# Satiation and Inequality*

Stefan Napel
University of Bayreuth
Department of Economics
Universitätsstraße 30
D-95440 Bayreuth
stefan.napel@uni-bayreuth.de

Wendelin Schnedler
University of Heidelberg
Department of Economics
Grabengasse 14
D-69117 Heidelberg
wendelin.schnedler@awi.uni-heidelberg.de

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**Abstract**

This paper studies trade of an indivisible good on a quasi-competitive market in which participants negotiate prices in random pairs. Costs and willingness to pay are identical for all sellers and all buyers, but individuals differ regarding their evaluation of the first units of any appropriated surplus. A *less satiated* agent places greater weight on these initial units (which generalizes greater risk aversion, impatience, immobility, etc.), and ends up trading at a less advantageous price in equilibrium. The least satiated population members may stay out of the market altogether. If satiation rises with wealth, prevailing inequality will be magnified.

**Keywords:** satiation; risk aversion; patience; bargaining; matching; inequality; hold-up; price floor; price ceiling

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# 1 Introduction

In perfectly competitive markets, the preferences of an individual have no bearing on the price this individual obtains. Real markets, however, are rarely perfectly competitive; prices are typically not determined by a central market maker but on a more disaggregate level. A seller is often only confronted with a single buyer (and vice versa), for example, because the buyer is the only customer present. There may be many other shops and many other interested customers but these alternatives are not immediately available. Obtaining such alternatives usually entails costs, e.g., the opportunity costs of time waiting for another customer or to walk to another shop. If costs to obtain these alternatives are small, the presence of these alternatives ensures that prices are not far from the Walrasian market price. We therefore refer to such markets as *quasi-competitive*. Still, there is a surplus from an immediate agreement that will be shared by buyer and seller. How much of the surplus is obtained by whom may well depend on the individual preferences of the involved agents. So, preferences may affect prices in quasi-competitive markets. In this paper, we examine this effect of individual preferences on prices in quasi-competitive markets.

In Rubinstein's seminal paper on bilateral bargaining (1982), a more patient player obtains the larger share of a pie. For the same setting, Roth (1985) showed that agents get a better deal when they are less risk-averse. We extend and these results in two ways. First, we consider a broader set of characteristics that, for example, also encompasses spatial preferences. Second, we analyze quasi-competitive markets.

Patience, risk attitude, mobility, etc. affect how an option $x$ that is immediately available to an agent compares to an alternative $x'$ which involves delay, risk, travel, and so on. Generally, we can ask how much an agent needs to be offered in order to forego the alternative $x'$. The higher this offer, the harder it is to satisfy the agent. In Section 2, we provide a formal definition of *hard-to-satisfy*. If preferences are represented as utilities, we obtain an alternative characterization: an agent is harder to satisfy, the lower the utility that he derives from $x$ in relation to the more attractive alternative $x'$ (Proposition 1). Put differently, an agent who is hard-to-satisfy is more *satiated*. We show that satiation nests patience, risk attitude and mobility of an agent (Proposition 2 and 3), and therefore provides a unified framework for the comparison of immediate and 'remote' options.

In Section 3, we analyze analyze how the prize that an agent obtains in a quasi-completive market is affected by his satiation. In particular, we take a market with decentralized price making in the tradition of Rubinstein & Wolinsky (1985). We suppose that agents on one market side, say the sellers, have different satiation and find that more satiated agents obtain more favorable prices (Theorem 1). The intuition is simple: given the same alternative $x'$, satiated sellers are more inclined to reject an offer $x$ than unsatiated sellers. If a buyer wants to realize at least part of the surplus, the seller must accept; in order to ensure this acceptance, the buyer must offer a satiated seller a higher price.

Depending on their satiation, agents obtain different prices. In Section 4, we examine the consequences of this finding on the functioning of quasi-competitive markets. We suppose that there are less potential sellers than buyers and that potential buyers as well as sellers must pay a fee to enter the market. These fees are the same for all buyers and all sellers; taken together, buyer and seller fee are smaller than the monetary surplus that is created within a buyer-seller match. Since there are less sellers, it is efficient that all sellers enter the market, are matched with some buyer and engage in trade that generates them jointly more than their entry fees. We find, however, that sellers with low satiation abstain from the market (Theorem Z). In principle, there is demand for the goods of these sellers and the generated surplus is enough to recover the entry fees for both, buyer and seller. At the time of negotiation, however, the entry fee of the seller is sunk. A seller with sufficiently low satiation will only receive proposals that are too low to recover her entry fee. She is held up by the buyers. Foreseeing this, she is not going to invest into the entry fee.

The fact that inefficiencies arise on quasi-competitive markets suggests scope for regulation of these markets or the creation of regulated secondary markets. In Section 5, we summarize and discuss our results and explore how governmental intervention may help.

# 2 Satiation

In this section, we develop a general tool to describe the preferences of agents that helps us later to assess how they will fare in negotiations.

In many situations, agents face the choice between an immediate bundle of goods and some other more attractive bundle under different conditions. Consider an investor who can choose between an investment with certain payoff $x$ and an uncertain investment with higher payoff $x'$. Alternative one may think of a buyer who can obtain an attractive good $x$ today but a more attractive good $x'$ tomorrow, a worker who commands a salary $x$ on a local labor market and a higher salary $x'$ on a labour market further afield, or a musician who can manage himself which means that $x$ people turn out at his concert or hand the management to a professional in which case $x' > x$ people appear but some of the revenue will be appropriated by the professional manager. In all these examples, there is a second dimension in addition to the actual good. In case of the investment this is the probability that the payoff $x'$ is obtained, in case of the buyer it is the time at which the good $x'$ is obtained, in case of the worker it is the commuting distance to the work place with salary $x'$ and in case of the musician it is the share of the revenue received. Preferences of the agent are hence defined over two dimensions: the actual good and some other dimension $z$ for which the agent prefers larger to small values $(x, z) \succ (x, z')$ if $z > z'$. We assume that if an agent prefers $x$ to $x'$ for given $z$, the agent also prefers $x$ to $x'$ for any other $z'$. Holding the probability constant, a higher payoff $x'$ is better than a lower payoff $x$ a good $x'$ is more attractive than $x$ irrespective whether it is received today or tomorrow; at the same location, a high salary $x'$ is better than a low salary $x$ and more people generate more revenue irrespective of who is in charge of advertisement.

Agents may differ in terms of their preferences. Some investors may be more other less risk-averse, buyers may be more or less patient, workers more or less mobile and musicians more or less popular. These differences affect how much of $x$ is required for an agent to be indifferent between $x$ in the here and now (at $z$) and the more attractive $x'$ under the inferior alternative condition ($z'$).

**Definition 1** (harder-to-satisfy)**.** *Consider an option $x'$ which agent 1 prefers to $x$ given condition $z$, i.e., $(x', z) \succ_1 (x, z)$, and an inferior condition $z'$ (satisfying $(y, z) \succ_i (y, z')$ for all $y$ and $i$). We then say that agent 1 is* harder to satisfy *than agent 2 if*

$$(x, z) \sim_1 (x', z') \Rightarrow (x, z) \succ_2 (x', z').$$

In order to move from preferences to a utility function that represents these preferences, a certain stationarity of preferences is helpful. For example, Fishburn & Rubinstein (1982) impose the following axiom:

$$\text{If } (x, z) \sim (x', z + \tau) \text{ then } (x, w) \sim (x', w + \tau).$$

In the context of probabilities, stationarity means that if agent $i$ is indifferent between winning $x$ and $x'$ with probability $z$, agent $i$ is also indifferent between winning $x$ and $x'$ with probability $z'$. Similar interpretations can be given for the other contexts by replacing $z$ by the respective quantity. Given this axiom and some other regularity axioms, it is possible to represent preferences of an arbitrary agent $i$ by a multiplicatively separable utility function of the following type (see Fishburn and Rubinstein 1982): $(x, z) \succ_i (x', z')$ if and only if $\kappa_i^z u_i(x) \geq \kappa_i^{z'} u_i(x')$. Moreover, it is straightforward to show that for each representation using factor $\tilde{\kappa} \in (0, 1)$ it is possible to derive a representation with an arbitrary other factor $\kappa \in (0, 1)$ by transforming the utility $\tilde{u}$ to $u = (\tilde{u})^k$ with $k = \log(\kappa)/\log(\tilde{\kappa})$. Using this result, we standardize the representation such that all agents have the same multiplicative factor $\kappa$ and heterogeneity is fully captured by $u_i$. Note that while any preferences that can be represented by varying discount factors may also be represented with a single discount factor and a respective concave transformation of the utility function, the opposite is not true: Appendix A gives an example of preferences that can be represented by a utility function and a discount factor but there is no discount factor that allows to represent the same preferences with a concave transformation of the utility function. This is why we standardize the discount factor rather than the utility.

The possibility to represent preferences by a multiplicative separable utility function[1] allows us to write down the indifference condition in Definition 1 in a particularly simple fashion: agent $i$ is indifferent between $x$ given $z$ and $x'$ given $z'$ if and only if $\kappa^z u_i(x) = \kappa^{z'} u_i(x')$. Accordingly, we formulate the following characterization of harder-to-satisfy agents.

**Proposition 1.** *Agent* 1 *is harder to satisfy than agent* 2 *if and only if*

$$\frac{u_1(x)}{u_1(x')} < \frac{u_2(x)}{u_2(x')}. \tag{1}$$

*Proof.* Suppose agent 1 is harder to satisfy than agent 2. Then, $\kappa^z u_1(x) = \kappa^{z'} u_1(x')$ and $\kappa^z u_2(x) > \kappa^{z'} u_2(x')$. Rewriting, we immediately get Equation (2). If Equation (2) holds and agent 1 is indifferent, then $\kappa^z u_2(x) > \kappa^{z'} u_2(x')$ follows and agent 1 is harder to satisfy. □

[1]Note that taking the logs of the multiplicative separable utility leads to an additive separable utility.

The characterization from this proposition offers a nice intuitive interpretation. The agent who is harder to satisfy derives a lower proportion of utility from good $x$ rather than $x'$. This brings us to a central concept in order to characterize utility functions.

**Definition 2** (Satiation)**.** *An agent with utility function* $u_1\colon \mathbb{R}_+ \to \mathbb{R}_+$ *is* more satiated *than an agent with* $u_2\colon \mathbb{R}_+ \to \mathbb{R}_+$ *or, in short,* $u_1 \succ_S u_2$ *if and only if*

$$\frac{u_1(x)}{u_1(x')} < \frac{u_2(x)}{u_2(x')}. \tag{2}$$

*for each* $x' \succ_1 x$*.*

Satiation provides us with a way to describe utility functions that is applicable in various contexts. Reconsider the case where the second dimension $z$ is a probability. In this setting, utility functions may be characterized in terms of risk-aversion in various equivalent ways—for an overview see page 191 in the textbook by Mas-Colell et al. (1995). One specific characterization is that the utility function of a more risk-averse agent is a concave transformation of the utility function of a less risk-averse agent. The following proposition shows that more satiation is identical to more risk aversion.

For the utility representation of preferences, we need v.N.-M. axioms as well as an objective probability.

Note that concavity is only applicable with one good, our definition also works for a bundle of goods.

**Proposition 2.** *Utility function* $u_1\colon \mathbb{R}_+ \to \mathbb{R}_+$ *is more satiated than* $u_2\colon \mathbb{R}_+ \to \mathbb{R}_+$ *if and only if* $u_2$ *is an increasing concave transformation of* $u_1$*, i.e.,*

$$u_1 \succ_S u_2 \iff \exists k \in \mathcal{C}_+\colon\ u_2 \equiv k(u_1),$$

*where* $\mathcal{C}_+$ *denotes the space of concave increasing functions* $f\colon \mathbb{R}_+ \to \mathbb{R}_+$*.*

*Proof.* As $u_1$ is monotone increasing, we can define an increasing function $g\colon \mathbb{R}_+ \to \mathbb{R}_+$ such that $u_2(x) = g(u_1(x))$ for all $x \in \mathbb{R}_+$. Rewriting inequality (2), we get

$$\frac{g(u_1(x))}{g(u_1(y))} > \frac{u_1(x)}{u_1(y)}$$

for all $y > x > 0$. Relabeling $u_1(x)$ as $\tilde{x}$ and $u_1(y)$ as $\tilde{y}$, we obtain

$$\frac{g(\tilde{x})}{\tilde{x}} > \frac{g(\tilde{y})}{\tilde{y}}$$

for all $\tilde{y} > \tilde{x}$. This, however, implies that $g$ is a concave function.

Conversely, if there is an increasing concave transformation such that $u_2(x) = k(u_1(x))$ for all $x \in \mathbb{R}_+$, it follows from concavity of $k$ that

$$\frac{k(u_1(x))}{k(u_1(y))} > \frac{u_1(x)}{u_1(y)}$$

for all $y > x > 0$. Using that $u_2(x) = k(u_1(x))$, it follows that $u_1$ is more satiated. □

If the utility functions are twice differentiable, then $u_2 \equiv k(u_1)$ for an increasing concave function $k$ is equivalent to $u_1$'s Arrow-Pratt measure of absolute risk aversion, $r_A(x, u_1) \equiv -u_1'(x)/u_1''(x)$, exceeding that of $u_2$ for every $x > 0$. We thus have

**Corollary 1.** *If utility functions $u_i$ are twice differentiable in $x$, then*

$$u_1 \succ_S u_2 \iff \forall x > 0\colon r_A(x, u_1) < r_A(x, u_2)$$

Hence, satiation reflects risk-aversion when the agent faces uncertainty. When the agent has inter-temporal preferences, more impatience is often represented by a lower discount factor and an otherwise identical preference representation.

**Proposition 3.** *Consider agent 1 and agent 2 which only differ with respect to discount factors $\kappa_1$ and $\kappa_2$ Then, agent 1 is more satiated than agent 2 if and only if $\kappa_1 > \kappa_2$.*

*Proof.* Let $u$ be the utility function of both agents. Standardize the discount factor of agent 2 to be $\kappa_1$ using the transformation idea by Fishburn & Rubinstein (1982). The respective utility functions is $u_2 = (u)^k$ with $k = \log(\kappa_1)/\log(\kappa_2)$. By definition agent 1 is more satiated than agent 2 if and only if $\frac{u(x)}{u(x')} < \frac{u_2(x)}{u_2(x')}$ for all $x, x'$. This is equivalent to $\frac{u(x)}{u(x')} < (\frac{u(x)}{u(x')})^k$ for all $x, x'$, which in turn is identical to $(u(x))^{k-1} > (u(x'))^{k-1}$ for all $x, x'$. The latter holds if and only if $k < 1 \Leftrightarrow \kappa_1 > \kappa_2$. □

Similar to this case of inter-temporal preferences, it is possible to reflect different regional preferences and the share remaining after paying a concert manager by discount factors. Accordingly, this proposition also allows us to identify mobility and popularity with satiation in the respective settings. Satiation thus proves a rather general concept to describe utility functions. In the next section, we examine how this concept can be employed to determine which agents get better deals on quasi-competitive markets.

## 3 Quasi-competitive markets

In this section, we want to describe a decentralized market mechanism where prices are determined in a bilateral negotiation. Neither party is locked in with the same trading partner but can switch to another partner. In this model, there is some (potentially small) friction as the alternative trading opportunity is not immediately available. Agents thus find themselves weighing an immediate $x$ against a larger $x'$ that is available in the future, at a different locality or only with some probability—this is exactly the type of situation discussed in the previous section. We then examine whether and what satiation can tell us about prices.

Our model heavily draws on decentralized market models suggested by Rubinstein & Wolinsky (1985) and Gale (1987). More specifically, we investigate a variation of Rubinstein (1989, Model A). This variation differs from Rubinstein's model in three ways: (i) agents here may have non-linear utility functions, (ii) sellers have heterogeneous preferences that can be ranked according to their satiation, (iii) there is a continuum of buyers and sellers rather than a finite number. The first two differences are crucial because they allow us to study the effect of satiation on prices. The third difference simplifies the model when dealing with heterogeneous agents.

Consider a market for a single indivisible good which is traded for a divisible good (money). Time is discrete and at the start of any given period $t \in \{0, 1, 2, \ldots\}$ agents are divided into two groups according to whether they own one unit of the indivisible good (potential sellers) or would like to purchase one in exchange for part of their monetary wealth (potential buyers). Buyers' monetary wealth is assumed to exceed the sellers' common reservation price, which is normalized to 0. Buyers' reservation price for the good, corresponding to the monetary value of the gains from trade, is 1. There is a continuum of $B$ potential buyers and $S$ potential sellers. We assume that sellers are on the short side of the market $|S| < |B|$. So, sellers are matched in any period with a buyer, while buyers have the same probability to be matched of $\alpha = |S|/|B|$.

When a buyer and a seller are matched, one of them is selected as the *proposer* with probability one half. He quotes a price. His potential trading partner, the *responder*, must then either accept or reject the suggested trade. If the responder accepts the proposal, both traders leave the market at the end of the period and are replaced by new agents of the same type.[2] If the responder refuses the proposal, the match continues with probability $\gamma$ or is resolved with probability $1-\gamma$. If the match is resolved, buyer and seller enter the pool of agents who are available for a new match at the beginning of the next trading period.

Buyers and sellers have preferences that satisfy the von Neumann-Morgenstern axioms regarding lotteries in any given period (so, in particular, lotteries can be replaced by their certainty equivalent) and, moreover, the Fishburn-Rubinstein axioms regarding intertemporal comparisons. A buyer trading at price $p$ in the $t$-th period, receives a surplus of $y^b \equiv 1-p$ which yields him a Bernoulli utility $U^b(y^b,t) = \delta^t u_i^b(y^b)$, where $\delta \in (0,1)$ is a fixed discount factor and $u_i^b(y^b)$ is assumed to be continuous and interpreted as the instantaneous utility from $y^b$ units of surplus for a buyer with satiation $i$. The seller's utility from the same deal is $U_\theta^s(y^s,t) = \delta^t u_\theta^s(y^s)$ where $y^s \equiv p$ and $\theta$ is the satiation of the seller. Suppose that the population of sellers $S$ is ordered according to their satiation and let the probability measure that describes distribution of satiation $\theta$ on $S$ be $\mu$. All agents are taken to be indifferent between any time $t$ in which they receive zero units of surplus, i.e., $u^b(0) = u_\theta^s = 0$.

Denote the offer of a seller with satiation $\theta$ to a buyer as $(x_\theta^b, x_\theta^s)$ and the offer of a buyer to a seller of satiation $\theta$ by $(y_\theta^b, y_\theta^s)$, where the superscript indicates the recipient of the offer. The continuation value of a seller of satiation $\theta$ at stage $t$ is

$$\tilde{u}_\theta^s(t) \;=\; \delta\left[\gamma\frac{u_\theta^s(x_\theta^s(t+1)) + u_\theta^s(y_\theta^s(t+1))}{2} + (1-\gamma)\tilde{u}_\theta^s(t+1)\right],$$

where we assumed that proposals only depend on the satiation of the seller and neither on the identity of agents nor on the history in the rounds up to $t$. If in addition, we suppose that in equilibrium proposals remain constant over time, then we can drop the time argument $t$. Solving for the continuation value $\tilde{u}_\theta^s$ yields:

$$\tilde{u}_\theta^s = \frac{\delta\gamma}{1-\delta(1-\gamma)}\,\frac{u_\theta^s(x_\theta^s) + u_\theta^s(y_\theta^s)}{2}. \tag{3}$$

We denote the current payment that yields the same utility as the continuation value for a seller $s$ of type $\theta$ by $v_\theta^s$: $u(v_\theta^s) = \tilde{u}_\theta^s$. Note that just like

[2] A plausible interpretation suggested by Rubinstein (1989, p. 248) is that "the numbers of sellers and buyers are kept approximately steady, but the fluctuations are small enough to give agents the impression that numbers are constant."

the continuation value $\tilde{u}^s_\theta$, this payment $v^s_\theta$ depends on the stationary offers to type $\theta$: $v^s_\theta(x^s_\theta, y^s_\theta)$. Moreover, $v^s_\theta(1,1) < 1$ since $\kappa < 1$. The buyer chooses his offer $y^s_\theta$ such that the seller is indifferent between continuing to search and accepting this offer: $u^s_\theta(y^s_\theta) = \tilde{u}^s_\theta$. We can use equation (3) to replace the continuation payoff and sort terms to get the following indifference condition for the seller:

$$u^s_\theta(y^s_\theta) = \frac{\delta\gamma/2}{1-\delta(1-\gamma)-\delta\gamma/2} \cdot u^s_\theta(x^s_\theta). \tag{4}$$

This condition implicitly defines an offer function $y^s_\theta$ that depends on $x^s_\theta$: $y^s_\theta(x^s_\theta)$. Let us derive some properties of this offer function. Given that instantaneous utility was assumed to be continuous and strictly increasing, its inverse is continuous and strictly increasing and hence $y^s_\theta$ is continuous and strictly increasing. Since $u^s_\theta(0) = 0$, it follows that $y^s_\theta(0) = 0$. Moreover, as $v^s_\theta(1,1) < 1$ and $y^s_\theta = v^s_\theta(1,1)$, it holds that $y^s_\theta(x) < 1$. Next, we compute the continuation value of the buyer, who is matched with a seller of satiation $\theta$:

$$\begin{aligned}\tilde{u}^b(t) = \quad & \delta\Bigg[\gamma\frac{u^b(x^b_\theta)+u^b(y^b_\theta)}{2} \\ & +(1-\gamma)\left(\alpha\frac{\int u^b(x^b_\tau)d\mu(\tau)+\int u^b(y^b_\tau)d\mu(\tau)}{2}+(1-\alpha)\tilde{u}^b(t+1)\right)\Bigg]\end{aligned}$$

Since $u^b$ is a continuous function, we can define a $\hat{x}^b$ and a $\hat{y}^b$ such that $u^b(\hat{x}^b) = \int u^b(x^b_\tau)d\mu(\tau)$ and $u^b(\hat{y}^b) = \int u^b(y^b_\tau)d\mu(\tau)$. Note that the integrals and hence $\hat{x}^b$ and a $\hat{y}^b$ may depend on $x^b_\theta$ and $y^b_\theta$. Using the new notation, assuming stationarity and suppressing the time index, we get:

$$\tilde{u}^s_\theta = \kappa\left[\frac{\gamma}{2}\left(u^b(x^b_\theta)+u^b(y^b_\theta)\right)+\frac{1-\gamma}{2}\left(u^b(\hat{x}^b(x^b_\theta))+u^b(\hat{y}^b(y^b_\theta))\right)\right], \tag{5}$$

where $\kappa = \frac{\delta}{1-\delta(1-\gamma)(1-\alpha)} < 1$. Now, we can define a payment $v^b$ that leaves the seller indifferent:

$$\kappa\left[\frac{\gamma}{2}\left(u^b(x^b_\theta)+u^b(y^b_\theta)\right)+\frac{1-\gamma}{2}\left(u^b(\hat{x}^b(x^b_\theta))+u^b(\hat{y}^b(y^b_\theta))\right)\right] = u^b(v^b(x^b_\theta, y^b_\theta)). \tag{6}$$

¿From this definition of $v^b$, it follows that $v^b(0,0) = 0$ and $v^b(1,1) < 1$. In any stationary sub-game perfect equilibrium, the seller will offer the lowest acceptable share to the buyer:

$$x^b_\theta = v^b(x^b_\theta, 1-y^s_\theta(1-x^b_\theta)) \equiv g_\theta(x^b_\theta). \tag{7}$$

Based on this indifference condition, we can derive the following lemma:

**Lemma 1.** *Suppose there is a unique stationary sequential equilibrium. Denote the equilibrium offer by the seller as $x_\theta^{b*}$ and let $g(x_\theta^b) \equiv v^b(x_\theta^b, 1-y_\theta^s(1-x^b))$. Then $g$ intersects the $45°$-degree line at $x_\theta^{b*}$* from above*, i.e., for any $x \in [0,1]$*

$$g_\theta(x) < x \Longleftrightarrow x > x_\theta^{b*}.$$

*Proof.* Since the seller's offer $y_\theta^s$ and the buyer's offer $v^b$ are continuous (see Lemma 3), $g$ is continuous. ¿From $v^b(0,0)=0$ and $y_\theta^s(1) < 1$, it follows that $g(0) = v^b(0, 1-y_\theta^s(1)) > 0$. Observe that $g(1) = v^b(1,1)$ since $y_\theta^s(0) = 0$. Moreover, $v^b(1,1) < 1$ so that $g(1) < 1$. Given that there is a unique point such that $g(x_\theta^{b*}) = x_\theta^{b*}$, the function $g$ has to be larger to the left and smaller to the right. □

It follows that an upward shift in the continuation value for the seller $g$ must result in an increase of $x_\theta^{b*}$. We exploit this observation to show that sellers are better off if they are more satiated:

**Theorem 1.** *Suppose that the matching model described above has a unique equilibrium outcome $(x_\theta^{s*}, y_\theta^{s*})$ characterized by the indifference conditions $x_\theta^b = g_\theta(x_\theta^b)$ and $u_\theta^s(y_\theta^s) = \gamma u_\theta^s(x_\theta^s)$. Then, more satiated sellers receive higher shares:*

$$\theta > \tilde{\theta} \Longrightarrow x_\theta^{s*} > x_{\tilde{\theta}}^{s*} \;\; and \;\; y_\theta^{s*} > y_{\tilde{\theta}}^{s*}.$$

*Proof.* By using the indifference condition (7) and applying Proposition 1, we get that a more satiated seller receives a higher offer for a given outside option: $\theta > \tilde{\theta} \Rightarrow y_\theta^s(x) > y_{\tilde{\theta}}^s(x)$ for arbitrary $x$. In particular, the inequality holds for the equilibrium share $x_{\tilde{\theta}}^{s*} = 1 - x_{\tilde{\theta}}^{b*}$ that the seller can keep when proposing $x_{\tilde{\theta}}^{b*}$: $y_\theta^s(1-x_{\tilde{\theta}}^{b*}) > y_{\tilde{\theta}}^b(1-x_{\tilde{\theta}}^{b*})$. Because the buyer's present value function $v^b$ is increasing in $y_\theta^b = 1 - y_\theta^s$, we get: $g_\theta(x_{\tilde{\theta}}^{b*}) < g_{\tilde{\theta}}(x_{\tilde{\theta}}^{b*})$. Moreover, since $x_{\tilde{\theta}}^{b*}$ is the fixed point of $g_{\tilde{\theta}}$, we have $g_{\tilde{\theta}}(x_{\tilde{\theta}}^{b*}) = x_{\tilde{\theta}}^{b*}$ and hence $g_\theta(x_{\tilde{\theta}}^{b*}) < x_{\tilde{\theta}}^{b*}$. Applying Lemma 1, we obtain that $x_{\tilde{\theta}}^{b*} > x_\theta^{b*}$. Consequently, $x_{\tilde{\theta}}^{s*} = 1 - x_{\tilde{\theta}}^{b*} < 1 - x_\theta^{b*} = x_\theta^{s*}$. This together with the indifference condition $u_\theta^s(y_\theta^s) = \gamma u_\theta^s(x^s)$ allows us to conclude that $y_\theta^{s*} > y_{\tilde{\theta}}^{s*}$. □

So in quasi-competitive markets, satiated agents obtain better deals.

# 4 Market entry costs and inefficiency

In this section, we introduce market entry costs, study their interaction with satiation and examine their consequences on the efficiency of markets. Quasi-competitive markets where agents are homogeneous with respect to satiation are efficient (Rubinstein & Wolinsky, 1985). It will be demonstrated here that this is no longer the case if agents' satiation differs.

Take the quasi-competitive market from the previous section where sellers satiation $\theta$ differs while buyers all have the same preferences. As before, there is an interval $B$ of buyers and an interval $S$ of sellers arriving at the doors of the market where $|S| < |B|$. In order to enter the market sellers incur entry costs $\epsilon^s$ and buyers $\epsilon^b$. Like Rubinstein & Wolinsky (1985), we assume that the sum of the entry costs is below the surplus that a buyer and seller create: $\epsilon^s + \epsilon^b < 1$. Accordingly, it is efficient that all sellers enter the market.

Buyers enter the market as long as the value of being in the market, $\tilde{u}^b/\delta$, exceeds the utility from consuming the entry fee: $\tilde{u}^b/\delta > u^b(\epsilon^b)$. Conversely, they leave whenever $\tilde{u}^b/\delta < u^b(\epsilon^b)$. Hence, the value of being in the market for buyers is $u^b(\epsilon^b)$. A proposing seller makes an offer $x^b$ that keeps the latter indifferent between an immediate $x^b$ or waiting: $u^b(x^b) = \delta u^b(\epsilon^b)$.

The expected utility of being in the market for a seller of satiation $\theta$ is: $\bar{u}^s_\theta = \frac{\gamma}{1-\delta(1-\gamma)} \frac{u^s_\theta(x^s_\theta)+u^s_\theta(y^s_\theta)}{2}$. A seller with satiation $\theta$ enters if and only if this utility is larger than the utility from consuming the entrance fee: $\bar{u}^s_\theta \geq u^s_\theta(\epsilon^s)$. Are there any sellers who abstain from the market?

We first examine an example. Suppose that the utility of buyers from consuming $x$ is $u^b(x) = \log(x+1)$ while the utility of sellers amounts to $\log(\frac{x+\theta}{\theta})/\log(\log(\frac{\theta+1)}{\theta})$, where $\theta$ is the satiation.[3] Further, we assume that any match is resolved after a proposal is rejected $\gamma = 0$. Given these assumptions, we compute the equilibrium proposal of a seller to a buyer, $x^b = (\epsilon^b + 1)^\delta$, and of a buyer to a seller with satiation $\theta$:

$$y^s = \theta \cdot \left( \left(1 + \frac{1}{\theta}\right)^{\frac{\delta/2}{1-\delta/2} \cdot \log\left(1 + \frac{1-x^b}{\theta} / \log(1+\frac{1}{\theta})\right)} - 1 \right).$$

By replacing $x^b$ in this formula, we get the equilibrium offer to a seller of satiation $\theta$ and compute the expected surplus. The gain from market entry

[3] A larger $\theta$ renders the utility of sellers less concave. Recall that by Proposition 2, this implies more satiation.

can be expressed in terms of the argument of the utility function by asking for which $\bar{y}^s_\theta$, the seller of satiation $\theta$ is indifferent between entering the market or staying out. Figure 1 compares this gain with the entry fee: for low satiation, the entry fee exceeds $\bar{y}^s_\theta$ and sellers abstain from the market. We thus observe an inefficiency.

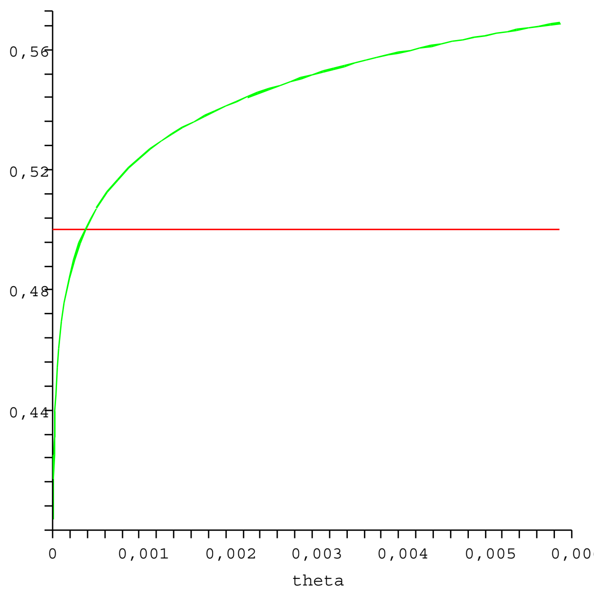


Figure 1: Entry fee (straight line) and gain from market entry for different levels of satiation

For a more general examination when inefficiency occurs, we introduce the following characterization of the population:

**Definition 3** (Population with desperate agents)**.** *A population encompasses desperate agents if and only if there is at least one agent of satiation $\theta$ who derives an arbitrarily large share of his utility from the first $\epsilon$ units of a good:*

$$\forall \epsilon > 0 \colon \forall \xi < 1 \colon \forall \kappa < 1 \colon \exists \theta \colon \frac{u_\theta(\epsilon)}{u_\theta(\xi)} > \kappa.$$

This definition gives us a sufficient condition for sellers to abstain from the market.

**Proposition 4.** *If the population of sellers encompasses desperate agents, these agents abstain from the market and an inefficiency occurs.*

*Proof.* For buyers to be indifferent between entering or abstaining from the market, $u^b(x^b) = \delta u^b(\epsilon^b)$ they must obtain some positive $x^b$ in some states of the world. Accordingly, the value $\bar{y}^s_\theta$, which is implicitly defined by $u^s_\theta(\bar{y}^s_\theta) = \bar{u}^s_\theta$ must be smaller than one: $\bar{y}^s_\theta < 1$. Consequently, there is some $\kappa < 1$ such that

$$\kappa u^s_\theta(1) > u^s_\theta(\bar{y}^s_\theta). \tag{8}$$

The presence of desperate sellers implies that there is some $\tilde{\theta}$ such that:

$$\frac{u^s_{\tilde{\theta}}(\epsilon^s)}{u^s_{\tilde{\theta}}(1)} > \kappa \text{ or } u^s_{\tilde{\theta}}(\epsilon^s) > \kappa u^s_{\tilde{\theta}}(1). \tag{9}$$

Taking equation (8) and (9) together, we get: $u^s_{\tilde{\theta}}(\epsilon^s) > \kappa u^s_{\tilde{\theta}}(1) > u^s_{\tilde{\theta}}(\bar{y}^s_{\tilde{\theta}})$. Consequently, the seller with satiation $\tilde{\theta}$ abstains. □

This proposition gives us a sufficient condition when quasi-competitive markets suffer from inefficiency because agents with low satiation do not enter. This condition is not necessary: it suffices that sellers with low saturations get offers below the market fee. While their entry allows a Pareto-improvement, it requires an investment in form of the entry fee. This investment is sunk when they enter negotiation. In the negotiation, buyers exploit the weak bargaining power of low satiated sellers; they hold up these sellers so that sellers cannot recoup their investment. Anticipating this, they abstain from the market. The underlying mechanism that results in the inefficiency is thus much more general than the examined setting might suggest. Whenever rents are shared by some mechanism in which agents are kept indifferent between an immediate $x$ and an $x'$ under alternative conditions, lower satiation will result in a lower share of the rent. If the production of the rent requires some up-front investment, agents who are hampered by their own preferences in the negotiation may choose not to invest.

# 5 Concluding remarks

With satiation, we have introduced a powerful notion that captures various characteristics of utility functions such as impatience and risk-aversion. We have seen that more satiated agents obtain better prices in quasi-competitive markets. In the presence of market entry fees, this meant that agents with low satiation may obtain to little of the generated surplus to recoup the entry fee and abstain from the market. Agents with low satiation prefer not to invest in the fee because they are held-up by their trading partner.

If these agents are on the short side of the market, an inefficiency arises. This inefficiency may be resolved in different ways. One possibility is that a price floor or ceiling is imposed exogenously (e.g., a minimum wage or rent control). It can be chosen such that agents on the short side can at least recover the costs of entering. This implies that *all* available surplus is realized in equilibrium. Agents from the long side of the market break even. Those satiated members of the short side who would have entered anyway do not lose either: offers that they make still correspond to the immediate value of their trade partner's entry costs, and offers which they receive are determined only by their own unchanged satiation (see equation (7)). So no further redistribution is needed in order to achieve a Pareto improvement.

Somewhat less intrusive than restricting the price on an existing market is the creation of a secondary marketplace by the government, on which the above-mentioned price controls apply. Members of the long side of the market are indifferent which market to enter. The least satiated members of the short side will profitably enter the secondary market, whilst those able to secure prices above the floor or below the ceiling, respectively, are indifferent between the unregulated and regulated one.

In fact no government intervention may be needed at all if the involved actors can overcome possible coordination and commitment problems: market participants on the short side could form a union or cooperative which negotiates a price for all members using an internal decision rule such as simple majority voting. This would replicate the prices otherwise received only by the agent with median satiation for all members. The agents who are benefiting from this (now finding it profitable to participate or simply receiving more beneficial prices) are able, at least in principle, to compensate those who would achieve more attractive deals on their own. The problem, however, is that if institutions like an effective cooperative or union can be created by the short side of the market, the long side might set one up, too – raising its own members' payoffs by voiding the disadvantageous quasi-competitive market structure.

Interestingly, the discussed inefficiency is not driven by non-convexities, liquidity constraints or credit market imperfections, which feature prominently in the development literature on inequality. Providing the least satiated agents with an interest-free loan to cover market entry costs would not change anything. Provided that their satiation is observable, they suffer the same strategic disadvantage as before. They receive a surplus share which is insufficient to cover entry costs and pay back the loan, i.e., they prefer not to accept the loan.

It should be emphasized, however, that our model involves inefficiency and thereby gives a rationale, e.g., for positive employment effects of a minimum wage or for rent control that actually improves the housing market, only in very specific contexts: market entry costs either need to be high relative to the gross surplus, or they have to be born by an extremely heterogeneous population on the short side of the market. The practical relevance of our inefficiency finding thus may be limited.

In contrast to this, the predicted magnification of absolute (possibly also relative) wealth inequality in quasi-competitive markets is very robust and general if satiation increases in wealth. Matching markets with a little bit of friction seem to provide the best available micro-foundation for the standard competitive model in economic textbooks. Unless a central clearing agency mitigates the conspicuous absence of the Walrasian auctioneer, agents have no choice but to engage in some sort of decentralized trade. As our model points out, this second-best institution gives an advantage to more satiated types. We thus identify a strategic reason for markets' occasional tendency to reward yesterday's winners again today (and tomorrow). Adam Smith pointed out that "Money, says the proverb, makes money. When you have got a little, it is often easy to get more. The great difficulty is to get that little." (Wealth of Nations, 'On the Profits of Stock'). Other properties of real markets certainly contribute to this. But we conjecture that amongst the poor in the developing world (and possibly also in more developed countries), strategic disadvantages associated with high degrees of non-satiation have an impact similar in scale and scope to credit restrictions or the general incompleteness of contracts.

# References

Fishburn, P. C. & Rubinstein, A. (1982). Time preference. *International Economic Review*, *23*(3), 677–94.

Gale, D. (1987). Limit theorems for markets with sequential bargaining. *Journal of Economic Theory*, *43*, 20–54.

Mas-Colell, A., Whinston, M. D., & Green, J. R. (1995). *Microeconomic Theory*. Oxford: Oxford University Press.

Roth, A. E. (1985). A note on risk aversion in a perfect equilibrium model of bargaining. *Econometrica*, *53*(1), 207–11.

Rubinstein, A. (1982). Perfect equilibrium in a bargaining model. *Econometrica*, *50*(1), 97–109.

Rubinstein, A. (1989). Competitive equilibrium in a market with decentralized trade and strategic behavior: (an) introduction. In G. R. Feiwel (Ed.), *The Economics of Imperfect Competition and Employment: Joan Robinson and Beyond* (pp. 243–259). Basingstoke: MacMillan Press.

Rubinstein, A. & Wolinsky, A. (1985). Equilibrium in a market with sequential bargaining. *Econometrica*, *53*(5), 1133–1150.

# A Representation of preferences

Fishburn & Rubinstein (1982) prove that preferences over $(y,t) \in [0,1] \times \{0,1,2,\ldots\}$ represented by $U^j(y,t) = \delta^t\, u^j(y)$ can equivalently be represented using a different discount factor $\tilde{\delta} > (<)\delta$ when the instantanous utility is transformed using a suitable concave (convex) transformation: $U^j(y,t) = \tilde{\delta}^t\, \tilde{u}^j(y)$ with and $\tilde{u}^j = T(u^j)$ and $T$ being a concave function. The converse is not generally true.

**Lemma 2.** *There are preferences which can be represented by $U^j(y,t) = \delta^t\, u^j(y)$ such that there is no discount factor $\tilde{\delta}$ such that a concave transformation $T$ of the instantanous utility $\tilde{u}^j = T(u^j)$ yields the same preferences: $\delta^t\, u^j(y) \neq \tilde{\delta}^t\, \tilde{u}^j(y)$.*

*Proof.* An example suffices. Take the linear instantanous utility $u^j(y) = y$ and consider the quantity-time pairs $(e^1, 1)$, $(e^2, 2)$ and $(e^3, 3)$. Assume $\delta = e^{-1}$. Then all pairs yield the same utility $U(x,t) = e^4$ and the player is indifferent. Next transform the utility to get $\tilde{u}^j(y) = T(u^j(y)) = \log(y)$. Now, we have to choose $\tilde{\delta} = \frac{3}{2}$ for the first two pairs to yield the same utility. This, however, implies that the first and second pair is preferred to the third pair which yields utility $\frac{3}{4}$ rather than one. Note that this example works in continuous and discrete time. $\square$

That greater satiation results in a greater surplus share hence implies that a greater discount factor results in a greater surplus share, but the reverse is not true. Satiation is thus the more general concept to describe which surplus share is appropriated.

# B Continuity of immediate value

**Lemma 3.** *$v^b(x, y)$ is continuous in $x$ and $y$.*

*Proof.* First, show that the left-hand side in Equation (6) is continuous in $x^b_\theta$. The first summand is continuous in $x^b_\theta$ since $u^b$ is continuous. If the second summand depends on $x^b_\theta$, it can only be through $\hat{x}^b$. The integral $\bar{u}^b := \int_\Omega u^b(x^b_\tau)d\mu(\tau)$ can be written as $\int_{\Omega\setminus\{\theta\}} u^b(x^b_\tau)d\mu(\tau) + u^b(x^b_\theta) \cdot \mu(\{\theta\})$. The integral $\bar{u}^b$ is thus continuous in $x^b_\theta$ and –since $(u^b)^{-1}$ is continuous– $\hat{x}^b = (u^b)^{-1}(\bar{u}^b)$ is also continuous in $x^b_\theta$. With a completely analogous argument, we can show that $\hat{y}^b$ is continuous in $y^b_\theta$. As the left-hand side is an additive function of continuous functions of $x^b_\theta$ and $y^b_\theta$, it is continuous. Moreover, applying $(u^b)^{-1}$ to the left-hand side yields a continuous function and so $v^b$ is continuous. □