Endogenous Human Capital Risk and Public Policy

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Abstract

We apply a model, where individuals are faced with risky human capital formation. They can influence their success probability in education by their learning effort. After realization of risk, they either work as skilled or as unskilled worker. We show that optimal public policy contains income-independent tuition fees, lump-sum transfers/taxes, and public funding of the educational sector. Contrary to standard models in case of income risk, it is not optimal to use a proportional lump-sum tax, because tuition fees and public education spending provide simultaneously insurance and redistribution. The wage tax is only optimal, if tuition fees are not available.

JEL-Classification: H21, I2, J2

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1 Introduction

The importance of fostering the investment into (higher) education has been emphasised by many branches of the economic literature. Most prominent is the literature on human capital and economic growth (Bils and Klenow, 2000). A major focus of this literature is, whether (higher) education should be provided privately or publicly (Glomm and Ravikumar, 1992). The public investment into higher education is usually motivated by any kind of market failure and positive externalities. With credit market imperfections the income distribution is a decisive determinant of economic growth (cf. Benabou, 1997). In this case, publicly supplied education is able to promote economic growth because credit constraints of potential students are slackened.

Educational risk is a salient feature of human capital investments. Education risk can be twofold, the most obvious risk is the risk to fail graduation, that means that most of the resources invested are lost. The other type of risk is the uncertainty about future wages or employment opportunities (Kodde, 1986, 1988). Analytically, the first case of failed graduation can be described in the very same way as uncertain future wages, if the probability of failure is exogenous. However, to assume that the probability of successful graduation is exogenous for individuals seems to be implausible. It seems reasonable, that to some degree this probability is the result of individual choices such as learning effort. Obviously, the effort chosen by individuals will depend on the educational system and public resources spend on education. This is also suggested by a recent political debate about failure rates at universities, e.g. in Germany.

Endogenizing learning effort then opens another channel, through which governmental intervention both via public spending and tax revenue collection influence market outcome. Taxation can not only create distortions in labor supply or in occupational choice, but also has negative effects on learning efforts, and therefore increases the risk of failure in education and with it income risk. Moreover, the insurance effect of labor taxation must not only balance distortions in resource allocation, but also in learning effort.

In order to analyze these topics, we apply a two-period model, where households decide on their learning effort and therefore on their probability of getting graduated. We show that the introduction of a proportional wage tax is never optimal, if the government can use income-independent tuition fees. Induced negative incentive effects in learning can be counteracted by (increased) funding of a public education system.

Our model closes a gap in the literature on human capital accumulation, risky labor income, and effects of taxation.

It is well known from the work by Eaton and Rosen (1980a,b), and an extended model by Hamilton (1987) that it is optimal to implement a distorting wage tax, because the insurance provided outweighs the excess burden, if wage income is due to (idiosyncratic) risk. A similar result is derived in Kanbur (1980), where households have to decide whether to work in a risky entrepreneur sector or to earn deterministic wage income as employee. There are no redistributive motives, because labor market equilibrium implies that the expected utilities of all households are equalized, but differentiated taxation provides insurance. The result is extended by Boadway et al. (1991) to a indirect progressive labor tax scheme.

More recent papers, dealing with risky human capital formation and risky skilled labor income, are, e.g., García-Peñalosa and Wälde (2000), Wigger and von Weizsäcker (2001), and Jacobs and van Wijnbergen (2007). In a nutshell, they show that a graduate tax accompanied by some direct education subsidies are optimal in order to insure against income risks. However, they neglect distortions in labor supply.

Anderberg and Andersson (2003) show that education itself can have an insurance effect and should in this case be overprovided, because this also increases tax revenue.

Common to all these papers is that they treat the risk as exogenous. There is no choice on learning effort, and therefore no effect of taxation on the probability distribution itself.[1]

In our model, the individuals first decide on their learning effort, and determine thereby their success probability in higher education. Then risk realizes and the individuals choose their labor supply either as skilled worker or as unskilled one. The government can use a proportional wage tax, and tuition fees in order to finance a general lump-sum transfer, and public funding of the education system. Public educational spending is assumed to increase the success probability.

We show, as mentioned above, that it is not optimal to use the distortionary wage tax, if the government can apply tuition fees and a general lump-sum tax. Arising negative incentive effects in learning effort are counteracted by an improved endowment in the learning technology. The combination of tuition fees and public funding of the educational sector thereby achieves redistribution and insurance simultaneously.

The proceeding is as follows. In section 2, we present the model, and examine

[1]The exception is Wigger and von Weizsäcker (2001), who briefly examine the case of ex-ante moral hazard. However, they restrict to two possible effort levels, and the government cannot influence the learning technology by public educational spending.

household behavior in the third section. Section 4 then introduces public policy, and section 5 determines the optimal tax and education policy. The paper closes with some conclusions.

2 The Model

We consider an overlapping generations economy in which a mass one of individuals of each generation live for two periods of time and die at the end of the second period. In the second period each individual gives birth to one child so that the population remains constant over time; total population is two. In each period individuals are endowed with one divisible unit of time. At the beginning of the first period individuals invest into higher education and start working in the second period.[2] Following Glomm and Ravikumar (1992), we assume that both, education in the first period and working in the second period are time consuming activities which generate disutility. When entering higher education individuals have to decide about their time effort $e \in [0, 1]$ devoted to learning; at the beginning of the second period individuals decide about their individual labor supply.

However, while entering the university a successful graduation is not guaranteed. The effort invested into education e determines the probability p to pass the educational process successfully, and to acquire a degree as skilled worker. We assume the probability function to be a concave function of learning effort, thus e has a positive, but diminishing marginal productivity. Beside individual effort, the success probability also depends positively on the public funding E of the educational sector, and we assume that private effort and public funding

[2] Implicitly, we assume that individuals already attended compulsory schooling.

are complements, whereby an increase in public funding also increases the marginal productivity of each time unit invested. Thus, we have $p = p(e, E)$, and $\frac{\partial p}{\partial e} = p_e > 0$, $\frac{\partial p}{\partial E} = p_E > 0$, $\frac{\partial^2 p}{\partial e^2}$, $\frac{\partial^2 p}{\partial E^2} < 0$, $\frac{\partial^2 p}{\partial e \partial E} = p_{eE} > 0$. To simplify the analysis we assume that both the effort invested and total public investment into higher education only alters the probability of a successful graduation, and have no effect on the stock of human capital acquired in case of success.

At the beginning of the second period, individuals who graduated from university start working as skilled worker, while those individuals who fail enter the labor market as unskilled worker. Again, the household is endowed with one divisible unit of time, which is divided between second-period leisure and labor supply. Total wage income is spent on total family consumption. All individuals have identical preferences over leisure in period one and two, l_1 and l_2, and over total family consumption C in period two.

Formally, the preferences are described by a von Neumann-Morgenstern expected utility function which is additively separable in its intertemporal components. Thus, we have

$$\mathrm{E}[U] = U_1(1-e) + p(e, E) \cdot U_2(C_H, 1-H) + [1-p(e, E)] \cdot U_2(C_L, 1-L), \quad (1)$$

where $H = 1 - l_{2H}$ denotes labor supplied by a skilled worker in the second period, and $L = 1 - l_{2L}$ denotes labor supplied by an unskilled worker in the second period.[3] In order to ensure inner solutions, especially for the learning effort $e = 1 - l_1$, we assume that the utility function fulfills the Inada conditions, and hence:

[3] Subscripts H and L denote the respective values for the different skill groups.

Assumption 1. *First and second period utility exhibits the following properties:*

$$\frac{\partial U_i}{\partial l_i}, \frac{\partial U_2}{\partial C} > 0, \frac{\partial^2 U_i}{\partial l_i^2}, \frac{\partial^2 U_2}{\partial C^2} < 0 \qquad i = 1, 2$$

$$\lim_{l_i \to 0} \frac{\partial U_i}{\partial l_i} = \lim_{C \to 0} \frac{\partial U_2}{\partial C} = \infty, \lim_{l_i \to 1} \frac{\partial U_i}{\partial l_i} = \lim_{C \to \infty} \frac{\partial U_2}{\partial C} = 0 \qquad i = 1, 2.$$

Wages for both skill groups are exogenously given and denoted by w_H and w_L respectively. The government uses an indirectly progressive income tax scheme consisting of a tax rate t and a lump-sum transfer T. Moreover, households which successfully graduated and work as skilled have to pay back-loaded tuition fees f_B.[4] These tuition fees can be deducted against taxable income. The budget constraint of a skilled household can then be written as

$$C_H = (1 - t) \cdot [w_H \cdot H - f_B] + T, \tag{2}$$

whereas consumption of an unskilled household is given by

$$C_L = (1 - t) \cdot w_L \cdot L + T. \tag{3}$$

The education risk is assumed to be idiosyncratic, hence, there are ex-post $p(e, E)$ skilled workers and $1-p(e, E)$ unskilled in each generation. The government then uses its instruments in order to maximize the utility of a representative steady-state generation. Consequently, the government faces a trade-off between efficient financing of public expenditure and optimal redistribution between successful and unsuccessful students as well as optimal insurance against the risk of education.

[4] Note that we do not require the tuition fees to cover all public expenses for higher education. Instead, the government can use a mix of instruments to finance higher education.

In a nutshell, the timing structure and the model can be summarized as follows: First, the government decides on public funding of the educational sector, and on the tax instruments. Second, the young generation will choose the learning effort given the wages and the governmental decisions. This in turn determines the success probability $p(e, E)$, and with it the fraction of skilled and unskilled workers. At the beginning of the second period each individual knows whether it graduated or failed and will then decide on its labor supply. In the following, we will solve the model by backward induction.

3 Household Behavior

The complete maximization problem of a representative household can be written as

$$\begin{aligned} \max_{\{e,H,C_H,L,C_L\}} \mathrm{E}[U] &= U_1(1-e) \\ &+ p(e,E)\cdot U_2(C_H,1-H) + [1-p(e,E)]\cdot U_2(C_L,1-L) \\ &\text{s.t. (2) and (3)} \end{aligned} \tag{4}$$

Substitution of (2) and (3) for C_H and C_L in (4) yields the following first order conditions

$$\frac{\partial \mathrm{E}[U]}{\partial H} = U_{2C}\cdot(1-t)w_H - U_{2l_2} = 0, \tag{5}$$

$$\frac{\partial \mathrm{E}[U]}{\partial L} = U_{2C}\cdot(1-t)w_L - U_{2l_2} = 0, \tag{6}$$

$$\frac{\partial \mathrm{E}[U]}{\partial e} = -U_{1l_1} + p_e\cdot[U_2(C_H,1-H) - U_2(C_L,1-L)] = 0 \tag{7}$$

The system of first order conditions (5)-(7) is block recursive such that optimal labor supply H^*, L^* and with it optimal consumption C_H^*, C_L^* are separately defined by (5) and (6) respectively.[5] Note that optimal consumption and labor supply of the respective skill group is conditional on the policy mix used by the government (t, T) as well as on the wage rate w_H, w_L. Additionally, back-loaded tuition fees f_B are only relevant for labour supply and consumption of skilled workers. Inserting optimal labor supply and consumption into the second period utility function gives the indirect utility function for both types of workers: $V^H = U_2(C_H^*, 1-H^*)$, $V^L = U_2(C_L^*, 1-L^*)$. Using the respective indirect utility function V^H and V^L in (7) results in the optimal effort $e^* = e(t, T, f_B, E, w_H, w_L)$. Evaluating first period utility at the optimal effort e^* gives the first period indirect utility function $V = U(1-e^*)$.

Given the properties of the utility functions stated in assumption 1 and the block recursive form of the first order conditions, it is sufficient to check the second order conditions of (4) for each separate variable:

$$\begin{aligned}
\left.\frac{\partial^2 \mathrm{E}[U]}{\partial H^2}\right|_{H=H^*} &= SOC(H) \\
&= U_{2CC}(1-t)^2 w_H^2 - U_{2l_2}\left(1+(1-t)w_H\right) + U_{2l_2l_2} < 0, \qquad (8)\\
\left.\frac{\partial^2 \mathrm{E}[U]}{\partial L^2}\right|_{L=L^*} &= SOC(L) \\
&= U_{2CC}(1-t)^2 w_L^2 - U_{2l_2}\left(1+(1-t)w_L\right) + U_{2l_2l_2} < 0, \qquad (9)\\
\left.\frac{\partial^2 \mathrm{E}[U]}{\partial e^2}\right|_{e=e^*} &= SOC(e) \\
&= U_{1l_1l_1} + p_{ee}\left(V^H - V^L\right) < 0. \qquad (10)
\end{aligned}$$

[5] Throughout the paper, asterisks denote optimal values. To simplify the notation, we drop the functional arguments t, T, f_B, w_H, w_L when this causes no confusion.

The inequality in equation (10) is given by decreasing marginal utility of leisure, and decreasing marginal productivity of learning, and by the fact that a skilled worker must have higher utility in the second period than a unskilled one, $V^H > V^L$, because else there will be no learning effort at all.

In the next section we derive the optimal policy mix. For that reason, we need to derive the comparative statics of the individual choice variable with respect to the different instruments. We start by calculating the comparative statics of the labour supply of both skill groups:

$$\begin{aligned}
\frac{\partial H^*}{\partial t} &= \frac{-U_{2CC}(1-t)w_H^2 + U_{2Cl_2}w_H}{\|SOC(H)\|}, \\
\frac{\partial H^*}{\partial T} &= -\frac{\partial H^*}{\partial f_B} \cdot \frac{1}{1-t} = \frac{U_{2CC}(1-t)w_H - U_{2Cl_2}}{\|SOC(H)\|}, \\
\frac{\partial L^*}{\partial t} &= \frac{-U_{2CC}(1-t)w_L^2 + U_{2Cl_2}w_L}{\|SOC(L)\|}, \\
\frac{\partial L^*}{\partial T} &= \frac{U_{2CC}(1-t)w_L - U_{2Cl_2}}{\|SOC(L)\|},
\end{aligned}$$

By the same reasoning we get comparative static results for the learning effort e^* with respect to lump sum transfer T:

$$\frac{\partial e^*}{\partial T} = \frac{p_e \cdot \left(\alpha^H - \alpha^L\right)}{\|SOC(e)\|} < 0, \tag{11}$$

with $\alpha^j = \frac{\partial V^j}{\partial C} > 0, \quad j = H, L$ denoting the marginal utility of income. The inequality in equation (11) stems from the fact that we assume agent monotonicity and the single crossing property (Mirrlees, 1976) to hold. These imply that a skilled household always commands a higher income than an unskilled worker, and hence $\alpha^H < \alpha^L$. The intuition is straightforward: any increase in lump-sum income T decreases the learning intensity e, because an educational degree gets marginally less attractive.

An increase in tuition fees change the learning effort according to

$$\frac{\partial e^*}{\partial f_B} = \frac{-p_e \cdot \alpha^H \cdot (1-t)}{\|SOC(e)\|} < 0, \tag{12}$$

while increased public spending in education E changes the effort according to

$$\frac{\partial e^*}{\partial E} = \frac{p_{eE} \cdot \left(V^H - V^L\right)}{\|SOC(e)\|} > 0. \tag{13}$$

Learning effort is unambiguously reduced if tuition fees rise because this directly reduces the returns to education, whilst increased spending in education increases the productivity of learning, and therefore learning effort.

Contrary to these effects, the effect of an increase in the wage tax t is less clear. Increasing *ceteris paribus* the tax burden on skilled wage income, decreases learning effort, because the returns to schooling decrease. Increasing *ceteris paribus* the wage tax for unskilled worker increase the returns to schooling, and increases the learning intensity. Combining both effects, we end up with

$$\frac{\partial e^*}{\partial t} = \frac{p_e \left[\alpha^L \cdot w_L L^* - \alpha^H \cdot (w_H H^* - f_B)\right]}{\|SOC(e)\|}. \tag{14}$$

If the labor supply of skilled worker is not significantly higher than labor supply of unskilled ones, and given the single crossing property, an increase in the tax rate increases the learning intensity, because $\alpha^L \cdot w_L L^* > \alpha^H \cdot (w_H H^* - f_B)$. The intuition is twofold: First, our assumption imply that the taxation of unskilled outweighs taxation of skilled, and second, a higher tax rate decreases the income risk of time investment in education, and therefore provides an insurance effect.

Evaluating the expected utility function in (4) at the optimal labor supplies, H^*, L^*, and the optimal learning effort, e^*, the indirect expected utility function of the household can be written as

$$\mathrm{E}[V^*(t, T, f_B, E)] = V(t, T, f_B, E) + p(e^*, E) \cdot V^H(t, T, f_B) + [1 - p(e^*, E)] \cdot V^L(t, T). \tag{15}$$

It is important to note, that $\mathrm{E}[V^*]$ is a function of the policy mix chosen by the government. This policy mix is exogenously given for the households. By using the envelope-theorem we can derive the marginal impact of a policy change on the expected utility of household:

$$\frac{\partial \mathrm{E}[V^*]}{\partial f_B} = -p^* \cdot \alpha^H \cdot (1-t) < 0, \tag{16}$$

$$\frac{\partial \mathrm{E}[V^*]}{\partial T} = p^* \cdot \alpha^H + (1-p^*) \cdot \alpha^L > 0 \tag{17}$$

$$\frac{\partial \mathrm{E}[V^*]}{\partial t} = -p^* \cdot \alpha^H \cdot [w_H H^* - f_B] - (1-p^*) \cdot \alpha^L \cdot w_L L^* < 0 \tag{18}$$

$$\frac{\partial \mathrm{E}[V^*]}{\partial E} = p^*_E \cdot \left[V^H - V^L\right] > 0. \tag{19}$$

4 Public Policy

The benevolent government aims to maximize social welfare. Therefore, it can influence the quality of the education system by choosing the public spending in education E, and it can grant a lump-sum transfer T. Overall expenditure $E + T$ must be financed by tuition fees, f_B, and by a proportional wage tax at rate t. We should stress again that the educational risk is idiosyncratic, and therefore there is no aggregate risk. From the government's perspective, there are $p(e^*, E)$

skilled workers supplying $p \cdot H$ efficiency units of skilled labour and $[1-p(e^*, E)]$ unskilled workers supplying $(1-p^*) \cdot L$ efficiency units of unskilled labour.

Thus, the governmental budget constraint can be written as

$$E + T = p^* \cdot [tw_H H + (1-t) f_B] + (1-p^*) \cdot tw_L L. \tag{20}$$

Using E, the government can directly influence the percentage of skilled worker, and using the tax instruments both redistributes income between skilled and unskilled households and affects indirectly the shares of skilled and unskilled workers via incentives for learning effort. Last but not least, the wage tax t provides a partial insurance against income fluctuations, and therefore against the educational risk.

We are now able to state some first results. Let us assume for a moment that all expenditure E is financed by a lump-sum tax $T < 0$.

Corollary 1. *It is not optimal to finance the education system by a pure lump-sum tax. Introducing (i) a wage tax or (ii) tuition fees and simultaneously reducing the lump-sum tax burden is welfare improving.*

Proof. Assume that initially $E = -T$ and $t = f_B = 0$. Let us then enact a balanced-budget policy reform with an introduction of either a wage tax $t > 0$ or tuition fees $f_B > 0$, and a simultaneous reduction in the lump-sum tax, such that in both cases total spending remains constant $dE = 0$. Implicit differentiation in

(20) with respect to t respective to f_B then yields:

$$\left.\frac{\partial T}{\partial f_B}\right|_{t=f_B=dE=0} = p(e^*, E), \tag{21}$$

$$\left.\frac{\partial T}{\partial t}\right|_{t=f_B=dE=0} = p(e^*, E) \cdot w_H H^* + [1 - p(e^*, E)] \cdot w_L L^*. \tag{22}$$

The welfare effect of introducing tuition fees respective a wage tax can then be derived by taking the derivative of (15) with respect to f_B respective t and observing that T is changed according to (21) and (22):

$$\begin{aligned} \left.\frac{d\mathrm{E}[V^*]}{df_B}\right|_{t=f_B=dE=0} &= \left.\frac{\partial \mathrm{E}[V^*]}{\partial f_B} + \frac{\partial \mathrm{E}[V^*]}{\partial T}\,\frac{\partial T}{\partial f_B}\right|_{t=f_B=dE=0} \\ &= p^* \cdot (1 - p^*) \cdot \left(\alpha^L - \alpha^H\right) > 0, \end{aligned} \tag{23}$$

$$\begin{aligned} \left.\frac{d\mathrm{E}[V^*]}{dt}\right|_{t=f_B=dE=0} &= \left.\frac{\partial \mathrm{E}[V^*]}{\partial t} + \frac{\partial \mathrm{E}[V^*]}{\partial T}\,\frac{\partial T}{\partial t}\right|_{t=f_B=dE=0} \\ &= p^* \cdot (1 - p^*) \cdot (w_H \cdot H^* - w_L L^*) \left(\alpha^L - \alpha^H\right) > 0, \end{aligned} \tag{24}$$

whereby we have used the Envelope results (16) – (18). □

Financing public expenditure partly by tuition fees creates not only an income effect on learning intensity and on labor supply, but also raise a substitution effect in learning, because being skilled gets relatively less attractive. However, around $f_B = 0$, for the first euro of tuition fees, the negative effect of this distortion is overcompensated by the fact that now the skilled workers pay more for their education than unskilled, who failed in getting a degree. The latter effect implies a welfare enhancing redistribution of income from the 'rich' to the 'poor.'

Introducing a wage tax not only affects the learning intensity, but also creates distortions in both skilled and unskilled labor supply. However, the wage tax simultaneously reduces the income risk of educational effort on the individual level, because the gap between high-skilled and low-skilled income is narrowed, and achieves a welfare enhancing redistribution of incomes from a society's point of view. Starting at $t = 0$, the insurance effect (in combination with the redistribution) dominates the induced distortions and especially compensates the negative incentive effects in labor supply.

Hence, the effect of a positive wage tax can be seen as reproducing or extending the seminal results of Eaton and Rosen (1980a,b) in our model of endogenous human capital risk.

The questions, which now force on, are: (i) What is the optimal combination of wage taxes, lump-sum elements and tuition fees in such an environment? (ii) What determines the optimal values of the tax rate t and the fee f_B? These questions will be tackled in the next section.

5 Optimal Taxation and Tuition Fees

The government seeks to maximize social welfare $\mathrm{E}[V^*(E, f_B, t, T)]$ by choosing public spending in education E as well as the financing scheme f_B, t and T. Formally, the problem can be written as:

$$\max_{\{E, f_B, t, T\}} \mathrm{E}[V^*(E, f_B, t, T)] \;\; \text{s.t.} \; [t w_H H^* + (1-t) f_B]\, p + t \cdot w_L L^* (1-p) = E + T \tag{25}$$

Note that the government takes the optimal choice of households as granted and anticipates the reaction of households while making its choice of the policy mix. Forming the Lagrangian, $\mathcal{L}$, and introducing the lagrange multiplier, λ, first order conditions read as follows:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial f_B} &= -p^* \cdot \alpha^H \cdot (1-t) + \lambda \left(p^*(1-t) + tw_H \frac{\partial H^*}{\partial f_B} \right) \\
&+ \lambda \left[tw_H H^* + (1-t) f_B - tw_L L^* \right] p_e^* \frac{\partial e^*}{\partial f_B} = 0 \qquad (26) \\
\frac{\partial \mathcal{L}}{\partial T} &= p^* \alpha^H + (1-p^*)\alpha^L + \lambda \left(t \left[p^* w_H \frac{\partial H^*}{\partial T} + (1-p^*) w_L \frac{\partial L^*}{\partial T} \right] - 1 \right) \\
&+ \lambda \left[tw_H H^* + (1-t) f_B - tw_L L^* \right] p_e^* \frac{\partial e^*}{\partial T} = 0 \qquad (27) \\
\frac{\partial \mathcal{L}}{\partial t} &= -p^* \alpha^H \cdot (w_H H^* - f_B) - (1-p^*)\alpha^L w_L L^* + \lambda \cdot p^* t w_H \frac{\partial H^*}{\partial t} \\
&+ \lambda \left((1-p^*) t w_L \frac{\partial L^*}{\partial t} + \left[tw_H H^* + (1-t) f_B - tw_L L^* \right] p_e^* \frac{\partial e^*}{\partial t} \right) \\
&+ \lambda \left(p^* \left[w_H H^* - f_B \right] + [1-p^*] w_L L^* \right) = 0 \qquad (28) \\
\frac{\partial \mathcal{L}}{\partial E} &= p_E^* \left[V^H - V^L \right] \\
&+ \lambda \left(\left[tw_H H^* + (1-t) f_B - tw_L L^* \right] \left[p_e^* \frac{\partial e^*}{\partial E} + p_E^* \right] - 1 \right) = 0 \qquad (29)
\end{aligned}$$

Solving (26) for $p^* \cdot \alpha^H$, respective solving (27) for $-(1-p^*) \cdot \alpha^L$, and substituting both rearranged expressions into the first order condition (28) results in

$$\begin{aligned}
&(w_L L^* - w_H H^* + f_B) \cdot \lambda \cdot \left\{ p^* + p_e^* \cdot A \cdot \frac{\partial e / \partial f_B}{1-t} + p^* \cdot t w_H \frac{\partial H^* / \partial f_B}{1-t} \right\} \\
&+ w_L L^* \cdot \lambda \cdot \left\{ (-1) + p_e^* \cdot A \cdot \frac{\partial e}{\partial T} + p^* \cdot t w_H \cdot \frac{\partial H^*}{\partial T} + (1-p^*) \cdot t w_L \cdot \frac{\partial L^*}{\partial T} \right\} \qquad (30) \\
&+ \lambda \cdot \{ p^* \cdot (w_H H^* - f_B) + (1-p^*) \cdot w_L L^* \} \\
&+ \lambda \cdot \left\{ p_e^* \cdot A \cdot \frac{\partial e}{\partial t} + p^* \cdot t w_H \cdot \frac{\partial H^*}{\partial t} + (1-p^*) \cdot t w_L \cdot \frac{\partial L^*}{\partial t} \right\} = 0,
\end{aligned}$$

where $A = [tw_H H^* + (1-t) f_B - tw_L L^*]$. After collecting terms and simplifying, whenever possible, we apply the Slutsky equations for the derivatives of labor supplies H^* and L^*. Note thereby that the derivatives of decision variables for the lump-sum transfer T are pure income effects, and that $\frac{\partial H^*/\partial f_B}{1-t} = -\frac{\partial H^*}{\partial T}$. Cancelling income effects, and rearranging then gives

$$p_e^* \cdot A \cdot \left\{ (w_L L^* - w_H H^* + f_B) \cdot \frac{\partial e/\partial f_B}{1-t} + w_L L^* \cdot \frac{\partial e}{\partial T} + \frac{\partial e}{\partial t} \right\} - t \cdot \left[p^* \cdot w_H^2 \cdot S_{HH} + (1-p^*) w_L^2 \cdot S_{LL} \right] = 0, \quad (31)$$

with S_{jj}, $j = H, L$, as substitution effect of labor supply, when its wage changes.

Applying the comparative-static results from equations (11), (12), and (14), we find that

$$(w_L L^* - w_H H^* + f_B) \cdot \frac{\partial e/\partial f_B}{1-t} + w_L L^* \cdot \frac{\partial e}{\partial T} + \frac{\partial e}{\partial t} = 0. \quad (32)$$

If we then define $\epsilon_{HH} = \frac{w_H}{H^*} \cdot S_{HH}$ as the compensated wage elasticity of skilled labor supply, and $\epsilon_{LL} = \frac{w_L}{L^*} \cdot S_{LL}$ as the one of unskilled labor supply, equation (31) then reduces to

$$t \cdot [p^* \cdot w_H H^* \cdot \epsilon_{HH} + (1-p^*) \cdot w_L L^* \cdot \epsilon_{LL}] = 0. \quad (33)$$

Therefore, we can conclude

Proposition 1. *If the government can use tuition fees, which do not depend on income, and have access to an unconstraint lump-sum transfer, it is not optimal to use a proportional wage tax, hence $t = 0$.*

Proof. Unconstraint lump-sum transfer implies that this transfer can turn negative, and can be used in order to finance public educational spending. In this case, we can apply the above calculations, and get from (33) directly $t = 0$, because the compensated elasticities $\epsilon_{jj},\ j = H, L$ are negative, whereas the wage bills of skilled and unskilled worker must be positive, hence the squared bracket in (33) is negative. □

Contrary to standard models featuring risky human capital and taxation (e.g. Eaton and Rosen (1980b), Hamilton (1987)), the distortionary wage tax is not used, although it would provide simultaneously insurance against income risk, and redistribution of resources to households with a higher weight in the social welfare function. As will be seen in the following, the reason is that tuition fees, which do not depend on income, are a superior instrument for redistribution, although they distort learning effort. The latter distortion can then be countered by public spending in the educational sector.

Applying $t = 0$ in the first order condition (26) gives

$$p^* \cdot \left[\lambda - \alpha^H\right] + \lambda \cdot p_e^* \cdot f_B \cdot \frac{\partial e}{\partial f_B} = 0, \tag{34}$$

where $\frac{\partial e}{\partial f_B} < 0$ from (12).

Adding equations (26) and (27), evaluating at $t = 0$, and substituting the comparative static effects (11) and (12), we get

$$(1 - p^*) \cdot \left(\alpha^L - \lambda\right) + \frac{\lambda \cdot \alpha^L \cdot p_e^{*2} \cdot f_B}{SOC(e)} = 0 \tag{35}$$

If we moreover define $\epsilon_{pe} = \frac{p^*(e,E)}{e} \cdot p_e^* > 0$, and $\epsilon_{pE} = \frac{p^*(e,E)}{E} \cdot p_E^* > 0$ as the elasticity of the success probability with respect to a change in learning effort, e, respective public spending for the education system, E, and $\eta_{e,E} = \frac{e}{E} \cdot \frac{\partial e}{\partial E} > 0$ as the elasticity of learning effort with respect to public educational expenditure, we find from rearranging equation (29)

$$E = p^* \cdot \left[\epsilon_{pE} \cdot \frac{V^H - V^L}{\lambda} + f_B \cdot (\epsilon_{pE} + \epsilon_{pe} \cdot \eta_{eE}) \right], \tag{36}$$

and can state

Proposition 2. *The optimal financing scheme includes tuition fees $f_B > 0$ for successful students. Induced distortions in the learning effort are mitigated by an positive public spending in the educational sector, $E > 0$.*

Proof. Assume $f_B < 0$. Because of $p_e > 0$, $\frac{\partial e}{\partial f_B} < 0$ and $SOC(e) < 0$, we then must have $\lambda < \alpha^H$ from (34), respective $\lambda > \alpha^L$ from (35). Taken together, this implies $\alpha^H > \lambda > \alpha^L$. However, this contradicts our assumption of agent monotonicity, because the low-skilled household would have the higher income. Thus $f_B < 0$ is not possible.

$f_B = 0$ would require $\alpha^H = \lambda = \alpha^L$ for the same reasoning as above. However, this is also not possible, because of agent monotonicity and the fact that, in case of $t = f_B = 0$, the government can only apply a general lump sum tax.

Only if $f_B > 0$, this implies $\alpha^H < \lambda < \alpha^L$ from equations (34) and (35), and fits with the assumption of agent monotonicity.

In case of $f_B > 0$, it follows at once from equation (36) that the optimal public spending in the education sector must be positive, because all elasticities and the marginal costs of tax revenue λ are positive, and an inner solution for learning effort e requires $V^H > V^L$.[6] □

Here, redistribution is executed by tuition fees, which have to be paid by suc-

[6] $V^H \leq V^L$ cannot appear, because this would imply $e = 0$, and $p^*(0, E) = 0$, which cannot be socially optimal as long as $w_H > w_L$.

cessful students. The advantage of tuition fees is that they do not distort labor supply, and that they are very efficient in redistributing from the 'rich' to the 'poor.' However, they induce a substitution effect in learning effort, because getting graduated gets less attractive. This inefficiency can be partly countered by public funding of the education sector. The more the government spends on education, the higher will be a) the probability of each student to graduate, and b) – ceteris paribus – the private learning effort.

As tuition fees reduce the income gap between skilled and unskilled worker, and public spending increases the likelihood of getting graduated, the combination of both instruments also has an insurance effect, because the income risk is reduced.

Taken together, efficient redistribution via tuition fees, and the insurance function of the combined instruments, discussed above, allow the government to abstain from the wage tax.

However, public expenditure in the education sector not only depends on tuition fees:

Corollary 2. *Optimal public expenditure for education increases in*

(i) the level of tuition fees f_B,

(ii) the effectivity of the learning technology,

(iii) the complementarity of private learning effort and public spending,

(iv) and in the skill premium, measured in utility, $V^H - V^L$.

Public expenditure is, however, decreasing in the marginal costs of creating tax revenue.

Proof. The proof to this Corollary follows directly from equation (36). (i), (iv), and the decrease in marginal costs λ are straightforward. The effectivity of the learning technology can be measured by the elasticities ϵ_{pe}, and ϵ_{pE}, whereas the complementarity of e and E is an increasing function of η_{eE}. From (36) it follows that the optimal E^* increases in all these elasticities, which proofs parts (ii) and (iii). □

The intuitions behind these results are as follows: The higher tuition fees are, the higher are the distortions in learning effort. This requires higher public spending for education. In fact, this result is similar to the result in Bovenberg and Jacobs (2005). In order to avoid major inefficiencies, when redistributing from skilled to unskilled, subsidies are necessary. Whilst in Bovenberg and Jacobs (2005) direct subsidies are granted, in our model the government subsidizes education indirectly via improved learning technologies.

The higher the effectivity of the learning technology and the more elastic learning effort, the more students can get graduated via educational spending – which can be seen as a kind of redistribution. Last but not least, the greater the difference in utilities of skilled and unskilled worker, the higher the welfare gain, when more students get graduated by public spending.

To close the model, we have to determine the optimal lump-sum transfer. For $t = 0$, the governmental budget constraint reduces to

$$E + T = p^* \cdot f_B. \tag{37}$$

Substituting for $p^* \cdot f_B$ in equation (36), we end up with

$$T = \frac{1 - (\epsilon_{pE} + \epsilon_{pe} \cdot \eta_{eE})}{\epsilon_{pE} + \epsilon_{pe} \cdot \eta_{eE}} \cdot E - p^* \cdot \epsilon_{pE} \cdot \frac{V^H - V^L}{\lambda}. \tag{38}$$

Obviously, the optimal lump-sum transfer turns out to be a lump-sum tax, unless the success probability is very inelastic, and hence unless the learning technology is very inefficient.

Proposition 3. *Some part of public expenditure is financed by a general lump sum tax,* $T < 0$*, if the success probability of a student with respect to public spending is elastic,* $\epsilon_{pE} \geq 1$.

Proof. Proposition 3 follows directly from (38), and recognizing that $V^H > V^L$. □

Thus, if the learning technology is not too inefficient, the educational system will be financed by both the skilled and the unskilled worker. Tuition fees are therefore not used to redistribute income directly to the unskilled, but are used in order to provide better chances in the educational systems. Moreover, the lump-sum tax is increasing in public spending E, and – at least in some range – in the effectivity of public spending ϵ_{pE}, because the second term in (38) tends to infinity, if $\epsilon_{pE} \to \infty$, whereas the first terms tends to nil, if ϵ_{pE}, and the sum tends to infinity.

To-Do-List

- closed form solution for tuition fees f_B ??

- apply a graduate tax (income-dependent tuition fees), and examine, if a wage tax with $t > 0$ gets optimal, then

6 Conclusions

We examine the effects of endogenous human capital risk, where the probability of getting graduated is endogenously determined by individuals, and depends therefore also on tax instruments. We apply a model, where households first choose their learning effort and after realization of risk, they choose their labor supply. We show that a distorting wage tax will not be used, although it would be optimal, if tuition fees are not available. Thus, the standard trade-off between distortions in labor supply and insurance against income risk does not apply. Tuition fees can achieve redistribution between skilled and unskilled households, and grant some insurance. The distortions in learning effort, induced by tuition fees, are mitigated by public spending in the educational sector. In addition, this public education funding is another instrument for redistribution.

The absence of a wage tax depends on the fact that the lump-sum transfer can turn negative, and unskilled worker also have to pay for the education sector, and on income-independent tuition fees. If tuition fees depend on income of the skilled worker (thus are a graduate tax), and distort labor supply, the well-known trade-off between insurance and distortions should apply, and a proportional wage tax with $t > 0$, paid by both skilled and unskilled workers, might be optimal.

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