

# Monetary Policy Implementation under Sovereign Default<sup>1</sup>

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**Preliminary version:** February 15, 2007

## **Abstract**

This paper shows that monetary policy can be conducted in an efficient way, even if revenues from distortionary taxation do not ensure government solvency. When the government holds income tax rates constant, sovereign default can occur. Monetary policy can then be implemented in a (constrained) optimal way, if it either controls the money growth rate or the exchange rate. Setting the interest rate on bonds is however ineffective, since the equilibrium rate of return on savings depends on the endogenous default premium. While low inflation reduces welfare costs, it tends to raise sovereign default.

*JEL classification:* E52, E63.

*Keywords:* Fiscal-monetary policy interactions, fiscal theory of sovereign default, distortionary taxation, money growth policy, exchange rate policy

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<sup>1</sup>The author would like to thank Roel Beetsma, Wouter den Haan, Mark Gertler, Andre Kurmann, Marcos Poplawski Ribeiro, Andrea Tambalotti, Leopold von Thadden, Sweder van Wijnbergen and other seminar participants at the Federal Reserve Bank of New York, the Free University Berlin, and the University of Amsterdam for helpful comments. The usual disclaimer applies.

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# 1 Introduction

Can monetary policy be conducted in an efficient way when fiscal policy is not committed to debt repayment? The concept of fiscal dominance suggests that monetary policy can be severely constrained, when fiscal policy fails to guarantee government solvency, i.e., debt repayment. In particular, the fiscal theory of the price level (FTPL) has forcefully demonstrated that monetary policy is required to passively accommodate fiscal policy, when the government decides on primary surpluses irrespective of outstanding debt. Monetary debt revaluation can then render the conduct of efficient monetary policy impossible, as emphasized by Woodford (1996) or Sims (2005).<sup>3</sup> This paper demonstrates that monetary policy can be conducted in a (constrained) optimal way, when full debt repayment is not guaranteed such that sovereign default might occur. This however requires *i.*) the central bank not to apply the interest rate as its instrument, and *ii.*) costs of sovereign default are negligible.

As shown in Uribe's (2006) "Fiscal Theory of Sovereign Risk", where the primary surplus has been assumed to be exogenous, sovereign default can emerge due to intertemporal insolvency. In contrast to his set-up, the equilibrium default rate depends on future revenues from distortionary taxes and from seigniorage. Non-distortionary means of government financing are not available and the fiscal authority is assumed to smooth income tax rates. Repayment of initial debt obligations is then not guaranteed, and sovereign default can occur if endogenous revenues from income taxation are too low.<sup>4</sup> Like in Uribe (2006) we do not consider the case where the fiscal authority strategically defaults on debt, and commits to equally distribute available revenues to lenders in case of (partial) default. We further accounts for a substantial role of money and monetary policy due to transactions frictions (a cash-constraint) that induces money to be valued in equilibrium. As a consequence, monetary policy affects the equilibrium allocation and the equilibrium default rate. A low inflation policy is, for example, welfare enhancing (since it reduces the inflation tax on consumption), while it raises the default rate.

The analysis in the main part of the paper further shows that the means of monetary policy implementation is decisive for the existence and the uniqueness of an equilibrium. Since bonds are associated with default risk, which is taken into account by investors (households) in a perfect foresight equilibrium, the contractual interest rate on public debt does not directly affect the saving and consumption decision of households. An interest rate policy thus becomes ineffective such that the equilibrium allocation and the associated price system are indetermined. If however the central bank controls the money

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<sup>3</sup>Benigno and Woodford (2006) qualify this conclusion and show that an inflation targeting regime can be implemented even under an "exogenous" fiscal policy regime. Nevertheless, monetary policy cannot prevent social welfare from being reduced compared to the case where fiscal policy guarantees solvency.

<sup>4</sup>The property that tax revenues from distortionary taxation might not be sufficient to ensure government solvency has been stressed by Woodford (1998) and Schabert and von Thadden (2006).

growth rate, the equilibrium allocation and the associated price system can uniquely be determined, while sovereign default is in general not ruled out.

In the last part of the paper, the analysis is extended to the case of a small open economy. Here, the same principles apply. The novel contribution is that an exchange rate policy is shown to be favorable to an interest rate policy under sovereign default risk, just like a money growth policy in the closed economy case. In particular, the equilibrium allocation and the associated price system are uniquely determined under a fixed depreciation rate. The default rate on government debt, which is denominated in domestic currency, then rises with a lower nominal value of the domestic currency and a higher rate of depreciation. Concisely, when sovereign default is possible, policy regimes that are known to provide a nominal anchor (in a default-free economy) are preferable to a regime that controls the interest rate.

The remainder of this paper is organized as follows. Section 2 develops the model. In section 3 examines the perfect foresight equilibrium under different policy regimes in a closed economy. Section 4 extends the analysis to the case of a open economy. Section 5 concludes.

## 2 The model

This section presents a closed economy monetary model, with two distortions, i.e., Lucas' (1982) cash-credit good distortion and a tax distortion. The analysis focusses on the case where the fiscal authority does not to have access to non-distortionary taxation, and that it does not raise tax revenues with the aim to guarantee intertemporal solvency. Current and future revenues from distortionary taxation might therefore be insufficient to finance government debt obligations, such that sovereign default might occur in equilibrium. Following Uribe (2006), the fiscal authority is assumed to be committed to equally distribute available revenues from a particular tax policy to lenders in case of (partial) default. Thus, strategic default on sovereign debt (like in Eaton and Gersovitz, 1981) is disregarded.

### 2.1 The private sector

There exists a continuum of infinitely lived and identical households of mass one. Their utility increases in consumption  $c_t$  and decreases in working time  $l_t$ , the latter variable being bounded by some finite value  $\bar{l}$  such that  $l_t \in (0, \bar{l})$ . The objective of a representative household is given by

$$\max \sum_{t=0}^{\infty} \beta^t \left[ \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{l_t^{1+\sigma_l}}{1+\sigma_l} \right], \quad \text{with } \sigma \geq 0, \quad \sigma_l \geq 0, \quad \beta \in (0, 1), \quad (1)$$

where  $\beta$  denotes the discount factor,  $\sigma$  represents the inverse of the intertemporal elasticity of substitution in consumption and  $\sigma_l$  denotes the inverse of the Frisch elasticity of labour supply.

Households enter a period  $t$  with two types of nominal assets, money balances  $M_{t-1}$  and interest bearing government debt  $B_{t-1}$ . The latter is issued in the form of one-period riskless bonds, earning a net interest rate  $i_{t-1}$  in period  $t$ . Labor income is taxed with a tax rate  $\tau_t \in (0, 1)$ . The budget constraint reads

$$P_t c_t + \frac{B_t}{R_t} + M_t \leq (1 - \tau_t) P_t w_t l_t + (1 - \delta_t) B_{t-1} + M_{t-1},$$

where we take into account (by  $\delta_t$ ) that government debt might not be fully repaid. We further assume that households have to satisfy a no-Ponzi game condition,  $\lim_{k \rightarrow \infty} (M_{t+k} + B_{t+k}/R_{t+k}) \prod_{i=1}^k \frac{\pi_{t+i}}{(1 - \delta_{t+i}) R_{t+i-1}} \geq 0$ , and cannot issue money,  $M_t \geq 0$ . After they leave the asset markets, households enter the goods market. We assume that they rely on cash as a means of payment in both markets. Thus, they are restricted by the following cash-in-advance constraint, which accords to the specification in Lucas (1982):

$$P_t c_t \leq M_{t-1} + (1 - \delta_t) B_{t-1} - B_t R_t^{-1} \quad (2)$$

Maximizing life-time utility with respect to the budget constraint, non-negativity constraints, the cash constraint, taking prices and initial asset endowments  $M_{-1} > 0$  and  $B_{-1} > 0$  as given, leads to the following first order conditions  $\forall t \geq 0$

$$c_t^{-\sigma} = \lambda_t + \psi_t, \quad l_t^{\sigma_l} = \lambda_t (1 - \tau_t) w_t, \quad (3)$$

$$(\lambda_t + \psi_t) R_t^{-1} = \beta (1 - \delta_{t+1}) \pi_{t+1}^{-1} (\lambda_{t+1} + \psi_{t+1}), \quad (4)$$

$$\lambda_t = \beta \pi_{t+1}^{-1} (\lambda_{t+1} + \psi_{t+1}), \quad (5)$$

(2) and  $\psi_t (m_{t-1} \pi_t^{-1} + (1 - \delta_t) b_{t-1} \pi_t^{-1} - b_t R_t^{-1} - c_t) \geq 0$ , where  $\psi_t \geq 0$  is the multiplier on the cash-constraint. The no-Ponzi game condition further holds with equality, while the non-negativity constraint on money will only be binding for  $t \rightarrow \infty$ . As a consequence, the household's optimum will be characterized by the following transversality conditions (TVCs) for money and bond holdings

$$\lim_{k \rightarrow \infty} \frac{b_{t+k}}{R_{t+k}} \prod_{i=1}^k \frac{\pi_{t+i}}{(1 - \delta_{t+i}) R_{t+i-1}} = 0 \quad \text{and} \quad \lim_{k \rightarrow \infty} m_{t+k} \prod_{i=1}^k \frac{\pi_{t+i}}{(1 - \delta_{t+i}) R_{t+i-1}} = 0. \quad (6)$$

Combining the first order conditions (4) and (5), gives  $\psi_t = [(1 - \delta_{t+1}) R_t - 1] \lambda_t$ . Eliminating the multiplier  $\lambda_t$  and  $\psi_t$  in (3) then leads to the following conditions

$$c_t^{\sigma} l_t^{\sigma_l} = \frac{(1 - \tau_t) w_t}{R_t (1 - \delta_{t+1})}, \quad \text{and} \quad \frac{c_{t+1}^{\sigma}}{c_t^{\sigma}} = \beta \frac{(1 - \delta_{t+1}) R_t}{\pi_{t+1}}, \quad (7)$$

Note that consumption depends – due to the cash-credit good distortion – on the difference between the rate return on money and the rate of return on debt, i.e., on  $(1 - \delta_{t+1}) R_t$ .<sup>5</sup>

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<sup>5</sup>For the case where utility is logarithmic in consumption,  $\sigma = 1$ , and linear in labor,  $\sigma_l = 0$ , which will

Moreover, perfectly competitive firms produce the final good  $y_t$  with a simple linear technology  $y_t = l_t$ , leading to a profit maximizing real wage equal to one:  $w_t = 1$ .

## 2.2 The public sector

The public sector consists of a central bank and a fiscal authority. The central bank (CB) either sets the nominal interest rate on bonds  $R_t$  or the money growth rate  $\mu_t = M_t/M_{t-1} = m_t\pi_t/m_{t-1}$ , where  $m_t$  denotes real balances  $m_t = M_t/P_t$ . It transfers seigniorage revenues to the fiscal authority. The budget of the CB reads

$$M_t - M_{t-1} = P_t\tau_t^m$$

$\Leftrightarrow (\mu_t - 1)m_t/\mu_t = \tau_t^m$ , where  $\tau_t^m$  denotes a transfer to the fiscal authority. We consider the realistic case, where the government does not have access to lump-sum taxation. It issues debt, raises revenues by taxing labor income, and purchases the amount  $g_t$  of the final good in each period. The consolidated (government) budget constraint is given by

$$B_t R_t^{-1} + P_t \tau_t w_t l_t + P_t \tau_t^m = P_t g_t + (1 - \delta_t) B_{t-1}$$

$\Leftrightarrow b_t R_t^{-1} + s_t = (1 - \delta_t) b_{t-1}/\pi_t$ , where  $b_t = B_t/P_t$  and  $\pi_t = P_t/P_{t-1}$  and  $s_t$  denotes the tax/transfer revenues net of expenditures  $s_t = \tau_t w_t l_t + \tau_t^m - g_t$ . Solving the flow budget constraint forward, leads to the following intertemporal government budget constraint (IGB)

$$(1 - \delta_t) B_{t-1}/P_t = \sum_{k=0}^{\infty} s_{t+k} \prod_{i=1}^k \frac{\pi_{t+i}}{(1 - \delta_{t+i}) R_{t+i-1}} + \lim_{k \rightarrow \infty} b_{t+k} R_{t+k}^{-1} \prod_{i=1}^k \frac{\pi_{t+i}}{(1 - \delta_{t+i}) R_{t+i-1}} \quad (8)$$

Below it will be assumed that the share of government expenditures  $g_t = \gamma c_t$  is exogenous. Following Uribe (2006), we assume that the fiscal authority decides on the tax rates and on government spending without considering initial outstanding debt. It firstly chooses a tax policy in a way that is consistent with the minimization of welfare-reducing tax distortions, and secondly commits to fulfill debt repayment obligations as far as possible for any equilibrium sequence of primary surpluses. Hence, sovereign default can occur when current and future revenues from income taxation and from seigniorage are too low.

## 2.3 Perfect foresight equilibrium

Prices adjust to clear markets for goods, labor, and assets. Households' initial asset endowments were assumed to be positive, implying that the government is initially indebted. When the labor income tax is not set in a way that ensures government solvency for any

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be examined below, these conditions read  $\frac{c_{t+1}}{c_t} = \beta \frac{(1 - \delta_{t+1}) R_t}{\pi_{t+1}}$  and  $c_t = \frac{1 - \tau_t}{R_t (1 - \delta_{t+1})}$ .

equilibrium allocation and price system, the TVCs given in (6) become relevant equilibrium conditions, which has been exploited for price level determination in the "Fiscal Theory of the Price Level" (FTPL) (see Kocherlakota and Phelan, 1999). Combining the TVC on bonds and the IGB constraint, yields

$$(1 - \delta_t) \frac{b_{t-1}}{\pi_t} = \sum_{k=0}^{\infty} (\tau_{t+k} w_{t+k} l_{t+k} - g_{t+k} + \tau_{t+k}^m) \prod_{i=1}^k \frac{\pi_{t+i}}{(1 - \delta_{t+i}) R_{t+i-1}} \quad (9)$$

In accordance with Uribe's (2006) "Fiscal Theory of Sovereign Risk" and in contrast to the FTPL, we do not restrict equilibria to be characterized by full repayment of government debt which would imply  $\delta_t = 0$  (see 8). Here, sovereign default  $\delta_t > 0$  is possible, when tax policy does not guarantees intertemporal solvency for any equilibrium allocation and price system. We further assume that the possibility of default is take into account by lenders, which accords to the arguments on equilibrium consistency in Buiter (2002) or Niepelt (2004).

A PFE is thus a set of sequences  $\{c_t, l_t \in [0, \bar{l}], w_t, \tau_t^m, m_t \geq 0, \pi_t \geq 0, \tau_t \in [0, 1], b_t \geq 0, \delta_t < 1, R_t \geq 1\}_{t=0}^{\infty}$  satisfying (7),  $m_t \geq c_t + g_t - \tau_t w_t l_t$ ,  $c_t + g_t = l_t$ ,  $b_t R_t^{-1} + \tau_t w_t l_t + \tau_t^m - g_t = (1 - \delta_t) b_{t-1} / \pi_t$ ,  $w_t = 1$ , and (9) where  $\tau_t^m = \frac{\mu_t - 1}{\mu_t} m_t$ , a fiscal policy setting  $\{\tau_t\}_{t=0}^{\infty}$  and  $\{g_t\}_{t=0}^{\infty}$ , and a monetary policy setting  $\{R_t\}_{t=0}^{\infty}$  or  $\{\mu_t\}_{t=0}^{\infty}$  where  $\mu_t = m_t \pi_t / m_{t-1}$ , the TVC  $\lim_{k \rightarrow \infty} m_{t+k} \prod_{i=1}^k \frac{\pi_{t+i}}{(1 - \delta_{t+i}) R_{t+i-1}}$ , and the initial asset endowments  $M_{-1} > 0$  and  $B_{-1} > 0$ .

It should be noted that we restrict the equilibrium default rate not to exceed one,  $\delta_t \leq 1$ , since we focus on the case where the government does not have access to non-distortionary taxation. However, we allow for the case where the government redistributes revenues (for example from seigniorage or income taxation) by subsidizing bond holdings,  $\delta_t < 0$ .

### 3 Macroeconomic policy and equilibrium default

In this section we assess the *PFE* under endogenous sovereign default. In the first part we discuss some equilibrium properties for constant income tax rates. In the second part of the section we examine monetary policy implementation. In the third and last part, we briefly discuss the case where default is not socially costless.

It should be noted the Friedman rule has to be reinterpreted when sovereign default is possible. In particular, the product  $(1 - \delta_{t+1}) R_t$  measures the opportunity costs of cash, such that the elimination of the money demand distortion does not necessarily imply the nominal interest rate  $R_t$  to equal one. Taking a look at the Ramsey problem for this economy shows that the Ramsey plan is characterized by the following condition for the

policy instruments  $\tau_t$  and  $R_t$  (see Appendix)

$$\frac{1 - \tau_t}{(1 - \delta_{t+1})R_t} = \frac{1 + \Phi(1 - \sigma)}{1 + \Phi(1 + \sigma_l)} \quad \forall t \geq 1, \quad (10)$$

where  $\Phi \geq 0$  is the multiplier on the implementability constraint, which equals zero only if government expenditures  $g_t$  and real value of initial public liabilities  $((1 - \delta_0)B_{-1} + M_{-1})/P_0$  equal zero. Evidently, the first best allocation can only then be implemented, which requires  $(1 - \delta_{t+1})R_t = 1$  and  $\tau_t = 0$ . Given that we assume that initial public liabilities and government expenditures,  $\gamma > 0$ , are in general not equal to zero, the multiplier on the implementability constraint will be strictly positive  $\Phi > 0$  in this economy. Since higher initial liabilities tightens the restriction on feasible policy regimes, the multiplier  $\Phi$  decreases with the default rate for given values for  $B_{-1}$ ,  $M_{-1}$ , and  $P_0$ . Further using that the RHS of (10) falls with  $\Phi$ , it follows that the opportunity costs of money holdings under the Ramsey plan tend to lower when the default rate is not restricted to equal zero.

Hence, social welfare can be increased when the default rate exceeds zero. In any case, there are infinitely many combinations of the effective nominal return on debt  $(1 - \delta_{t+1})R_t$  and the tax rate  $\tau_t$  that are consistent with the Ramsey plan.<sup>6</sup> In what follows we consider the case where the CIA constraint is binding,  $(1 - \delta_{t+1})R_t > 1 \Rightarrow \psi_t > 0$ .

### 3.1 Constant income tax rates

The analysis in this paper focuses on the case where monetary and fiscal policy is not conducted on a coordinated way. In contrast, we examine the case, which corresponds the case usually considered in studies on "fiscal dominance" (see e.g. Loyo, 1999), where the fiscal authority sets tax rates irrespective of its outstanding liabilities and of monetary policy. In particular, we assume that the fiscal authority holds the tax rate constant. This assumption can evidently been justified with the minimization of tax distortions (like in Woodford, 1998).<sup>7</sup> To further facilitate the derivation of closed form solutions, we assume that households' contemporaneous utility is logarithmic in consumption and linear in labor; the latter is often used in studies on monetary policy and equilibrium determination (see e.g. Carlstrom and Fuerst, 2005).

**Assumption 1** *Household preferences satisfy  $\sigma = 1$  and  $\sigma_l = 0$ .*

Since we assumed that initial household bond holdings are strictly positive,  $B_{-1} > 0$ , the fiscal authority has to raise tax revenues to repay outstanding debt. Throughout the remainder of the paper, we focus on the case where the tax rate is held constant. Moreover,

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<sup>6</sup>See Chari and Kehoe (1999) for a discussion of this type of indeterminacy of policy instruments under the Ramsey plan.

<sup>7</sup>The assumption of a constant distortionary tax rate has recently also been considered in Benigno and Woodford (2006) for the analysis of the implementation of an inflation targeting policy under different fiscal policy regimes.

we assume that the central bank sets its instrument in a way that is consistent with a binding cash constraint,  $\psi_t > 0$ . We further assume that the government expenditure share is constant.

**Assumption 2** *The fiscal authority holds the income tax rate  $\tau_t = \tau \in [0, 1]$  and the expenditure share constant  $g_t/c_t = \gamma \in (0, 1)$ , while monetary policy is consistent with a binding cash-in-advance constraint.*

Under assumption 1 and 2 a *PFE* for a binding cash-in-advance constraint can be reduced to a system in  $c_t$ ,  $m_t$ ,  $\pi_t$ ,  $R_t$ ,  $\delta_t$ , and  $b_t$  satisfying

$$c_t = \frac{1 - \tau}{R_t(1 - \delta_{t+1})}, \quad (11)$$

$$\frac{c_{t+1}}{c_t} = \beta \frac{(1 - \delta_{t+1}) R_t}{\pi_{t+1}}, \quad (12)$$

$$m_t = c_t (1 - \tau) (1 + \gamma) \quad (13)$$

$$b_t/R_t = [\gamma - \tau(1 + \gamma)] c_t - \tau_t^m + (1 - \delta_t) b_{t-1}/\pi_t \quad (14)$$

$$(1 - \delta_t) \frac{b_{t-1}}{\pi_t} = \sum_{k=0}^{\infty} ([\tau(1 + \gamma) - \gamma] c_{t+k} + \tau_{t+k}^m) \prod_{i=1}^k \frac{\pi_{t+i}}{(1 - \delta_{t+i}) R_{t+i-1}} \quad (15)$$

where  $\tau_t^m(m_{t-1}, \pi_t) = \frac{\mu_t - 1}{\mu_t} m_t$ , a monetary policy, and the TVC for money holdings, taking as given the initial values  $M_{-1} > 0$ , and  $B_{-1} > 0$ . Note that (14) is the consolidated flow budget constraint of the fiscal authority, whereas (15) stems from TVC on bonds (see 6) combined with (8). Condition (11) relates the marginal utility of consumption to the marginal disutility of labor. This condition further features the constant tax rate  $\tau$  and the time-varying distortion from the cash-credit good friction  $(1 - \delta_{t+1}) R_t$ . Since the equilibrium price of debt depends on the growth rate of consumption (12), the allocation and nominal price of debt cannot separately be pinned down. Using that (12) implies  $\prod_{i=1}^k \frac{\pi_{t+i}}{(1 - \delta_{t+i}) R_{t+i-1}} = \beta^k \frac{c_t}{c_{t+k}}$  and eliminating real balances with (13), the IGB (15) can be rewritten as

$$\begin{aligned} (1 - \delta_t) \frac{b_{t-1}}{\pi_t} &= \sum_{k=0}^{\infty} \beta^k \frac{c_t}{c_{t+k}} \left( [\tau(1 + \gamma) - \gamma] c_{t+k} + \frac{\mu_{t+k} - 1}{\mu_{t+k}} c_{t+k} (1 - \tau) \right) \\ &= c_t \sum_{k=0}^{\infty} \beta^k \left( \tau(1 + \gamma) - \gamma + \frac{\mu_{t+k} - 1}{\mu_{t+k}} (1 - \tau) \right) \end{aligned} \quad (16)$$

It should be noted that there are six unknowns and six equilibrium conditions, (11)-(13) and (16) plus a monetary policy specification. Nonetheless, one cannot uniquely determine the sequence of real debt in equilibrium, i.e., the equilibrium allocation and the associated price system is consistent with infinitely many sequences for real debt. To see this, consider an equilibrium sequence for consumption  $\{c_t\}_{t=0}^{\infty}$ . Given that the tax rate is constant, it is by (11) associated with a unique sequence for  $R_t(1 - \delta_{t+1}) \forall t \geq 0$ , for real balances

$\forall t \geq 0$  (see 13), and for the inflation rate  $\forall t \geq 1$  (see 12).

Under full debt repayment,  $\delta_0 = 0$ , (16) for period  $t = 0$  reads

$$B_{-1}/P_0 = c_0 \sum_{k=0}^{\infty} \beta^k \left( \tau (1 + \gamma) - \gamma + \frac{\mu_k - 1}{\mu_k} (1 - \tau) \right),$$

and can be using  $\tau_t^m = \frac{m_t P_t M_{t-1}^{-1} - 1}{m_t P_t M_{t-1}^{-1}} m_t$  be written as

$$B_{-1}/c_0 = P_0 \frac{\tau (1 + \gamma) - \gamma}{1 - \beta} + \frac{m_0 P_0 M_{-1}^{-1} - 1}{m_0 M_{-1}^{-1}} (1 - \tau) + P_0 \sum_{k=0}^{\infty} \beta^{k+1} \frac{m_{t+k+1} \pi_{t+k+1} m_{t+k}^{-1} - 1}{m_{t+k+1} \pi_{t+k+1} m_{t+k}^{-1}} (1 - \tau)$$

For a particular equilibrium allocation (and thus for given sequences  $\{R_t(1 - \delta_{t+1})\}_{t=0}^{\infty}$ ,  $\{m_t\}_{t=0}^{\infty}$ , and  $\{\pi_{t+1}\}_{t=1}^{\infty}$ ) the TVC pins down a unique value  $P_0$ , for given initial values  $B_{-1}$  and  $M_{-1}$ , and (with  $\{\pi_{t+1}\}_{t=1}^{\infty}$ ) the entire price level sequence. The flow government budget constraint (14) for  $t = 0$ ,

$$b_0/R_0 = [\gamma - \tau (1 + \gamma)] c_0 - \tau_0^m (m_0, P_0 M_{-1}^{-1}, \tau) + B_{-1}/P_0,$$

determines the subsequent real value of debt  $b_0$ , since the sequence  $\{R_t\}_{t=0}^{\infty}$  is given for  $\delta_t = 0$ . Thus, under full debt repayment a particular equilibrium allocation is consistent with exactly one sequence for real debt  $\{b_t\}_{t=0}^{\infty}$ .

When the fiscal authority is not committed to full debt repayment, this logic of price level and real debt determination immediately breaks down. Since  $\delta_0$  and  $P_0$  cannot simultaneously be determined from (16) for  $t = 0$  for a given equilibrium allocation. Thus, there are infinitely many pairs of sequence for real debt and the default rate, which a consistent with a particular equilibrium allocation. This property corresponds to the well-established real bonds indeterminacy property of an equilibrium in an infinite horizon framework under lump-sum taxation (see e.g. Canzoneri and Diba, 2005).

**Proposition 1** *A PFE under assumption 1 and 2 is consistent with infinitely many sequences of real debt.*

Taking this real debt indeterminacy property as given, we restrict our attention to the period  $t = 0$  version of (15) and on the determination of the remaining macroeconomic variables. Eliminating consumption in (11) and (12), the set of equilibrium conditions can be reduced, leading to the following equilibrium definition.

**Definition 1** *A PFE under assumption 1 and 2 and a binding cash-in-advance constraint is a set of sequences  $\{m_t \geq 0, \pi_t \geq 0, R_t \geq 1, \delta_t < 1\}_{t=0}^{\infty}$  and an initial price level  $P_0 > 0$*

satisfying

$$m_t = \frac{(1 - \tau)^2 (1 + \gamma)}{R_t(1 - \delta_{t+1})} \quad (17)$$

$$\beta \frac{R_t(1 - \delta_{t+1})}{\pi_{t+1}} = \frac{m_{t+1}}{m_t} \quad (18)$$

$$(1 - \delta_0) B_{-1}/P_0 = \frac{m_0}{1 + \gamma} \sum_{k=0}^{\infty} \beta^k \left( \frac{\tau(1 + \gamma) - \gamma}{1 - \tau} + \frac{\mu_{t+k} - 1}{\mu_{t+k}} \right) \quad (19)$$

where  $\mu_t = \frac{m_t \pi_t}{m_{t-1}}$ , and a monetary policy, taking as given the initial values  $M_{-1} > 0$ , and  $B_{-1} > 0$ .

In contrast to the case where sovereign default is neglected, the opportunity costs of money are not equal to the nominal interest rate. Due to sovereign default the nominal interest rate on bonds becomes ineffective. As a consequence, the equilibrium allocation  $\{m_t\}_{t=0}^{\infty}$  depends on the effective rate of return  $R_t(1 - \delta_{t+1})$  and thus on the default rate. Since the sequence of debt is except of  $B_{-1}$  unknown, one can at best determine the initial default rate in equilibrium. When the equilibrium allocation and the equilibrium price level sequence  $\{P_t > 0\}_{t=0}^{\infty}$  are given,  $\delta_0$  is uniquely determined by initial nominal debt  $B_{-1}$ . Yet, all subsequent default rates  $\{\delta_t\}_{t=1}^{\infty}$  and contractual interest rates  $\{R_t \geq 1\}_{t=0}^{\infty}$  are indetermined. The following proposition summarized the main equilibrium properties.

**Proposition 2** *Suppose that assumption 1 and 2 hold.*

1. *For any equilibrium allocation  $\{m_t > 0\}_{t=0}^{\infty}$  (17) uniquely determines the sequence of the effective nominal rate of return  $\{(1 - \delta_{t+1}) R_t\} \forall t \geq 0$ , and (18) determines a sequence of future inflation rates  $\{\pi_t > 0\} \forall t \geq 1$ .*
2. *For any equilibrium allocation  $\{m_t > 0\}_{t=0}^{\infty}$ , a sequence of money growth rates  $\{\mu_t\} \forall t \geq 0$  and an initial price level  $P_0 > 0$ , (19) determines the initial equilibrium default rate  $\delta_0$ , while there exists infinitely many sequences of contractual interest rates  $R_t \geq 1 \forall t \geq 0$  and of equilibrium default rates  $\delta_t \forall t \geq 1$  consistent with a PFE.*

It should be noted that one can in principle derive a unique equilibrium sequence for the market value of newly issued debt  $b_t/R_t$  from (14), which will be shown below. Nevertheless, since there exist infinitely many nominal interest rates that are consistent with the equilibrium allocation  $\{m_t\}_{t=0}^{\infty}$ , real bonds  $b_t$  are indetermined for all periods  $t \geq 0$  (see proposition 1).

### 3.2 Monetary policy implementation

In this section we examine the implementation of monetary policy and its impact on the determination of the equilibrium allocation and the associated price system. The central bank either sets the contractual interest rate  $R_t$  or the money growth rate  $\mu_t$ . Like in Schabert (2006), where the default rate is restricted to equal zero and taxes are lump-sum,

the choice of the monetary policy instrument will be decisive for equilibrium determination when the fiscal authority sets the tax rate irrespective of intertemporal solvency.

### 3.2.1 Interest rate policy

Suppose that the central bank uses the nominal interest rate  $R_t$  as its instrument. Before we turn to the case with equilibrium default, we assume – to facilitate comparisons with the FTPL – that there is no default.

**The case of full debt repayment** Suppose that sovereign default is ruled out by assumption  $\delta_t = 0$  and that the central bank pegs the nominal interest rate  $R_t = R$ . This set-up relates to the well-known "Fiscal Theory of the Price Level" (see Woodford, 1994), where the price level has to jump in a way that the intertemporal government budget constraint is satisfied in equilibrium when the primary surplus is exogenous. As shown by Woodford (1998) and Schabert and von Thadden (2006), the same logic of price level determination can apply when distortionary taxes lead to insufficiently low revenues.

Then, the equilibrium allocation is immediately determined by  $m_t = (1-\tau)^2 (1+\gamma) R^{-1}$  and inflation is pinned down by  $\pi = \beta R \forall t \geq 1$ . Further, the initial price level can be determined by using the IGB constraint with  $\mu_t = \frac{m_t}{m_{t-1}} \pi_t = \beta R$ :

$$P_0^{-1} = \frac{1}{B_{-1}} \frac{m_0}{1+\gamma} \frac{1}{1-\beta} \left( \frac{\tau(1+\gamma) - \gamma}{1-\tau} + \frac{\beta R - 1}{\beta R} \right) = \frac{\Theta(\tau, R)}{B_{-1}},$$

where  $\Theta(\tau, R) = \frac{\tau + R\beta(1-\gamma(1-\tau)) - 1}{\beta(1-\beta)R^2} (1-\tau)$

Hence, an equilibrium exists only if the interest rate is sufficiently high  $R > \frac{1-\tau}{\beta(1-\gamma(1-\tau))} \Rightarrow P_0 > 0$ . The partial derivatives of  $\Theta$  are given by  $\Theta_\tau(\tau, R) = -\frac{R\beta(1-2\gamma(1-\tau))-2(1-\tau)}{(1-\beta)R^2\beta}$  and  $\Theta_R(\tau, R) = -(1-\tau) \frac{R\beta(1-\gamma(1-\tau))-2(1-\tau)}{(1-\beta)R^3\beta}$ . Hence, the initial price level tends to be lower for higher tax rates and interest rates, as long as the tax rate is sufficiently high.

Actually, the price level has to jump in a way that ensures debt repayment. Given that the inflation rate  $\pi_1 = P_1/P_0$  is known, the entire price level sequence is also determined, like in the standard FTPL case with an exogenous primary surplus.

**Proposition 1 (FTPL)** *Suppose that debt is fully repaid,  $\delta_t = 0$ , and the central bank pegs the interest rate,  $R_t = R$ . Then, the equilibrium allocation and the price level are uniquely determined only if  $R > \frac{1-\tau}{\beta(1-\gamma(1-\tau))}$ .*

Note that the equilibrium allocation would also be uniquely determined under an interest rate peg, if taxes were lump-sum and sufficiently high to endogenously ensure solvency  $\delta_t = 0$  (see Schabert and von Thadden, 2006). Yet, under such a policy regime, which often called a "Ricardian policy", the equilibrium exhibits nominal indeterminacy, i.e., it is consistent with infinitely many price level sequences.

**Equilibrium under sovereign default** We now return to the case where full debt repayment is not guaranteed by price level jumps. Applying the corollaries from above immediately implies that any monetary policy regime which sets  $R_t$  exogenously or contingent on macroeconomic indicators, like  $\pi_{t+i}$ ,  $c_{t+i}$ , or  $y_{t+i}$  for  $i \lesseqgtr 0$ , will leave the equilibrium allocation and the price level indetermined. Thus, the classical nominal indeterminacy problem under interest rate policy is aggravated when sovereign default is taken into account.

**Proposition 2 (IR)** *If the central bank sets the nominal interest rate  $R_t$  in an exogenous way or as a function of macroeconomic indicators, the equilibrium allocation and the equilibrium price level cannot be determined.*

If one would assume that the central bank is able to set the effective rate of return on bonds,  $(1 - \delta_{t+1}) R_t$ , then it is evidently able to uniquely implement an equilibrium allocation and future inflation rates. Yet, there would exist infinitely many consistent pairs of initial default rates  $\delta_0$  and initial price level  $P_0$  (see 19).

### 3.2.2 Constant money growth

Now suppose that the central bank supplies money according to a constant money growth rate  $\mu = M_t/M_{t-1} = m_t\pi_t/m_{t-1} \forall t \geq 0$ . A money growth policy is consistent with a binding CIA if the money growth rate exceeds the Friedman rule  $\mu > \beta \Rightarrow \psi_t = [(1 - \delta_{t+1}) R_t - 1] \lambda_t > 0$  (see 18). The transversality condition on money will then further be satisfied in equilibrium. For a constant money growth rate (19) simplifies to

$$\begin{aligned} (1 - \delta_0) B_{-1}/P_0 &= \frac{m_0}{1 + \gamma} \sum_{k=0}^{\infty} \beta^k \left( \frac{\tau(1 + \gamma) - \gamma}{1 - \tau} + \frac{\mu - 1}{\mu} \right) \\ \Leftrightarrow \delta_0 &= 1 - \frac{P_0}{B_{-1}} \frac{1}{1 - \beta} \left( \frac{\tau(1 + \gamma) - \gamma}{1 - \tau} + \frac{\mu - 1}{\mu} \right) \frac{m_0}{1 + \gamma} \end{aligned}$$

The conditions (17)-(18) further imply that real balances, consumption, inflation, and the effective rate of return  $R_t(1 - \delta_{t+1})$  are constant and given by

$$\begin{aligned} m_t &= \beta(1 - \tau)^2(1 + \gamma)/\mu, & c_t &= \beta(1 - \tau)/\mu \quad \forall t \geq 0 \\ R_t(1 - \delta_{t+1}) &= \mu/\beta \quad \forall t \geq 0, & \pi_t &= \mu, \quad \forall t \geq 1 \end{aligned}$$

Given that monetary policy sets the sequence of nominal balances  $\{M_t\}_{t=0}^{\infty}$  and that real balances are constant, the price level  $\{P_t\}_{t=0}^{\infty}$  is uniquely determined. Rewriting (13) for the initial period and using  $P_0 = \frac{M_{-1}\mu}{m_0}$  shows that the default rate is a function of initial liabilities, the tax rate, and the money growth rate

$$\delta_0 = 1 - \frac{M_{-1}}{B_{-1}} \frac{1}{1 - \beta} \frac{\mu(1 - \gamma + \tau\gamma) - (1 - \tau)}{(1 - \tau)(1 - \beta)(1 + \gamma)} \quad (20)$$

Since we restrict fiscal policy not to have access to non-distortionary taxation  $\delta_t \leq 1$ , the existence of an equilibrium requires the money growth to be sufficiently large,  $\mu > (1 - \tau) / (1 - \gamma + \tau\gamma)$ . The initial default rate  $\delta_0$  is uniquely determined, while the subsequent default rates  $\{\delta_t\}_{t=0}^{\infty}$  and contractual interest rates  $\{R_t \geq 1\}_{t=0}^{\infty}$  are indetermined.

**Proposition 3 (MG)** *Suppose that the fiscal authority sets the tax rate equal to a constant  $\tau \in [0, 1]$  and that the central bank holds the money growth rate constant with  $\mu > \beta$ . Then, the existence of an equilibrium requires  $\mu > (1 - \tau) / (1 - \gamma + \tau\gamma)$ . The equilibrium allocation  $m_t \forall t \geq 0$  and the price level  $P_t \forall t \geq 0$ , the effective nominal rate of return  $(1 - \delta_{t+1}) R_t \forall t \geq 0$ , and the initial default rate  $\delta_0$  are then uniquely determined, while the nominal interest rates  $R_t \forall t \geq 0$  and the default rates  $\delta_t \forall t \geq 1$  are indetermined.*

Condition (20) reveals that a higher money growth rate unambiguously lowers the initial default rate,  $\delta'_0(\mu) < 0$ , due to its impact on the initial price level and seigniorage revenues. Yet, a higher money growth rate also increases the nominal rate of return on debt  $R_t(1 - \delta_{t+1})$  (see 18) and thereby raises the cash-credit good distortion.

**Proposition 4 (MGII)** *Suppose that the fiscal authority sets the tax rate equal to a constant  $\tau \in [0, 1]$  and that the central bank holds the money growth rate constant with  $\mu > \max\{1/\beta, (1 - \tau) / (1 - \gamma + \tau\gamma)\}$ . Then, a higher money growth rate reduces the initial default rate, while it aggravates the cash-credit good distortion.*

Though, the level of debt cannot be determined in equilibrium (see proposition 1) it is possible to compute the end-of-period value of debt at market prices. To see this, we use the government budget constraint under a constant money growth policy, which is given by  $B_t R_t^{-1} + P_t \tau w_t l_t - P_t g_t + \frac{\mu-1}{\mu} M_t = (1 - \delta_t) B_{t-1}$ . Using that  $w_t = 1$ ,  $m_t = l_t(1 - \tau)$ ,  $m_t = c_t(1 - \tau)(1 + \gamma)$ , and that the equilibrium default rate in all periods  $t \geq 0$  satisfies

$$\delta_t = 1 - \frac{\pi_t}{b_{t-1}} \frac{\mu(1 - \gamma + \tau\gamma) - (1 - \tau)}{(1 - \beta)\mu^2} (1 - \tau)\beta$$

one can further compute the discounted real value of government bonds for each period  $t \geq 0$ :

$$\frac{b_t}{R_t} = [(\gamma(1 - \beta) - \beta)(1 - \tau) + \mu(\beta - \gamma(1 - \tau))] \frac{(1 - \tau)\beta}{(1 - \beta)\mu^2}$$

Hence, the government issues new debt in all periods  $t \geq 0$  if the money growth rate satisfies  $\mu(\beta - \gamma(1 - \tau)) > (\beta - \gamma(1 - \beta))(1 - \tau)$ . In this case, the government remains indebted for all periods  $b_t > 0 \forall t \geq 0$ , implying that sovereign default is always possible.

Once the central bank can uniquely implement an equilibrium allocation under a given constant tax rate, it can also select one that is consistent with the Ramsey plan, i.e., with the condition (10). This is evidently impossible under an interest rate policy, where the allocation cannot be uniquely implemented. Thus, sovereign default risk does not render implementation of efficient policy impossible. Moreover, sovereign default alleviates the implementability constraint on public policy (measured by  $\Phi$ ) as demonstrated at the

beginning of this section, and enables the central bank to implement equilibria that Pareto-dominate those under full debt repayment. It should however be noted this property crucially relies on the assumption that default is costless.

### 3.3 Costly default

Suppose that the enforceability of governmental debt contracts is imperfect, such that there will be debt repayment negotiations in case of sovereign default. Further suppose that these negotiations are costly for the lenders, i.e., the households, while the negotiation costs  $\Psi_t$  increase with the default rate,  $\Psi_t = \Psi(\delta_t)$ , where  $\Psi' > 0$  and  $\Psi(0) = 0$ . Consider for example the following simple linear specification for real negotiation costs

$$\Psi_t = \phi\delta_t, \quad \phi > 0. \quad (21)$$

This assumption on debt negotiation costs is in fact closely related to the assumption of (linear) monitoring costs in the case of bankruptcy, which is commonly assumed for the derivation of standard debt contracts (see e.g. Bernanke et al., 1999).

When default is costless, a welfare maximizing central bank would avoid choosing a high money growth rate. The reason is that higher money growth and thus higher inflation raises the cash-credit good distortion, such that consumption becomes more costly compared to leisure. The optimal money growth rate thereby depends on the tax rate and on initial debt (see 10). In order to implement a policy that is consistent with the Ramsey plan, the central bank does not take into account that default rises for smaller money growth rates.

If, however, default is associated with negotiation costs (21) the policy trade-off summarized by the first-order condition (10) changes. Since negotiation costs represent real social resource losses, the aggregate resource constraint would then read  $l_t = c_t + g_t + \phi\delta_t$ , and with  $g_t = \gamma/c_t$ .

$$l_t = (1 + \gamma) c_t + \phi\delta_t$$

For a given equilibrium sequence for consumption  $\{c_t\}$ , sovereign default immediately raises working hours and thereby reduces utility (see 1). Hence, when costs on default (like in 21) exists, an optimizing central bank will choose a higher money growth rate, which will reduce consumption compared to the case of costless default. A demonstration of this principle can be found in the appendix.

## 4 Sovereign default in an open economy

In this section we extend the analysis to the case of a small open economy. Domestic households consume domestically produced goods and foreign goods. Further they have access to internationally traded foreign bonds, domestic government bonds and money. Since domestic public liabilities are assumed to be held solely by domestic households so

that the specification of the domestic public sector is unchanged.

#### 4.1 The model

In what follows we will restrict our attention to the differences to the previous (closed economy) model. Consumption is an aggregate of domestically produced goods  $c_H$  and foreign goods  $c_F$ :

$$c_t = \gamma c_{H,t}^{1-\vartheta} c_{F,t}^{\vartheta},$$

where  $0 \leq \vartheta \leq 1$  and  $\gamma = [\vartheta^{\vartheta}(1-\vartheta)^{1-\vartheta}]^{-1}$ . For a given level of aggregate consumption, the cost minimizing demand for the goods of home and foreign origin are given by

$$c_{H,t} = (1-\vartheta) \left( \frac{P_{H,t}}{P_t} \right)^{-1} c_t, \quad c_{F,t} = \vartheta \left( \frac{P_{F,t}}{P_t} \right)^{-1} c_t, \quad (22)$$

where  $P_{H,t}$  and  $P_{F,t}$  are the price indices of the domestically produced and foreign consumption goods, respectively. The price index of the aggregate consumption good (CPI) is defined as

$$P_t = P_{H,t}^{1-\vartheta} P_{F,t}^{\vartheta}. \quad (23)$$

Households' preferences are given by (1). They are initially endowed with  $F_{-1}$  internationally traded foreign bonds measured in units of the foreign currency and  $B_{-1} + M_{-1}$ . The period  $t$  price of the former is given by  $1/R_t^*$ , such that the budget constraint of a representative household in terms of domestic currency (consumption) reads

$$B_t R_t^{-1} + S_t F_t (R_t^*)^{-1} + M_t \leq (1-\delta_t) B_{t-1} + S_t F_{t-1} + M_{t-1} + (1-\tau_t) P_t w_t l_t - P_t c_t,$$

where  $S_t = q_t P_t / P_t^*$  denotes the nominal exchange rate,  $q_t$  the real exchange rate,  $P_t^*$  the foreign currency price of the foreign consumption basket,  $\pi_t^*$  the foreign inflation rate  $\pi_t^* = P_t^* / P_{t-1}^*$ , and  $f_t$  real foreign bonds measured in the foreign consumption basket,  $f_t = F_t / P_t^*$ . The household maximizes lifetime utility (1) subject to the budget constraint, the cash-in advance constraint (2), no-Ponzi game conditions for domestic and foreign borrowing, and a non-negativity constraint on money holdings, taking prices, taxes, dividends, the default probability and the initial wealth endowment  $F_{-1}$  and  $B_{-1} > 0$  as given. Its first order conditions are (3)-(5), and

$$\beta c_{t+1}^{-\sigma} q_{t+1} / (\pi_{t+1}^*) = q_t c_t^{-\sigma} / R_t^*, \quad (24)$$

Further, the budget constraint holds with equality and the transversality conditions on domestic assets (6) and on foreign bonds

$$\lim_{k \rightarrow \infty} \frac{q_{t+k} f_{t+k}}{R_{t+k}^*} \prod_{i=1}^k \frac{\pi_{t+i}}{(1-\delta_{t+i}) R_{t+i-1}} = 0 \quad (25)$$

are satisfied. Combining (3), (5), and (24) leads to the arbitrage freeness condition  $\frac{q_{t+1}}{q_t} \frac{R_t^*}{\pi_{t+1}^*} = \frac{(1+\delta_{t+1})R_t}{\pi_{t+1}}$ .

Perfectly competitive firms produce a consumption good with the linear technology  $y_{H,t} = n_t$ , leading to the following profit-maximizing labor demand condition

$$w_t = P_{H,t}/P_t. \quad (26)$$

The home country is assumed to be small in the sense that its exports are negligible for the foreign prices. The foreign producer price level  $P_{F,t}^*$  is then identical to the foreign consumption price index  $P_t^*$ ,

$$P_t^* = P_{F,t}^*. \quad (27)$$

The law of one price holds (separately) for each good such that  $P_{H,t} = S_t P_{H,t}^*$  and  $P_{F,t} = S_t P_{F,t}^*$ , where  $P_{H,t}^*$  is the price of home produced goods expressed in foreign currency. Let the terms of trade  $z_t$  be defined as  $z_t = P_{H,t}/P_{F,t}$ . Thus, we get the following relation between the terms of trade and the real exchange rate,  $z_t = \frac{1}{q_t} \frac{P_{H,t}}{P_t}$ . Note that the ratio of domestic producer prices to the consumer price index,  $P_{H,t}/P_t$ , affects the marginal costs  $mc_H$  (see 26).

We assume that preferences of foreign households exhibit the same qualitative structure as of domestic households. Hence, foreign demand for domestically produced consumption goods  $c_{H,t}^*$  and the foreign consumption goods  $c_{F,t}^*$  satisfy  $c_{H,t}^* = \vartheta^* \left( P_t^*/P_{H,t}^* \right) c_t^*$  and  $c_{F,t}^* = (1 - \vartheta^*) \left( P_t^*/P_{F,t}^* \right) c_t^*$ , where  $\vartheta^* \in (0, 1)$  and  $c_t^*$  is aggregate foreign consumption. Using (27), foreign demand for domestic consumption goods can be rewritten as  $c_{H,t}^* = \vartheta^* z_t^{-1} c_t^*$ .

Foreign households have access to foreign bonds denominated in domestic currency. We assume that the instantaneous utility function of foreign households is similar to the one of domestic households (see 1) and that they exhibit the same discount factor  $\beta$ . Their investments in foreign bonds is then characterized by

$$\beta R_t^*/\pi_{t+1}^* = (c_{t+1}^*/c_t^*)^{\sigma^*} \quad (28)$$

where  $\sigma^*$  denotes the inverse of foreign households' intertemporal elasticity of substitution. Combining (28) and (24) leads to the following relation between domestic and foreign consumption growth

$$\frac{c_{t+1}^\sigma}{c_t^\sigma} = \frac{q_{t+1}}{q_t} \left( \frac{c_{t+1}^*}{c_t^*} \right)^{\sigma^*}, \quad \forall t \geq 0 \quad (29)$$

The CPI definition (23) together with  $P_{F,t} = S_t P_t^*$  imply  $z_t = (1/q_t)^{\frac{1}{1-\vartheta}}$  and

$$P_{H,t}/P_t = q_t^{\frac{\vartheta}{\vartheta-1}}, \quad (30)$$

Goods market clearing for domestically produced final goods requires  $y_{H,t} = c_{H,t} + c_{H,t}^*$ . Using that domestic and foreign demand for the domestically produced final good sat-

isfy  $c_{H,t} = (1 - \vartheta) (P_t/P_{H,t}) c_t$  and  $c_{H,t}^* = \vartheta^* (P_t^*/P_{H,t}^*) c_t^*$ , and (30), the goods market equilibrium for the domestically produced final good can be summarized by

$$y_{H,t} = (1 - \vartheta) q_t^{\frac{\vartheta}{1-\vartheta}} c_t + \vartheta^* q_t^{\frac{1}{1-\vartheta}} c_t^*, \quad (31)$$

Factor and asset market clearing implies that the net foreign debt position is determined by  $P_{H,t} y_{H,t} - P_t c_t \leq S_t [(F_t/R_t^*) - F_{t-1}]$ , or in real terms, by

$$q_t (f_t/R_t^*) = q_t^{\frac{\vartheta}{\vartheta-1}} y_{H,t} - c_t + q_t (f_{t-1}/\pi_t^*), \quad (32)$$

In a perfect foresight equilibrium the first order conditions of domestic households and firms, and of foreign households have to be satisfied for a domestic monetary and fiscal policy, and given sequences for the starred variables as well as initial asset endowments.

## 4.2 Equilibrium properties under default risk

### 4.2.1 PFE

Since foreign and domestic households share the same pay-off from internationally traded foreign debt, their consumption growth rates are linked by (29), which can be integrated to

$$c_t^\sigma = \xi q_t (c_t^*)^{\sigma^*}, \quad \text{where } \xi > 0 \quad (33)$$

Note that the constant  $\xi$  cannot be determined without any further restriction. Using (33) to replace  $c_t^*$  in (31) gives the demand for domestically produced goods solely as a function of domestic consumption and the real exchange rate,  $y_{H,t} = (1 - \vartheta) q_t^{\frac{\vartheta}{1-\vartheta}} c_t + \vartheta^* q_t^{\frac{1}{1-\vartheta}} (c_t^\sigma / (\xi q_t))^{1/\sigma^*}$ . Throughout the remainder of this section we abstract from government expenditures and set  $g_t = 0$ , for convenience.

The equilibrium definition can be found in the appendix. Counting equations and unknowns shows that there is one degree of freedom: Like in the closed economy case, the sequence of debt cannot be determined in equilibrium for conventional monetary policy specifications. Further, the equilibrium allocation is again indetermined under a standard interest rate policy, like in the closed economy (see proposition 2).

Combining the asset pricing conditions for domestic and foreign bonds leads to the arbitrage freeness condition  $\frac{q_t}{q_{t+1}} \frac{\pi_{t+1}^*}{\pi_{t+1}} = \frac{R_t^*}{(1-\delta_{t+1})R_t}$  and with  $q_t = \frac{S_t P_t^*}{P_t}$  to a "risk" adjusted uncovered interest rate parity UIP condition

$$\frac{S_{t+1}}{S_t} = \frac{(1 - \delta_{t+1}) R_t}{R_t^*} \quad (34)$$

The adjusted UIP condition (34) evidently implies that the nominal rate of return on bonds is pinned down for a particular growth rate of the exchange rate and a given foreign interest rate.

## 4.2.2 Balanced trade equilibrium

As in the closed economy case we assume that the assumptions 1 and 2 hold. Then, demand for domestically produced goods can further be simplified to  $y_{H,t} = (1-\vartheta)q_t^{\frac{\vartheta}{1-\vartheta}}c_t + \frac{\vartheta^*}{\xi}q_t^{\frac{\vartheta}{1-\vartheta}}c_t \Leftrightarrow$

$$y_{H,t} = \kappa_0 q_t^{\frac{\vartheta}{1-\vartheta}} c_t,$$

where  $\kappa_0 = 1 - \vartheta + \vartheta^*/\xi \in [0, 1]$ . We further introduce a restriction on the constant  $\xi$  in (33). In particular, we assume that it takes a value that is consistent with balanced trade, i.e.  $\xi = \vartheta^*/\vartheta$  implying  $\kappa_0 = 1$  and  $f_t/R_t^* = f_{t-1}/\pi_t^*$ . Hence, with an initial asset endowment satisfying  $F_{-1} = 0$ , the stock of net foreign assets will always equal to zero,  $F_t = 0 \forall t \geq 0$ , and private sector default is not an issue.<sup>8</sup>

**Assumption 3** *The ratio of real initial consumption levels is consistent with balanced trade,  $\xi = \vartheta^*/\vartheta$  ( $\kappa_0 = 1$ ) and  $F_{-1} = 0$ , and foreign variables are constant.*

We will now examine the equilibrium properties for the case where the assumptions 1-3 hold. For this we redefine the equilibrium as follows.

**Definition 5** *A PFE under assumption 1, 2, and 3, a binding cash-in-advance constraint, and  $g_t = 0$  is a set of sequences  $\{c_t, S_t, R_t \geq 1, \delta_t \leq 1, \pi_t, P_t \geq 0, B_t \geq 0\}_{t=0}^{\infty}$  satisfying*

$$c_t^{\frac{1}{1-\vartheta}} = (c^*\xi)^{\frac{\vartheta}{1-\vartheta}} \frac{1-\tau}{R_t(1-\delta_{t+1})} \quad (35)$$

$$\frac{c_{t+1}}{c_t} = \beta \frac{R_t(1-\delta_{t+1})}{\pi_{t+1}} \quad (36)$$

$$\frac{S_t}{S_{t+1}} = \frac{R^*}{(1-\delta_{t+1})R_t} \quad (37)$$

$$S_t = \frac{c_t P_t}{c^* \xi P^*} \quad (38)$$

$$(1-\delta_t) \frac{B_{t-1}}{P_t} = \sum_{k=0}^{\infty} \left( \beta^k \frac{c_t}{c_{t+k}} \right) (\tau c_{t+k} + \tau_{t+k}^m) \quad (39)$$

where  $\tau_t^m = \frac{(m_t \pi_t / m_{t-1}) - 1}{(m_t \pi_t / m_{t-1})} m_t$  and  $m_t = (1-\tau)c_t$ , the transversality conditions, and a monetary policy, for given initial values  $B_{-1} > 0$ , and  $M_{-1} > 0$ .

Like in the case of a closed economy, the equilibrium allocation cannot be pinned down under an interest rate policy, since the interest rate only enters the set of equilibrium conditions in combination with the default rate. As shown in the previous section, the latter and the allocation (due to the transactions friction) can then not be determined.

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<sup>8</sup>In the appendix we briefly discuss the case of  $\kappa_0 \neq 1$ .

### 4.2.3 Exchange rate policy

In the closed economy case, the control of the money growth rate has been shown to be a way out of this fundamental indeterminacy. Here, in the open economy case, we will examine the control of the exchange rate as an alternative means of monetary policy implementation. In particular, we consider the case where the central bank sets the nominal exchange rate according to a constant growth rate

$$\{S_t\}_{t=0}^{\infty} : S_{t+1}/S_t = \varepsilon, \quad S_0 > 0, \quad \varepsilon > 0, \quad \forall t \geq 0. \quad (40)$$

Evidently, one example for this policy regime is a constant nominal interest rate  $S_t = S \Rightarrow \varepsilon = 1$ . The risk adjusted UIP condition (37) implies that the nominal rate of return on domestic bonds is constant under the exchange rate policy (40) and given by

$$(1 - \delta_{t+1}) R_t = \varepsilon R^* \quad \forall t \geq 0,$$

Since the nominal rate of return on domestic bonds is constant, (35) implies that domestic consumption is also constant  $\forall t \geq 0$  and equals  $c_t^{\frac{1}{1-\vartheta}} = (c^* \xi)^{\frac{\vartheta}{1-\vartheta}} \frac{1-\tau}{R_t(1-\delta_{t+1})} \Rightarrow$

$$c_t = (c^* \xi)^{\vartheta} [(1 - \tau) / (\varepsilon R^*)]^{1-\vartheta}$$

Domestic CPI inflation can then be determined for all periods  $t \geq 1$  by substituting out the nominal rate of return on domestic bonds in the consumption Euler equation (36), to give

$$\pi_t = \beta \varepsilon R^*, \quad \forall t \geq 1$$

Finally, we use that real money balances are then also constant and given by  $m_t = (1 - \tau) (c^* \xi)^{\vartheta} \left(\frac{1-\tau}{\varepsilon R^*}\right)^{1-\vartheta}$ ,  $\forall t \geq 0$  such that the price level can be pinned down for a given nominal exchange rate by  $P_t = (c^* \xi P^*) \frac{S_t}{c_t} \Rightarrow$

$$P_t = S_t \left( \frac{c^* \xi \varepsilon R^*}{1 - \tau} \right)^{1-\vartheta} P^*, \quad \forall t \geq 0$$

**Equilibrium default** In order to examine equilibrium default, we use that seigniorage satisfies  $\tau_t^m = \frac{\mu_t - 1}{\mu_t} m_t$  and that  $\mu_t = \frac{m_t \pi_t}{m_{t-1}} = \frac{c_t (1-\tau) \pi_t}{c_{t-1} (1-\tau)} = \beta R_{t-1} (1 - \delta_t)$  for  $t \geq 1$ , implying  $\tau_t^m = \frac{\beta R_{t-1} (1-\delta_t) - 1}{\beta R_{t-1} (1-\delta_t)} c_t (1 - \tau)$  for  $t \geq 1$ . Under the exchange rate policy (40), seigniorage thus satisfies in equilibrium for any period  $t + 1$  :

$$\tau_{t+1}^m = \frac{\beta \varepsilon R^* - 1}{\beta \varepsilon R^*} c_{t+1} (1 - \tau) \quad (41)$$

while for period  $t$  we use that it satisfies  $\tau_t^m = m_t - M_{t-1}/P_t$ . Rewriting the IGB constraint (39) such that  $\tau_t^m$  and  $\tau_{t+k+1}^m$  for  $k \geq 0$  are treated differently, leads to

$$(1 - \delta_t) \frac{B_{t-1}}{P_t} = c_t \sum_{k=0}^{\infty} \beta^k \tau + c_t \sum_{k=0}^{\infty} \beta^{k+1} \frac{\tau_{t+k+1}^m}{c_{t+k+1}} + \tau_t^m$$

Substituting out seigniorage with (41) and  $\tau_t^m = (1 - \tau)c_t - M_{t-1}/P_t$  (where we used that real balances satisfy  $m_t = (1 - \tau)c_t$ ), and using  $\sum_{k=0}^{\infty} \beta^{k+1} = \beta/(1 - \beta)$  gives

$$\begin{aligned} (1 - \delta_t) \frac{B_{t-1}}{P_t} + \frac{M_{t-1}}{P_t} &= c_t \left( \frac{1}{1 - \beta} \tau + \frac{\beta}{1 - \beta} (1 - \tau) \frac{\beta \varepsilon R^* - 1}{\beta \varepsilon R^*} + (1 - \tau) \right) \\ &= c_t \frac{\varepsilon - (1 - \tau)/R^*}{(1 - \beta) \varepsilon} \end{aligned}$$

Hence, the existence of an equilibrium requires the growth rate of the nominal exchange rate to be sufficiently large,  $\varepsilon > (1 - \tau)/R^*$ . Otherwise,  $\delta_t$  would be negative (given that public liabilities are non-negative), which has been assumed to be unfeasible.

To see how the exchange rate policy (40) affects the equilibrium default rate we further replace  $c_t$  with  $c_t = (c^* \xi)^\vartheta \left(\frac{1-\tau}{\varepsilon R^*}\right)^{1-\vartheta}$  and the price level with  $P_t = S_t \left(\frac{c^* \xi \varepsilon R^*}{1-\tau}\right)^{1-\vartheta} P^*$  leading to  $(1 - \delta_t) B_{t-1} + M_{t-1} = P_t \cdot (c^* \xi)^\vartheta \left(\frac{1-\tau}{\varepsilon R^*}\right)^{1-\vartheta} \frac{\varepsilon - (1-\tau)/R^*}{(1-\beta)\varepsilon} \Rightarrow$

$$(1 - \delta_t) B_{t-1} + M_{t-1} = S_t \cdot (P^* c^* \xi) \frac{\varepsilon - (1 - \tau)/R^*}{(1 - \beta) \varepsilon}$$

Summarizing constant terms and solving for the default rate thus yields the following equilibrium relation between the default rate, the predetermined stocks of public liabilities, the exchange rate, and its rate of depreciation

$$\delta_t = 1 - \frac{S_t \Gamma(\varepsilon) - M_{t-1}}{B_{t-1}}, \quad \text{where } \Gamma(\varepsilon) = (P^* c^* \xi) \frac{\varepsilon - (1 - \tau)/R^*}{(1 - \beta) \varepsilon}$$

While the sequence of nominal money balances is for  $t \geq 0$  uniquely pinned down by  $M_t = P_t m_t = S_t P^* c^* \xi (1 - \tau)$  the sequence of government debt can again not be determined. Only the initial value for debt is given, which allows to determine the initial equilibrium default rate.

**Proposition 6 (EXR)** *Suppose that the fiscal authority sets the tax rate equal to a constant  $\tau \in [0, 1]$  and that the central bank sets the exchange rate according to (40). Then, the existence of an equilibrium requires  $\varepsilon > (1 - \tau)/R^*$ . The equilibrium allocation  $\{c_t\}_{t=0}^{\infty}$ , the price level  $\{P_t\}_{t=0}^{\infty}$ , the effective nominal rate of return  $\{(1 - \delta_{t+1}) R_t\}_{t=0}^{\infty}$ , and the initial default rate  $\delta_0$  are then uniquely determined, while the nominal interest rates  $R_t$   $t \geq 0$  and the default rates  $\delta_t \forall t \geq 1$  are indetermined.*

Taking a closer look at the equilibrium default rate, we want to assess the effect of monetary

(exchange rate) policy on the *initial* default rate:

$$\delta_0 = 1 - \frac{S_0 \Gamma(\varepsilon) - M_{-1}}{B_{-1}}$$

Since  $\Gamma'(\varepsilon) = \frac{1-\tau}{(1-\beta)R^*\varepsilon^2} > 0$ , the initial default rate decreases with the nominal exchange rate and the rate of depreciation. Since domestic inflation tends to rise with a higher depreciation rate it can be concluded that an inflationary policy reduces sovereign default, like in the closed economy.

**Proposition 7 (EXRII)** *Suppose that the fiscal authority sets the tax rate equal to a constant  $\tau \in [0, 1]$  and that the central bank sets the exchange rate according to (40) and  $\varepsilon > (1 - \tau) / R^*$ . Then, the initial default rate is reduced by a devaluation of the domestic currency and by a higher rate of depreciation (while the latter raises the cash-credit good distortion).*

Is debt issued in the subsequent periods? Using the period-by-period government budget constraint  $\frac{B_t}{R_t} + P_t \tau w_t l_t + M_t = M_{t-1} + (1 - \delta_t) B_{t-1}$ , one gets to the following condition on the market value of new debt

$$\frac{b_t}{R_t} = - \left( \tau q_t^{\frac{\vartheta}{\vartheta-1}} y_{H,t} + (1 - \tau) c_t \right) + c_t \frac{\varepsilon - (1 - \tau) / R^*}{(1 - \beta) \varepsilon} = c_t \frac{\beta \varepsilon - (1 - \tau) / R^*}{(1 - \beta) \varepsilon}$$

Hence, for a rate of depreciation satisfying  $\varepsilon > \frac{1-\tau}{R^*\beta}$  the government permanently remains indebted. Note that this threshold can easily be smaller than one (implying an appreciation). If  $\varepsilon > \frac{1-\tau}{R^*\beta}$ , the fiscal authority will issue debt in any subsequent period, while tax revenues still do not guarantee repayment of debt. The precise repayment rate is, however, unknown.

## 5 Conclusion

[Remains to be written.]

## Appendix

**Definition 8** A rational expectations equilibrium of the open economy model for  $g_t = 0$  is a set of sequences  $\{w_t, m_t \geq 0, c_t, l_t \in [0, \bar{l}], q_t, y_{H,t}, R_t \geq 1, f_t, b_{t-1} \geq 0, \delta_t \leq 1, \pi_t \geq 0\}_{t=0}^{\infty}$  satisfying for all periods  $t \geq 0$

$$\begin{aligned}
c_t^\sigma l_t^{\sigma l} &= w_t \frac{1 - \tau_t}{R_t(1 - \delta_{t+1})} \\
m_t &\geq c_t - \tau_t w_t l_t \\
\frac{c_{t+1}^\sigma}{c_t^\sigma} &= \beta \frac{R_t(1 - \delta_{t+1})}{\pi_{t+1}} \\
c_t^\sigma &= \xi q_t (c_t^*)^{\sigma^*} \\
y_{H,t} &= l_t \\
w_t &= q_t^{\frac{\vartheta}{\vartheta-1}} \\
y_{H,t} &= (1 - \vartheta) q_t^{\frac{\vartheta}{1-\vartheta}} c_t + \vartheta^* q_t^{\frac{1}{1-\vartheta}} (c_t^\sigma / (\xi q_t))^{1/\sigma^*} \\
q_t^{\frac{\vartheta}{\vartheta-1}} y_{H,t} &= c_t + q_t [(f_t/R_t^*) - (f_{t-1}/\pi_t^*)], \\
(1 - \delta_t) \frac{b_{t-1}}{\pi_t} &= \sum_{k=0}^{\infty} (\tau_{t+k} l_{t+k} + \tau_{t+k}^m) \prod_{i=1}^k \frac{\pi_{t+i}}{(1 - \delta_{t+i}) R_{t+i-1}}
\end{aligned}$$

where  $\tau_t^m = \frac{(m_t \pi_t / m_{t-1}) - 1}{(m_t \pi_t / m_{t-1})} m_t$  the transversality conditions, a fiscal (tax) policy  $\{\tau_t\}_{t=0}^{\infty}$  and a monetary policy, for given sequences  $\{c_t^*, R_t^*, \pi_t^*\}_{t=0}^{\infty}$  satisfying  $\beta R_t^* / \pi_{t+1}^* = (c_{t+1}^* / c_t^*)^{\sigma^*}$ , initial asset endowments  $F_{-1}, B_{-1} > 0$ , and  $M_{-1} > 0$ .

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