\log(Y)$  and  $\log(E)$  for the OECD sample. The result is:

$$\log E = -15.48 + 1.003 \cdot \log Y, R^2 = 0.96.$$

(1.25)      (0.053)

The association is depicted in Figure 2. Needless to say, this is not a test of the model; this would require data on parameters like  $1/v$  and  $e_h$ . Still, the above exercise does serve to illustrate that a core prediction of the (consumption augmented) model is not glaringly inconsistent with the data, under plausible assumptions.

**Figure 2:** Log energy consumption vs. Log PPP GDP, 1996. 21 OECD countries. Data: World Development Indicators 2005 & Caselli (2006).



## 5. EXTENSIONS

**5.1. Technological Change.** Although structurally identical to the Solow model our theory proposes quite a different interpretation of adjustment dynamics and steady-states. Holding the technology of energy distribution and consumption constant any growth is conceived as adjustment towards a steady-state for capital (and income). Only if the structure changes in terms

of energy costs (e.g.  $\nu$ ) or energy supply ( $\epsilon$ ), the system restarts and converges towards a new and hopefully higher steady-state. In other words, any technological progress with permanent impact on economic performance originates from an improvement in use and distribution of energy.

At first sight the view that all lasting progress requires improvements of energy use may appear odd. It is, however, quite intuitive at closer inspection. If we accept the fact that there is little utility gained from the mere existence at subsistence level (beyond the utility received from own reproduction, see Dalgaard and Strulik, 2006), i.e. by existing merely to sustain one's own metabolism, any progress has to be fueled by non-human energy and is thus limited by the current technology to use this energy. Human *economic* development can then be understood as a perpetual series of efficiency gains in appropriating non-human energy. This way we can think of, for example, the wheel (the wheeled plow) as a device that exploits kinetic energy more efficiently and of the system of three field crop rotation as a device that exploits solar energy more efficiently than previously available methods. Indeed Jared Diamond (1997) argues that the superiority of the Eurasian vs. the Latin-American technology was caused to a large extent by the Eurasian knowledge of how to employ animal energy for various purposes (and closely related, by the difference across continents in the availability of large mammals, which allowed themselves to be utilized by humans in this manner).

The emphasis of energy-use improving technologies is not meant to belittle the importance of several other pathbreaking inventions of mankind like, for example, the clock, eyeglasses, letter printing, or gunpowder. These and several other inventions have without question brought forward human *social* and *political* development a great deal. The point is that they are themselves, in scale and *economic* impact, limited by the currently available technology of energy use; think, for example, of the energy needed to print a book.

The notion of the importance of non-human energy might be helpful to understand why the introduction of some technologies did *not* initiate long-run growth. For instance, reading how Landes (1998) marvels at the efficiency benefits gained by the invention of the clock one wonders why the clock did not initiate industrialization, which instead seems to have awaited the arrival of the steam engine. Maybe the answer is that the clock predominantly improved the efficiency of using human energy? If so, the impact of the invention would be limited by the availability of human energy, which is somewhat modest in scope (see Section 3). Rather, for a *fundamental*

improvement of economic development, a fundamental improvement of the use of non-human energy, like that brought forth by the steam engine, was required.

The model from Section 2 also sheds a new light on the general purpose technology (GPT) phenomenon. GPT innovations are viewed as “fundamental” innovation which tend to “reset” the economy, and instigates (ultimately) a growth “spurt”. The process, however, may involve a non-monotonous adjustment process, with an initial slump of productivity levels while the GPT forces a replacement of old machines with new ones that employ the new basic technology.

Bresnahan and Trajtenberg (1996) who where among those who initiated GPT research asked (p. 84): “Could it be that a handful of technologies had a dramatic impact on growth over extended periods of time? What is it in the nature of the steam engine, the electric motor, or the silicon wafer, that make them prime suspects of having played such a role?” They gave a very broad answer which is still used in the literature (see e.g., Jovanovich and Rousseau, 2005): The technology must be pervasive (spread to most sectors), there must be scope for improvement over time (lowering the costs of its use) and it must be innovation spawning, i.e. it enables the production of new products. The following “handful” of technologies are usually referred to as GPT’s: the waterwheel, the steam engine, electricity, railways, motor vehicles, and IT.

Based on the theory developed above we can suggest a more precise answer to Bresnahan and Trajtenberg’s question. A GPT must improve either the use of energy (waterwheel, steam) or its delivery through a network (railways, cars) or both (electricity, IT). Interestingly, while not all proposals of GPT candidates available in the literature coincide perfectly, electricity and IT, the technologies that revolutionized both the use and distribution of energy, are always on the lists. Speculating about what could possibly be the next GPT experts usually come up with nano-technology, again a new system for distributing energy at a new (finer) level of network. With our theory at hand it becomes intuitive why other, seemingly equally fundamental, innovations (e.g. the decoding of the DNA) are not GPT’s: they do not (much) improve the distribution and use of non-human energy.

An ad hoc way to mimic a GPT within the standard Solow model is to simultaneously vary general productivity ( $A$ ) and the depreciation rate ( $\delta$ ). The first parameter change is meant to capture the long-run increase in productivity, and the second captures the initial slump, originating from obsolescence of machines embodying the old technology (see Aghion and Howitt,

1998, Ch. 8.4). The problem is that both measures move the steady-state in opposite directions and some fine-tuning is needed to create the desired transitional and long-run effects.

In the current model, we have an – while admittedly equally ad hoc – more elegant way to produce the desired growth trajectory: a decrease of  $\nu$ . A lower value for  $\nu$  means that machines can be produced at lower energy costs, which, for example, could have been initiated through the transistor replacing the energy-intensive vacuum tube in electronic devices. From inspection of the reduced form of the model (9) we see that a lower  $\nu$  has a double effect. It raises both the first term, “productivity”, and the second term, “depreciation”. From Proposition 1 we also see that it unambiguously raises the steady-state level. Starting at the original steady-state (using tube technology)  $\dot{k}$  equals zero initially. Evaluating the RHS of (9) after the fall of  $\nu$ , we see (since  $a < 1$ ) that initially the negative effect through the depreciation channel dominates. In conclusion, energy saving technological progress causes GPT-like adjustment dynamics with an initial slump, recovery, and convergence towards a higher steady-state level.

**5.2. Growth in the Very Long Run.** It seems widely conceded that human societies at large enjoy income and consumption levels of historically unprecedented magnitudes (*e.g.* Galor, 2006). A key implication of the model above is that such increases is inescapably linked to the ability of human societies to expand energy supply. The derivations above assume  $e = \epsilon k^a$ , which implicitly imposes that the “supply of energy” can rise to match elevated needs from an expanding network. This may be a reasonable assumption for the last century or so.

However, from a very long-run perspective this is unlikely to always have been the case. Early humans, living off hunting and gathering, were undoubtedly limited by their own metabolism as the source of energy. In such a regime it might be a reasonable approximation to put  $e = e_h$ , implying the following transition equation for capital (tools):

$$\dot{k} = \frac{1}{\nu} e_h - \left( \frac{\mu}{\nu} + n \right) k,$$

which suggest a maximum level of equipment per person of  $\underline{k} = [(e_h/\nu) / (\frac{\mu}{\nu} + n)]$ , implying investment per capita of  $(\frac{\mu}{\nu} + n) \underline{k} = (\frac{\mu}{\nu} + n) [(e_h/\nu) / (\frac{\mu}{\nu} + n)]$ . Assuming consumption per capita is constant at some level of subsistence,  $\underline{c} > 0$ , output per capita can be written (by national accounts identity):

$$\underline{y} = \frac{e_h}{v} + \underline{c},$$

i.e. proportional to  $e_h$ , human metabolism. This state of affairs may not be a bad description of society up until the on-set of the Neolithic revolution.

After the advent of the Neolithic revolution energy supply would rise, mainly as a result of humans ability to harness the energy supply (metabolism) of animals in the production process. Under the model this fundamental innovation should be associated with a “step-increase” in living standards, or, in a Malthusian setting, an expanding population size.

Further advances in the energy supply of humans awaited the harnessing of natural forces. In this respect a major step forward occurred during classical antiquity in the mastery of water-power.<sup>19</sup> As argued by Greene (2000) the use of water-mills was pervasive during the Roman era; a technology which in all likelihood derives from around the mid-3rd century B.C. But the harnessing of water-power was employed in many more activities than milling grain during Roman times. For example, evidence exist of water-powered marble saws, and of the use of water on a seemingly rather spectacular scale in mining activities (Wilson, 2002). This innovation led to a remarkable intensification in the extraction of silver, lead and copper, the evidence of which can be found by analyzing samples of the ice core from Greenland. Indeed a similar level of activity, measured by the concentration of lead and copper aerosols in the northern hemisphere, is not attained until the industrial revolution (Wilson, 2002, Figure 5 and 6). From this perspective the Roman aqueduct technology becomes very significant. Aside from being useful in supplying Roman bathes and households with fresh water, a commonly cited virtue of the technology, aqueducts were also critical in channelling water to mining sites for final use in the context of the extraction of mineral ore. The aqueducts can therefore be regarded as an early example of an energy transporting network system; reminiscent of the power-grid, albeit naturally more modest in coverage and application. Unfortunately, the knowledge and use of this network declined with the fall of the Roman Empire.

In spite of these important advances throughout history, the more recent harnessing of electricity during the 19th century must be regarded as sowing the seeds of a dramatic change in human societies. First, it is the period during which the modern day energy transport network

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<sup>19</sup>The use of wind-power pre-dates water-power, and goes back (at least) to the ancient Egyptians who used sail-ships to navigate the Nile.

is created. That is, this period represents the genesis of  $e(t) = \epsilon k(t)^a$ . Second, as a result, these innovations allowed for investment growth, and thus income growth, of unprecedented scale, by removing the constraint on capital accumulation previously imposed by the metabolism of humans and animals.

In current-day US average energy consumption is about 11.000 Watts, which corresponds to the metabolism of a 30.000 kg primate (Moses and Brown, 2003). Accordingly, if we assume an average human weights 80 kg, and assume energy per capita is proportional to GDP per capita (as the consumption-augmented model would predict), this implies that the income of a modern day American is comparable to the combined “income” of 375 distant ancestors on the savannah. Under the approach taken in the present paper, this would be the ball-park estimate of total growth in per capita income over the last roughly 50.000 years.

## 6. CONCLUSION

The fundamental notion that economic growth originates from (and is limited by) energy has a long intellectual history, going back to Herbert Spencer’s (1862) *First Principles*. According to Spencer the evolution of societies depends on their ability to harness increasing amounts of energy for the purpose of production. Differences in stages of development can be accounted for by energy: the more energy a society consumes the more advanced it is. Chemist and Nobel prize winner Wilhelm Ostwald (1907) developed the Spencerian ideas further. Ostwald emphasized that it is not the sheer use of energy, but the degree of efficiency by which raw energy is made available for human purposes that defines the stage of economic (and according to Ostwald also cultural) development of society.<sup>20</sup>

The theory developed above demonstrates that this notion of development, when given a modern network interpretation, is compatible with neoclassical growth theory. Indeed, it coincides with the structural form of the economist’s core model of economic growth, the Solow growth model. At the same time, the theory developed above does not require the existence of an aggregate production function. Rather, the theory relies on thermodynamics and efficiency laws pertaining to the distribution of energy through networks.

Empirically, we find that cross-country data support the network vision. Specifically, when we confine attention to rich areas (such as the OECD) we find support for a log-linear association

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<sup>20</sup>Further refinements were made by several natural and social scientists, among them Frederick Soddy, Alfred Lotka, and Fred Cottrell.

between energy use and capital, with a coefficient of about  $3/4$ . The latter is predicted by the theory, provided the network is three-dimensional and efficient. With an inefficient network, the theory predicts a lower coefficient, which is detected on the African continent, or more generally among the poorest countries in the world. Contingent on the collection of data on energy costs of production, use and maintenance of capital, a fully fledged test of the steady-state predictions of the model will be possible. This next step is left open to future research.

The proposed theory allows for some reconciliation between neoclassical growth theory and the work of some of its staunchest critics. Ecological and biophysical economists, most prominently Nicolas Georgescu-Roegen (1976) and Herman Daly (1977), reject neoclassical growth theory for not taking the laws of thermodynamics into account. The central charge is that energy, or “natural capital” in the terminology of the authors, is introduced into the models in an unsatisfactory way (if not ignored altogether). That is, by including energy in the aggregate production function as a separate input, which can be substituted for by capital. This approach is fundamentally flawed, the argument goes, because it does not take into account that any capital good is itself produced by means of energy. Moreover, this energy use cannot be avoided because any transformation of material of a low degree of order (raw material) to a high degree of order (capital goods) needs a certain amount of energy for thermodynamic reasons. Accordingly, sceptics maintain that it is difficult to see how energy can be substituted for by capital.

Here, we have explicitly taken the thermodynamic argument into account, by assuming that all (capital) goods are created, employed, and maintained through human and non-human energy use. Interestingly, however, after performing the network calculations we come up with a law of motion for capital which is structurally identical to that implied by the Solow model.

The theory also have bearing on the fundamental “limits to growth” debate. In particular, while conceding the importance of energy for growth, the theory also highlights the crucial importance of human ingenuity. As shown above, absent technological change, growth will come to a halt even with unlimited supplies of energy, since energy dissipation increases as the economic network (appliances and machines connected) becomes larger. This result therefore implies that technology, associated with the harnessing and use of energy, is as important for growth prospects as the supply of energy itself; energy and technology are equal partners in development. Indeed, as argued above, “major” innovations (which usually are referred to as GPTs) can be seen as rare instances of progress, which in a profound way improves the harnessing, transformation,

and/or distribution of energy. Integrating the literature on endogenous technological change, with the present model of capital accumulation, would therefore seem like another useful topic for future research.

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