On the Optimal Combination of Promotion Tournaments and Individual Performance Pay*

Anja Schöttner[†] and Veikko Thiele[‡]

This version: February 2007

Abstract

We analyze the optimal combination of promotion tournaments and individual performance pay in an employment relationship. Agents' efforts are non-observable and they have private information about their suitability for promotion. We find that the principal does not provide individual incentives if it is sufficiently important to promote the most able candidate. Thus, we give a possible explanation for why individual performance schemes is less often observed than theory predicts. Furthermore, optimally trading off incentive and selection issues causes a form of the Peter Principle to arise: The less able agent has an inefficiently high probability of promotion.

JEL Classification: D82, D86, M52.

Keywords: Promotion Tournaments, Piece Rates, Hidden Characteristics, Hidden Action

*We would like to thank Dominique Demougin, Oliver Fabel, Carsten Helm, Jenny Kragl, and Dorothee Schneider.

[†]Humboldt University at Berlin, School of Business and Economics, Ziegelstr. 13a, D-10099 Berlin, Germany, phone: +49 30 2093-1345, fax: +49 30 2093-1343, e-mail: schoettner@wiwi.hu-berlin.de.

[‡]University of British Columbia, Sauder School of Business, 2053 Main Mall, Vancouver, BC, Canada V6T 1Z2, phone: +1 604 822-8491, fax: +1 604 822-8477, e-mail: veikko.thiele@sauder.ubc.ca

1 Introduction

Within firms, incentive schemes often serve not only as incentive but also as selection devices. This is particularly true for promotion tournaments, which provide effort incentives and help assign employees to the jobs they are best suited for (Baker et al. 1988, Milgrom and Roberts 1992). Individual performance pay may as well lead to incentive and selection effects. For example, Lazear (2000) shows that the introduction of a simple piece rate scheme in a US autoglass company increased output and attracted more able workers. There is a large literature that analyzes either relative incentive schemes (e.g., promotion tournaments) or individual performance pay, or shows when one contractual form dominates the other.[1] Most of this literature focuses on the provision of incentives. By contrast, we examine how relative and individual compensation schemes should be combined when incentive and selection issues arise simultaneously. Our purpose is not to characterize an optimal mechanism, but rather to look at two incentive schemes that are of high practical relevance: piece rates and promotion tournaments.

In particular, we consider a relationship between the owner of a manufacturing firm (principal) and two workers (agents). The principal first employs both agents as production workers but later wishes to promote one of them to a position in middle management. In the production stage, workers perform a manufacturing task. Effort in this task is not observable. However, due to the availability of a contractible performance measure, the principal is able to establish an individual incentive scheme. In the management job, effort is also non-observable. Moreover, since middle managers usually perform difficult-to-measure tasks as supervising subordinates or organizing the workflow in production, performance measures are not available. Therefore, a manager receives a fixed salary.[2]

Agents share the same abilities in production but may differ in their skills for the management task. They learn their management skills after signing the employment contract and entering the firm. The principal never observes these skills but benefits from selecting a high-

[1] See, e.g., Lazear and Rosen (1981), Green and Stokey (1983), Nalebuff and Stiglitz (1983), or Rosen (1986).

[2] This assumption is not crucial for our results but greatly simplifies the analysis. We discuss the extension to incentive contracts at the management stage in section 5.

skilled worker for promotion. She therefore has to design a contract that optimally balances incentive and selection issues. This contract is composed of a fixed wage and an individual performance scheme for for the manufacturing task, and a tournament prize, which is the fixed salary for the management position.

We find that individual performance pay and the promotion tournament are substitutes in the provision of incentives. On which of these instruments the principal puts more emphasis depends on how critical the selection decision is for firm performance. Under our modeling assumptions, the interplay is as follows: The more important it is to promote the high-skilled worker, the higher the management wage and the lower-powered are individual incentives. Moreover, if the selection decision is sufficiently crucial, the optimal contract does not incorporate individual incentives. Thus, even though there is a verifiable, non-distorting performance measure available at the production stage, the principal may decide not to use it in an individual incentive scheme.

The rationale for this result is as follows. Given the fixed payment at the management stage, a manager exerts a minimum required level of effort. We assume that, given any particular effort level, a high-skilled agent makes not only a higher contribution to firm value but also has lower effort cost than a low-skilled agent.[3] Therefore, under any management salary, the more able worker has a higher net benefit from being promoted. As a consequence, whenever agents are heterogeneous, the high-skilled worker exerts more effort than the low-skilled one at the production stage. The performance measure thus serves as a signal about agents' abilities: The agent who performed better in production is more likely to be high-skilled at the management stage and should hence be promoted. However, enhancing individual performance pay dilutes the informativeness of this signal. The reason is that the harder working high-skilled agent responds less strongly to intensified individual incentives than the agent less suited for the management task. As a result, the latter's promotion probability increases. If this effect

[3]Thus, in our model, being of a superior type means to have higher marginal productivity *and* lower marginal effort cost. Usually, only one of these assumptions is made to model different abilities of agents. We require both of them because we only allow for a fixed wage at the management stage. Refer to section 5 for a detailed explanation.

is sufficiently detrimental from the principal's point of view, she refrains from applying an individual performance scheme.

This result provides a possible explanation for the fact that individual incentive schemes are less often observed than theory predicts (Parent 2002). Alternative explanations can be found in the literature. Holmström and Milgrom (1991) show that it may be optimal not to implement an incentive scheme if the agent's task has different dimensions of which some are easier to measure than others. In Bernheim and Whinston (1998), contracting parties might want to leave some verifiable aspects of performance unspecified when there are other important but non-verifiable aspects of performance, since this may allow to punish undesired behavior. Another explanation originally comes from psychology. It says that monetary incentive payments may crowd out intrinsic motivation (Deci 1971). Frey and Oberholzer-Gee (1997), Benabou and Tirole (2003), and Sliwka (2006) give economic explanations for the occurrence of crowding-out.

Furthermore, in our paper, when two homogeneous agents join the firm, selection is not an issue. Hence, since agents do not differ in their abilities for the manufacturing task, efficiency dictates that effort levels should be identical in the two possible homogeneous matches of agents. However, since the principal cannot offer contracts tailored to agents' types, she is obliged to implement inefficient effort levels. In particular, if two high-skilled agents compete for promotion, they are induced to work too hard. By contrast, two low-skilled agents exert too little effort. This is a consequence from the fact that, under any given contract, high-skilled workers are more highly motivated since they benefit more strongly from promotion than those with low skills.

If workers are heterogeneous, the principal gains from increasing the promotion probability of the employee who is better suited for the management position. Taking into account this benefit from differentiating effort levels, the more able agent puts in too little effort and the less able agent too much. Thus, a form of the Peter Principle (Peter and Hull 1969) arises: The promotion probability of the less-skilled agent is inefficiently high. The principal accepts this inefficiency as an outcome of optimally trading off incentive and selection issues.

Fairburn and Malcomson (2001) derive a similar result when agents are risk averse. By contrast, the agents in our model are risk-neutral. Lazear (2004) uses a quite different approach to explain the Peter Principle. In his paper, the observation that employees' performance often declines after receiving promotion is a necessary consequence of a statistical process. Agents who received promotion experienced exceptionally good random influences in the period before they got promoted. However, on average, they will be less lucky in the next period.

Selection in tournaments is also analyzed by Rosen (1986), Meyer (1991), Clark and Riis (2001), Hvide and Kristiansen (2003), and, in the context of sabotage, by Lazear (1989), Chen (2003), and Münster (2006). In contrast to these authors, we focus on the selection effect of a promotion tournament when it may be combined with a piece rate scheme. Furthermore, in our model, agents are heterogeneous in the tournament stage only because they differ in their valuations for the tournament prize. In the aforementioned papers, agents' heterogeneity is due to different abilities in the tournament stage.

The paper is organized as follows. The model is introduced in the following section. In section 3, we derive the effort chosen at the production stage given the tournament prize and individual performance pay. The optimal combination of the tournament prize and the individual incentive scheme is characterized in section 4. Section 5 discusses the impact of some our assumptions on the results. Finally, section 6 concludes.

2 The model

We consider an employment relationship between a principal and two agents. All parties are risk-neutral. The principal owns a firm in which two different kinds of jobs need to be performed: conducting a pure manufacturing task (production stage) and exerting a management task (management stage).

In the economy, there are two different types of agents, denoted A and B. They share the same abilities for the manufacturing task, but differ in their abilities for the management job.[4]

[4] In section 5, we discuss the case in which distinct types also exhibit different abilities at the production

In particular, agents of type A can conduct the management task more efficiently than agents of type B. It is common knowledge that an agent is of type A with probability p and with probability $1-p$ of type B. Prior to the contracting stage, agents' types are unknown. An agent learns his and the coworker's type after accepting the contract offered by the principal and entering the employment relationship. The principal never observes agents' types.

In the management stage, for the reasons discussed in the introduction, there are no feasible incentive contracts. Therefore, a manager receives a fixed payment and exerts only some minimum required effort level. However, since agents of type A have a higher ability for conducting the management task, their expected contribution to firm value under the minimum effort level is higher than that of type B agents. Letting Π_k, $k \in \{A, B\}$, denote type k's contribution to firm value on the management stage, it holds that $\Pi_A > \Pi_B$. Moreover, we also assume that type A has lower cost for implementing the minimum effort level than type B. To reduce the notational burden, we normalize type B's effort cost to zero, while agent A's cost are $-\delta$, where $\delta > 0$. Thus, type A obtains a higher payment net of effort cost from being employed as a manager.

Since $\Pi_A > \Pi_B$, the principal prefers the selection of type A agents for pursuing the management task. However, she has no means to screen agents with respect to their types. The only thing she can do to favor the assignment of type A agents to the management position is to take advantage of the fact that these agents have a higher valuation for being promoted. To do so, the principal designs the following promotion tournament: In the first period, both agents are assigned to the manufacturing task. At the end of this period, the better performing agent is promoted to the management position. Under this promotion rule, performance in the production stage serves as a signal about skills for the management job: Whenever the agents employed are heterogeneous, the type A agent exerts higher effort than the type B agent because the former has a higher valuation of being promoted.[5] Consequently, the better performing agent is more likely to be of type A.

stage.

[5] We prove this statement in section 3.

Note that this promotion tournament causes an inefficiency to arise: Although both types of agents are equally skilled in production, they exert different effort levels in the manufacturing task. However, we assume that the promotion decision is sufficiently crucial for firm performance so that it is always in the principal's interest to improve her information about agents types by implementing a promotion tournament.[6]

In the production stage, agent i, $i = 1, 2$, chooses a non-obervable effort level $e_i \geq 0$ leading to the verifiable output

$$q_i = e_i + \mu_i, \tag{1}$$

where μ_1 and μ_2 are identically and independently distributed random variables with zero mean. An agent's effort cost function $c(e_i)$ satisfies the standard assumptions of being strictly convex and increasing in e_i. Moreover, we also assume that marginal effort costs increase disproportionately, i.e. $c''' > 0$. We discuss the impact of this assumption on the results in sections 3 and 5. To guarantee the existence of a pure-strategy equilibrium in the production stage, we also suppose that $\inf_{e>0} c''(e) > 0$.

For simplicity, we assume that the reservation utility of both types of agents is zero throughout the game. However, the framework could easily be extended to a situation where the types differ in their reservation utilities as long as type A still has a higher valuation for promotion within the firm than type B.

In the production stage, the agents' wage is composed of a piece rate scheme $r \geq 0$ conditioned on individual output q_i and a fixed payment w_1. In the management position, an agent obtains a fixed salary $w_2 \geq 0$.[7] At the beginning of the first period, the principal offers a contract specifying r, w_1, and w_2. As it is common in tournament models, we assume that the principal can commit not to renegotiate the tournament prize w_2 at the end of the first period.[8]

[6] Alternatively, the principal could announce to randomly assign an agent to the management task at the end of the first period, e.g., by flipping a fair coin. This induce efficient effort at the production stage at the expense of not enhancing the selection decision.

[7] Since agents' reservation utilities are zero, restricting w_2 to be nonnegative ensures interim participation.

[8] Otherwise, an opportunism problem arises since the principal could ex post lower the wage such that it just meets an agent's reservation utility. Since agents anticipate such a behavior, a tournament can then not be used as an incentive device.

The timing of this game is as follows. First the principal offers two randomly chosen agents a contract (r, w_1, w_2). After accepting the contract, both agents learn their types and are assigned to the manufacturing task. Afterwards, they choose their effort levels in the production stage. Once their respective outputs are realized, agents are paid and the agent with the higher output is promoted to the management level. The other agent leaves the firm and obtains his reservation utility.[9]

3 Effort in the Production Stage

In this section, we derive agents' effort choices in the production stage for a given contract. There are three possible matches of agents: Two homogeneous matches, where agents are either both of type A or both of type B, and a heterogeneous match with a type A and a type B agent.

Agent i's effort choice affects his wage in the production stage as well as his promotion probability to the management position, which is

$$\text{Prob}[q_i > q_j] = \text{Prob}[e_i - e_j > \mu_j - \mu_i] \equiv G(e_i - e_j), \tag{2}$$

where we define $G(\cdot)$ as the cumulative distribution function of the random variable $\mu_j - \mu_i$. Let $g(\cdot)$ denote the corresponding density. We assume that $g(.)$ is differentiable and single-peaked at zero. Furthermore, since μ_i and μ_j are identically distributed, $g(\cdot)$ is symmetric around zero.

We first consider the case where both agents are of type A. Agent i, $i = 1, 2$, maximizes his expected payments from the next two periods taking the effort of agent j, $i \neq j$, $j = 1, 2$, as

[9] Alternatively, one can assume that the losing agent stays in the firm and competes in the next period with a newly employed agent. However, since this does not provide any additional insights, we decided for the simplifying assumption.

given.[10] Thus, agent i solves the problem

$$\max_{e_i} w_1 + G(e_i - e_j)(w_2 + \delta) + re_i - c(e_i). \tag{3}$$

Analogously, agent j chooses e_j to solve

$$\max_{e_j} w_1 + [1 - G(e_i - e_j)](w_2 + \delta) + re_j - c(e_j). \tag{4}$$

In a pure-strategy Nash-equilibrium, both agents choose the same effort level, denoted by e_{AA}, which is characterized by the first-order condition

$$g(0)(w_2 + \delta) + r = c'(e_{AA}). \tag{5}$$

Similarly, for the case where both agents are of type B, we obtain that equilibrium effort e_{BB} is characterized by

$$g(0)w_2 + r = c'(e_{BB}). \tag{6}$$

The following two conditions are sufficient for (e_{AA}, e_{AA}) and (e_{BB}, e_{BB}) being Nash-equilibria since they ensure concavity of agents' objective functions:

$$g'(e_i - e_j)(w_2 + \delta) - c''(e_i) < 0 \text{ for all } e_i, e_j \geq 0, \tag{7}$$

$$\text{and} \qquad g'(e_i - e_j)w_2 - c''(e_i) < 0 \text{ for all } e_i, e_j \geq 0. \tag{8}$$

We assume that these conditions are satisfied for the highest w_2 that the principal might want to choose.[11] Since $\inf_{e>0} c'' > 0$, this is the case whenever random influences on output are significant enough, i.e., $g(.)$ is sufficiently "flat".

[10] For simplicity, we assume that there is no discounting.

[11] Agents' cost are convexly increasing in effort, while the principal's expected profit is assumed to be concave in effort. Consequently, the principal, who has to compensate agents for their effort cost, does not wish to implement arbitrarily high effort level. Thus, it is possible to derive an upper bound on w_2.

Now we turn to the case of heterogeneous agents. Without loss of generality, assume that agent 1 is of type A and agent 2 is of type B. Then, type A's and type B's respective optimization problems are:

$$\max_{e_1} w_1 + G(e_1 - e_2)(w_2 + \delta) + re_1 - c(e_1) \tag{9}$$

$$\max_{e_2} w_1 + [1 - G(e_1 - e_2)]w_2 + re_2 - c(e_2) \tag{10}$$

In equilibrium, type A's effort level e_A and type B's effort level e_B are characterized by the following two first-order conditions:

$$g(e_A - e_B)(w_2 + \delta) + r = c'(e_A) \tag{11}$$

$$g(e_A - e_B)w_2 + r = c'(e_B) \tag{12}$$

The second-order conditions are identical to (7) and (8) and are thus satisfied.

From (11) and (12) we infer that $\Delta e \equiv e_A - e_B > 0$. Because type A's benefit from getting promoted is higher, he is motivated to work harder than type B for each given incentive scheme. Consequently, the agent of type A has a higher probability of winning the promotion tournament, i.e., $G(\Delta e) > 0.5$.

It can be easily verified that e_A and e_B are increasing in r and w_2.[12] Besides this *incentive effect*, enhancement of either r or w_2 also has a *selection effect*. The latter arises from the fact that modifying r or w_2 affects the effort difference Δe and thus agents' promotion probabilities. As the following proposition shows, providing higher incentives lowers type A's chances for promotion and is therefore detrimental for selection.

Proposition 1 *Type A's probability of winning the promotion tournament is decreasing in r and w_2.*

All proofs are given in the appendix.

[12] Refer to the proof of proposition 1 in the appendix.

When the principal strengthens incentives by raising either r or w_2, both agents are motivated to exert more effort. However, the harder working type A agent responds less strongly to intensified incentives than the agent of type B. From the proof of proposition 1 it becomes clear that this result completely relies on our assumption that marginal costs increase disproportionately, i.e., $c''' > 0$. If, in contrast, $c''' < 0$, type A's winning probability increases in r and w_2. Thus, in our model, assuming $c''' > 0$ is equivalent to the presumption that, in a tournament with heterogeneous agents, the harder working agent's effort choice is less sensitive to enhanced incentives. We consider this as plausible but are not aware of any empirical evidence.

4 The principal's problem

Now we turn to the principal's problem of choosing w_1, w_2, and r to maximize her expected profit. Her optimization problem can be stated as follows:

$$\begin{aligned} \max_{w_1,w_2,r,e_A,e_B,e_{AA},e_{BB}} \; & 2p(1-p)[(1-r)(e_A+e_B)+G(\Delta e)\Pi_A+(1-G(\Delta e))\Pi_B] \\ & +p^2[2(1-r)e_{AA}+\Pi_A]+(1-p)^2[2(1-r)e_{BB}+\Pi_B]-2w_1-w_2 \end{aligned} \tag{13}$$

s.t. (5), (6), (11), (12) and

$$\begin{aligned} & w_1+p(1-p)[G(\Delta e)(w_2+\delta)+re_A-c(e_A)]+(1-p)p[(1-G(\Delta e))w_2+re_B-c(e_B)] \\ & +p^2[0.5(w_2+\delta)+re_{AA}-c(e_{AA})]+(1-p)^2[0.5w_2+re_{BB}-c(e_{BB})]\geq 0 \end{aligned} \tag{14}$$

The principal maximizes her expected profit subject to the incentive compatibility constraints for each possible tournament matching and agents' participation constraint (14).

First observe that, for any fixed r and w_2, the principal chooses w_1 such that (14) is binding. Consequently, we can eliminate w_1 from the optimization problem and simplify the principal's problem to

$$\max_{r,w_2,e_A,e_B,e_{AA},e_{BB}} \Pi := \pi_{AB} + \pi_{AA} + \pi_{BB} \qquad \text{s.t. } (5),(6),(11),(12) \tag{15}$$

were π_{kl} denotes the expected profit of a tournament match where one agent is of type k and the other agent of type l, weighted by this tournament's probability of occurrence, i.e.,

$$\pi_{AB} := 2p(1-p)\left[e_A + e_B + \Pi_B + G(\Delta e)(\Pi_A - \Pi_B + \delta) - c(e_A) - c(e_B)\right], \tag{16}$$

$$\pi_{AA} := p^2\left[2e_{AA} + \Pi_A + \delta - 2c(e_{AA})\right], \tag{17}$$

$$\pi_{BB} := (1-p)^2\left[2e_{BB} + \Pi_B - 2c(e_{BB})\right]. \tag{18}$$

From a closer inspection of (16)-(18) it becomes clear that, under each tournament match, the principal receives all the surplus from the relationship. Since agents have no private information before contracting with the principal, they are just compensated for their effort costs and, consequently, earn no rents.

Given a particular tournament match, the corresponding surplus maximizing effort levels e_A^*, e_B^*, e_{AA}^*, and e_{BB}^* are characterized by:

$$c'(e_{AA}^*) = c'(e_{BB}^*) = 1 \tag{19}$$

$$c'(e_A^*) = 1 + g(\Delta e)(\Pi_A - \Pi_B + \delta) \tag{20}$$

$$c'(e_B^*) = 1 - g(\Delta e)(\Pi_A - \Pi_B + \delta) \tag{21}$$

By (19), agents in AA-matches should exert the same effort as agents in BB-matches. The reason is that both agents share the same abilities for conducting the manufacturing task and there is no benefit from promoting a particular agent to the management position. In contrast, in the AB-match, it is beneficial from the principal's perspective that a type A agent works harder than an agent of type B, i.e., $e_A^* > e_B^*$. Implementing such an effort difference increases the probability of promoting the more able agent.

The principal, however, cannot observe the agents' respective types and is thus not able to tailor incentives for the different tournament matches. As a result, she cannot induce the surplus maximizing effort levels. In particular, from the incentive compatibility constraints (5) and (6), we can infer that the effort levels in an AA-match always exceeds the effort in a BB-match. Furthermore, by (11) and (12),

$$c'(e_A) - c'(e_B) = g(\Delta e)\delta. \tag{22}$$

Together with (20) and (21), it follows that

$$c'(e_A) - c'(e_B) < c'(e_A^*) - c'(e_B^*). \tag{23}$$

Therefore, the principal is not able to implement the efficient effort levels e_A^* and e_B^* for a AB-match either. The following proposition characterizes agents effort levels under the optimal contract.

Proposition 2 *Suppose the optimal contract comprises $r^*, w_2^* > 0$. Then, agents in an AA-match exert too much and agents in a BB-match exert too little effort, i.e., $e_{AA} > e_{AA}^* = e_{BB}^* > e_{BB}$. By contrast, in an AB-match, type A works too little while type B works too hard, i.e., $e_A < e_A^*$ and $e_B^* < e_B$. Thus, agent B's promotion probability is inefficiently high.*

Regarding the heterogeneous tournament, note that condition (23) does not necessarily rule out the possibility of implementing the efficient effort difference $e_A^* - e_B^*$. However, to do so, both agents' effort levels would have to be inefficiently low. Proposition 2 shows that trading off incentive and selection issues in the heterogeneous tournament demands for implementing an inefficiently low effort difference, making type A work too little and type B work too hard. Thus, a form of the Peter Principle arises: The type B agent is promoted "too often". The principal deliberately accepts this inefficiency as a necessary consequence of compromising on selection and incentive effects.

Since r and w_2 are substitutes with respect to incentive provision, there is an infinite number of combinations of r and w_2 that implement a given pair of effort levels in the AB-match at the same cost for the principal. The principal picks the one that maximizes her expected profit from the homogeneous tournaments. The marginal benefit from lowering incentives and thus decreasing e_{AA} must equal the marginal benefit from increasing incentives and thereby increasing e_{BB}. Clearly, this cannot be the case if agents work either too hard or too little in both matches. Consequently, agents exert too much effort in AA-matches and too less effort in BB-matches.[13]

The next proposition establishes how the optimal contract changes when selection issues become more important.

Proposition 3 *Let (r^*, w_2^*) denote the contract elements that solve (15).*

(i) If the optimal contract specifies $r^* > 0$, $w_2^* > 0$, we have

$$\frac{\partial r^*}{\partial(\Pi_A - \Pi_B)} = -g(0)\frac{\partial w_2^*}{\partial(\Pi_A - \Pi_B)} \tag{24}$$

and $\frac{\partial r^*}{\partial(\Pi_A - \Pi_B)} < 0$. As a consequence, e_A and e_B decrease in $\Pi_A - \Pi_B$ while $\Delta e = e_A - e_B$ increases. The effort levels e_{AA} and e_{BB}, however, remain constant.

(ii) If $\Pi_A - \Pi_B$ is sufficiently large, the principal sets $r^* = 0$.

If it becomes more important to the principal to promote the more able agent A, she decreases the piece rate r and increases the tournament prize w_2. The overall effect on effort in the tournament with heterogeneous agents is such that A's winning probability increases. Furthermore, by (24), r and w_2 are adapted such that incentives do not change when the competing agents are homogeneous.

The intuition for this result is as follows. Proposition 1 showed that both decreasing r and w_2 improves selection. The reason why the principal lowers the piece rate while increasing

[13] Formally, from equations (72) and (73) in the appendix, the principal chooses r and w_2 such that $\frac{\partial \pi_{AB}}{\partial r} = \frac{\partial \pi_{AB}}{\partial w_2} = 0$ and $\frac{\partial \pi_{AA}}{\partial r} + \frac{\partial \pi_{BB}}{\partial r} = \frac{\partial \pi_{AA}}{\partial w_2} + \frac{\partial \pi_{BB}}{\partial w_2} = 0$.

promotion incentives stems from the optimal incentive structure for the tournaments with homogeneous agents. Clearly, since selection is irrelevant in these tournaments, effort should not change, which leads to condition (24). Given this condition and the incentive compatibility constraints (11) and (12), the change in r determines the overall effect on e_A and e_B, respectively. This is due to the fact that marginal changes in w_2 have a weaker effect on the effort choices of heterogeneous agents compared to those of homogeneous agents ($g(\Delta e) < g(0)$). Thus, the reduction in e_A and e_B aimed at improving the selection effect can only be accomplished by lowering the piece rate.

5 Discussion

In this section, we discuss the impact of some of our assumptions on the results derived in proposition 1 and 3.[14]

So far, we analyzed a situation where both types of agents have identical skills for production. Now suppose instead that type A has effort costs $\alpha c(e_i)$, $\alpha > 0$, at the production stage, whereas type B's effort costs are still $c(e_j)$. If $\alpha < 1$, type A is not only the better manager, but also has a comparative advantage at the production stage. By contrast, if $\alpha > 1$, agents of type B perform the manufacturing task more efficiently than type A agents. We assume that this advantage is not too strong so that, in equilibrium, agent A still chooses a higher level of effort.[15] Then, in a heterogeneous match, agent A's winning probability is decreasing in w_2 and r if and only if[16]

$$c''(e_B) - \alpha c''(e_A) < 0. \tag{25}$$

Assuming that $c''' > 0$, this condition is always satisfied if $\alpha > 1$. Since type A's marginal costs of effort are then higher than those of type B, enforcing incentives leads to an even stronger deterioration of the selection effect than under identical abilities in production. Thus, the

[14] For all extensions considered in this section, the findings in proposition 2 remain valid. The only exception is that, if agents differ in their abilities in the production stage, $e^*_{AA} \neq e^*_{BB}$.

[15] Otherwise, the principal would always want to promote the agent with the lower effort level.

[16] Compare formula (30) in the appendix.

results of proposition 1 and 3 remain valid.

By contrast, if type A's marginal effort costs are lower than those of B, condition (25) may be violated. Intensifying incentives then improves the selection effect so that the result in proposition 1 is reversed. As a consequence, r^* is then increasing and w_2^* decreasing in $\Pi_A - \Pi_B$. The same is true when we return to the case of identical agents in production ($\alpha = 1$), but assume that marginal effort costs are no longer disproportionately increasing, i.e., $c''' < 0$.

Finally, in our model, being of a superior type with respect to the management task means to have a higher productivity *and* lower effort cost. Usually, only one of these assumptions is made to differentiate between types of agents. However, to ensure that the principal wishes to promote a type A agent *and* that a type A agent has a higher valuation for promotion than a type B agent, we require both assumptions. The reason is that we only allow for a fixed wage at the management stage. In a richer framework where incentive contracts are feasible at the management stage, it would be sufficient that type A is either more productive or more cost efficient. Then, at the end of the first period, the worker receiving promotion chooses from a menu of incentive contracts as in a standard adverse selection model. Regardless of being more productive or more cost efficient, a high-skilled type earns a higher rent than a low-skilled type under their, respectively, preferred contracts. Thus, type A still benefits more strongly from being employed as a manager. Also, the principal still profits from implementing a promotion tournament and thereby increasing the chance of selecting a type A agent for promotion. Even though this type receives a higher rent, he makes a higher contribution to firm value.

6 Conclusion

The previous literature thoroughly analyzed the provision of individual incentive schemes and the use of promotion tournaments. As a next logical step, this paper examines the optimal combination of these incentive devices aimed at motivating high effort (*incentive effect*) and matching agents with the jobs their are most suitable for (*selection effect*).

One important observation is that both instruments are substitutes in the provision of incentives. In particular, the principal puts more emphasis on the tournament scheme relative to individual performance payments when selecting the more able candidate for promotion becomes more important. If selection is sufficiently crucial, the principal even refrains from implementing individual performance pay. The reason is that individual rewards dilute the selection effect of the promotion tournament. Thus, we provide a possible explanation for the observation that individual incentive schemes are less often applied in practice than theory predicts.

Furthermore, we find that, in a heterogeneous tournament match, the agent less suited for promotion exerts too much effort while the more able agent does not work hard enough. As a consequence, the promotion probability of the less able agent is inefficiently high, causing a form of the Peter Principle to arise. This is a consequence from the fact that the optimal contract has to trade off incentive and selection issues.

7 Appendix

Proof of proposition 1. From (11) and (12) we obtain

$$H \begin{pmatrix} \frac{\partial e_A}{\partial r} \\ \frac{\partial e_B}{\partial r} \end{pmatrix} = \begin{pmatrix} -1 \\ -1 \end{pmatrix}, \tag{26}$$

where

$$H := \begin{pmatrix} g'(\Delta e)(W_2 + \delta) - c''(e_A) & -g'(\Delta e)(W_2 + \delta) \\ g'(\Delta e) W_2 & -g'(\Delta e) W_2 - c''(e_B) \end{pmatrix}. \tag{27}$$

Since $\Delta e > 0$, we have $g'(\Delta e) < 0$. With (7) and (8) it follows that $\det(H) > 0$. Applying Cramer's Rule to (26) yields

$$\frac{\partial e_A}{\partial r} = \frac{-g'(\Delta e)\delta + c''(e_B)}{\det(H)} > 0 \tag{28}$$

$$\frac{\partial e_B}{\partial r} = \frac{-g'(\Delta e)\delta + c''(e_A)}{\det(H)} > 0. \tag{29}$$

It follows that

$$\frac{\partial \Delta e}{\partial r} = \frac{c''(e_B) - c''(e_A)}{\det(H)}. \tag{30}$$

By $c''' > 0$, $c''(e_B) < c''(e_A)$ and hence Δe is decreasing in r. Furthermore,

$$H \begin{pmatrix} \frac{\partial e_A}{\partial w_2} \\ \frac{\partial e_B}{\partial w_2} \end{pmatrix} = \begin{pmatrix} -g(\Delta e) \\ -g(\Delta e) \end{pmatrix}, \tag{31}$$

and we therefore obtain:

$$\frac{\partial e_A}{\partial w_2} = g(\Delta e)\frac{\partial e_A}{\partial r} \tag{32}$$

$$\frac{\partial e_B}{\partial w_2} = g(\Delta e)\frac{\partial e_B}{\partial r} \tag{33}$$

Consequently, $\frac{\partial \Delta e}{\partial w_2} < 0$. □

Proof of proposition 2. Denoting the Lagrangian of problem (15) by $\mathcal{L}$ and the Lagrange multipliers for the constraints (5), (6), (11), (12) by $\lambda_1, \ldots, \lambda_4$, respectively, we obtain the following first-order conditions:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial r} &= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 0 && (34)\\
\frac{\partial \mathcal{L}}{\partial w_2} &= [\lambda_1 + \lambda_2]\, g(\Delta e) + [\lambda_3 + \lambda_4]\, g(0) = 0 && (35)\\
\frac{\partial \mathcal{L}}{\partial e_A} &= 2p(1-p)\,[1 + g(\Delta e)(\Pi_A - \Pi_B + \delta) - c'(e_A)] \\
&\qquad + \lambda_1\,[g'(\Delta e)(w_2 + \delta) - c''(e_A)] + \lambda_2 g(\Delta e) w_2 = 0 && (36)\\
\frac{\partial \mathcal{L}}{\partial e_B} &= 2p(1-p)\,[1 - g(\Delta e)(\Pi_A - \Pi_B + \delta) - c'(e_B)] \\
&\qquad - \lambda_1 g'(\Delta e)(w_2 + \delta) - \lambda_2\,[g'(\Delta e) w_2 + c''(e_B)] = 0 && (37)\\
\frac{\partial \mathcal{L}}{\partial e_{AA}} &= 2p^2\,[1 - c'(e_{AA})] - \lambda_3 c''(e_{AA}) = 0 && (38)\\
\frac{\partial \mathcal{L}}{\partial e_{BB}} &= 2(1-p)^2\,[1 - c'(e_{BB})] - \lambda_4 c''(e_{BB}) = 0. && (39)
\end{aligned}$$

From (34) and (35) it follows that, since $g(\Delta e) < g(0)$, we must have $\lambda_1 + \lambda_2 = \lambda_3 + \lambda_4 = 0$. If $\lambda_3 = \lambda_4 = 0$, it follows from (38) and (39) that $c'(e_{AA}) = c'(e_{BB})$, which is a contradiction to (5) and (6). Thus, $\lambda_3 = -\lambda_4 \neq 0$. Consequently, since $c'' > 0$, one of the first terms of the derivatives in (38) and (39) must be negative while the other is positive. From (5) and (6) we know that $c'(e_{AA}) > c'(e_{BB})$. Therefore $1 - c'(e_{AA}) < 0$ and $1 - c'(e_{BB}) > 0$, implying that $e_{AA} > e^*_{AA}$ and $e_{BB} < e^*_{BB}$.

If $\lambda_1 = \lambda_2 = 0$, it follows from (36) and (37) in combination with (5) and (6) that

$$\begin{aligned}
1 + g(\Delta e)(\Pi_A - \Pi_B + \delta) &= g(\Delta e)(w_2 + \delta) + r && (40)\\
1 - g(\Delta e)(\Pi_A - \Pi_B + \delta) &= g(\Delta e) w_2 + r. && (41)
\end{aligned}$$

Subtracting the second from the first equation yields $2(\Pi_A - \Pi_B) + \delta = 0$, which is a contradiction to $\Pi_A > \Pi_B$ and $\delta > 0$. Thus, $\lambda_1 = -\lambda_2 \neq 0$. By using this relationship, (36) and (37) can be transformed to

$$2p(1-p)\left[1+g(\Delta e)(\Pi_A-\Pi_B+\delta)-c'(e_A)\right]+\lambda_1\left[g'(\Delta e)\delta-c''(e_A)\right]=0 \tag{42}$$

$$2p(1-p)\left[1-g(\Delta e)(\Pi_A-\Pi_B+\delta)-c'(e_B)\right]-\lambda_1[g'(\Delta e)\delta-c''(e_B)]=0. \tag{43}$$

Since one of the second terms on the left-hand side of these two equations is negative while the other is positive, so must be the first terms. Transforming these terms by applying (11) and (12) yields

$$1+g(\Delta e)(\Pi_A-\Pi_B+\delta)-c'(e_A) \;=\; 1+g(\Delta e)(\Pi_A-\Pi_B-w_2)-r \qquad \equiv \phi_1, \tag{44}$$

$$1-g(\Delta e)(\Pi_A-\Pi_B+\delta)-c'(e_B) \;=\; 1-g(\Delta e)(\Pi_A-\Pi_B+w_2+\delta)-r \equiv \phi_2. \tag{45}$$

Since $\phi_1 > \phi_2$ it follows that

$$1+g(\Delta e)(\Pi_A-\Pi_B+\delta)-c'(e_A) \;>\; 0, \tag{46}$$

$$1-g(\Delta e)(\Pi_A-\Pi_B+\delta)-c'(e_B) \;<\; 0. \tag{47}$$

As a result, $e_A < e_A^*$ and $e_B > e_B^*$. □

Proof of proposition 3 (i) The principal's problem (15) can be further simplified to

$$\max_{r,w_2}\;\left[\Pi(r,w_2)\equiv \pi_{AB}(r,w_2)+\pi_{AA}(r,w_2)+\pi_{BB}(r,w_2)\right] \tag{48}$$

where $\pi_{AB}(r,w_2)$, $\pi_{AA}(r,w_2)$, and $\pi_{BB}(r,w_2)$ are defined as in (16)- (18) with the only difference that e_A, e_B, e_{AA}, e_{BB} are now considered as functions of r and w_2 implicitly given by $(5), (6), (11)$. We assume that the functional forms are such that $\Pi(r,w_2)$ is concave for all $p \in (0,1)$. Then, the first-order conditions that characterize the optimal contract, provided that $r^*, w_2^* > 0$ are

$$\frac{\partial \Pi}{\partial r} = \frac{\partial \pi_{AB}}{\partial r} + \frac{\partial \pi_{AA}}{\partial r} + \frac{\partial \pi_{BB}}{\partial r} = 0, \tag{49}$$

$$\frac{\partial \Pi}{\partial w_2} = \frac{\partial \pi_{AB}}{\partial w_2} + \frac{\partial \pi_{AA}}{\partial w_2} + \frac{\partial \pi_{BB}}{\partial w_2} = 0, \tag{50}$$

where, for $y \in \{r, w_2\}$

$$\begin{aligned}\frac{\partial \pi_{AB}}{\partial y} &= 2p(1-p)\Bigg[(1-c'(e_A))\frac{\partial e_A}{\partial y} + (1-c'(e_B))\frac{\partial e_B}{\partial y} \\ &\qquad + g(\Delta e)\frac{\partial(\Delta e)}{\partial y}(\Pi_A - \Pi_B + \delta)\Bigg]\end{aligned} \tag{51}$$

$$\frac{\partial \pi_{AA}}{\partial y} = 2p^2[1-c'(e_{AA})]\frac{\partial e_{AA}}{\partial y} \tag{52}$$

$$\frac{\partial \pi_{BB}}{\partial y} = 2(1-p)^2[1-c'(e_{BB})]\frac{\partial e_{BB}}{\partial y} \tag{53}$$

From (49) and (50) we obtain that

$$K \begin{pmatrix} \frac{\partial r}{\partial(\Pi_A - \Pi_B)} \\ \frac{\partial w_2}{\partial(\Pi_A - \Pi_B)} \end{pmatrix} = \begin{pmatrix} -2p(1-p)g(\Delta e)\frac{\partial(\Delta e)}{\partial r} \\ -2p(1-p)g(\Delta e)\frac{\partial(\Delta e)}{\partial w_2} \end{pmatrix}, \tag{54}$$

where

$$K := \begin{pmatrix} \frac{\partial^2 \Pi}{\partial r^2} & \frac{\partial^2 \Pi}{\partial r \partial w_2} \\ \frac{\partial^2 \Pi}{\partial r \partial w_2} & \frac{\partial^2 \Pi}{\partial w_2^2} \end{pmatrix}. \tag{55}$$

Since we assume that Π is concave, K must be negative definite, and thus $\det(K) > 0$. Using that, by (32) and (33), $\frac{\partial(\Delta e)}{\partial w_2} = g(\Delta e)\frac{\partial(\Delta e)}{\partial r}$ and applying Cramer's Rule to (54) yields:

$$\frac{\partial r^*}{\partial(\Pi_A - \Pi_B)}\det(K) = 2p(1-p)g(\Delta e)\frac{\partial(\Delta e)}{\partial r}\left[g(\Delta e)\frac{\partial^2 \Pi}{\partial r \partial w_2} - \frac{\partial^2 \Pi}{\partial w_2^2}\right] \tag{56}$$

$$\frac{\partial w_2^*}{\partial(\Pi_A - \Pi_B)}\det(K) = 2p(1-p)g(\Delta e)\frac{\partial(\Delta e)}{\partial r}\left[\frac{\partial^2 \Pi}{\partial r \partial w_2} - g(\Delta e)\frac{\partial^2 \Pi}{\partial r^2}\right] \tag{57}$$

As a next step, we transform the expression in square brackets on the right-hand side of

(56). To do so, we can use the following relationships, where $i = A, B$:

$$\frac{\partial e_i}{\partial w_2} = g(\Delta e)\frac{\partial e_i}{\partial r} \tag{58}$$

$$\frac{\partial e_{ii}}{\partial w_2} = g(0)\frac{\partial e_{ii}}{\partial r} \tag{59}$$

$$\frac{\partial^2 e_{ii}}{\partial r \partial w_2} = g(0)\frac{\partial^2 e_{ii}}{\partial r^2} = \frac{1}{g(0)}\frac{\partial^2 e_{ii}}{\partial w_2^2} \tag{60}$$

Equation (58) is identical to (32) and (33), respectively. Equation (59) is easily verified by applying the implicit function theorem to (5) and (6), respectively. The last equation can be proved as follows: From (5) and (6) we obtain $\frac{\partial e_{ii}}{\partial r} = \frac{1}{c''(e_{ii})}$ and thus $\frac{\partial^2 e_{ii}}{\partial r^2} = -\frac{c'''(e_{ii})}{[c''(e_{ii})]^2}\frac{\partial e_{ii}}{\partial r}$. Therefore,

$$\frac{\partial^2 e_{ii}}{\partial r \partial w_2} = -\frac{c'''(e_{ii})}{[c''(e_{ii})]^2}\frac{\partial e_{ii}}{\partial w_2} = -\frac{c'''(e_{ii})}{[c''(e_{ii})]^2}g(0)\frac{\partial e_{ii}}{\partial r} = g(0)\frac{\partial^2 e_{ii}}{\partial r^2}. \tag{61}$$

Analogously, $\frac{\partial e_{ii}}{\partial w_2} = \frac{g(0)}{c''(e_{ii})}$ and thus $\frac{\partial^2 e_{ii}}{\partial w_2^2} = -g(0)\frac{c'''(e_{ii})}{[c''(e_{ii})]^2}\frac{\partial e_{ii}}{\partial w_2}$. Consequently,

$$\frac{\partial^2 e_{ii}}{\partial r \partial w_2} = -\frac{c'''(e_{ii})}{[c''(e_{ii})]^2}\frac{\partial e_{ii}}{\partial w_2} = \frac{1}{g(0)}\frac{\partial^2 e_{ii}}{\partial w_2^2}. \tag{62}$$

We are now able to make the following transformation:

$$g(\Delta e)\frac{\partial^2 \Pi}{\partial r \partial w_2} - \frac{\partial^2 \Pi}{\partial w_2^2} \tag{63}$$

$$= g(\Delta e)\left(\frac{\partial^2 \pi_{AB}}{\partial r \partial w_2} + \frac{\partial^2 \pi_{AA}}{\partial r \partial w_2} + \frac{\partial^2 \pi_{BB}}{\partial r \partial w_2}\right) - \left(\frac{\partial^2 \pi_{AB}}{\partial w_2^2} + \frac{\partial^2 \pi_{AA}}{\partial w_2^2} + \frac{\partial^2 \pi_{BB}}{\partial w_2^2}\right) \tag{64}$$

$$= g(\Delta e)\frac{\partial^2 \pi_{AB}}{\partial r \partial w_2} - \frac{\partial^2 \pi_{AB}}{\partial w_2^2} + \left(\frac{g(\Delta e)}{g(0)} - 1\right)\left(\frac{\partial^2 \pi_{AA}}{\partial w_2^2} + \frac{\partial^2 \pi_{BB}}{\partial w_2^2}\right) \tag{65}$$

$$= g(\Delta e)\frac{\partial^2 \pi_{AB}}{\partial r \partial w_2} - \frac{\partial^2 \pi_{AB}}{\partial w_2^2} - g(0)\left(g(0) - g(\Delta e)\right)\left(\frac{\partial^2 \pi_{AA}}{\partial r^2} + \frac{\partial^2 \pi_{BB}}{\partial r^2}\right) \tag{66}$$

For the second and third equation, we used that

$$\frac{\partial^2 \pi_{AA}}{\partial r \partial w_2} + \frac{\partial^2 \pi_{BB}}{\partial r \partial w_2} = \frac{1}{g(0)}\left[\frac{\partial^2 \pi_{AA}}{\partial w_2^2} + \frac{\partial^2 \pi_{BB}}{\partial w_2^2}\right] = g(0)\left[\frac{\partial^2 \pi_{AA}}{\partial r^2} + \frac{\partial^2 \pi_{BB}}{\partial r^2}\right], \tag{67}$$

which is easily verified by applying (59) and (60). Furthermore, using (58), it can be shown that:

$$g(\Delta e)\frac{\partial^2 \pi_{AB}}{\partial r \partial w_2} - \frac{\partial^2 \pi_{AB}}{\partial w_2^2} = 2p(1-p)\left\{[1 - c'(e_A) + g(\Delta e)(\Pi_A - \Pi_B + \delta)]\left[g(\Delta e)\frac{\partial^2 e_A}{\partial r \partial w_2} - \frac{\partial^2 e_A}{\partial w_2^2}\right]\right.$$
$$\left. + [1 - c'(e_B) - g(\Delta e)(\Pi_A - \Pi_B + \delta)]\left[g(\Delta e)\frac{\partial^2 e_B}{\partial r \partial w_2} - \frac{\partial^2 e_B}{\partial w_2^2}\right]\right\} \quad (68)$$

Since, by (32) and (33), $\frac{\partial e_i}{\partial w_2} = g(\Delta e)\frac{\partial e_i}{\partial r}$, $i = A, B$, it holds that

$$\frac{\partial^2 e_i}{\partial w_2^2} = g'(\Delta e)\frac{\partial(\Delta e)}{\partial w_2}\frac{\partial e_i}{\partial r} + g(\Delta e)\frac{\partial^2 e_i}{\partial r \partial w_2}. \quad (69)$$

It follows that (68) can be rewritten as:

$$g(\Delta e)\frac{\partial^2 \pi_{AB}}{\partial r \partial w_2} - \frac{\partial^2 \pi_{AB}}{\partial w_2^2} = 2p(1-p)g'(\Delta e)\frac{\partial(\Delta e)}{\partial w_2}\left\{[1 - c'(e_A) + g(\Delta e)(\Pi_A - \Pi_B + \delta)]\frac{\partial e_A}{\partial r}\right.$$
$$\left. + [1 - c'(e_B) - g(\Delta e)(\Pi_A - \Pi_B + \delta)]\frac{\partial e_B}{\partial r}\right\} \quad (70)$$
$$= 2p(1-p)g'(\Delta e)\frac{\partial(\Delta e)}{\partial w_2}\frac{\partial \pi_{AB}}{\partial r} \quad (71)$$

Using (58) and (59), the first-order conditions (49) and (50) can be rewritten as:

$$\frac{\partial \Pi}{\partial r} = \frac{\partial \pi_{AB}}{\partial r} + \frac{\partial \pi_{AA}}{\partial r} + \frac{\partial \pi_{BB}}{\partial r} = 0 \quad (72)$$
$$\frac{\partial \Pi}{\partial w_2} = g(\Delta e)\frac{\partial \pi_{AB}}{\partial r} + g(0)\left(\frac{\partial \pi_{AA}}{\partial r} + \frac{\partial \pi_{BB}}{\partial r}\right) = 0 \quad (73)$$

As a result, since $g(\Delta e) < g(0)$, we must have $\frac{\partial \pi_{AB}}{\partial r} = 0$ and $\frac{\partial \pi_{AA}}{\partial r} + \frac{\partial \pi_{BB}}{\partial r} = 0$ at any interior solution. Together with (71) it follows that

$$g(\Delta e)\frac{\partial^2 \pi_{AB}}{\partial r \partial w_2} - \frac{\partial^2 \pi_{AB}}{\partial w_2^2} = 0. \quad (74)$$

Therefore, by (66), we finally obtain

$$g(\Delta e)\frac{\partial^2\Pi}{\partial r\partial w_2} - \frac{\partial^2\Pi}{\partial w_2^2} = -g(0)\left(g(0) - g(\Delta e)\right)\left(\frac{\partial^2\pi_{AA}}{\partial r^2} + \frac{\partial^2\pi_{BB}}{\partial r^2}\right) \tag{75}$$

Very similarly, we can compute

$$\begin{aligned}\frac{\partial^2\Pi}{\partial r\partial w_2} - g(\Delta e)\frac{\partial^2\Pi}{\partial r^2} &= \frac{\partial^2\pi_{AB}}{\partial r\partial w_2} - g(\Delta e)\frac{\partial^2\pi_{AB}}{\partial r^2} + \left(g(0) - g(\Delta e)\right)\left\{\frac{\partial^2\pi_{AA}}{\partial r^2} + \frac{\partial^2\pi_{BB}}{\partial r^2}\right\} &(76)\\ &= \left(g(0) - g(\Delta e)\right)\left\{\frac{\partial^2\pi_{AA}}{\partial r^2} + \frac{\partial^2\pi_{BB}}{\partial r^2}\right\}, &(77)\end{aligned}$$

where the second equation follows since one can show that $\frac{\partial^2\pi_{AB}}{\partial r\partial w_2} - g(\Delta e)\frac{\partial^2\pi_{AB}}{\partial r^2} = 0$. Combining (56), (57), (75), and (77) yields (24). Furthermore, since π_{AA} and π_{BB} equal Π for $p = 1$ and $p = 0$, respectively, concavity of Π for all $p \in (0, 1)$ implies concavity of π_{AA} and π_{BB}. Thus, $\frac{\partial^2\pi_{ii}}{\partial r^2} < 0$. Together with $\frac{\partial\Delta e}{\partial r} < 0$, we obtain $\frac{\partial r^*}{\partial(\Pi_A - \Pi_B)} < 0$.

Furthermore, $\frac{\partial e_A}{\partial(\Pi_A - \Pi_B)}, \frac{\partial e_B}{\partial(\Pi_A - \Pi_B)} < 0$, $\frac{\partial(e_A - e_B)}{\partial(\Pi_A - \Pi_B)} > 0$, and $\frac{\partial e_{AA}}{\partial(\Pi_A - \Pi_B)} = \frac{\partial e_{BB}}{\partial(\Pi_A - \Pi_B)} = 0$ are easily verified by using (24) in a comparative statics analysis of (11), (12); and (5), (6), respectively.

(ii) From (49) and (51)-(53), since $\frac{\partial\Delta e}{\partial r} < 0$, one can find a pair $\hat{\Pi}_A, \hat{\Pi}_B$ such that $\max_{w_2} \frac{\partial\Pi}{\partial r}\Big|_{r=0} < 0$ for all $\Pi_A - \Pi_B > \hat{\Pi}_A - \hat{\Pi}_B$. Since Π is concave, $\frac{\partial\Pi}{\partial r}$ is decreasing in r for all w_2. Thus, $\frac{\partial\Pi}{\partial r} < 0$ for all $r > 0$ and $\Pi_A - \Pi_B > \hat{\Pi}_A - \hat{\Pi}_B$. Consequently, $r^* = 0$ for all $\Pi_A - \Pi_B > \hat{\Pi}_A - \hat{\Pi}_B$. □

References

Baker, G. P., M. C. Jensen, and K. J. Murphy (1988). Compensation and incentives: Practice vs. theory. *Journal of Finance 43*, 593–616.

Benabou, R. and J. Tirole (2003). Intrinsic and extrensic motivation. *Review of Economic Studies 70*, 489–520.

Bernheim, D. and M. D. Whinston (1998). Incomplete contracts and strategic ambiguity. *American Economic Review 88*, 902–932.

Chen, K.-P. (2003). Sabotage in promotion tournaments. *Journal of Law, Economics, and Organization 19*, 119–140.

Clark, D. J. and C. Riis (2001). Rank-order tournaments and selection. *Journal of Economics 73*, 167–191.

Deci, E. (1971). Effects of externally mediated rewards on intrinsic motivation. *Journal of Personality and Social Psychology 18*, 105–115.

Fairburn, J. A. and J. M. Malcomson (2001). Performance, promotion, and the peter principle. *Review of Economic Studies 68*, 45–66.

Frey, B. S. and F. Oberholzer-Gee (1997). The cost of price incentives: An empirical analysis of motivation crowding-out. *American Economic Review 87*, 746–755.

Green, J. R. and N. L. Stokey (1983). A comparison of tournaments and contracts. *Journal of Political Economy 91*, 349–364.

Holmström, B. and P. Milgrom (1991). Multitask principal-agent analyses: Incentive contracts, asset ownership, and job design. *Journal of Law, Economics, and Organization 7, Special Issue*, 24–52.

Hvide, H. K. and E. G. Kristiansen (2003). Risk taking in selection contests. *Games and Economic Behavior 42*, 172–179.

Lazear, E. P. (1989). Pay equality and industrial politics. *Journal of Political Economy 97*, 561–580.

Lazear, E. P. (2000). Performance pay and productivity. *American Economic Review 90*(5), 1346–1361.

Lazear, E. P. (2004). The peter principle: A theory of decline. *Journal of Political Economy 112*, S141–S163.

Lazear, E. P. and S. Rosen (1981). Rank-order tournaments as optimum labor contracts. *Journal of Political Economy 89*, 841–864.

Meyer, M. A. (1991). Learning from coarse information: Biased contests and career profiles. *Review of Economic Studies 58*, 15–41.

Milgrom, P. and J. Roberts (1992). *Economics, organizations, and management*. New Jersey: Prentice Hall.

Münster, J. (2006). Selection tournaments, sabotage, and participation. Discussion Paper SP II 2006-08, Wissenschaftszentrum Berlin.

Nalebuff, B. and J. Stiglitz (1983). Prizes and incentives: Towards a general theory of compensation and competition. *Bell Journal of Economics 14*, 21–43.

Parent, D. (2002). Incentive pay in the united states: Its determinants and its effects. In *M. Brown and J. Heywood (Ed.) Paying for Performance: An International Comparison*, pp. 17–51. M.E. Sharpe.

Peter, L. J. and R. Hull (1969). *The peter principle: why things always go wrong*. New York: Morrow.

Rosen, S. (1986). Prizes and incentives in elimination tournaments. *American Economic Review 76*, 701–715.

Sliwka, D. (2006). Trust as a signal of a social norm and the hidden costs of incentive schemes. IZA Discussion Paper No. 2293.