Creditor Coordination with Social Learning and Endogenous Timing of Credit Decisions

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Abstract

In case of multiple source lending even solvent firms may be forced into bankruptcy due to uncoordinated credit withdrawals of lenders. This paper analyzes whether a debtor firm can thwart such inefficient liquidations by offering creditors the option to delay their foreclosure decision rather than obliging them to simultaneous actions as suggested by Morris and Shin (2004). With this option, lenders endogenously determine the timing of their credit decisions, trading off the informational benefit from waiting against the associated cost of delay. We find that granting lenders the option to delay mitigates coordination failures whenever the borrowing firm is fundamentally distressed.

Keywords: Coordination; Coordination Failures; Equilibrium Selection; Global Games; Creditor Coordination; Multiple Source Financing; Uncertainty; Social Learning; Incomplete Information; Option to Delay

JEL classification: D82, D83, G32, G33

1 Introduction

Particularly in Europe, the business sector is characterized by a large number of small and medium-sized firms, which typically resort to bank debt financing when procuring capital for their investment projects. Since these firms are usually financed by a multitude of bank lenders, it is hard to overstate the importance of *creditor coordination failures.*[1] Banks may decide to foreclose their loans because they fear that others will also withdraw, even if the borrowing firm's project is viable so that the value at maturity is enough to refund all credits. Eventually, such uncoordinated withdrawals of bank loans can lead to inefficient project liquidations, forcing even economically solvent firms into bankruptcy.

Despite its considerable relevance, the problem of inefficient creditor coordination has received scant attention from the previous economic literature, which certainly can be ascribed to the drawbacks of multiple equilibria generally inherent in traditional coordination games. Only recently, the risk of creditor coordination failures has been analyzed more elaborately, building on the *theory of global games.* Global games, as introduced by CARLSSON AND VAN DAMME (1993) and generalized by MORRIS AND SHIN (2003) and FRANKEL ET AL. (2003), assume that each player noisily observes the game's payoff structure, which itself is determined by a random draw from a given class of games. Under certain conditions, this stochastic environment enables the derivation of a unique equilibrium, so that the incidence of inefficient project liquidations arising from the coordination problem among lenders can be quantified.

The concept of global games has first been applied to credit markets by HUBERT AND SCHÄFER (2002) and MORRIS AND SHIN (2004). They analyze coordination failures among a continuum of homogeneous creditors in a static model, where all lenders are supposed to decide simultaneously at an interim stage of the debtor firm's investment project whether or not to foreclose their loans. Credit decisions are made based on imperfect information regarding a fundamental state which can

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[1] In a notable empirical study analyzing cross-section data of 1079 firms from 20 European countries, ONGENA AND SMITH (2001) provide evidence on the prevalence of multiple bank lending.

be interpreted as a measure of project quality, and even economically sound projects may be doomed to failure if too many creditors foreclose. In this context, MORRIS AND SHIN (2004) and BANNIER AND HEINEMANN (2002) propose that firms can mitigate the risk of inefficient project liquidations by adjusting their degree of information dissemination. Other studies do not focus on information policy as an instrument of creditor coordination, but analyze to what extent a debtor firm can avoid coordination failures by choosing a heterogeneous creditor structure. These models assume that over time the borrowing firm has established close business relations to a particular bank which is therefore willing to finance a sizeable fraction of the firm's project. TAKEDA (2003) shows that the incidence of inefficient project liquidations is reduced if such a "relationship bank" jointly finances the project with a continuum of small "arm's length banks" and SCHÜLE AND STADLER (2005) demonstrate that creditor coordination may be even more efficient if the relationship bank is able to signal its credit decision to the small lenders.[2] Combining both perceptions regarding a debtor's capability to diminish creditor coordination failures - information policy and relationship lending - BANNIER (2006) analyzes a debtor's optimal information dissemination strategy in a model with multiple heterogeneous bank lenders.

However, at least in some situations it has to be doubted whether these existing strands of literature provide satisfying policy advice for debtor firms affected by creditor coordination failures. Firstly, it seems questionable if borrowers virtually can control for the precision of their lenders' information and therewith for the incidence of uncoordinated credit terminations. Secondly, especially young and small firms often do not dispose of long-term relations to a particular bank, so that they have to rely exclusively on arm's length debt financing. In this respect, we introduce a new aspect to the debate on applicable instruments of creditor coordination by dealing with the question to what extent borrowing firms can mitigate the risk of coordination failures via *debt contracting.* Particularly when bearing in mind the multitude of different forms of loan agreements consisting across the world, it appears incongruous that this issue has so far been neglected in the context of creditor coordination games.

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[2] See BOOT (2002) for an elaborate survey on the vices and virtues of relationship banking.

In scrutinizing the effects of debt contracting on the uncoordinated behavior of multiple lenders, this paper concentrates on two specific types of loan agreements a firm might offer to thwart coordination failures. As a benchmark, we delineate the static global game by Morris and Shin (2004), who assume that financing is undertaken via a *standard debt contract*, obliging all lenders to simultaneous credit decisions at an interim stage of the firm's investment project. Introducing an inherently dynamic setting, we then consider the debtor firm's possibility to offer a *leniency debt contract* which grants lenders the option to delay their roll over or foreclosure decisions. Provided with such a leniency debt contract, waiting rather than withdrawing the credit early generates an informational benefit via social learning. Namely, lenders who delay their credit decision are able to observe how many creditors have stopped lending before and use this additional information to update their prior beliefs regarding the quality of the firm's project. However, making a better informed decision late in the game may be associated with a cost of delay since foreclosing late rather than early yields a lower payoff. Thus, if the debtor firm decides to offer a leniency debt contract, lenders endogenously determine the timing of their credit decisions, trading off the informational benefit from waiting against the ensuing cost of delay.

Methodically, our dynamic creditor coordination game with endogenous timing of credit decisions and costs of delay builds on the global games framework analyzed by Dasgupta (2006) who considers a continuum of players with the option to delay investment in a risky project.[3] In his model, investing late rather than early reduces a player's uncertainty regarding the project quality, but involves a cost of delay by generating a lower payoff if the project succeeds. Hence, our approach differs from Dasgupta (2006) insofar as we assume that the *risky* action (to roll over) is reversible, while the *safe* action (to foreclose) is irreversible. Moreover, our application to creditor coordination failures requires that deferring the *safe* action to withdraw the credit is associated with a cost of delay, whereas in Dasgupta (2006) delaying the *risky* action (to invest) is costly.

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[3] Alternative global games with endogenous timing of actions and learning have been analyzed by Heidhues and Melissas (2003), Xue (2003), and Brindisi (2005).

In a limiting case where the incidence of creditor coordination failures can be derived explicitly, these modifications of the payoff structure also affect our qualitative results. As the investment project's level of risk approaches infinity, the incidence of inefficient liquidations remains unaltered in our dynamic creditor coordination game compared to the static benchmark model by Morris and Shin (2004), whereas efficiency increases with the option to delay in Dasgupta's investment game. However, for the more relevant case of substantial but finite levels of project risk, our findings indicate that debt contracting describes a very effective device for borrowing firms to enhance the efficiency of their creditors' actions. Thereby it essentially depends on the investment project's commonly expected quality which kind of debt contract proves optimal in minimizing the incidence of creditor coordination failures. Relying on numerical calculations, we demonstrate that offering a standard debt contract à la Morris and Shin (2004) is optimal if the expected project quality is sufficiently high. In contrast, for sufficiently low values of expected project quality, the risk of inefficient project liquidations can be mitigated by granting lenders the option to delay their credit decisions.

The remainder of this paper is organized as follows. Section 2 sets up the model, introducing the timing of events and the information available at all stages in the static benchmark game and in the dynamic game with the option to delay. In Section 3 we briefly discuss the incidence of coordination failures when creditors are provided with a standard debt contract. Section 4 solves for the equilibrium of the dynamic creditor coordination game, when creditors obtain a leniency debt contract. Comparing the risk of inefficient project liquidations in both games, we provide implications on the optimal debt contract in Section 5. Finally, Section 6 concludes.

2 The model

The model considers a simple economy consisting of a debtor firm and a continuum of ex ante identical, risk neutral arm's length lenders. Resorting to debt financing from the continuum of creditors, the firm decides to set up a risky investment project which matures in period T. Whether the project succeeds and loans can be repaid at maturity decisively depends on a fundamental state $\theta \in \mathbb{R}$, to which we refer as project quality, and on the credit decisions of lenders. Each creditor finances a loan which is secured on collateral and has a face value normalized to 1. Provided with a standard debt contract, creditors have the option to foreclose their loans prematurely in period $t_1 < T$ and seize the collateral $\kappa_1 \in (0, 1)$. Alternatively, the debtor firm can offer a leniency debt contract, granting the option to delay credit decisions. Then, lenders may withdraw their credit either in t_1 or in a later period $t_2 \in (t_1, T)$, if they are willing to foreclose at all. As foreclosing a loan in t_2 merely generates a payoff of $\kappa_2 \in (0, \kappa_1)$, protracting the decision to stop lending is associated with a cost of delay.[4]

Whether a lender decides to withdraw his credit prematurely or to roll over is determined by his expectations of the project quality θ, which is drawn from the commonly known distribution $N(\mu, \frac{1}{a})$ and is not revealed until the project matures in period T. In addition to the expected project quality $\mu \in \mathbb{R}$ and the project's level of risk $\frac{1}{a} \in \mathbb{R}^+$, every creditor i observes a noisy private signal x_i regarding θ previous to his credit decision in t_1:

$$x_i = \theta + \frac{\varepsilon_i}{\sqrt{b}}, \tag{1}$$

where ε_i is a random variable distributed i.i.d. $N(0, 1)$ and independent of θ, and $b > 0$ is a scale factor reflecting the precision of $x_i | \, \theta$. Provided with the option to delay, lenders who have rolled over their loans in t_1 receive an additional private signal y_i before making their final credit decision in period t_2:

$$y_i = \Phi^{-1}(1 - \ell_1) + \frac{\eta_i}{\sqrt{c}}, \tag{2}$$

where $c > 0$ is a constant and the idiosyncratic random variable $\eta_i \sim N(0, 1)$ is i.i.d. across creditors and independent of ε_i. As ℓ_1 denotes the fraction of creditors who decided to withdraw their credit in t_1, the signal y_i can be interpreted as a

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[4] Note that all payoffs are stated in expected terminal wealth, so that discounting does not complicate our analysis. Hence, the restriction $\kappa_2 < \kappa_1$ is equivalent to the assumption that creditors discount and receive a constant payoff at the time they withdraw their credit.

noisy statistic based on the proportion of ”active lenders”.[5] Hence, exercising the option to delay generates an informational benefit which has to be balanced with the potentially incorporated costs of delaying the foreclosure decision.

By assumption, lenders withdrawing their loans prematurely cause disruption to the debtor firm’s investment project, such that the project is doomed to failure whenever $\ell > \theta$, where $\ell \in [0,1]$ denotes the total mass of foreclosing creditors. In this case, the firm is forced into bankruptcy, implying that the loans of creditors who have rolled over cannot be refunded. In contrast, if the project succeeds ($\ell \leq \theta$), the firm remains in operation and is able to repay all loans at full face value. Thus, a creditor’s payoff from extending his credit until maturity in period T is given by

$$u(\theta,\ell) = \begin{cases} 1 & \text{if } \theta \geq \ell \\ 0 & \text{if } \theta < \ell. \end{cases}$$

Clearly, the unconstrained efficient outcome of this creditor coordination game would have all lenders withdrawing their credit in t_1 whenever $\theta < 0$ and not at all otherwise, independent of the debt contract offered by the firm. However, with imperfect information on θ creditors cannot coordinate on this efficient equilibrium, so that even economically sound projects with $\theta \geq 0$ may be liquidated. Below, we examine whether the debtor firm can mitigate the risk of such inefficient project liquidations by offering lenders a leniency debt contract with the option to delay rather than a standard debt contract as proposed by Morris and Shin (2004).

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[5] We assume that creditors observe the statistic y_i with some idiosyncratic noise in order to ensure the existence of a unique equilibrium. However, when determining the incidence of creditor coordination failures, we focus on the case of perfect observation of the past ($c \to \infty$) as is common in the literature on herds and cascades (see e.g. Bikhchandani et al. (1992)). In this limit, the monotone transformation of ℓ_1, $\Phi^{-1}(1-\ell_1)$, is equivalent to observing ℓ_1 and thus without loss of generality.

3 The standard debt contract with simultaneous credit decisions

To set a benchmark, we first discuss the static creditor coordination game in which the debtor firm offers a standard debt contract, obliging all lenders to simultaneous credit decisions in period t_1. Morris and Shin (2004) show that this global game has a unique equilibrium, provided that the firm's investment project is sufficiently risky relative to the precision of creditors' private information. The equilibrium is then characterized by trigger strategies, such that each lender rolls over his loan whenever he obtains a private signal x_i greater than a trigger value x^* and withdraws credit otherwise. Since private signals are correlated with θ, this implies that the project fails whenever a quality lower than the fundamental threshold θ^* is realized.

As a necessary condition for an equilibrium in trigger strategies, the marginal creditor who receives the critical signal x^* must be indifferent between foreclosing his loan in t_1 and rolling over, i.e.

$$\kappa_1 = Pr(\theta \geq \theta^* | x^*).$$

Since the posterior beliefs of a lender i who has observed the realization of the private signal x_i are given by

$$\theta | x_i \sim N\left(\frac{a\mu + bx_i}{a+b}, \frac{1}{a+b}\right), \tag{3}$$

the indifference condition for the marginal creditor can be rewritten as

$$x^* = \frac{a+b}{b}\theta^* - \frac{a}{b}\mu + \frac{\sqrt{a+b}}{b}\Phi^{-1}(\kappa_1). \tag{4}$$

The second condition necessary to derive the equilibrium thresholds x^* and θ^* reflects that the investment project is on the margin of success and failure at the state θ for which $\theta = \ell$. Due to the assumed independence of private signals and the continuum of creditors, the mass of foreclosing lenders ℓ is equivalent to the probability that an individual lender withdraws his credit, i.e.

$$\ell = Pr(x_i < x^* | \theta) = \Phi\left(\sqrt{b}(x^* - \theta)\right).$$

Thus, the critical project quality θ^* is implicitly given by the *critical mass condition*

$$\theta^* = \Phi\left(\sqrt{b}(x^* - \theta^*)\right). \tag{5}$$

Finally, substituting the creditors' *cutoff condition* (4) into the *critical mass condition* (5) delivers an equation purely in terms of θ^*:

$$\theta^* = \Phi\left(\frac{a}{\sqrt{b}}(\theta^* - \mu) + \sqrt{\frac{a+b}{b}}\Phi^{-1}(\kappa_1)\right). \tag{6}$$

As a sufficient condition for the uniqueness of equilibrium, consider that the expression on the right-hand side of Eq. (6) must have a slope of less than 1 everywhere. Deriving the right-hand side with respect to θ^*, it can easily be seen that a sufficient condition for a unique equilibrium is $a < \sqrt{2\pi b}$. Hence, as long as the creditors' prior information regarding the project quality θ is sufficiently diffuse relative to their private signals x_i, Eq. (6) delivers a unique $\theta^* \in (0,1)$, quantifying the risk of inefficient project liquidations by the set of states $\theta \in [0, \theta^*)$.

4 The leniency debt contract with the option to delay credit decisions

We now augment the static creditor coordination game by Morris and Shin (2004) as analyzed above to examine how efficiency is affected if creditors are provided with a leniency debt contract, granting each lender the option to delay his credit decision. The information system of this dynamic game, given by (1) and (2), implies that deferring the credit decision rather than withdrawing the credit in period t_1 generates an informational benefit which may offset the costs associated with delaying the foreclosure decision. As a creditor i who waits until t_2 observes a private signal y_i in addition to his first period signal x_i, the information held by this creditor in t_2 can be specified by a sufficient statistic $s_i(x_i, y_i)$. We can thus look for equilibria where lenders act according to trigger strategies around thresholds (x_D^*, s_D^*) such that:

- A creditor i forecloses his loan in t_1 if and only if $x_i < x_D^*$. Otherwise he chooses to wait.

- A creditor i who has exercised the option to delay forecloses his loan in t_2 if and only if $s_i < s_D^*$.

Assuming such trigger strategies, the proportion of lenders who withdraw their credit in period t_1 at any state θ is given by

$$\ell_1 = Pr(x_i < x_D^* | \theta) = \Phi\left(\sqrt{b}(x_D^* - \theta)\right).$$

Hence, substituting ℓ_1 into Eq. (2) demonstrates that the second period signal y_i actually provides a creditor i with additional information regarding the unknown project quality θ:

$$y_i = \sqrt{b}\,(\theta - x_D^*) + \frac{\eta_i}{\sqrt{c}}.$$

Note that in equilibrium observing y_i is equivalent to observing an exogenous signal $z_i = \frac{y_i}{\sqrt{b}} + x_D^*$, where z_i can be rewritten as

$$z_i = \theta + \frac{\eta_i}{\sqrt{bc}}.$$

Since $z_i | \theta$ is distributed $N(\theta, \frac{1}{bc})$, applying Bayes' Rule to update the creditors' previous beliefs $\theta | x_i$ as given by (3) delivers

$$\theta | x_i, z_i \sim N\left(\frac{a\mu + bx_i + bcz_i}{a + b + bc}, \frac{1}{a + b + bc}\right).$$

Finally, resubstituting $z_i = \frac{y_i}{\sqrt{b}} + x_D^*$, we get

$$\theta | x_i, y_i \equiv \theta | s_i \sim N\left(s_i, \frac{1}{a + b + bc}\right), \tag{7}$$

where

$$s_i = \frac{a\mu + bx_i + \sqrt{b}cy_i + bcx_D^*}{a + b + bc}$$

denotes the sufficient statistic for (x_i, y_i).

Having derived the posterior beliefs of creditors who exercise their option to delay, we are now in a position to establish necessary conditions for an equilibrium in trigger strategies. Provided that lenders follow trigger strategies as outlined above, the total mass of creditors who foreclose their loans prematurely at any fundamental state θ is given by $Pr(x_i < x_D^*|\theta) + Pr(x_i \geq x_D^*, s_i < s_D^*|\theta)$. Thus, the debtor firm's project succeeds if and only if

$$\theta \geq Pr(x_i < x_D^*|\theta) + Pr(x_i \geq x_D^*, s_i < s_D^*|\theta).$$

However, since the decisions of a creditor to withdraw his credit or to roll over in the two periods are not independent in the dynamic game with the option to delay, it is not apparent that there exists a critical θ_D^* above which the investment project succeeds and below which it fails. Lemma 1 verifies that such a threshold θ_D^* really exists.

Lemma 1. *Define*

$$G(\theta) = Pr(x_i < x_D^*|\theta) + Pr(x_i \geq x_D^*, s_i < s_D^*|\theta) - \theta.$$

Then, $G(\theta)$ is strictly decreasing and crosses zero exactly once.
Proof. See the Appendix.

Given Lemma 1, we can express the *critical mass condition* of the dynamic creditor coordination game as

$$\theta_D^* = Pr(x_i < x_D^*|\theta_D^*) + Pr(x_i \geq x_D^*, s_i < s_D^*|\theta_D^*). \tag{8}$$

The *cutoff condition* for creditors considering to exercise their option to delay in t_1 states that lenders trade off the proceeds from foreclosing early against the expected benefit of waiting and then acting optimally:

$$\kappa_1 = Pr(s_i < s_D^*|x_D^*)\kappa_2 + Pr(\theta \geq \theta_D^*, s_i \geq s_D^*|x_D^*). \tag{9}$$

Finally, the marginal creditor who has rolled over his loan in t_1 must be indifferent between withdrawing his credit in period t_2 and continuing lending until the project matures:

$$\kappa_2 = Pr(\theta \geq \theta_D^*|s_D^*).$$

Using (7), this *cutoff condition* for lenders in t_2 can be rewritten as

$$s_D^* = \theta_D^* + \frac{\Phi^{-1}(\kappa_2)}{\sqrt{a+b+bc}}\,. \tag{10}$$

As a first step to solve the system of equations (8) - (10), note that substituting the threshold s_D^* as given by (10) into the *critical mass condition* (8) yields an equation merely in x_D^* and θ_D^*:

$$\theta_D^* = Pr(x_i < x_D^* | \theta_D^*) + Pr(x_i \geq x_D^*, s_i < \theta_D^* + M | \theta_D^*),$$

where $M = \frac{\Phi^{-1}(\kappa_2)}{\sqrt{a+b+bc}}$. Lemma 2 states that as long as the debtor firm's investment project is sufficiently risky, this equation implicitly defines θ_D^* as a smooth increasing function of x_D^* with a bounded derivative.

Lemma 2. *Assume* $a < \frac{\sqrt{2\pi b(1+c)}}{1+\sqrt{1+c}}$. *Then, for any* x_D^*, *there is a unique* $\hat{\theta}(x_D^*)$, *such that* $G(\hat{\theta}, x_D^*) = 0$, *where*

$$G(\theta, x_D^*) = Pr(x_i < x_D^* | \theta) + Pr(x_i \geq x_D^*, s_i < \theta_D^* + M | \theta) - \theta.$$

Moreover, $\frac{d\hat{\theta}}{dx_D^*} \in \left(0, \frac{b}{a+b}\right)$.
Proof. *See the Appendix.*

Using Lemma 2 and Eq. (10), the *cutoff condition* (9) of creditors in period t_1 can be expressed purely in terms of x_D^*:

$$\kappa_1 = Pr(s_i < \theta_D^*(x_D^*) + M | x_D^*)\kappa_2 + Pr(\theta \geq \theta_D^*(x_D^*), s_i \geq \theta_D^*(x_D^*) + M | x_D^*).$$

As we show in the Appendix, this equation has a unique solution, provided that $a < \frac{\sqrt{2\pi b(1+c)}}{1+\sqrt{1+c}}$. By means of Lemma 2, this implies that there also exists a unique solution $(x_D^*, s_D^*, \theta_D^*)$ to the system (8) - (10). We can thus state:

Proposition 1. *When creditors act according to trigger strategies, the dynamic game with the option to delay credit decisions has a unique equilibrium provided that* $a < \frac{\sqrt{2\pi b(1+c)}}{1+\sqrt{1+c}}$.
Proof. *See the Appendix.*

While this uniqueness result holds for general values of c, we have to focus on the limiting case when creditors in t_2 observe the fraction of active lenders with vanishing noise ($c \to \infty$) in order to identify the incidence of inefficient project liquidations. In this limit, lenders who exercise the option to delay their credit decision essentially face no uncertainty regarding the project quality θ in period t_2. The *critical mass condition* (8) then reduces to

$$1 - \theta_D^* = (1 - \kappa_2)\, \Phi \left(\sqrt{b}(\theta_D^* - x_D^*) \right), \tag{11}$$

whereas Eq. (9) can be rewritten as[6]

$$1 - \kappa_1 = (1 - \kappa_2)\, \Phi \left(\sqrt{a + b} \left(\theta_D^* - \frac{a\mu + b x_D^*}{a + b} \right) \right). \tag{12}$$

Rearranging (12),

$$x_D^* = \frac{a + b}{b} \theta_D^* - \frac{a}{b} \mu + \frac{\sqrt{a + b}}{b} \Phi^{-1} \left(\frac{\kappa_1 - \kappa_2}{1 - \kappa_2} \right), \tag{13}$$

and substituting into Eq. (11), the critical project quality θ_D^* is implicitly given by

$$1 - \theta_D^* = (1 - \kappa_2)\, \Phi \left(\sqrt{\frac{a + b}{b}} \Phi^{-1} \left(\frac{1 - \kappa_1}{1 - \kappa_2} \right) - \frac{a}{\sqrt{b}} (\theta_D^* - \mu) \right). \tag{14}$$

Similar to the benchmark static game, in case of leniency debt contracting the risk of inefficient project liquidations is specified by the interval $\theta \in [0, \theta_D^*)$, where $\theta_D^* \in (0, 1)$.

5 Implications on the optimal debt contract

Having derived implicit solutions for the equilibrium project quality thresholds θ^* and θ_D^*, this section analyzes what kind of debt contract the borrowing firm should offer in order to minimize the incidence of creditor coordination failures. As mentioned above, the risk of inefficient project liquidations is given by the intervals $[0, \theta^*)$ and $[0, \theta_D^*)$, respectively. Offering a leniency debt contract with the option to delay credit decisions instead of a standard debt contract à la Morris and Shin (2004) therefore reduces the risk of uncoordinated credit withdrawals if and only if $\theta_D^* < \theta^*$.

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[6] See the Appendix for a formal derivation of the equations (11) and (12).

5.1 Comparison of coordination failures in the limit as $a \to 0$

Both, the static creditor coordination game and the dynamic game with the option to delay required the debtor firm's investment project to be sufficiently risky in order to ensure the uniqueness of equilibrium. We now focus on the extreme case where the project's level of risk approaches infinity ($a \to 0$), so that the prior distribution $\theta \sim N(\mu, \frac{1}{a})$ converges to an improper uniform prior over the real line. This property allows for a characterization of the thresholds θ^* and θ^*_D of the respective games in closed form.

Provided with a standard debt contract, uncoordinated credit withdrawals of lenders lead to a failure point θ^* as implicitly defined by Eq. (6). In the limit as $a \to 0$, the right-hand side of this equation simplifies to $\Phi(\Phi^{-1}(\kappa_1))$, implying that

$$\theta^* = \kappa_1.$$

If the firm offers a leniency debt contract instead, the critical state θ^*_D at which the project is on the margin of failure and success is implicitly given by Eq. (14), which reduces to

$$\theta^*_D = \kappa_1$$

in the limit as $a \to 0$. Hence, whenever the investment project conducted by the debtor firm is arbitrarily risky, the incidence of inefficient project liquidations is determined by $\theta \in [0, \kappa_1)$ in the static benchmark game as well as in our dynamic creditor coordination game, and therefore independent of the debt contract offered by the firm.

Let us examine this limiting result in more detail by providing some insight into the decision strategies of creditors in the static game and in the dynamic game with the option to delay, respectively. If lenders are provided with a standard debt contract and thus simultaneously decide on rolling over or foreclosing their loans, we know from Eq. (4) that

$$x^* = \frac{a+b}{b}\theta^* - \frac{a}{b}\mu + \frac{\sqrt{a+b}}{b}\Phi^{-1}(\kappa_1).$$

Clearly, a higher expected project quality μ shifts the trigger signal x^* to the left as it increases the lenders' incentives to extend their credit. Considering the critical signal x^*_D of the dynamic game as given by Eq. (13),

$$x^*_D = \frac{a+b}{b}\theta^*_D - \frac{a}{b}\mu + \frac{\sqrt{a+b}}{b}\Phi^{-1}\left(\frac{\kappa_1 - \kappa_2}{1-\kappa_2}\right),$$

it is easy to see that the same intuition applies to the credit decisions of lenders in period t_1 if they are provided with a leniency debt contract. However, creditors who exercise their option to delay and additionally observe the second period signal y_i essentially face no uncertainty regarding the project quality θ in the limit as $c \to \infty$, and thus follow strategies independent of the prior mean μ. Hence, for a finite level of project risk $1/a$, the strategies of all lenders in the static game are affected by the ex ante expected project quality μ, whereas some creditors provided with a leniency debt contract follow mean independent strategies. As $a \to 0$, however, the strategies of *all* creditors are mean independent in *both* games, finally implying that the risk of inefficient project liquidations does not depend on the debt contract offered by the firm.

In order to illustrate how mean independence leads to an identical risk of inefficient project liquidations in both games, let us compare the mass of lenders withdrawing their credit prematurely at the critical state θ^* in the static game with the mass of foreclosing creditors at the critical state θ^*_D in the dynamic game. First consider the case of standard debt contracting, obliging lenders to simultaneous credit decisions in period t_1. Using the definition of the trigger signal x^* as given by (4), the mass of creditors who decide to foreclose their loans can be expressed as

$$Pr(x_i < x^*|\theta^*) = \Phi\left(\frac{a}{\sqrt{b}}(\theta^* - \mu) + \sqrt{\frac{a+b}{b}}\Phi^{-1}(\kappa_1)\right). \tag{15}$$

Similarly, using Eq. (13), the mass of lenders who withdraw their credit in period t_1 in the dynamic game with the option to delay is given by

$$Pr(x_i < x^*_D|\theta^*_D) = \Phi\left(\frac{a}{\sqrt{b}}(\theta^*_D - \mu) + \sqrt{\frac{a+b}{b}}\Phi^{-1}\left(\frac{\kappa_1 - \kappa_2}{1-\kappa_2}\right)\right). \tag{16}$$

On the one hand, we expect this mass to be lower than the mass of creditors foreclosing in t_1 in the static game since creditors provided with a leniency debt contract have another opportunity to foreclose their loans in t_2 and seize the collateral κ_2. On the other hand, in case of leniency debt contracting a creditor mass of

$$Pr(x_i \geq x_D^*, s_i < s_D^* | \theta_D^*) = Pr(x_i \geq x_D^* | \theta_D^*)\kappa_2, \tag{17}$$

exercises its option to delay and stops lending in period t_2, thus causing additional disruption to the debtor firm's project.[7] Whether the total mass of foreclosing lenders at the critical state in the dynamic game, given by the sum of (16) and (17), exceeds the total mass of foreclosing lenders at the critical state of the static game, given by (15), obviously depends on the parameters of the prior distribution $\theta \sim N(\mu, \frac{1}{a})$.

However, in the limit as $a \to 0$ the decision strategies of all creditors are independent of the prior mean μ, so that the total mass of lenders who withdraw their credit is merely dependent on the payoffs of the respective games. Considering the benchmark static game, Eq. (15) reduces to

$$\lim_{a \to 0} Pr(x_i < x^* | \theta^*) = \kappa_1,$$

whereas the mass of creditors who do not exercise their option to delay in the dynamic game is given by

$$\lim_{a \to 0} Pr(x_i < x_D^* | \theta_D^*) = \frac{\kappa_1 - \kappa_2}{1 - \kappa_2}\ .$$

Consequently, the debtor firm "gains" a creditor mass of

$$\kappa_1 - \frac{\kappa_1 - \kappa_2}{1 - \kappa_2} = \frac{1 - \kappa_1}{1 - \kappa_2}\,\kappa_2$$

in period t_1 by offering a leniency debt contract instead of a standard debt contract. As $a \to 0$, however, Eq. (17) becomes

$$\lim_{a \to 0} Pr(x_i \geq x_D^*, s_i < s_D^* | \theta_D^*) = \left(1 - \frac{\kappa_1 - \kappa_2}{1 - \kappa_2}\right)\kappa_2 = \frac{1 - \kappa_1}{1 - \kappa_2}\,\kappa_2,$$

implying that the benefits from offering a leniency debt contract in t_1 are just balanced by the loss of creditors in period t_2. It is no coincidence then, that the thresholds θ^* and θ_D^* of the respective games coincide when lenders follow mean independent strategies due to diffuse prior information regarding the project quality θ.

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[7] The decomposition of the product term in Eq. (17) arises because as $c \to \infty$, $Cov(x_i, s_i | \theta) \to 0$, since $s_i \to \theta$. A formal derivation is given in section A.4 of the Appendix.

5.2 Comparison of coordination failures away from the limit

While the result of debtor firms not being able to affect the incidence of creditor coordination failures by debt contracting in the limit as $a \to 0$ is rather discontenting, the above discussion indicates that offering a leniency debt contract instead of a standard debt contract à la Morris and Shin (2004) may well have an effect for the more relevant case of substantial but finite levels of project risk ($a \nrightarrow 0$). Using numerical methods, we thus examine to what extent the incidence of inefficient project liquidations is influenced by a decreasing level of project risk in the static as well as in the dynamic creditor coordination game. For all numerical calculations, let $\kappa_1 = 0.5$ and $\kappa_2 = 0.3$, implying that the lenders' cost of delay, given by $k \equiv \kappa_1 - \kappa_2$, amounts to 0.2.[8] Since the idea of debtor firms being able to control for the precision of creditors' private signals x_i is indeed precarious and our results rather depend on the ratio of a to b than on the absolute values of these precisions, we fix $b = 1$ while varying the prior precision a in the range where the uniqueness of equilibrium can be guaranteed, $a \in (0, \sqrt{2\pi})$.

Figure 1:

High expected project quality $\mu = 1.5$

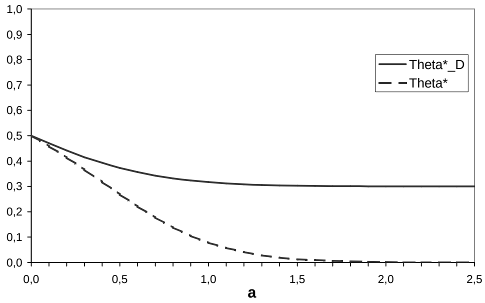
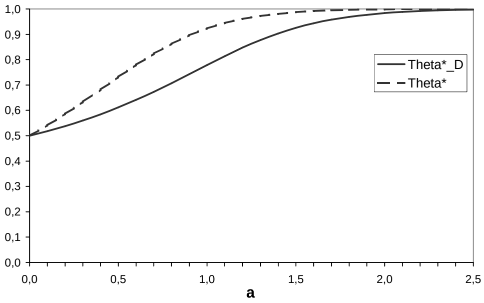


Figure 2:

Low expected project quality $\mu = -0.5$



Figures 1 and 2 depict the results for low ($\mu = -0.5$) and high ($\mu = 1.5$) expected project quality, whereby our choice of means is determined by their distance from the crucial region of θ, $\theta \in (0, 1)$. Evidently, both figures approve our result that in the limit as $a \to 0$, the risk of inefficient project liquidations is given by $\theta \in [0, \kappa_1)$, independent of the debt contract offered by the firm. However, as the project's

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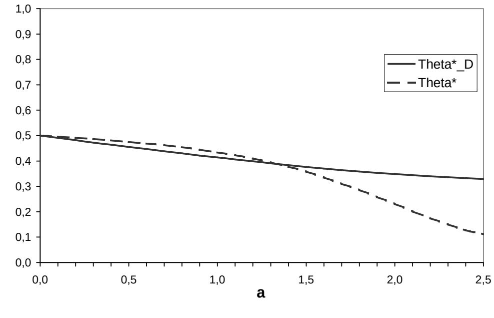
[8] We have checked many different values of $\kappa_1 \in (0, 1)$ and $k \in (0, \kappa_1)$, but did not find evidence that our results are affected qualitatively by varying costs of delay and collateral values.

level of risk $1/a$ decreases, creditors in period t_1 put more weight on their prior information regarding θ relative to the information contained in their private signals x_i. For high values of the expected project quality (Figure 1), this implies that all lenders in the static benchmark game become more optimistic as a increases, leading to a declining mass of foreclosing creditors and thus to a decrease in θ^*. The same intuition applies to the decision strategies of lenders in the first period of the dynamic game. But provided with a leniency debt contract, creditors who exercise their option to delay follow mean independent strategies in t_2 and thus are less optimistic, causing more disruption to the firm's investment project overall ($\theta^*_D > \theta^*$). Hence, a firm conducting a project with high expected quality μ benefits from offering a standard debt contract instead of a leniency debt contract with the option to defer credit decisions, independent of the cost of delay and the project's level of risk. In contrast, similar arguments imply that granting creditors the option to delay rather than providing them with a standard debt contract is beneficial whenever the debtor firm merely has access to projects with low expected quality (Figure 2). In this case, a declining level of project risk $1/a$ fosters the pessimism of creditors acting in period t_1 of the static benchmark game and therewith leads to an increase of θ^*. Whereas this intuition also applies to lenders in t_1 in the dynamic game, the strategies of creditors who choose to protract their credit decision are independent of the low prior mean μ, thus less pessimistic, implying that overall less creditors withdraw their loans if they are provided with a leniency debt contract ($\theta^*_D < \theta^*$).

Figure 3:

Intermediate expected project quality $\mu = 0.6$



For intermediate values of the expected project quality (e.g. $\mu = 0.6$), lying inside the crucial region $\theta \in (0, 1)$ where uncoordinated credit withdrawals may lead to inefficient project liquidations, Figure 3 illustrates that the effects of the prior mean μ may be ambiguous, so that the firm's choice of the optimal debt contract becomes a nontrivial decision.

6 Concluding remarks

This paper has introduced a new aspect to the debate on how debtor firms can mitigate the risk of inefficient project liquidations arising due to uncoordinated credit withdrawals of their lenders. Adopting a dynamic global game as analyzed by DASGUPTA (2006), it examines whether a firm can diminish creditor coordination failures by granting its lenders the option to delay their roll over or foreclosure decisions rather than obliging them to simultaneous credit decisions as suggested by MORRIS AND SHIN (2004). In this respect, our analysis of efficient debt contracting complements the previous literature which exclusively concentrates on a borrower's information policy and the possibility of relationship lending as applicable instruments of creditor coordination.

Our model implies that creditors endogenously determine the timing of their credit decisions, trading off the benefits from waiting and gathering more accurate information against the potentially incorporated cost of delaying the foreclosure decision whenever the debtor firm offers a leniency debt contract with the option to delay. Comparing this dynamic creditor coordination game to the static benchmark game of MORRIS AND SHIN (2004) who assume financing via a standard debt contract, binding creditors to simultaneous roll over or foreclosure decisions, enabled us to provide implications on the firm's choice of the optimal debt contract.

In the limit when the investment project conducted by the borrowing firm is arbitrarily risky, so that the equilibrium of the model can be analyzed in closed form, our results state that the risk of inefficient project liquidations remains unaffected of the debt contract offered by the firm. In contrast, resorting to numerical calculations for the more relevant case of substantial but finite levels of project risk, we have demonstrated that granting lenders the option to delay generally does exert decisive influence on their ability to coordinate credit decisions. Whenever the expected quality of the firm's investment project is sufficiently sound, providing lenders with an option to delay is detrimental for efficient creditor coordination, implying that the firm should adhere to a standard debt contract à la Morris and Shin (2004). When the debtor firm is expected to be severely in distress, however, it can reduce the incidence of uncoordinated credit withdrawals by offering its lenders a leniency debt contract with the option to delay credit decisions. Hence, just in those situations in which the issue of creditor coordination failures becomes most prominent our paper suggests an effective device to mitigate the risk of inefficient project liquidations, complementing the commonly discussed instruments in terms of information policy and relationship lending.

Appendix

A.1. Proof of Lemma 1

Using Eq. (1) and rewriting (2) as $y_i = \sqrt{b}(\theta - x^*) + \frac{\eta_i}{\sqrt{c}}$, the sufficient statistic $s_i = \frac{a\mu + bx_i + \sqrt{b}cy_i + bcx_D^*}{a+b+bc}$ can be expressed as $s_i = \frac{a\mu}{a+b+bc} + \frac{b(1+c)}{a+b+bc}\theta + \frac{\sqrt{bc}}{a+b+bc}\gamma$, where $\gamma = \frac{\varepsilon_i}{\sqrt{c}} + \eta_i$. Then, $s_i < s_D^*$ is equivalent to $\gamma < \frac{a+b+bc}{\sqrt{bc}}s_D^* - \frac{a\mu}{\sqrt{bc}} - \frac{b(1+c)}{\sqrt{bc}}\theta$ and we can rewrite:

$$G(\theta) = \Phi(A(\theta)) + \int_{A(\theta)}^{\infty}\int_{-\infty}^{B(\theta)} f(\varepsilon, \gamma)d\gamma d\varepsilon - \theta,$$

where $A(\theta) = \sqrt{b}(x_D^* - \theta)$ and $B(\theta) = \frac{a+b+bc}{\sqrt{bc}}s_D^* - \frac{a\mu}{\sqrt{bc}} - \frac{b(1+c)}{\sqrt{bc}}\theta$. Using Leibniz' rule, differentiation under the double integral delivers

$$G'(\theta) = A'(\theta)\phi(A(\theta)) - A'(\theta)\int_{-\infty}^{B(\theta)} f(A(\theta), \gamma)d\gamma + B'(\theta)\int_{A(\theta)}^{\infty} f(\varepsilon, B(\theta))d\varepsilon - 1.$$

Denoting by $\phi(\cdot)$ the standard normal PDF of ε, and by $\hat{\phi}(\cdot)$ the (non-standard) normal PDF of γ, we can express the joint densities as

$$f(\varepsilon = A(\theta), \gamma) = \phi(a(\theta))f(\gamma|\varepsilon = A(\theta))$$
$$f(\varepsilon, \gamma = B(\theta)) = \hat{\phi}(B(\theta))f(\varepsilon|\gamma = B(\theta)).$$

Since $A'(\theta) = -\sqrt{b}$ and $B'(\theta) = -\frac{b(1+c)}{\sqrt{bc}}$, we can now rewrite $G'(\theta)$ as

$$-\sqrt{b}\,\phi(A(\theta))\left[1 - \int_{-\infty}^{B(\theta)} f(\gamma|\varepsilon = A(\theta))d\gamma\right] - \frac{b(1+c)}{\sqrt{bc}}\,\hat{\phi}(B(\theta))\int_{A(\theta)}^{\infty} f(\varepsilon|\gamma = B(\theta))d\varepsilon - 1.$$

Clearly, $G'(\theta) < 0$. Furthermore, $\lim_{\theta\to-\infty} G(\theta) = \infty$ and $\lim_{\theta\to\infty} G(\theta) = -\infty$, which completes the proof of Lemma 1. ■

A.2. Proof of Lemma 2

As above, the sufficient statistic s_i can be rewritten as $s_i = \frac{a\mu}{a+b+bc} + \frac{b(1+c)}{a+b+bc}\hat{\theta} + \frac{\sqrt{bc}}{a+b+bc}\gamma$, where $\gamma = \frac{\varepsilon_i}{\sqrt{c}} + \eta_i$. Writing $s_D^* = \hat{\theta} + M$, $s_i < s^*$ implies that $\gamma < \frac{a}{\sqrt{bc}}(\hat{\theta} - \mu) + \frac{a+b+bc}{\sqrt{bc}}M$. Define

$$B(\hat{\theta}) = \frac{a}{\sqrt{bc}}(\hat{\theta} - \mu) + \frac{a+b+bc}{\sqrt{bc}}M.$$

Since $B'(\hat{\theta}) = \frac{a}{\sqrt{bc}}$, using the proof of Lemma 1 gives us

$$\begin{aligned}\frac{\partial G(\hat{\theta}, x_D^*)}{\partial \hat{\theta}} =& -\sqrt{b}\phi(A(\hat{\theta}, x_D^*))\left[1 - \int_{-\infty}^{B(\hat{\theta})} f(\gamma|\varepsilon = A(\hat{\theta}, x_D^*))d\gamma\right] \\ &+ \frac{a}{\sqrt{bc}}\hat{\phi}(B(\hat{\theta}))\int_{A(\hat{\theta}, x_D^*)}^{\infty} f(\varepsilon|\gamma = B(\hat{\theta}))d\varepsilon - 1.\end{aligned}$$

Now define

$$\begin{aligned}P_1 &= \int_{B(\hat{\theta})}^{\infty} f(\gamma|\varepsilon = A(\hat{\theta}, x_D^*))d\gamma \\ P_2 &= \int_{A(\hat{\theta}, x_D^*)}^{\infty} f(\varepsilon|\gamma = B(\hat{\theta}))d\varepsilon.\end{aligned}$$

Note that $P_2 < 1$ and $\hat{\phi}(\cdot) < \frac{\sqrt{c}}{\sqrt{2\pi}\sqrt{1+c}}$, since the variance of γ amounts to $\frac{1+c}{c}$. Thus, $a < \sqrt{2\pi b(1+c)}$ is a sufficient condition for $\frac{\partial G(\hat{\theta}, x_D^*)}{\partial \hat{\theta}} < 0$. In contrast,

$$\frac{\partial G(\hat{\theta}, x_D^*)}{\partial x_D^*} = \sqrt{b}\phi(A(\hat{\theta}, x_D^*))P_1 > 0.$$

Applying the implicit function theorem,

$$\frac{d\hat{\theta}(x_D^*)}{dx_D^*} = -\frac{\frac{\partial G(\hat{\theta}, x_D^*)}{\partial x_D^*}}{\frac{\partial G(\hat{\theta}, x_D^*)}{\partial \hat{\theta}}},$$

defining $Q = \frac{\partial G(\hat{\theta}, x_D^*)}{\partial x_D^*} > 0$, and rewriting

$$\frac{d\hat{\theta}(x_D^*)}{dx_D^*} = \frac{Q}{Q - \frac{a}{\sqrt{bc}}\hat{\phi}(B(\hat{\theta}))P_2 + 1},$$

it is easy to check that

$$\frac{d\hat{\theta}(x_D^*)}{dx_D^*} < \frac{b}{a+b}$$

holds whenever $a < \frac{\sqrt{2\pi b(1+c)}}{1+\sqrt{1+c}}$. Since this implies that $a < \sqrt{2\pi b(1+c)}$, the proof is complete. ■

A.3. Proof of Proposition 1

Using Lemma 2 and Eq. (10), we can write $s_D^* = \theta_D^*(x_D^*) + M$, where $M = \frac{\Phi^{-1}(\kappa_2)}{\sqrt{a+b+bc}}$. Thus, the creditors' *cutoff condition* in t_1 can be expressed purely in terms of x_D^*:

$$\kappa_1 = Pr(s_i < \theta_D^*(x_D^*) + M|x_D^*)\kappa_2 + Pr(\theta \geq \theta_D^*(x_D^*), s_i \geq \theta_D^*(x_D^*) + M|x_D^*)$$

Write x for x_D^* and let

$$G(x) = Pr(s_i < \theta_D^*(x) + M|x)\kappa_2 + Pr(\theta \geq \theta_D^*(x), s_i \geq \theta_D^*(x) + M|x) - \kappa_1.$$

Note that

$$Pr(\theta \geq \theta_D^*(x)|x) = 1 - \Phi\left(\sqrt{a+b}\left(\theta_D^*(x) - \frac{a\mu + bx}{a+b}\right)\right).$$

Define $A(x)$ as

$$A(x) = \sqrt{a+b}\left(\theta_D^*(x) - \frac{a\mu + bx}{a+b}\right)$$

and note that, given x,

$$s_i = \frac{a\mu + bx + bc\theta + \sqrt{bc}\eta_i}{a+b+bc}.$$

Rearranging terms, this can be rewritten as

$$s_i = \frac{a\mu + bx}{a+b} + \frac{bc}{a+b+bc}\left[\frac{z}{\sqrt{a+b}} + \frac{\eta_i}{\sqrt{bc}}\right],$$

where $z = \sqrt{a+b}\left(\theta - \frac{a\mu+bx}{a+b}\right)$ is distributed $N(0,1)$ conditional on x. Let $\gamma = \frac{z}{\sqrt{a+b}} + \frac{\eta_i}{\sqrt{bc}}$. Then, $s_i < \theta_D^*(x) + M$ is equivalent to

$$\gamma < \frac{a+b+bc}{bc\sqrt{a+b}}A(x) + \frac{\sqrt{a+b+bc}}{bc}\Phi^{-1}(\kappa_2).$$

Now let

$$B(x) = \frac{a+b+bc}{bc\sqrt{a+b}}A(x) + \frac{\sqrt{a+b+bc}}{bc}\Phi^{-1}(\kappa_2)$$

and rewrite

$$\begin{aligned} G(x) =& Pr(\gamma < B(x))\kappa_2 + Pr(z \geq A(x), \gamma \geq B(x)) - \kappa_1 \\ =& Pr(z < A(x), \gamma < B(x))\kappa_2 + Pr(z \geq A(x), \gamma < B(x))\kappa_2 \\ &+ Pr(z \geq A(x), \gamma \geq B(x)) - \kappa_1. \end{aligned} \tag{18}$$

Differentiating under the double integral and rearranging, we get:

$$G'(x) = B'(x)\hat{\phi}(B(x))[\kappa_2 - P_2] - A'(x)\phi(A(x))P_1,$$

where P_1 and P_2 are defined as follows:

$$P_1 = \int_{B(x)}^{\infty} f(\gamma|z = A(x))d\gamma$$
$$P_2 = \int_{A(x)}^{\infty} f(z|\gamma = B(x))dz.$$

Using standard computations to derive conditional distributions of normal random variables (see e.g. MITTELHAMMER (1996)), we know that:

$$z|\gamma = B(x) \sim N\left(A(x) + \frac{\sqrt{a+b}}{\sqrt{a+b+bc}}\Phi^{-1}(\kappa_2), \frac{a+b}{a+b+bc}\right).$$

Thus,

$$P_2 = \int_{A(x)}^{\infty} f(z|\gamma = B(x))dz = \kappa_2,$$

and therefore $G'(x)$ reduces to

$$G'(x) = -A'(x)\phi(A(x))P_1.$$

Under the conditions of the theorem, Lemma 2 states that $\frac{d\hat{\theta}(x)}{dx} < \frac{b}{a+b}$. Thus, $A'(x) < 0$, implying that $G'(x) > 0$. Moreover, note that $\lim_{x\to-\infty} G(x) = \kappa_2 - \kappa_1 < 0$ and $\lim_{x\to\infty} G(x) = 1-\kappa_1 > 0$. Hence, there exists a unique solution $(x_D^*, s_D^*, \theta_D^*)$ to the three necessary conditions (8) to (10) for an equilibrium in trigger strategies.

Finally, fixing θ_D^*, the indifference condition for creditors in t_1 as given by (18) depends on x only via the functions $A(x) = \sqrt{a+b}\left(\theta_D^* - \frac{a\mu+bx}{a+b}\right)$ and $B(x) = \frac{a+b+bc}{bc\sqrt{a+b}}A(x) + \frac{\sqrt{a+b+bc}}{bc}\Phi^{-1}(\kappa_2)$. If θ_D^* is fixed, $A(x, \theta_D^*)$ clearly is strictly decreasing in x for all $b > 0$, so that creditors who receive signals $x < x_D^*$ choose to foreclose in t_1, and they choose to delay the foreclosure decision whenever $x \geq x_D^*$. Therefore, the proof is complete. ■

A.4. Formal derivation of equations (11) and (12)

First consider the derivation of Eq. (11). Applying Lebesgue's theorem of dominated convergence, we can write:

$$Pr(x_i \geq x_D^*, s_i < s_D^*|\theta_D^*) = \int_{x_D^*}^{\infty} Pr(s_i < s_D^*|\theta_D^*, x_i)f(x_i|\theta_D^*)dx_i.$$

By definition $s_i = \frac{a\mu + bx_i + \sqrt{b}cy_i + bcx_D^*}{a+b+bc}$. Given x_i and θ_D^*, and substituting $y_i = \sqrt{b}(\theta_D^* - x_D^*) + \frac{\eta_i}{\sqrt{c}}$, this transforms to $s_i = \frac{a\mu + bx_i + bc\theta_D^* + \sqrt{bc}\eta_i}{a+b+bc}$. Using Eq. (10), $s_i < s_D^*$ then implies that

$$\eta_i < \frac{a+b}{\sqrt{bc}}\theta_D^* + \sqrt{\frac{a+b+bc}{bc}}\Phi^{-1}(\kappa_2) - \frac{a}{\sqrt{bc}}\mu - \sqrt{\frac{b}{c}}x_i.$$

As $c \to \infty$, the right-hand side converges pointwise to $\Phi^{-1}(\kappa_2)$ and therefore

$$Pr(s_i < s_D^* | \theta_D^*, x_i) \to \Phi\left(\Phi^{-1}(\kappa_2)\right) = \kappa_2.$$

Thus,

$$Pr(x_i \geq x_D^*, s_i < s_D^* | \theta_D^*) \to Pr(x_i \geq x_D^* | \theta_D^*)\kappa_2 = \Phi\left(\sqrt{b}(\theta_D^* - x_D^*)\right)\kappa_2,$$

implying that Eq. (8) reduces to

$$\theta_D^* = 1 - \Phi\left(\sqrt{b}(\theta_D^* - x_D^*)\right) + \Phi\left(\sqrt{b}(\theta_D^* - x_D^*)\right)\kappa_2.$$

Rearranging terms, we finally get

$$1 - \theta_D^* = (1 - \kappa_2)\,\Phi\left(\sqrt{b}(\theta_D^* - x_D^*)\right).$$

Now, consider the derivation of Eq. (12). By Lebesgue dominated convergence,

$$Pr(\theta \geq \theta_D^*, s_i \geq \theta_D^* | x_D^*) = \int_{\theta_D^*}^{\infty} Pr(s_i \geq s_D^* | \theta, x_D^*) f(\theta | x_D^*) d\theta.$$

Given x_D^* and θ, it is easy to see that $s_i = \frac{a\mu + bx_i + bc\theta + \sqrt{bc}\eta_i}{a+b+bc}$. Thus,

$$s_i \geq s_D^* \Leftrightarrow \frac{a\mu + bx_i + bc\theta + \sqrt{bc}\eta_i}{a+b+bc} \geq \theta_D^* + \frac{1}{\sqrt{a+b+bc}}\Phi^{-1}(\kappa_2),$$

which reduces to

$$\eta_i \geq \sqrt{bc}(\theta_D^* - \theta) + \frac{a+b}{\sqrt{bc}}\theta_D^* + \sqrt{\frac{a+b+bc}{bc}}\Phi^{-1}(\kappa_2) - \frac{a}{\sqrt{bc}}\mu - \frac{b}{\sqrt{bc}}x_i.$$

As $c \to \infty$, the right-hand side of this inequality tends to $-\infty$ if $\theta > \theta_D^*$, and to ∞ if $\theta < \theta_D^*$. Hence,

$$Pr(s_i \geq s_D^* | x_D^*) \to \begin{cases} 1 & \text{if } \theta > \theta_D^* \\ 0 & \text{if } \theta < \theta_D^*. \end{cases}$$

Thus,

$$Pr(\theta \geq \theta_D^*, s_i \geq s_D^* | x_D^*) \rightarrow Pr(\theta \geq \theta_D^* | x_D^*).$$

This implies that Eq. (9) reduces to $\kappa_1 = Pr(\theta < \theta_D^* | x_D^*)\kappa_2 + Pr(\theta \geq \theta_D^* | x_D^*)$, or in other words,

$$1 - \kappa_1 = (1 - \kappa_2)\Phi\left(\sqrt{a+b}\left(\theta_D^* - \frac{a\mu + bx_D^*}{a+b}\right)\right).$$

■

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