Social Preferences, Fuzzy Information and the Provision of Public Goods

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Abstract

This paper deals with the provision of a public good in an environment where people may exhibit social preferences and are uncertain about each other's contribution behavior. In contrast to classical theoretical results but consistent with numerous public goods experiments in the lab and in the field the presented model is able to explain participants' positive contributions. We combine a simple utility function allowing for conditional cooperation by taking social preferences into account and the theory of fuzzy sets to specify the decision maker's uncertainty with respect to other players' behavior. Crisp contribution decisions are derived by means of a defuzzification strategy based on the Hurwicz principle. The model clarifies the substantial relevance of preferences and beliefs for social interaction and is suitable to illustrate the provision of a public good on the basis of a realistic set-up. We obtain a multiplicity of Nash equilibria under social preferences and fuzzy information, many of them comprising positive contributions of all players. The well-known decay of cooperation in repeated public goods experiments is depicted by simple adjustment processes towards Nash equilibria with low or zero contributions.

Keywords: public goods, social preferences, fuzzy sets, Hurwicz criterion, Nash equilibrium under fuzzy information

JEL classification: C72, D80, H41

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1 Introduction

It is a well-established insight of economic theory at least since Samuelson (1954) that individually rational free rider behavior prevents the provision of the socially efficient amount of a public good by voluntary contributions of income maximizing and rational individuals. This conflict between individual and collective rationality crucially affects diverse important fields like environmental protection, tax compliance, the participation in collective actions like demonstrations and strikes, donations to charities, teamwork, collusion between firms, embargoes and many other.

While the classical economic assumptions induce the result that it is a dominant strategy for each potential participant not to contribute anything to a public good and thereby the socially efficient production is caused to remain undone, in recent time numerous economic experiments have shown that after all at least some economic agents do make contributions[1]. Moreover personal experience as well as common sense suggest that under certain circumstances people are not only interested in their material well-being but show contingent behavior taking interpersonal aspects into account. The results of many public goods experiments may be summed up by two stylized facts:[2]

1. People contribute substantially more to the provision of a public good than theoretically predicted.
2. The cooperation is often very fragile and tends to collapse with repeated interactions.[3]

To reconcile these conflicting theoretical and empirical aspects this paper deals with the provision of a public good in a realistic environment where

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[1] See e.g. the surveys by Ledyard (1995), Camerer (2003) or Gächter and Herrmann (2005).

[2] Cf. e.g. Croson et al. (2005) or Gächter (2006).

[3] In terms of experimental economics repeated interaction includes games played under the 'Stranger' condition where group members change randomly from round to round as well as under the 'Partner' condition where the group composition stays the same for all rounds (see Andreoni and Croson (1998) and Keser and van Winden (2000)). Note that only the 'Partner' condition may give rise to strategic behavior in the game theoretical sense of repeated games.

the decision makers may exhibit social preferences[4] and are uncertain about each other's contribution behavior. We adopt a utility function for the participants in a standard linear public goods game (or voluntary contribution mechanism) that integrates aspects in the tradition of the important papers of Rabin (1993) and Fehr and Schmidt (1999) reflecting the agents' concern for other players' contributions. These models include factors like kindness and intention-based reciprocity in the case of Rabin or aversion against inequality of payoff distributions in the case of Fehr and Schmidt in the players' utility functions. By taking such aspects into account, our model in addition to the usual material payoff considers the fact that (many) people do not want to be exploited by others. Moreover, the chosen approach incorporates the fact that (at least some) subjects often seem to be trying to avoid excessive free riding at the expense of fellow players perceived as nice or fair. In other words our approach may represent conditional cooperation, meaning that players make their contributions conditional on the contributions of other players.[5] Based on this assumption the deployed utility function leads ceteris paribus to a negative effect for a player's benefit if he falls significantly short of other players' contributions. This reflects preferences that may be caused by something like a guilty conscience.[6] Our model captures the two types of behavior observed most frequently in recent experiments, conditional cooperation and free riding and thus allows the analysis of public goods games played by subjects with heterogeneous preferences.[7] Corresponding with these empirical results our approach seems to depict a reasonable way to model actual behavior of many people in reality.

Since no one in a public goods game can be sure about other players' actual contributions, the maximization of the utility function takes place under

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[4] Following Fehr and Fischbacher (2002) we state that a player possesses social preferences when he is not just interested in his own monetary payoff, but also cares about the distribution of these payoffs between himself and other reference agents and/or cares about how this distribution emerges.

[5] Cf. e.g. Keser and van Winden (2000) and Fischbacher et al. (2001).

[6] Cf. Sugden (1984) who presents a model considering that people feel an 'obligation to contribute' if others contribute.

[7] See the empirical results of Fischbacher and Gächter (2006) and the survey by Gächter (2006).

uncertainty. This lack of knowledge builds the second important pillar of our paper. Usually uncertainty in economics is modeled within a stochastic framework. In our opinion this approach is quite - sometimes even inadequately - demanding with respect to the implied level of players' information. Instead of assuming that the agents dispose of exact knowledge regarding the distribution functions of the unknown contributions of other players we apply the theory of fuzzy sets where only vague information on part of the decision makers is supposed.[8] Given that the players are vaguely informed about the possible contributions of others and that this is represented by fuzzy sets, they cannot derive their optimal decision in a straightforward manner, since each own action leads to strongly differing outcomes depending on the other players' behavior. In order to remove the fuzziness which gives rise to this multivaluedness of the optimization problem we implement a so called defuzzification strategy based on the well-known Hurwicz principle.[9]

We obtain the result that depending on the magnitude of his social preferences, his degree of pessimism or optimism and his beliefs in reference to other players each agent possibly optimizes his utility by contributing a positive amount to the provision of the public good. Thus our model is suitable to account for the explanation of the first stylized fact.

For the case of two players we derive the conditions for the existence of so called Nash equilibria under fuzzy information introduced by Hauenschild and Stahlecker (2003). This concept firstly is characterized by the condition that the contributions considered most plausible by all participants are actually provided by their fellows. Secondly it implies the fact that all participants maximize their objective function with respect to these contributions. It is demonstrated that potentially differing positive contributions of the players may be equilibrium strategies in this sense and that for beneficial parameter constellations equilibrium solutions may converge towards the provision of the socially optimal quantity of the public good.

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[8] The theory of fuzzy sets is applied for the analysis of microeconomic questions e.g. by Greenhut et al. (1995), Mansur (1995), Goodhue (1998) and Hauenschild and Stahlecker (2001).

[9] Stahlecker et al. (1999) were the first to employ the Hurwicz criterion to remove the fuzziness from the decision problem.

As an important aspect regarding the second stylized fact it can be stated that the so called decay of cooperation shown in numerous repeated public goods experiments may be explained by adjustment processes[10] towards certain fuzzy Nash equilibria with low or zero contributions. In our model this phenomenon arises from the players' social interaction caused by preference heterogeneity and corresponding updating of beliefs.[11] For numerical examples and different styles to update beliefs we present adjustment paths towards different fuzzy Nash equilibria. Therefore our approach constitutes a plausible theoretical foundation for the occurence of both stylized facts by simultaneously capturing the important roles of heterogeneity in (social) preferences and (the updating of) uncertain beliefs.

The paper is organized as follows. In section 2 several approaches trying to explain positive contributions in public goods games are discussed. In section 3 we set up our model. We specify a utility function suitable to capture conditional cooperation and free riding. Moreover, after introducing the concept of fuzzy sets, optimal contribution decisions under fuzzy information are derived. In addition we depict some comparative statics. To illustrate our theoretical findings we present some numerical results. The concept of a Nash equilibrium under fuzzy information is applied to the public goods game in section 4. We identify a multiplicity of Nash equilibria and describe, by means of numerical examples, conceivable adjustment processes towards these equilibria. In section 5 some concluding remarks are presented.

2 Why do people cooperate?

In this section we briefly survey some existing theoretical approaches presented by different authors aiming at the explanation of the two stylized facts, i.e., positive contributions and the fragility of this voluntary cooperation in (repeated)

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[10] The difficulties regarding adjustment processes towards Nash Equilibria are discussed e.g in Varian (1992), pp. 265-266, 287-288, 302-303 and Mas-Colell et al. (1995), pp. 248-250. We further refer to this issue in section 4 of this paper.

[11] Fischbacher and Gächter (2006), p. 3, state that 'the decay in contributions results from learning about how others behave and the players' social preferences which determine the players' response to the observed behavior of other players.'

public goods games. The - according to recent economic experiments[12] - most plausible of these aspects constitute the fundament for the utility function presented in section 3.

An interesting approach to interpret subjects' contribution behavior in public goods games is given by the theory of 'warm-glow' saying that certain people may increase their utility simply by the act of contributing. A second plausible explanation suggests that people exhibit altruistic preferences which cause them to want to benefit others. Pure altruism implies that subjects reduce their contributions if they come to know that others contribute more, since they are only interested in the outcome but not in the division of contributions.[13] Both of these explanations may only account for the existence of positive contributions but cannot shed light on the dynamic issue of fragility or decay of cooperation. A third possible reason indicates that subjects commit decision errors, implying firstly that they cooperate by mistake and secondly that they may learn to behave according to their true income maximizing preferences from round to round in a repeated game and thus finally cease contributing.[14] To discriminate between these motives Palfrey and Prisbrey (1997) conduct some economic experiments and find evidence for warm-glow preferences and the existence of errors but not for altruism.

It should be pointed out however that neither warm-glow nor altruistic preferences cause people to make their behavior dependent from the cooperation attitude of others. In contrast to this, recent experiments show that a lot of subjects act as conditional cooperators, i.e. they make contributions if they believe that their counterparts will contribute also. Furthermore these studies identify many participants as free riders and therefore detect considerable heterogeneity in people's cooperation preferences. The coexistence of these two types of behavior and the social interaction between them leading to continuous updating of the beliefs on part of the conditional cooperators in repeated

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[12] See e.g. Keser and van Winden (2000), Fischbacher et al. (2001), Croson (2002), Frey and Meier (2004), Falk et al. (2004), Gächter and Thöni (2005), Gächter and Renner (2006) and Fischbacher and Gächter (2006).

[13] For a detailed description of the concepts of warm-glow and altruism see Andreoni (1989, 1990) and Croson (2002).

[14] See e.g. Anderson et al. (1998).

games[15] may contribute to the explanation of both stylized facts.

Keser and van Winden (2000) deliver an early analysis emphasizing the importance of conditional cooperation by finding that behavior in a public goods game is oriented towards the average behavior of the other group members in the previous period. Such behavior is in line with the principle of reciprocity, explicitly used by e.g. Axelrod (1984) to explain the cooperation observed in a prisoners' dilemma situation where individuals tend to reciprocate cooperation with cooperation and defection with defection. Fischbacher et al. (2001) and especially Fischbacher and Gächter (2006) arrange experimental designs which make it possible to demonstrate the positive correlation of beliefs and contributions for many players. By using a revealed preferences method the authors elicit people's beliefs about their fellows and obtain their contribution preferences as a function of the contributions of their counterparts. According to this contribution function subjects which prefer to contribute zero independent of other players' behavior are called free riders and people with contribution preferences which increase with contributions of others' are named conditional cooperators. Fischbacher and Gächter (2006) find in their experiments that 55% of the people behave like conditional cooperators and 23% act as free riders. Related studies come to similar conclusions detecting heterogeneous cooperation preferences with most of subjects qualified as free riders or conditional cooperators and the last-mentioned being the majority type.[16]

Based on these most recent results of empirical research a theoretical model of human decision processes and behavior should be able to capture the most frequently observed types of preferences resulting in conditional cooperation or free riding respectively. To meet this requirement we introduce an appropriate utility function related to the social preference approaches of Fehr and Schmidt (1999) and Bolton and Ockenfels (2000) in section 3.

Moreover to deal with the players' uncertainty regarding the preferences and

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[15] The free riders' beliefs are irrelevant since they exhibit not contributing as a dominant and therefore unconditional strategy.

[16] See e.g. Burlando and Guala (2005), Page et al. (2005) or Ones and Putterman (2004). These results from experiments in labs are supported by field experiments which examine people's behavior with regard to naturally occuring phenomena (see e.g. Frey and Meier (2004) or Carpenter and Seki (2005)).

contributions of their fellows we deploy the theory of fuzzy sets. After showing in section 3 that positive contributions may represent optimal behavior for certain types of subjects, we introduce the concept of Nash equilibria under fuzzy information and illustrate some possible methods to update beliefs explaining the phenomenon of changing voluntary contributions in repeated games in section 4.

3 The model

In this section we consider a standard linear public goods game. The utility function used for modeling the players' preferences is suitable to capture the most frequently observed behavior of subjects, i.e. conditional cooperation or free riding, described in section 2. After briefly showing the equilibrium results under perfect information we enhance the model to incorporate the players' uncertainty about their fellows' preference-based contributions by means of fuzzy sets. Finally employing an economically plausible defuzzification strategy the (in a sense)[17] optimal decisions of all participating subjects are derived and our theoretical findings are augmented by some illustrative numerical examples.

3.1 A simple public goods game

We examine a standard linear public goods game for the case of two players. The agents' utility functions are capable to reflect social preferences by taking a comparison of contributions into account. They are denoted by

$$u_i(g_i, g_j) = y + aG - g_i - e_i\,(g_i - g_j)^2 \;\text{ for } i, j \in \{1, 2\},\; i \neq j, \tag{1}$$

where $G = g_i + g_j$ specifies the aggregated quantity of the public good provided, y characterizes initial wealth normalized to 1 for both players in the following, $0 \leq g_i, g_j \leq y$ are the investment costs contributed by player i and j respectively, and $1/2 < a < 1$ represents the marginal return of the total investments.[18] Note that an alternative interpretation may regard player j as

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[17] The decisions are optimal with respect to the chosen defuzzification strategy, i.e. other methods may lead to other conditions for optimality.

[18] An empirical analysis regarding the influence of the value of the marginal return parameter a on the level of cooperation is conducted by Isaac and Walker (1988).

a representative or average player standing for a population of N players.[19]

The equality preference parameter e_i is describing player $i's$ concern about the equality of contributions.[20] Since we rule out the existence of subjects who like to contribute less (and, of course, more) than others per se, we adopt the restriction $e_i \geq 0$. For an equality preference parameter $e_i = 0$ the utility function represents pure income maximization. In this well-known standard case the optimal decision for player i is to behave as a free rider not contributing to the provision of the public good. The subjects' empirically revealed concern for equitable and contingent contributions to the provision of the public good described in section 2 is captured by values $e_i > 0$ reflecting conditional cooperation. Our set-up handles deviations from equal sharing symmetrically. Particularly falling short of the other player's investment causes player i to feel an obligation to contribute, implying (ceteris paribus) a positive relationship between his own contribution and the (expected) contribution of the fellow player.[21] To assess an economically sensible upper bound for e_i we postulate for the partial derivative

$$\frac{\partial u_i(\cdot)}{\partial g_j} = a + 2e_i\left(g_i - g_j\right) > 0$$

meaning that even socially oriented players independent of their own investment always obtain a higher utility u_i for higher contributions g_j of the other player. To meet this condition for all $g_i, g_j \in [0, 1]$ we assume in the following

$$e_i < a/2 \tag{2}$$

to avoid compensation of material interest by 'excessive' social preferences.[22]

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[19] In this case g_j would have to be seen as an average contribution and the total contribution would have to be adjusted to $G = g_i + g_j(N-1)$. Both approaches correspond to the partner and stranger condition mentioned above, respectively. An analysis of the N player scenario with individual instead of average treatment of other players is outlined in the appendix.

[20] Cf. Fehr and Schmidt (1999), p. 845 for a discussion about social preferences in the context of public goods games. As mentioned above, our approach corresponds to the fairness models of Fehr and Schmidt (1999) and Bolton and Ockenfels (2000) under mild assumptions.

[21] Cf. Sugden (1984) and Sobel (2005), p. 413.

[22] For the case of N players with g_j describing the average contribution of other players the condition would be $e_i < \frac{a(N-1)}{2}$.

For the first derivative of the utility function (1) with respect to g_i we obtain

$$\frac{\partial u_i(\cdot)}{\partial g_i} = a - 1 - 2e_i\left(g_i - g_j\right).$$

For pure income maximization represented by $e_i = 0$ this expression is always negative meaning that player i prefers to act as a free rider and chooses the lowest possible contribution of 0. For $e_i > 0$ the optimal contribution[23] of player i is given by

$$g_i^* = \max\left\{0,\, g_j - \frac{1-a}{2e_i}\right\}. \tag{3}$$

Potentially negative optimal solutions are regarded as lowest possible contributions $g_i^* = 0$.[24] Furthermore equation (3) shows that for $a + 2e_i > 1$ there exist values of $g_j \in [0,1]$ inducing player i to make a positive contribution to the public good, but also that each player exhibits g_i^* lower than g_j since $\left(1-a\right)/2e_i > \left(1-a\right)/a > 0$.[25] Thus, while an affinity to cooperate reflected by high values of the equality preference parameter e_i may cause subjects to contribute, the only existing Nash equilibrium still features contributions of 0.[26]

In a more realistic setting player i would not be able to anticipate the other player's contribution g_j perfectly and therefore would not have the possibility to choose his utility maximizing level g_i^* as given by (3). Moreover, it seems quite plausible that player i does not know player $j's$ utility function at all. To cope with this kind of uncertainty we assume in the subsequent analysis that player i possesses some vague information regarding the expected contribution of the other player, g_j, which can be represented by a fuzzy set.

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[23] Since we obtain $-2e_i < 0$ for the second derivative we find in fact the function's unique maximum.

[24] Since the utility function is strictly concave in g_i and thus monotonically decreasing in the feasible range between 0 and 1 for negative maxima, this procedure is appropriate.

[25] Consider that we imposed the restriction $e_i < a/2$ (cf. (2)).

[26] Note that the adjustment process towards the Nash equilibrium based on conjectural variations is similar to the process in the well-known Bertrand price competition.

3.2 Fuzzy sets

In what follows we assume that the participating players try to maximize their utility taking into account that they are not informed about the contributions delivered by their fellow players. In economic theory, situations of this kind are usually handled by assuming that the uncertain contributions can be described by a probability distribution from which the different types of players are drawn. A substantial drawback of this approach results from the sometimes unrealistic presumption that detailed knowledge on the part of the players - which is not available in many economically relevant situations with unknown counterparts - is postulated.

In the present case player i usually would not be in a position to assign definite probabilities to each possible contribution of player j. In consequence of this it seems more reasonable from our point of view to assume that the players do only hold some vague beliefs regarding the preferences and behavior of their fellow players. They perhaps can only classify certain contributions of other players as possible or more or less plausible. Since it appears to be difficult to capture such fuzziness in a traditional stochastic framework in the paper at hand the theory of fuzzy sets is employed to model the players' uncertainty with respect to potential contributions of their counterparts.

We adopt this concept to the two player public goods game with player i being uncertain concerning the contribution of player j and grasp this uncertainty by the fuzzy set $\mathcal{B}_i$ and the corresponding membership function[27]

$$m^i_{\mathcal{B}_i} : [0,1] \to [0,1]\,. \tag{4}$$

This function assigns different degrees of membership between 0 and 1 to the various possible contribution levels, $g_j \in [0,1]$, of player j where the degrees of membership represent the plausibility with which certain contributions belong to the fuzzy set.[28] Values that do not belong to the fuzzy set exhibit a degree of $m^i_{\mathcal{B}_i} = 0$, while the contributions considered most plausible are described by $m^i_{\mathcal{B}_i} = 1$. Furthermore there usually exist a lot of possible contributions with assigned membership degrees between 0 and 1 meaning that they belong 'to

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[27] A comprehensive introduction into the theory of fuzzy sets can be found e.g. in Rommelfanger (1994), Bandemer and Gottwald (1995) and Zimmermann (2001).

[28] A fuzzy set exhibiting a maximum degree of membership equal to 1 is called normalized.

some extent' to the fuzzy set $\mathcal{B}_i$. This gradual change from membership to non-membership characterizes the so called fuzziness regarding the contribution levels of player j. In contrast to this, stochastic uncertainty is based on classical set theory where different contributions may either belong or not belong to a set and where the probability of an event is given by the value of some probability measure on this set.[29]

The significant advantage of modeling with fuzzy sets arises from the possibility to represent the players' incomplete information in an intuitively sensible way. Fuzziness seems to be an appropriate concept to describe the level of information people usually face in reality and thus can be seen from a more formal perspective as a compromise between the more extreme assumptions of no information, perfect information and stochastic uncertainty.

To substantiate our approach let the vague information of player i with respect to player j (for $i, j \in \{1, 2\}$; $i \neq j$) be given by the membership function (4) with

$$m^i_{\mathcal{B}_i}(g_j) = \begin{cases} 0, \text{ if } g_j < \overline{g}_j - l_j \\ 1 - \frac{\overline{g}_j - g_j}{l_j}, \text{ if } \overline{g}_j - l_j \leq g_j \leq \overline{g}_j \\ 1 - \frac{g_j - \overline{g}_j}{r_j}, \text{ if } \overline{g}_j \leq g_j \leq \overline{g}_j + r_j \\ 0, \text{ if } g_j > \overline{g}_j + r_j, \end{cases} \tag{5}$$

where $l_j, r_j > 0$, and $\overline{g}_j$ represents the contribution with the highest possible membership value $m^i_{\mathcal{B}_i}(\overline{g}_j) = 1$.[30] Therefore player i considers $\overline{g}_j$ as the most plausible contribution simultaneously keeping in mind (for $l_j > 0$ or $r_j > 0$) that the unknown contribution may deviate from $\overline{g}_j$.

The membership function $m^i_{\mathcal{B}_i}(g_j)$ given by (5) is depicted (for an example with a relatively small most plausible value $\overline{g}_j$, a lowest possible contribution of 0 and a moderate highest possible value) in figure 1 and shows the following properties:

1. The membership function consists of linear sections and possesses a left and a right branch, which show the same degree of membership,

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[29] Cf. Klir and Wierman (1999) or Seising (1999) for details regarding the relationship and differences between fuzzy sets and probability theory.

[30] For an alternative approach based on a quadratic membership function see Arnold et al. (2000).

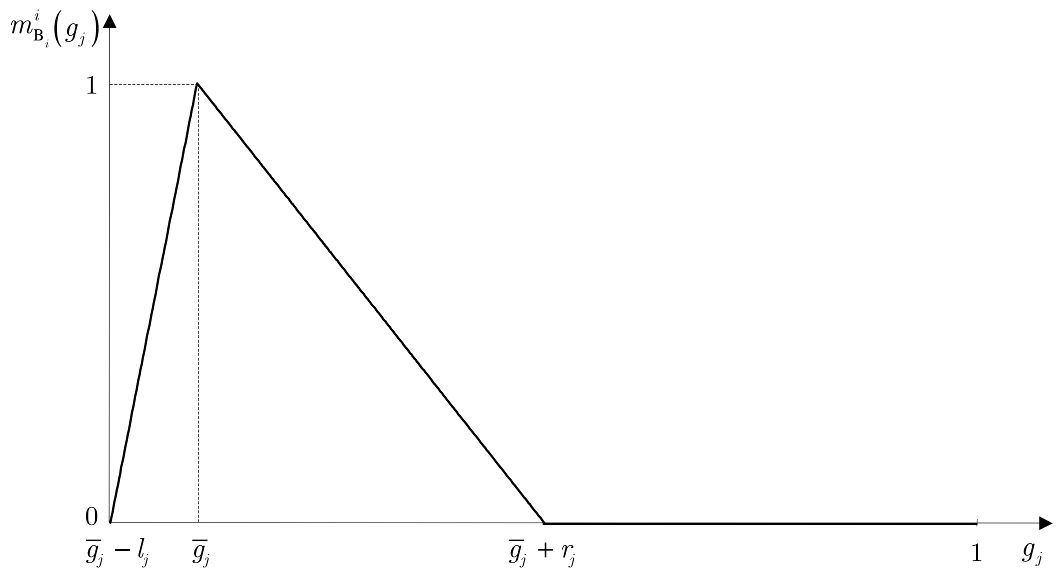

Figure 1: Membership function

$m^i_{\mathcal{B}_i}(\overline{g}_j) = 1$, for the most plausible value $\overline{g}_j$ in the function's unique maximum. Thus the membership function is triangular-shaped.

2. The degree of membership $m^i_{\mathcal{B}_i}(g_j)$ is decreasing with growing distance between g_j and $\overline{g}_j$.
3. Based on the assumptions $0 \leq g_j \leq y_j$ and $y_j = 1$ it follows $g_j \in [0, 1]$ and l_j and r_j are restricted by[31]

   $$\overline{g}_j - l_j \geq 0 \Leftrightarrow \overline{g}_j \geq l_j \tag{6}$$

   and

   $$\overline{g}_j + r_j \leq 1. \tag{7}$$

4. Higher values of l_j and r_j imply that player i is rather uninformed about the other player's contribution g_j.

Player $i's$ uncertainty regarding the other player's contribution modeled by the membership function in (5) builds a central aspect of our approach.[32] The

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[31] Note that the specified membership function, to keep things simple, only allows membership levels of 0 or 1 for $g_j = 0$. To relax this restriction further case differentiations would be required.

[32] Usually a player's membership function need not to be known by other players.

contribution g_j represents a parameter based on player $j's$ preferences and is only vaguely known by player i. The membership degrees assigned to different values may result from player $i's$ experience in similar situations, from personal knowledge with respect to the other player or from simple guessing.[33]

Given the assumption that the players' expectations are represented by fuzzy sets there arise some problems with respect to the derivation of their optimal behavior. Player $i's$ fuzzy information regarding player $j's$ contribution implies that i cannot derive his optimal decision in an easy way since there exists a continuum of contributions considered possible. Each of these contributions results in different amounts of the public good provided and therefore yields different associated utilities for i. This multivaluedness has to be tackled by means of a so called defuzzification strategy which allows the deduction of a unique crisp decision. In a stochastic framework such multivaluedness associated with the probability distribution is usually solved by using the mathematical expectation operator. As the membership degrees of a fuzzy set generally do not add up to 1 an averaging procedure like applying the mathematical expectation by weighting all possible contributions appears to be not reasonable.[34]

Within the model presented in this paper we deploy an economically reasonable defuzzification strategy instead which takes into account the different degrees of membership and the attitude towards deviations from the contribution levels considered most plausible simultaneously. Hence, in the next subsection we present a suitable approach which eliminates the uncertainty from the problem and thus generates a crisp and in some sense optimal decision. This defuzzification strategy crucially depends on the so-called α-cuts of the fuzzy set $\mathcal{B}_i$ defined by

$$\mathcal{B}_{i,\alpha} = \left\{ g_j \in [0,1] \,\middle|\, m^i_{\mathcal{B}_i}(g_j) \geq \alpha \right\}, \tag{8}$$

for all $\alpha \in (0,1]$. The crisp[35] set $\mathcal{B}_{i,\alpha}$ includes all contributions g_j exhibiting a

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[33] While in the first round of a repeated public goods game simple guessing may prevail, in the following rounds experience in similar situations (for changing fellows) or personal knowledge (for constant group composition) may emerge. See section 4.

[34] This aspect is especially meaningful if in some practical situations a normalization of the membership function is not possible or sensible.

[35] $\mathcal{B}_{i,\alpha}$ is a crisp set since the various, possible values of g_j may either be members or

degree of membership equal to or larger than α.

3.3 The defuzzification strategy

In this subsection we present a defuzzification strategy which eliminates the fuzziness in player $i's$ decision problem and leads to a transformed and crisp objective function.[36] This strategy comprises three steps and explicitly takes into account the decision maker's attitude towards possible deviations from the contribution considered most plausible. Since the information regarding g_j is fuzzy, each own contribution g_i may result in a variety of outcomes depending on the g_j actually realizing. Hence, to determine a unique decision concerning his contribution, player i necessitates a defuzzification strategy to remove the multivaluedness from the problem.

Firstly for every feasible own contribution g_i and every membership level $\alpha \in (0,1]$ the contributions $g_{j,\min}$ minimizing $i's$ utility and $g_{j,\max}$ maximizing $i's$ utility are determined over the set of all contributions with degree of membership $m^i_{\mathcal{B}_i}(g_j) \geq \alpha$, i.e., over $\mathcal{B}_{i,\alpha}$. Hence, we obtain the worst case and best case regarding $i's$ utility for given g_i and α.

By (1) in combination with (2) player $i's$ utility is an increasing function of the other player's contribution g_j. Thus, we can derive the worst case and best case regarding player $i's$ utility for each given α-cut by determining the borders of $\mathcal{B}_{i,\alpha}$ and using the membership function given in (5) as

$$g_{j,\min} = \overline{g}_j - (1-\alpha)\, l_j$$

and

$$g_{j,\max} = \overline{g}_j + (1-\alpha)\, r_j .$$

The corresponding utility levels for player i are given by

$$\min_{g_j \in \mathcal{B}_{i,\alpha}} u_i(g_i, g_j) = 1 + a\left(g_i + \left(\overline{g}_j - (1-\alpha)\, l_j\right)\right) - g_i - e_i\left(g_i - \left(\overline{g}_j - (1-\alpha)\, l_j\right)\right)^2 \tag{9}$$

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non-members of this set.

[36] The deployed defuzzification strategy was introduced by Stahlecker et al. (1999) and adopted by Arnold et al. (2000), Hauenschild and Stahlecker (2001, 2003) and Arnold et al. (2004).

and

$$\max_{g_j \in \mathcal{B}_{i,\alpha}} u_i(g_i, g_j) = 1 + a\left(g_i + \left(\overline{g}_j + (1-\alpha)\, r_j\right)\right) - g_i - e_i\left(g_i - \left(\overline{g}_j + (1-\alpha)\, r_j\right)\right)^2. \tag{10}$$

The important outcome of the first step of the chosen defuzzification strategy is the fact that the decision maker can concentrate on those contributions g_j minimizing or maximizing his utility depending on α. While this procedure causes a loss of information we regard it as very suitable to represent the process of human decision making in reality. Furthermore, it should be noted that the difference between $g_{j,\min}$ and $g_{j,\max}$ is decreasing with higher levels of α and for the membership function (5) completely vanishes for $\alpha = 1$ and therefore $g_{j,\min} = \overline{g}_j = g_{j,\max}$.

The second step of the defuzzification strategy aims at integrating the decision maker's preferences regarding the worst case and the best case scenarios in the objective function. Player i is characterized as pessimistic or optimistic depending on the weight he is putting on the lowest or highest possible contributions of the other player. At this, an absolutely pessimistic player would try to avoid being exploited by the other player's free rider behavior by maximizing his minimum utility resulting from $g_{j,\min}$ (what still depends on α). In contrast to this an perfect optimist would derive his contribution decision by considering the highest possible contributions of the other player. Hence an optimistic player is prepared to accept negative surprises with respect to the other player's behavior. Of course both cases are rather extreme and accordingly it seems more sensible to assume that player i is taking the worst case as well as the best case scenario into account to derive his optimal decision. A weighted average reflecting these considerations (for every α-cut) is given by

$$u^{\alpha}_{i,q_i}(g_i) = q_i \min_{g_j \in \mathcal{B}_{i,\alpha}} u_i(g_i, g_j) + (1-q_i) \max_{g_j \in \mathcal{B}_{i,\alpha}} u_i(g_i, g_j), \tag{11}$$

where $q_i \in [0,1]$ denotes a pessimism parameter representing player $i's$ preferences. The described cases of an absolutely pessimistic and optimistic player respectively are captured as special cases for $q_i = 1$ and $q_i = 0$. The parameter q_i is increasing with player $i's$ pessimism. Equation (11) rests upon the well-known Hurwicz principle for balancing worst case and best case.

The third step of the approach is adopted to eliminate the dependency on the α-cut still prevailing in equation (11). We remove the parameter α from the decision problem by aggregating over all α-cuts. To reach this goal we deploy a procedure which weights each $u^{\alpha}_{i,q_i}(g_i)$ with a function $w(\alpha)$. For this weighting function we assume $w : (0, 1] \rightarrow \mathbb{R}_+$ and $\int_0^1 w(\alpha)\,d\alpha = 1$.[37] Just like the pessimism parameter q_i the weighting function w can be chosen by the decision maker to reflect his attitude towards deviations from the contributions g_j considered most plausible. Implementing the three steps of the defuzzification strategy leads to the new, crisp objective function of player i given by[38]

$$\begin{aligned} Z_{i,q_i}(g_i) &= \int_0^1 w(\alpha) \cdot u^{\alpha}_{i,q_i}(g_i) d\alpha \\ &= \int_0^1 w(\alpha) \left(q_i \min_{g_j \in \mathcal{B}_{i,\alpha}} u_i(g_i, g_j) + (1 - q_i) \max_{g_j \in \mathcal{B}_{i,\alpha}} u_i(g_i, g_j) \right) d\alpha. \end{aligned}$$

By inserting equations (9) and (10) we obtain

$$Z_{i,q_i}(g_i) = \int_0^1 w(\alpha) \left(\begin{array}{c} q_i\Big(1+a\big(g_i+\big(\overline{g}_j-(1-\alpha)l_j\big)\big)-g_i-e_i\big(g_i-\big(\overline{g}_j-(1-\alpha)l_j\big)\big)^2\Big) \\ +(1-q_i)\Big(1+a\big(g_i+\big(\overline{g}_j+(1-\alpha)r_j\big)\big)-g_i-e_i\big(g_i-\big(\overline{g}_j+(1-\alpha)r_j\big)\big)^2\Big) \end{array} \right) d\alpha.$$

Based on the first derivative with respect to the decision variable g_i we get the first order condition

$$\begin{aligned} \frac{\partial Z_{i,q_i}(g_i)}{\partial g_i} &= -(1 - a) - 2e_i g_i + 2e_i \overline{g}_j \\ &\quad + \int_0^1 \left(-2q_i e_i (1 - \alpha) l_j + 2(1 - q_i) e_i (1 - \alpha) r_j\right) w(\alpha) d\alpha \overset{!}{=} 0. \end{aligned}$$

Notice again, as discussed in subsection 3.1, that for player i with $e_i = 0$ the first derivative of the objective function is always negative. Thus, the optimal feasible contribution for such a player is 0. In other words, since free riding is a dominant strategy, the fuzziness disappears from the decision problem. With $e_i > 0$ we obtain

$$g_i^* = \max \left\{ 0, \overline{g}_j - \frac{1 - a}{2e_i} + (-q_i l_j + (1 - q_i) r_j) \int_0^1 (1 - \alpha) w(\alpha) d\alpha \right\} \tag{12}$$

---

[37] For simplification we use the same weighting function $w(\alpha)$ for each player in the following.

[38] Strictly speaking we are dealing with improper integrals as the integrands are not defined at their lower limit of integration.

for the optimal contribution level of player i.[39] Note that if player i is exactly informed about the contribution of player j the fuzziness vanishes, meaning that the known value coincides with the value considered most plausible ($\overline{g}_j = g_j$) and $l_j = r_j = 0$ and it follows that (12) reduces to (3).[40] Moreover for a 'neutral' decision maker i ($q_i = 1/2$) and 'symmetric' uncertainty ($l_j = r_j$) the most plausible value $\overline{g}_j$ in (12) corresponds to the known value g_j in (3). If player j also exhibits one of these properties both constellations prevent the existence of a Nash equilibrium with positive contributions (see section 3.1). Regarding the comparative statics of the model we find that the optimal contribution g_i^* is increasing with the contribution of player j considered most plausible, $\overline{g}_j$, the marginal rate of return for contributing to the public good a, the equality preference parameter e_i and the highest value considered possible based on r_j and decreasing with the pessimism parameter q_i and the lowest value considered possible based on l_j.

If we define

$$\beta = \int_0^1 (1-\alpha)w(\alpha)d\alpha$$

and assume for both players a weighting function $w(\alpha) = 1$ which is putting the same weight on every α-cut,[41] we get $\beta = 1/2$ and can derive the optimal contribution[42] for this special case as

$$g_i^* = \max\left\{0,\, \overline{g}_j - \frac{1-a}{2e_i} + \frac{1}{2}\left(-q_i l_j + (1-q_i)r_j\right)\right\}. \tag{13}$$

To illustrate the theoretical findings we use (13) to create table 1 where we present some numerical examples showing optimal contributions for various parameter constellations.

---

[39] Due to the second derivative $Z''_{i,q_i}(g_i) = -2e_i$ the function $Z_{i,q_i}(g_i)$ is strictly concave (for $e_i > 0$). Hence, fulfilling the first order condition leads to the unique maximum.

[40] We assume $0 < \int_0^1 (1-\alpha)w(\alpha)d\alpha < \infty$.

[41] Alternatively w could be increasing in α, i.e. put more weight on higher degrees of membership. Otherwise for example in problems regarding insurance economics putting higher weights on less plausible events may be reasonable.

[42] Keep in mind that $e_i = 0$ leads to $g_i^* = 0$.

$\overline{g}_j$	l_j	r_j	q_i										
			0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0.0	0.0	0.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		0.1	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		0.2	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		0.3	0.025	0.010	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		0.4	0.075	0.055	0.035	0.015	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		0.5	0.125	0.100	0.075	0.050	0.025	0.000	0.000	0.000	0.000	0.000	0.000
		0.6	0.175	0.145	0.115	0.085	0.055	0.025	0.000	0.000	0.000	0.000	0.000
		0.7	0.225	0.190	0.155	0.120	0.085	0.050	0.015	0.000	0.000	0.000	0.000
		0.8	0.275	0.235	0.195	0.155	0.115	0.075	0.035	0.000	0.000	0.000	0.000
		0.9	0.325	0.280	0.235	0.190	0.145	0.100	0.055	0.010	0.000	0.000	0.000
		1.0	0.375	0.325	0.275	0.225	0.175	0.125	0.075	0.025	0.000	0.000	0.000
0.1	0.0	0.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		0.1	0.025	0.020	0.015	0.010	0.005	0.000	0.000	0.000	0.000	0.000	0.000
		0.2	0.075	0.065	0.055	0.045	0.035	0.025	0.015	0.005	0.000	0.000	0.000
		0.3	0.125	0.110	0.095	0.080	0.065	0.050	0.035	0.020	0.005	0.000	0.000
		0.4	0.175	0.155	0.135	0.115	0.095	0.075	0.055	0.035	0.015	0.000	0.000
		0.5	0.225	0.200	0.175	0.150	0.125	0.100	0.075	0.050	0.025	0.000	0.000
		0.6	0.275	0.245	0.215	0.185	0.155	0.125	0.095	0.065	0.035	0.005	0.000
		0.7	0.325	0.290	0.255	0.220	0.185	0.150	0.115	0.080	0.045	0.010	0.000
		0.8	0.375	0.335	0.295	0.255	0.215	0.175	0.135	0.095	0.055	0.015	0.000
		0.9	0.425	0.380	0.335	0.290	0.245	0.200	0.155	0.110	0.065	0.020	0.000
	0.1	0.0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		0.1	0.025	0.015	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
		0.2	0.075	0.060	0.045	0.030	0.015	0.000	0.000	0.000	0.000	0.000	0.000
		0.3	0.125	0.105	0.085	0.065	0.045	0.025	0.005	0.000	0.000	0.000	0.000
		0.4	0.175	0.150	0.125	0.100	0.075	0.050	0.025	0.000	0.000	0.000	0.000
		0.5	0.225	0.195	0.165	0.135	0.105	0.075	0.045	0.015	0.000	0.000	0.000
		0.6	0.275	0.240	0.205	0.170	0.135	0.100	0.065	0.030	0.000	0.000	0.000
		0.7	0.325	0.285	0.245	0.205	0.165	0.125	0.085	0.045	0.005	0.000	0.000
		0.8	0.375	0.330	0.285	0.240	0.195	0.150	0.105	0.060	0.015	0.000	0.000
		0.9	0.425	0.375	0.325	0.275	0.225	0.175	0.125	0.075	0.025	0.000	0.000
0.2	0.0	0.0	0.075	0.075	0.075	0.075	0.075	0.075	0.075	0.075	0.075	0.075	0.075
		0.1	0.125	0.120	0.115	0.110	0.105	0.100	0.095	0.090	0.085	0.080	0.075
		0.2	0.175	0.165	0.155	0.145	0.135	0.125	0.115	0.105	0.095	0.085	0.075
		0.3	0.225	0.210	0.195	0.180	0.165	0.150	0.135	0.120	0.105	0.090	0.075
		0.4	0.275	0.255	0.235	0.215	0.195	0.175	0.155	0.135	0.115	0.095	0.075
		0.5	0.325	0.300	0.275	0.250	0.225	0.200	0.175	0.150	0.125	0.100	0.075
		0.6	0.375	0.345	0.315	0.285	0.255	0.225	0.195	0.165	0.135	0.105	0.075
		0.7	0.425	0.390	0.355	0.320	0.285	0.250	0.215	0.180	0.145	0.110	0.075
		0.8	0.475	0.435	0.395	0.355	0.315	0.275	0.235	0.195	0.155	0.115	0.075
	0.1	0.0	0.075	0.070	0.065	0.060	0.055	0.050	0.045	0.040	0.035	0.030	0.025
		0.1	0.125	0.115	0.105	0.095	0.085	0.075	0.065	0.055	0.045	0.035	0.025
		0.2	0.175	0.160	0.145	0.130	0.115	0.100	0.085	0.070	0.055	0.040	0.025
		0.3	0.225	0.205	0.185	0.165	0.145	0.125	0.105	0.085	0.065	0.045	0.025
		0.4	0.275	0.250	0.225	0.200	0.175	0.150	0.125	0.100	0.075	0.050	0.025
		0.5	0.325	0.295	0.265	0.235	0.205	0.175	0.145	0.115	0.085	0.055	0.025
		0.6	0.375	0.340	0.305	0.270	0.235	0.200	0.165	0.130	0.095	0.060	0.025
		0.7	0.425	0.385	0.345	0.305	0.265	0.225	0.185	0.145	0.105	0.065	0.025
		0.8	0.475	0.430	0.385	0.340	0.295	0.250	0.205	0.160	0.115	0.070	0.025
	0.2	0.0	0.075	0.065	0.055	0.045	0.035	0.025	0.015	0.005	0.000	0.000	0.000
		0.1	0.125	0.110	0.095	0.080	0.065	0.050	0.035	0.020	0.005	0.000	0.000
		0.2	0.175	0.155	0.135	0.115	0.095	0.075	0.055	0.035	0.015	0.000	0.000
		0.3	0.225	0.200	0.175	0.150	0.125	0.100	0.075	0.050	0.025	0.000	0.000
		0.4	0.275	0.245	0.215	0.185	0.155	0.125	0.095	0.065	0.035	0.005	0.000
		0.5	0.325	0.290	0.255	0.220	0.185	0.150	0.115	0.080	0.045	0.010	0.000
		0.6	0.375	0.335	0.295	0.255	0.215	0.175	0.135	0.095	0.055	0.015	0.000
		0.7	0.425	0.380	0.335	0.290	0.245	0.200	0.155	0.110	0.065	0.020	0.000
		0.8	0.475	0.425	0.375	0.325	0.275	0.225	0.175	0.125	0.075	0.025	0.000

Table 1: Optimal contributions $g_i^*(a = 0.9,\, e_i = 0.4,\, w(\alpha) = 1)$

The contributions g_i^* depicted in table 1 are based on the values $a = 0.9$ and $e_i = 0.4$ which are in line with our assumptions stated in subsection 3.1. For various beliefs represented by different contributions of player j considered most plausible, $\overline{g}_j$, and feasible values of l_j and r_j (cf. conditions (6) and (7)) player $i's$ optimal contributions are calculated. Furthermore the optimal behavior, given these beliefs, is presented for different levels of pessimism, q_i, including the polar cases of pure optimism ($q_i = 0$) and pure pessimism ($q_i = 1$). The table clarifies the comparative statics and stresses (separated by a broken borderline) which beliefs cause an optimal contribution of 0, indicating a free riding attitude despite the existence of social preferences (since $e_i > 0$), and which beliefs give rise to positive contributions. The framed cells indicate constellations with $g_i^* = \overline{g}_j$ meaning that player i contributes exactly as much as he expects player j to contribute with highest plausibility. Moreover it should be noted that certain parameter combinations with high values of r_j or low values of q_i expressing confidence of player i lead to $g_i^* > \overline{g}_j$. These points can be found in the south-west of the solid line. The relevance of this border will be further explained in the next section where we examine conditions for the existence of Nash equilibria with positive contributions of both players.

4 Nash Equilibria

In this section the concept of a Nash equilibrium under fuzzy information introduced by Hauenschild and Stahlecker (2003) is applied to the public goods game. Firstly we present the characteristic equations of such a Nash equilibrium. The resulting equilibrium depends on the technical parameter a and the players' inherent characteristics, i.e. their equality preference parameter e_i and their degree of pessimism q_i. Furthermore the players' adjustable beliefs given by $\overline{g}_i$, l_i and r_i determine the outcome, meaning that the same group of players may end up with different equilibrium contributions. In a next step numerical examples for possible adjustment paths towards different Nash equilibria are depicted. These simulations are suitable to shed some light on the evolution of cooperation in repeated games.

4.1 Nash Equilibria under Fuzzy Information

In the following we postulate that both players' beliefs are of the same kind and thus can be represented by membership functions as given by (5). Furthermore, it is assumed that the two subjects deploy the same defuzzification strategy.[43] According to the definition of a traditional Nash equilibrium as an 'equilibrium of actions and beliefs'[44] the approach in a fuzzy environment postulates that the contributions considered most plausible by both participants, $\overline{g}_1$ and $\overline{g}_2$, must actually be realized by their fellows in an equilibrium. Only in such cases the players' beliefs are considered as satisfactorily correct, i.e. both players do not have an incentive to adjust their beliefs. Additionally, to complete the specification of the equilibrium, the usual requirement of mutual best responses has to be taken into account, meaning that g_1 and g_2 have to maximize the utility of the players for the given beliefs.

Taking this properties and (13) into account, a Nash equilibrium under fuzzy information for the case of two players (and for $\beta = 1/2$) is characterized by[45]

$$g_1^* = \max\left\{0,\, \overline{g}_2 - \frac{1-a}{2e_1} + \frac{1}{2}\left(-q_1 l_2 + (1-q_1) r_2\right)\right\} = \overline{g}_1 \text{ and} \tag{14}$$

$$g_2^* = \max\left\{0,\, \overline{g}_1 - \frac{1-a}{2e_2} + \frac{1}{2}\left(-q_2 l_1 + (1-q_2) r_1\right)\right\} = \overline{g}_2. \tag{15}$$

The contributions g_1^* and g_2^* firstly are mutual best response strategies and secondly match both players' expectations[46] in the sense that the contributions considered most plausible by the players are actually provided by their fellows ($g_1^* = \overline{g}_1$ and $g_2^* = \overline{g}_2$). With regard to Hauenschild and Stahlecker (2003) it should be stated that the equations (14) and (15) combine the requirements of n optimal contributions and $n(n-1)$ consistent beliefs for $n = 2$. Note

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[43] The properties of the membership and objective functions postulated by Hauenschild and Stahlecker (2003), p. 170 are satisfied for our model.

[44] Cf. Varian (1992), pp. 265-266, and Mas-Colell et al. (1995), pp. 248-250, for discussions concerning the concept of Nash equilibrium and possibilities for interpretation. We will refer to this issue in the next subsection.

[45] See Hauenschild and Stahlecker (2003), pp. 169.

[46] Remember that because of their dominant strategy the beliefs of pure income maximizers ($e_i = 0$) may be neglected. Thus, the condition of consistent expectations does not have to be considered for these players.

that, while actions and beliefs are pairwise corresponding, the players still keep the possibility in mind (l_1, l_2, r_1, and r_2 are still relevant in (14) and (15) respectively) that the other player may choose a contribution deviating from the one considered most plausible. This seems to be realistic since each player, even though having no incentive to change his own contribution, does not know if the other player will stick to his strategy. Consequently the uncertainty about the other player's decision is part of the equilibrium.[47]

Combining and reducing the equations (14) and (15) we obtain for a Nash equilibrium

$$g_1^* = \max\{0,\, g_2^* + D_1\}\,, \text{ with } D_1 = -\frac{1-a}{2e_1} + \frac{1}{2}\left(-q_1 l_2 + (1-q_1) r_2\right) \text{ and}$$

$$g_2^* = \max\{0,\, g_1^* + D_2\}\,, \text{ with } D_2 = -\frac{1-a}{2e_2} + \frac{1}{2}\left(-q_2 l_1 + (1-q_2) r_1\right).$$

Using these simplifications we can state that in a Nash equilibrium under fuzzy information with positive contributions of both players, i.e. $g_1^* = \overline{g}_1 > 0$ and $g_2^* = \overline{g}_2 > 0$, the condition

$$D_1 = -D_2 \tag{16}$$

must always hold. It should be emphasized that D_i depends on g_j^* since $0 \leq l_j \leq g_j^*$ and $0 \leq r_j \leq 1 - g_j^*$ due to (6) and (7).[48] For Nash equilibria with $g_i^* > g_j^* = 0$ we find that $D_i = g_i^*$. Furthermore, for no-contribution equilibria comprising $g_i^* = g_j^* = 0$ the condition (16) is not relevant.

In order to further specify the set of possible Nash equilibria we define the upper and the lower bounds for mutual best responses depending on players' inherent characteristics as

$$\begin{aligned}
\overline{g}_1^*(g_2^*) &= \max\left\{0,\, g_2^* - \frac{1-a}{2e_1} + \frac{1}{2}(1-q_1)(1-g_2^*)\right\}, && (17)\\
\overline{g}_2^*(g_1^*) &= \max\left\{0,\, g_1^* - \frac{1-a}{2e_2} + \frac{1}{2}(1-q_2)(1-g_1^*)\right\}, && (18)\\
\underline{g}_1^*(g_2^*) &= \max\left\{0,\, g_2^* - \frac{1-a}{2e_1} + \frac{1}{2}(-q_1 g_2^*)\right\} \text{ and} && (19)
\end{aligned}$$

---

[47] Cf. Hauenschild and Stahlecker (2003), p. 173.

[48] Especially, we can determine the upper limit of D_i for $a \to 1$, $e_i \to a/2$, $q_i = 0$ and $r_j = 1$ as $D_i \to 1/2$. The lower bound is given by the condition $D_i = -D_j$. Remember that $r_j = 1$ implies $\overline{g}_j = 0$.

$$\underline{g}_2^*(g_1^*) \quad = \quad \max\left\{0,\, g_1^* - \frac{1-a}{2e_2} + \frac{1}{2}\left(-q_2 g_1^*\right)\right\}. \tag{20}$$

At this, $\bar{g}_i^*(g_j^*)$ is the highest possible best response resulting from considering 'socially optimal' beliefs, i.e. a largest contribution of $g_j^* + r_j = 1$ as possible. Analogous $\underline{g}_i^*(g_j^*)$ represents the lowest best response in consequence of 'socially detrimental' beliefs of $g_j^* - l_j = 0$.

Combining (17) and (18) we find for both players' optimal contributions in a Nash equilibrium

$$g_1^* \quad \leq \quad \max\left\{0,\, 1 - \frac{(1+q_1)(1-a)}{e_2(3-q_1-q_2-q_1q_2)} - \frac{2(1-a)}{e_1(3-q_1-q_2-q_1q_2)}\right\}, \tag{21}$$

$$g_2^* \quad \leq \quad \max\left\{0,\, 1 - \frac{(1+q_2)(1-a)}{e_1(3-q_1-q_2-q_1q_2)} - \frac{2(1-a)}{e_2(3-q_1-q_2-q_1q_2)}\right\}. \tag{22}$$

Note that in an 'optimal society' defined by socially optimal beliefs, absolute optimistic attitudes of all players ($q_1 = q_2 = 0$) and widely social preferences ($e_1 = e_2 \rightarrow a/2$) the highest optimal contributions in a Nash equilibrium are given as $g_1^* = g_2^* \rightarrow 1 - \frac{2(1-a)}{a}$. For $a \rightarrow 1$ this expression converges towards the socially optimal contributions of $g_1^* = g_2^* = 1$.[49]

Figure 2 serves to illustrate two sets containing pairs of contributions which may appear in Nash equilibria for a marginal productivity parameter $a = 0.9$ and $\beta = 1/2$. The light set is restricted according to equations (17)-(20) for values of an optimal society (OS) defined above. In consequence of this all possible Nash equilibria in this set only depend on the players' beliefs given by r_i.[50] The huge differences in the optimal contributions and accordingly in the provided quantity of the public good clarify the crucial role of beliefs for outcomes in social interaction. The point OS denotes the best possible Nash equilibrium in an optimal society, which is realized if both players exhibit most confident beliefs of $r_i = 1 - g_i^*$. Since the point OS lies on the 45°-line it implies equal contributions ($D_1 = -D_2 = 0$) of both players. An increase of

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[49] The lowest possible mutual best responses in a Nash equilibrium are clearly zero.

[50] Since $q = 0$ the value of l_i, which influences the (ignored) worst case, is irrelevant (see (14) and (15)).

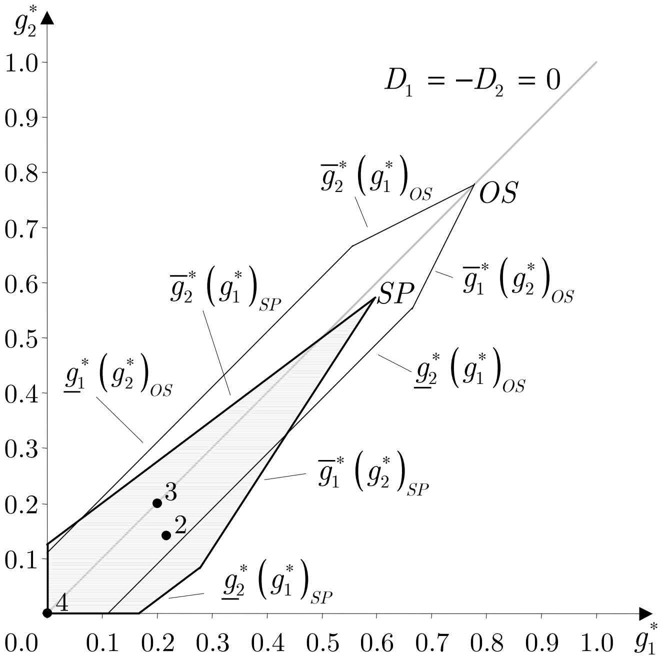

Figure 2: Sets of possible Nash equilibria: Specific players vs. optimal society ($a = 0.9$, $\beta = 1/2$)

the parameter a would shift this point to the north-east towards its maximum of $g_1^* = g_2^* \rightarrow 1$.

In contrast to this, the shaded area depicts a situation with two specific players (SP) showing inherent characteristics of $e_1 = e_2 = 0.4$ as well as $q_1 = 0.3$ and $q_2 = 0.5$. Hence, the boundaries of this set are determined by plugging these values into (17)-(20). The asymmetry of the shaded set results from the players different attitude towards worst and best cases, since player 1 is more optimistic. The actually resulting Nash equilibrium again is determined by the players beliefs l_i and r_i. Since the restriction given by $\underline{g}_1^*(g_2^*)_{SP}$ is already covered by $\overline{g}_2^*(g_1^*)_{SP}$ it is not presented. The reason for this is the rather pessimistic attitude of player 2, which induces him to be quite reluctant with respect to his contribution. Regarding the comparative statics it can be stated that altering q_i leads to a changing slope of the boundaries, while varying values of a and e_i shift them. The point SP characterizes the highest contribution Nash equilibrium for the specific players under consideration. It reflects their heterogeneity with the more optimistic player 1 making the higher contribution for $D_1 = -D_2 \neq 0$. The points in the shaded area mark certain equilibria to which we will refer in the next subsection.

Furthermore it is notable that the light set is mostly bounded by the lowest possible best responses, while the shaded set is primarily restricted by the highest possible contributions. This reflects the fact that an optimal society is defined by the highest possible preferences for equality which causes players to refrain from exploiting their fellows too much. Thus, the light set is close to the 45°-line. Actually, there exist two areas (where the shaded set lies outside of the light set) for which the provided quantity of the public good for the specific players is possibly higher than in the optimal society. This is, since in the optimal society no one of the players wants to react with too low contributions for a given decision of his fellow. Summing it up, the figure impressively illustrates the importance of beliefs for the selection of actual results from the set of possible equilibria.

In a further step we reconsider - in the light of the definition of a Nash equilibrium under fuzzy information - the framed cells in table 1 of section 3 showing, e.g., an optimal contribution of $g_i^* = \overline{g}_j = 0.200$ (for $i, j = 1, 2$). Such a situation (as all constellations in framed cells) represents a Nash equilibrium under fuzzy information with positive and equal contributions of both players satisfying the condition (16) since $D_i = -D_j = 0$. All constellations with $D_i > D_j$ lie in the south-west of the solid line. Notice that not all of these contributions and beliefs may be part of a Nash equilibrium.

It is shown that our model may explain positive contributions of both players as Nash equilibria under fuzzy information and thus is suitable to account for the first stylized fact appearing in public goods games. In the following we consider some possible adjustment processes towards different Nash equilibria aiming to explain the evolution of cooperation in repeated public goods games.

4.2 Simulation of simple adjustment processes

There is no reason why the players' contributions and beliefs in a public goods game under fuzzy information should comply with the requirements of a Nash equilibrium defined in equations (14) and (15). Moreover, we even cannot assure that the outcomes converge towards a Nash equilibrium in repeated social interaction. In traditional game theory the implementation of strategies and beliefs being consistent with Nash equilibria is usually explained by conjectural variations of the players and corresponding implicit adjustment processes.[51] In our model it is questionable how the players will react if their beliefs are incorrect and their contributions are not maximizing their utility. Since we do not assume common knowledge regarding utility functions and membership functions and deal with the provision of a public good in a fuzzy environment, there seems to be no striking argument to model the players' conjectures or adaption behavior in a definite way. Therefore, we assume in this subsection that the players' initial behavior deviates from the equilibrium and propose simple adjustment procedures based on rules of thumb. These procedures seem to be suitable to replicate peoples' decision processes in reality.[52] In all of the following examples the adjustment processes result in rest points, i.e. Nash equilibria.[53] However, one should keep in mind that players' interactions do not need to arrive at any rest point.

We examine different adjustment processes leading from arbitrary non-equilibrium initial points to different Nash equilibria after five rounds of adaption in a two-player game. The players' reactions after having observed that their beliefs were incorrect are modeled in two alternative ways. Firstly we assume that the players move their somewhat inaccurate membership functions (including $\bar{g}_i \neq g_i^*$) gradually in the direction of the lastly observed foreign contribution. Alternatively we adopt a procedure in a second step, which instantly turns the observation of this round into the most plausible expectation for the next round. In this way we manage on the one hand to emphasize the crucial importance of the interaction of preferences and beliefs, and on the other hand

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[51] See e.g. Varian (1992), pp. 302-303.

[52] Note that we do not postulate that every step of the adjustment process meets the requirements of correct beliefs and mutual best responses. Especially if player j is regarded as a representative agent reflecting the average behavior of $n-1$ participants, it seems difficult to set limits to the shape and evolution of player $i's$ beliefs. This approach is suitable for experiments using the stranger condition, i.e. the group composition changes from round to round. It remains an open question, how a 'rational' adjustment procedure may be defined in a fuzzy environment.

[53] Cf. Mas-Colell et al. (1995), p. 249: 'The notion of an equilibrium as a rest point for some dynamic adjustment process underlies the use and the traditional appeal of equilibrium notions in economics.'

to depict processes identified in repeated public goods experiments.

The presented examples capture the well-known decay of cooperation as well as persisting and increasing positive contributions in public goods games. We assume again $a = 0.9$ and $\beta = 0.5$ (for both players) as well as $e_1 = 0.4$ (cf. also table 1 and figure 2) and $q_1 = 0.3$ for the inherent characteristics of player 1 throughout this subsection.

Consider now table 2 which points out a first example of an adjustment process towards a Nash equilibrium. This scenario depicts the result that convergence to the standard outcome of no contributions may occur even if certain subjects (player 1 in this case) are very strongly interested in social behavior.[54] It is highlighted that player 1 'learns' that player 2 exhibits free rider behavior.[55] Due to this information feedback player 1 has an incentive to reduce his contributions until ending with equilibrium investments of 0 in step 5.[56]

	Player 1				Player 2			
Step	g_1^*	$\overline{g}_2$	l_2	r_2	g_2^*	$\overline{g}_1$	l_1	r_1
1	0.19	0.20	0.20	0.40	0.00	-	-	-
2	0.13	0.15	0.15	0.35	0.00	-	-	-
3	0.07	0.10	0.10	0.30	0.00	-	-	-
4	0.01	0.05	0.05	0.25	0.00	-	-	-
5	0.00	0.00	0.00	0.20	0.00	-	-	-

Table 2: Free riding vs. conditional cooperation, gradual adjustment of beliefs and the social dilemma

Player 1 gradually reduces his beliefs regarding the contribution of player 2 considered most plausible, $\overline{g}_2$, as well as the possibility of lower (l_2) and higher (r_2) contributions in constant steps of 0.05. Based on this evolution of beliefs he derives his own contributions g_1^* according to (13). In step 5 the resulting Nash

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[54] Cf. Fehr and Schmidt (2003), p. 212.

[55] Remember that free rider behavior results from a dominant strategy. Thus, in this case player $2's$ beliefs and degree of pessimism do not influence his decision and can be ignored.

[56] As a possible interpretation of this process, one may say that player 1 loses his 'team spirit' in consequence of his disappointment regarding player $2's$ behavior. Bewley (1999) notes that similar reasoning may cause companies to fire shirkers to maintain the work morale of the remaining workers.

equilibrium featuring no contributions by both players is depicted. Since player 1 is interested in conditional cooperation his contribution stepwise corrodes in the light of player $2's$ permanent free riding.

The second scenario shows an adjustment process towards a Nash equilibrium with positive contributions of both players. Player 2 is neither optimistic nor pessimistic oriented ($q_2 = 0.5$) and features the same attitude towards the equality of contributions as player 1 ($e_2 = 0.4$). Hence, both players can be characterized as conditional cooperators. We maintain these characteristics, which correspond to the shaded set of Nash equilibria in figure 2, in the remainder of this section. The membership functions representing the initial beliefs of the players differ in all three parameters. As before, the players adjust their beliefs gradually in the direction of the last observation. The distance between l_i and r_i is decreasing or constant from step to step. This reflects the sharpening of the players' beliefs leading to less uncertainty. Going on with this process both players alter their contributions, even though due to different belief adjustment processes (cf. table 3). Finally an asymmetric Nash equilibrium with $g_1^* = \overline{g}_1 = 0.21$ and $g_2^* = \overline{g}_2 = 0.14$ arises and (16) is satisfied for $D_1 = -D_2 = 0.07$. The resulting equilibrium is denoted by point 2 in the shaded area of figure 2.[57]

	Player 1				Player 2			
Step	g_1^*	$\overline{g}_2$	l_2	r_2	g_2^*	$\overline{g}_1$	l_1	r_1
1	0.26	0.18	0.18	0.66	0.12	0.17	0.17	0.46
2	0.25	0.17	0.17	0.65	0.12	0.18	0.18	0.45
3	0.24	0.16	0.16	0.64	0.13	0.19	0.19	0.44
4	0.22	0.15	0.15	0.63	0.13	0.20	0.20	0.43
5	0.21	0.14	0.14	0.62	0.14	0.21	0.21	0.42

Table 3: Conditional cooperative players, gradual adjustment of beliefs and positive contributions

In scenario 3 we consider instant instead of gradual adjustment of expectations meaning that both players update their beliefs by setting $\overline{g}_i$ in the current

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[57] The first scenario is not depicted in figure 2 since in this case the characteristics of the players deviate from the ones underlying both sets.

step equal to g_i^* observed in the previous step. In addition their membership functions are stepwise updated with decreasing distance between l_i and r_i and thus decreasing uncertainty of both players. Furthermore, during the whole process the players' contributions and beliefs differ only in r_i. After starting with initial contributions of 0.16 the cooperative atmosphere in this situation induces both players to bring in increasing contributions. Thus, their 'like-minded' attitude results in an equilibrium showing positive contributions of $g_1^* = \overline{g}_2 = 0.200 = \overline{g}_1 = g_2^*$ and $D_1 = -D_2 = 0$ (cf. table 4 and point 3 on the 45°-line in figure 2).[58]

	Player 1				Player 2			
Step	g_1^*	$\overline{g}_2$	l_2	r_2	g_2^*	$\overline{g}_1$	l_1	r_1
1	0.16	0.13	0.12	0.49	0.16	0.13	0.12	0.73
2	0.18	0.16	0.12	0.46	0.18	0.16	0.12	0.69
3	0.19	0.18	0.11	0.44	0.19	0.18	0.11	0.66
4	0.20	0.19	0.11	0.42	0.20	0.19	0.11	0.63
5	0.20	0.20	0.10	0.40	0.20	0.20	0.10	0.60

Table 4: Conditional cooperative players, instant adjustment of beliefs and increasing positive contributions

Although both players exhibit an attitude to cooperate conditionally, a spiral decline in contributions may emerge. In the last example this is shown for different initial expectations and a slightly modified process to update beliefs. Note that player 1 in the initial situation exhibits exactly the same contribution and beliefs as in the Nash equilibrium in scenario 3. Therefore all deviations from this result have to be caused by the influence of beliefs. While we still assume an instant adjustment as described in scenario 3, the whole range of contributions considered as possible develops more erratic as before (see table 5). The remarkable outcome of this situation is that plausible adjustment processes of two players with an extensive affinity towards cooperation may

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[58] Note that the equilibrium contributions but not the players' characteristics are equal. This outcome is obtained since player 2 compensates his higher degree of pessimism ($q_2 = 0.5 > 0.3 = q_1$) through higher contributions considered possible ($r_2 > r_1$). Therefore, a kind of trade-off between uncertainty and the attitude towards this uncertainty is clarified.

end up in an equilibrium with no contributions at all. Such a result may be seen as a consequence of a misunderstanding between the players and is illustrated by point 4 in figure 2.

	Player 1				Player 2			
Step	g_1^*	$\overline{g}_2$	l_2	r_2	g_2^*	$\overline{g}_1$	l_1	r_1
1	0.20	0.20	0.10	0.40	0.13	0.20	0.20	0.42
2	0.11	0.13	0.10	0.33	0.11	0.20	0.20	0.34
3	0.06	0.11	0.10	0.26	0.02	0.11	0.11	0.26
4	0.00	0.02	0.02	0.19	0.00	0.06	0.06	0.18
5	0.00	0.00	0.00	0.12	0.00	0.00	0.00	0.10

Table 5: Conditional cooperative players, instant adjustment of beliefs, misunderstanding and the social dilemma

As highlighted by the four presented scenarios a dynamic interpretation of our approach may capture the possible evolution of cooperation in different environments. In scenario 1 the cooperative attitude of player 1 is destroyed by player $2's$ ongoing free rider behavior and the public good is not provided. The second scenario shows an asymmetric equilibrium with positive contributions. Player 2 is less optimistic and therefore makes a lower contribution in equilibrium. Scenario 3 illustrates for instant adjustment of beliefs the possibility of increasing contributions by both players and ceases in a Nash equilibrium with positive and equal contributions. The final scenario 4 presents a situation where uncoordinated updating of beliefs leads to a social dilemma in a cooperative environment. The examples 2, 3 and 4 demonstrate that social interaction of like-minded and social oriented players may lead to highly different equilibrium outcomes. This phenomenon is reflected by the shaded set in figure 2. Summing things up one may state that a dynamic interpretation of our model may add to the understanding of the second stylized fact, i.e. changes in cooperation behavior in repeated interaction.

5 Conclusion

This paper complements the literature on decision processes underlying the voluntary contribution to public goods. We present a model which is suitable to capture conditional cooperation and free riding, two heterogeneous types of preferences emerging most frequently in recent public goods experiments. Moreover we take into account the players' uncertainty regarding the contribution behavior of their counterparts by means of fuzzy sets.

It is shown in an economically plausible way how a utility function capturing social preferences in combination with uncertainty regarding other players' behavior and an individual pessimistic or optimistic attitude may give rise to optimal decisions implying positive contributions. Thereby the employed defuzzification strategy serving to eliminate the multivaluedness from the decision problem is based on the Hurwicz principle. We complement our results through a numerical analysis which in addition illustrates some comparative statics aspects.

In a further step we analyze possible Nash equilibria under fuzzy information and conceivable adjustment paths towards them. At this, the realistic set-up of our approach clarifies the substantial relevance of preferences and beliefs for decision processes in social interaction in repeated games. It is demonstrated, consistent with empirical evidence, that besides the Nash equilibrium representing the famous social dilemma there may result many Nash equilibria comprising positive contributions of the players. Even the socially optimal fully cooperative solution may be reached as a borderline case of Nash equilibria under fuzzy information and social preferences.

The multitude of contribution strategies possibly being part of Nash equilibria under fuzzy information suggests the crucial importance of beliefs in groups as well as in whole societies. Depending on the atmosphere with respect to beliefs and trust in social interaction very different situations implying a wide scope of resulting social welfare may arise. Frey and Torgler (2004) emphasize the important role of beliefs regarding other people's behavior for tax morale, meaning that potential Nash equilibria may comprise most of the people paying their taxes honestly or alternatively a widespread culture of tax evasion in the population. Another example is related to the productivity of workers which seems to be influenced by their perception of management and co-workers. Ichino and Maggi (2000) as well as Falk and Ichino (2006) present empirical evidence saying that subjects adapt their work morale to those of their peers in social interaction. In such cases policy makers or managers may have the possibility to alter people's behavior by modifying their beliefs in an attempt to realize a more preferable Nash equilibrium.

With respect to empirical research clear-cut and testable hypotheses about the interaction between preferences and beliefs in the economically meaningful decision process of contributing to the provision of a public good can be deduced from our model. It seems possible to identify the relevant parameters in the style of Fischbacher and Gächter (2006) who try to elicit the players' affinity to conditional contribution (approximated by e_i in our approach) using the strategy method in a one-shot standard public goods game. Moreover they ask for subjects' beliefs about other players' contributions (approximated by $\bar{g}_j$ in our model). Inquiring in addition uncertainty regarding $\bar{g}_j$ (e.g. by asking for something like l_j and r_j in this paper) and an attitude towards this uncertainty (e.g. a pessimism parameter q_i) would meet all requirements to test our model of human behavior in decision processes empirically.

Appendix

In this appendix we enhance our approach to the case of n players. The utility function of player i changes to

$$u_i(g_1, ..., g_n) = y + aG - g_i - \sum_{j=1, j \neq i}^{n} e_{ij} \left(g_i - g_j\right)^2 \text{ for } i, j \in \{1, ..., n\}, i \neq j \tag{23}$$

with $G = \sum_{i=1}^{n} g_i$. Under the assumption that the contributions can be assigned to the players ex-post,[59] player i is in the position to relate individual equality preference parameters e_{ij} to each single player j. We still assume $e_{ij} \geq 0$ and $e_{ij} < a/2$.[60] Furthermore the assumptions $0 \leq g_i \leq y = 1$ (for $i \in \{1, ..., n\}$) as well as $1/n < a < 1$ should hold.

By maximizing (23) we derive the optimal contribution of player i under perfect information (and for at least one $e_{ij} > 0$) as

$$g_i^* = \max\left\{0, \frac{\sum_{j \neq i} e_{ij} g_j}{\sum_{j \neq i} e_{ij}} - \frac{1-a}{2\sum_{j \neq i} e_{ij}}\right\}.$$

Similarly to the two player game, it can be shown that there exist values of $g_j \in [0, 1]$ (for $j \in \{1, ..., n\}, i \neq j$) which induce player i to undertake positive contributions.[61] However, for $g_i^* > 0$ the contribution g_i^* is lower than the weighted average of the other players' contributions, $\sum_{j \neq i} e_{ij} g_j / \sum_{j \neq i} e_{ij}$, which implies that there exists a fellow player j with $g_j > g_i^*$.[62] Since this result holds for all players, the only existing Nash equilibrium once again exhibits contributions of zero by all players.

In the following we reconsider the players' uncertainty about the other players' contributions. To depict these vague information we assume that player i

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[59] If only the total amount of the public good is observable ex-post, the situation can be modeled in the two-player framework with one representative player (cf. subsection 3.1).

[60] We postulate $\partial u_i(\cdot)/\partial g_j = a + 2e_{ij}\left(g_i - g_j\right) > 0$ resulting in $e_{ij} < a/2$ for $j \in \{1, ..., n\}$, $i \neq j$ for identical economic reasons as presented in section 3.

[61] Especially player i may only be motivated to contribute to the public good, if another player contributes $g_j > 0$, and this player j is relevant for $i's$ social comparison (reflected by $e_{ij} > 0$).

[62] Suppose instead that $g_i^* \geq g_j$ for all j. That would imply $g_i^* \sum_{j \neq i} e_{ij} \geq \sum_{j \neq i} e_{ij} g_j$ which is equivalent to $g_i^* \geq \frac{\sum_{j \neq i} e_{ij} g_j}{\sum_{j \neq i} e_{ij}}$. But this is impossible since $(1-a)/(2\sum_{j \neq i} e_{ij}) > 0$.

possesses $n-1$ membership functions $m^i_{\mathcal{B}_{ij}} : [0,1] \rightarrow [0,1]$ based on the appropriate fuzzy sets $\mathcal{B}_{ij}$. These membership functions represent player $i's$ beliefs with respect to each single player's level of contribution by

$$m^i_{\mathcal{B}_{ij}}(g_{ij}) = \begin{cases} 0, \text{ if } g_{ij} < \overline{g}_{ij} - l_{ij} \\ 1 - \frac{\overline{g}_{ij} - g_{ij}}{l_{ij}}, \text{ if } \overline{g}_{ij} - l_{ij} \leq g_{ij} \leq \overline{g}_{ij} \\ 1 - \frac{g_{ij} - \overline{g}_{ij}}{r_{ij}}, \text{ if } \overline{g}_{ij} \leq g_{ij} \leq \overline{g}_{ij} + r_{ij} \\ 0, \text{ if } g_{ij} > \overline{g}_{ij} + r_{ij}, \end{cases} , i \neq j. \tag{24}$$

The properties of (24) remain unchanged compared to the membership function given by (5).

Furthermore we apply the three-stage defuzzification strategy introduced in section 3. For the first step we examine α-cuts for every $\mathcal{B}_{ij}$ as defined by (8). Since player $i's$ utility is increasing in each g_j we derive worst and best cases regarding $u_i(\cdot)$ for all $n-1$ membership functions considered by player i as

$$\begin{aligned} g_{ij,\min} &= \overline{g}_{ij} - (1-\alpha)\, l_{ij} \text{ and} \\ g_{ij,\max} &= \overline{g}_{ij} + (1-\alpha)\, r_{ij}, \text{ for } j \in \{1, ..., n\}, i \neq j. \end{aligned}$$

Therefore, taking (23) into account, the corresponding utility levels for player i are given by

$$\begin{aligned} \min_{g_{ij} \in \mathcal{B}_{ij,\alpha}} u_i(\cdot) &= 1 + a\left(g_i + \textstyle\sum_{j \neq i} \left(\overline{g}_{ij} - (1-\alpha)\, l_{ij}\right)\right) - g_i \\ &\quad - \textstyle\sum_{j \neq i} e_{ij} \left(g_i - \left(\overline{g}_{ij} - (1-\alpha)\, l_{ij}\right)\right)^2 \end{aligned} \tag{25}$$

and

$$\begin{aligned} \max_{g_{ij} \in \mathcal{B}_{ij,\alpha}} u_i(\cdot) &= 1 + a\left(g_i + \textstyle\sum_{j \neq i} \left(\overline{g}_{ij} + (1-\alpha)\, r_{ij}\right)\right) - g_i \\ &\quad - \textstyle\sum_{j \neq i} e_{ij} \left(g_i - \left(\overline{g}_{ij} + (1-\alpha)\, r_{ij}\right)\right)^2 . \end{aligned} \tag{26}$$

Note that (26) and (25) imply that player i considers the same α-cut to evaluate the worst and the best case for each j.[63] In the second step we use equation (11) for every $\mathcal{B}_{ij,\alpha}$ which enables us to apply the Hurwicz criterion including

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[63] This approach seems reasonable since, independent of the respective membership function, player i only considers α-cuts representing the same plausibility simultaneously.

the pessimism parameter q_i. The last step rests upon a weighting function[64] $w(\alpha)$ to eliminate the dependency on the α-cuts from the decision problem. Combining (11), (25) and (26) for every $\mathcal{B}_{ij,\alpha}$ and integrating over all α, we derive the crisp objective function of player i as

$$Z_{i,q_i}(g_i) = \int_0^1 w(\alpha) \left(q_i \min_{g_{ij} \in \mathcal{B}_{ij,\alpha}} u_i(\cdot) + (1 - q_i) \max_{g_{ij} \in \mathcal{B}_{ij,\alpha}} u_i(\cdot) \right) d\alpha. \tag{27}$$

Calculating the first derivative[65] of (27) leads to the first order condition

$$\begin{aligned} \frac{\partial Z_{i,q_i}(g_i)}{\partial g_i} &= -(1-a) - 2g_i \sum_{j \neq i} e_{ij} + 2 \sum_{j \neq i} e_{ij} \overline{g}_{ij} \\ &\quad +2 \int_0^1 \Big(- q_i \sum_{j \neq i} e_{ij}(1-\alpha) l_{ij} + (1-q_i) \sum_{j \neq i} e_{ij}(1-\alpha) r_{ij} \Big) w(\alpha) d\alpha \\ &\overset{!}{=} 0, \end{aligned}$$

which results in the optimal contribution level[66] for player i

$$g_i^* = \max \left\{ 0, \frac{\sum_{j \neq i} e_{ij} \overline{g}_{ij}}{\sum_{j \neq i} e_{ij}} + D_i(\beta) \right\} \text{ with} \tag{28}$$

$$D_i(\beta) = -\frac{1-a}{2 \sum_{j \neq i} e_{ij}} + \beta \left(-q_i \frac{\sum_{j \neq i} e_{ij} l_{ij}}{\sum_{j \neq i} e_{ij}} + (1-q_i) \frac{\sum_{j \neq i} e_{ij} r_{ij}}{\sum_{j \neq i} e_{ij}} \right) \tag{29}$$

Player $i's$ optimal contribution depends on his personal concern regarding each other player j. While low interest represented by small values of e_{ij} leads to little impact on his decision, higher e_{ij} characterizing ‘reference’ subjects influence $i's$ behavior more significantly. Thus, player i may discriminate between fellows being more or less important for his optimal contribution.

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[64] We maintain the same weighting function $w(\alpha)$ for all players and the assumptions $w : (0,1] \to \mathbb{R}_+$ with $\int_0^1 w(\alpha) = 1$.

[65] Notice that for pure income maximization, $e_{ij} = 0$ (for $i, j \in \{1, ..., n\}, i \neq j$), the first derivative is always negative inducing free riding as dominant strategy with $g_i^* = 0$. Clearly, for a problem with a dominant strategy, fuzziness is not relevant (see also subsection 3.3).

[66] The second derivative $Z''_{i,q_i}(g_i) = -2 \sum_{j \neq i} e_{ij}$ is always negative (for at least one $e_{ij} > 0$). Thus, the function $Z_{i,q_i}(g_i)$ is strictly concave and fulfilling the first order condition leads to the unique maximum. Moreover we still assume $\beta = \int_0^1 (1-\alpha) w(\alpha) d\alpha$ and $0 < \beta < \infty$.

Furthermore (28) is applied to derive necessary conditions for the existence of a Nash equilibrium under fuzzy information. We suppose again that the contributions considered most plausible must actually be realized in an equilibrium. In addition each g_i has to maximize the utility of player i (for $i \in \{1, ..., n\}$) for given beliefs, meaning that the contributions undertaken in equilibrium have to represent mutual best responses. Thus, for the case of n players (and for $\beta = 1/2$) we obtain the conditions

$$g_i^* = \max \left\{ 0, \frac{\sum_{j \neq i} e_{ij} \overline{g}_{ij}}{\sum_{j \neq i} e_{ij}} + D_i \left(\beta = \frac{1}{2} \right) \right\} = \overline{g}_{ji} \text{ for all } i \neq j. \tag{30}$$

It is clarified by the conditions given by (30) that depending on the technical parameter a, the players' inherent characteristics reflected by e and q and their beliefs given by $\bar{g}$, l and r an infinite number of possible Nash equilibria under fuzzy information results.

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