Two-stage group rent-seeking with relative preferences

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Abstract

The purpose of this paper is to examine the rent dissipation in a group rent-seeking contest in which rent seeking activities take place in two stages compared to the rent dissipation of an one-stage individual contest. Especially the effect of the model extention through the introduction of rent-seeker with relative preferences is analyzed. Focusing on the relationship between contest-structure and preference-type we find that rent dissipation given absolute seekers is in a two-stage contest lower than in a one-stage contest and given relative seekers it is vice versa. We also show that relative preferences acts to increase rent dissipation.

Keywords: rent-seeking; two-stage group contest; relative preferences
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1 Introduction

Various economic, political and social interactions can be viewed as rent seeking contests. Examples are political competition, lobbying and R&D competition. These examples show the enormous number and the importance of rent seeking contests in the real life. One component in the field of rent seeking tournaments are group oriented rent seeking contests. For example struggles for government support (request for subsidies, lobbying for relaxed regulations, imposing tax reductions, etc.) betweeen different industries clearly involve group decisions, because it affects several firms rather than an individual unit. Our paper examines group rent seeking contest in which rent seeking activities take place in two stages.[1] In the first stage two groups of homogenous players compete for a single rent V. Aggregate group effort determines the probability that the group wins the rent. There exist no predetermined distribution rule in the groups like in Nitzan (1991). Instead the members of the winning group rent-seek in a second stage for the prize, which may but need not be divisible in this stage. Individual spendings determine the probability that a particular individual receives the rent or the proportion of the rent allocated to each member. The results derived in this paper apply in both situations.

Earlier work in the literature for two-stage group rent seeking is done by Katz and Tokatlidu (1996). In a model with symmetric rent valuation they focus on the relation between group size and aggregate rent seeking and show that social waste depends not only on total numbers but also on the distribution across groups. Group size asymmetry acts to reduce rent dissipation. The model of Katz and Tokatlidu is basis for Stein and Rapoport (2004) as well as for this paper here. Stein and Rapoport compare two variations of a two-stage contest. They consider more than two groups and focusing on asymmetries between groups and players. Our paper extend the model by introducing a technology parameter and players with relative preferences. In the literature little has been done so far to characterize the potential impact of relative preferences. Porac, Wade and Pollock (1999) and Alexopolos and Sapp (2006) analyse the impact on the observed economic behavior of firms. Executive compensation in firms is based on a comparison of firm performance to the performance of other, similar firms.

[1] For an overview of the literature on group rent seeking see Nitzan (1994).

Thus manager will focus their actions on improving the performance of their firm relative to a set of peer/competitor firms. Therefore many firms appear to maximize their market share at the expense of profits, which symbolize relative behavior. Also individuals may care about how their payoffs compared to those of others and not just about the level of their payoff. These individuals have relative preferences. Empirical evidence of relative preferences are given for example in Sobel (2005) and in Solnik and Hemenway (1998). Solnik and Hemenway ask over 250 persons of faculty, staff and graduate students of the Havard School of Public Health, if they would prefer case A, their yearly income is 50$ and all other earn 25$ or case B, yearly income 100$and other earn 200$ (in Thousand $). More than 50% of the asked people preferred the positional answer. Solnik and Hemenways result indicates that a lot individuals have relative preferences. Relative preferences exist in a different strong appearance. Guse and Hehenkamp (2006) analyse rent-seeking contest with a heterogeneous population where part of the player are absolute maximizer while others are additionally concerned with their relative position. In this paper we just focus on the two extreme cases; absolute preferences and total relative preferences to find their influence on the contest-structure choice.

Our basic model is presented in Section 2. In Section 3 we analyse the model under the assumption of absolute payoff maximizers following Katz and Tokatlidu (1996) and in Section 4 we assume relative payoff maximizers. The focus is as in Kockesen, Ok and Sethi (2000) on intragroup symmetric equilibria, where all players choose the same effort level in each group. Afterwards in Section 5 we compare the results of the two-stage contest with absolute-seekers with those of the two-stage contest with relative-seekers and in addition with the results of an one-stage contest. The theoretical results are applied to the case of formation of jurisdiction, unification and federalism, in Section 6.

2 The Model

Assume that the government intends to adopt a policy that will provide a rent V to group X or to group Y. The rent V can be monetary or nonmonetary. Groups X and Y consist respectively of n and m homogenous risk neutral members, which induces that each player has the same valuation for the rent. A member of a group is given a chance to compete directly for the rent V only if its group is awarded the rent. In the first round members of each group attempt to obtain the rent for their group. The two groups play an inter-group-Tullock contest, which means that a group of player can make an investment in order to participate in a lottery. The higher a group‘s investment the higher is the chance of winning the prize. Hence, their initial rent seeking is group oriented. Denote the first round rent seeking done by member i of group X and member j of group Y by x_{1i} and y_{1j} respectively, and denote the probability that group X (Y) is awarded the rent by p_X (p_Y). We assume the probability that a given group will win the rent depends on the value of this groups outlays relative to the outlays of both groups: Following Tullock (1980), the probability that group X wins the rent is given by the ratio

$$p_X = \frac{\sum_{i=1}^{n} x_{1i}^r}{\sum_{i=1}^{n} x_{1i}^r + \sum_{j=1}^{m} y_{1j}^r}$$

where r is the technology parameter. It determines if there is decreasing, constant or increasing returns to scale technology.

The probability that group Y wins the rent p_Y is equal to $1 - p_X$. After the completion of the first round, each member of the winning group engages in rent seeking activities in order to win the rent V for itself. The members of the winning group play an intra-Tullock contest. The rent is divisible and has private good characteristics. Denote the second round rent seeking done by member i of group X and member j of group Y by x_{2i} and y_{2j} respectively. If group X won the first round, then its ith member will win in the second round with a probability

$$p_{xi}(x_1, \ldots, x_n) = \frac{x_{2i}^r}{\sum_{l=1}^{n} x_{2l}^r}$$

And if group Y won the first round, its jth member will win the rent in the second round with a probability

$$p_{yj}(y_1, \ldots, y_m) = \frac{y_{2j}^r}{\sum_{k=1}^{m} y_{2k}^r}$$

The parameter r can be interpreted as a measure of how decisive relative effort is. For instance, when r tends to zero, the probability to win the rent tend to $1/n$ respective to $1/m$, indicating that the designation of the winner is independent of effort. When r tends to infinity, the contest becomes fully discriminatory. Then the prize is always awarded to the contestant who exerts the most effort, and the random elements of the contest outcome disappear.

If we see p_{xi} and p_{yj} not as winning probabilities of the whole rent but instead as shares of the rent, we can construct the case, where the rent is divided in the second round into shares for the different participants of this stage. This feature do not change the analysis.

The model follows Katz and Tokatlidu (1996), but we assume a technology parameter r in the winning probability function and in addition agents with relative preferences. We restrict the technology parameter to decreasing returns to scale technologies, which means $r < 1$. This is the interesting case because otherwise the relative-seeker would not offer an effort in the first stage of the contest. The relative maximizers aim is to beat the average payoff and this is not only furthered by higher own payoff, but also by a lower payoff of other player. Behavior that lowers rivals payoff has been termed spiteful (Hamilton (1971)). It can occur in the present contest via lowering the opponents (group) probability of winning the contest. This extends the analysis of two-stage group rent-seeking contest to an additional strategic component.

The first round of the contest does not offer individuals an ultimate payoff; it only determines the probability that a contestant will participate in the second round. The marginal benefit of an individuals first round spending is the increased probability to win the ultimate rent. Hence, the rent seeking done by individuals in the first round is determined by optimizing their expected payoff, given that all individuals act rationally and optimally in the second round.

First we analyze the model under the assumption that players are absolute payoff maximizers following Katz and Tokatlidu (1996). In a second step we assume relative payoff maximizers. The focus is as in Kockesen, Ok and Sethi (2000) and Guse (2006) on intragroup symmetric equilibria, where all players choose the same effort level in each group. We solve for a subgame perfect equilibrium outcome of this game via backward induction. Therefore we start the analysis with the second stage. All previous rent seeking expenditures are sunk in this stage.

3 Analysis with absolute maximizers

Assume without loss of generality that group X with n players wins the contest in the first stage. Therefore we just have to look in the second stage of the contest on the player of group X. For group Y it works equivalently.

3.1 The second stage

In the second round the ith member of group X solves the following maximization-problem

$$\begin{aligned} max \quad \Pi_{2i}^{abs} &= \Pi_{2i}(x_1, \ldots, x_n) \\ &= p_{xi} V - x_{2i} \\ &= \frac{x_{2i}^r}{\sum_{l=1}^n x_{2l}^r} V - x_{2i} \end{aligned} \tag{1}$$

where Π_2^{abs} is the payoff of member i of the second stage and p_{xi} is the success probability of player i

$$p_{xi}(x_1, \ldots, x_n) = \frac{x_{2i}^r}{\sum_{l=1}^n x_{2l}^r} \tag{2}$$

Assuming a symmetric Nash equilibrium within each group and a regular interior solution, the first-order condition for member i of group X is

$$\frac{\partial \Pi_{2i}^{abs}}{\partial x_{2i}} = \frac{r x_{2i}^{r-1} \sum x_{2l}^r - x_{2i}^r r x_{2i}^{r-1}}{(\sum x_{2l}^r)^2} V - 1 = 0 \tag{3}$$

Since we assume identical agents and search for symmetric equilibria, we set $x_{2i} = x_{2l} = x_2$. This gives $\frac{r x_2^{2r-1}(n-1)}{(n x_2^r)^2} V - 1 = 0$ which is solved by

$$x_2^{abs} = \frac{n-1}{n^2} rV \tag{4}$$

.

This is our only interior solution candidate. To proof it is really a maximum we calculate the second order condition in the appendix. So we found for our model with absolute payoff maximizers a global unique Nash optimum. It is given by $x_2^{abs} = \frac{n-1}{n^2} rV$.

Substituting the optimal effort $\frac{n-1}{n^2} rV$ of each member of group X in Eq.(1), we find that the individuals valuation of the benefit of entering round two is $\frac{1-r}{n}V + \frac{r}{n^2}V = \frac{n-nr+r}{n^2}V$ for a member of group X. If a player enters the second round of the contest he can earn this amount of the rent. The amount is just the share of the rent, which he gets, reduced of the effort in this stage. To act rational respective to find an optimal effort level in the first stage every player has to bear this in mind. Hence in the first round each group-member solves the rent seeking maximization for a reduced rent:
$V_n^{abs} = \frac{n-nr+r}{n^2}V$ for group X and $V_m^{abs} = \frac{m-mr+r}{m^2}V$ for group Y.

The value of entering the second round is lower for a member of the larger group, because the prize is diluted by the number of members.[2]

3.2 The first stage

Group sizes n of group X and m of group Y are given in this stage.

The payoff of an absolute maximizing individuum i of group X is Π_i^{abs} and p_X the success function of group X

$$p_X = \frac{\sum_{i=1}^n x_{1i}}{\sum_{i=1}^n x_{1i} + \sum_{j=1}^m y_{1j}} \tag{5}$$

With these we get the maximization problem

$$max \quad \Pi_i^{abs} = \frac{\sum x_{1k}^r}{\sum x_{1k}^r + \sum y_{1j}^r} V_n^{abs} - x_{1i} \tag{6}$$

Calculating the first order condition gives

$$\frac{\partial \Pi_i^{abs}}{\partial x_{1i}} = \frac{r x_{1i}^{r-1}(\sum x_{1k}^r + \sum y_{1j}^r) - (\sum x_{1k}^r) r x_{1i}^{r-1}}{(\sum x_{1k}^r + \sum y_{1j}^r)^2} V_n^{abs} - 1 \overset{!}{=} 0 \tag{7}$$

[2]See Münster (2005) for more information to this group cohesion effect.

Following our assumption that members in each group are identical, we can write that $\sum x_{1k}^r = nx_1^r$ and $\sum y_{1j}^r = my_1^r$. Substituting this in the equation, we yield

$$\frac{rx_1^{r-1}(nx_1^r + my_1^r) - nx_1^r rx_1^{r-1}}{(nx_1^r + my_1^r)^2} V_n^{abs} - 1 = 0$$

Equivalent for a player of group Y:

$$\frac{ry_1^{r-1}(nx_1^r + my_1^r) - my_1^r ry_1^{r-1}}{(nx_1^r + my_1^r)^2} V_m^{abs} - 1 = 0$$

After some calculations and equating both equations we get

$$\begin{aligned} x_1 &= \frac{m}{n} y_1 \frac{V_n^{abs}}{V_m^{abs}} \\ &= \frac{m^3(nr - n - r)}{n^3(mr - m - r)} y_1 \end{aligned}$$

Substituting this in the above equations we yield

$$x_1^{abs} = \frac{rmV_n^{abs}(\frac{mV_n^{abs}}{nV_m^{abs}})^r}{(n(\frac{mV_n^{abs}}{nV_m^{abs}})^r + m)^2} \quad (8)$$

$$= -\frac{rmV(\frac{m^3(nr-n-r)}{n^3(mr-m-r)})^r(nr - n - r)}{n^2(n\frac{m^3(nr-n-r)}{n^3(mr-m-r)})^r + m)^2} \quad (9)$$

This is the optimal effort of a player of group X in the first stage, if all palyer have absolute preferences.

3.3 The overall absolute two-stage contest

If we take a look on the contest from a welfare perspective, we focus on the aggregate expenditures of the player in the whole contest. Therefore we aggregate the expenditures of the single stages and add them multiplied by their probability to occure.

$$x_1^{abs} = -\frac{rmV(\frac{m^3(nr-n-r)}{n^3(mr-m-r)})^r(nr - n - r)}{n^2(n\frac{m^3(nr-n-r)}{n^3(mr-m-r)})^r + m)^2}$$

$$y_1^{abs} = -\frac{rnV(\frac{n^3(mr-m-r)}{m^3(nr-n-r)})^r(mr-m-r)}{m^2(m\frac{n^3(mr-m-r)}{m^3(nr-n-r)})^r+n)^2}$$

$$x_2^{abs} = \frac{n-1}{n^2}rV$$

$$y_2^{abs} = \frac{m-1}{m^2}rV$$

$$totaleffort = nx_1^{abs} + my_1^{abs} + \frac{nx_1^{abs}}{nx_1^{abs}+my_1^{abs}}nx_2^{abs} + \frac{my_1^{abs}}{nx_1^{abs}+my_1^{abs}}my_2^{abs} \quad (10)$$

In the case of absolute payoff-maximizers and decreasing returns to scale technology ($r < 1$) there is always underdissipation of the rent. With increasing number of players the aggregate expenditures converge to rV from below, but still it never reaches rV.

4 Analysis with relative maximizers

Assume again without loss of generality that group X with n players wins the contest in the first stage. Therefore we just have to look in the second stage of the contest on the player of group X.

4.1 The second stage

In the second round the ith member of group X solves the maximization-problem

$$\begin{aligned} max \quad \Pi_{2i}^{rel} &= \Pi_{2i}(x_1,\ldots,x_n) - \bar{\Pi}_2(x_1,\ldots,x_n) \\ &= \frac{x_{2i}^r}{\sum_{l=1}^n x_{2l}^r}V - x_{2i} - \frac{1}{n-1}\left((1-\frac{x_{2i}^r}{\sum_{l=1}^n x_{2l}^r})V - \sum_{\substack{l=1\\l\neq i}}^n x_{2l}^r\right) \quad (11) \end{aligned}$$

where $\bar{\Pi}_2$ is the population mean payoff of the second stage

$$\bar{\Pi}_2(x_1, \ldots, x_n) = \frac{1}{n-1}\left((1 - \frac{x_{2i}^r}{\sum_{l=1}^n x_{2l}^r})V - \sum_{\substack{l=1\\l\neq i}}^n x_{2l}^r\right)$$

and p_{xi} is the success probability of player i

$$p_{xi}(x_1, \ldots, x_n) = \frac{x_{2i}^r}{\sum_{l=1}^n x_{2l}^r}$$

Assuming a symmetric Nash equilibrium within each group and a regular interior solution, the first-order condition for member i of group X is

$$\frac{\partial \Pi_{2i}^{rel}}{\partial x_i} = \frac{r x_{2i}^{r-1} \sum x_{2l}^r - x_{2i}^r r x_{2i}^{r-1}}{(\sum x_{2l}^r)^2} V - 1 + \frac{V}{n-1} \frac{r x_{2i}^{r-1} \sum x_{2l}^r - x_{2i}^r r x_{2i}^{r-1}}{(\sum x_{2l}^r)^2} = 0$$

Since we assume identical agents and search for symmetric equilibria, we set $x_{2i} = x_{2l} = x_2$. This gives $\frac{r x_2^{2r-1}(n-1)}{(n x_2^r)^2} V - 1 + \frac{V}{n-1} \frac{r x_2^{2r-1}(n-1)}{(n x_2^r)^2} = 0$ which is solved by

$$x = \frac{r}{n} V \tag{12}$$

.

This is our only interior solution candidate of the second stage.

The second order condition for a local maximum becomes

$$\begin{aligned} \frac{\partial \partial \Pi_{2i}^{rel}}{\partial \partial x_{2i}} &= (V - \frac{V}{n-1}) \frac{(r(r-1) x_{2i}^{r-2} \sum x_{2l}^r + r^2 x_{2i}^{2r-2} - x_{2i}^{2r-2}(2r-1)r)}{(\sum x_{2l}^r)^2} \\ &- \frac{2(r x_{2i}^{r-1} \sum x_{2l}^r - x_{2i}^{2r-1} r) x_{2i}^{r-1} r}{(\sum x_{2l}^r)^3} < 0 \end{aligned}$$

Simplification under the assumption that all player are identical lead to

$$\frac{(1 - \frac{1}{n-1}) V r (r n^2 - n^2 - 3 r n + n + 2r)}{n^3 x_2^2} < 0$$

this holds in the symmetric solution $x_2^{rel} = \frac{r}{n} V$ if $\frac{n(rn - n - 2r)(n - 1 + V)}{rV} < 0$ which is negativ if $rn - 2r - n < 0$ holds. Thus the SOC for local maximization holds for

$r < \frac{n}{n-2}$.

Also note that SOC implies global concavity of the relative payoff function, if $r \leq 1$ holds: The bracketed term in the numerator is always negative, the one in the denominator always positive. hence the local optimum is a global one for all $r \leq 1$.[3]

To hold the analysis easy and interesting, we concentrate on the case of decreasing returns to scale technology, which means $r < 1$. Therefore the SOC for local maximazation is always fullfilled.

$r < 1 < \frac{n}{n-2}$

We found a global unique Nash optimum with realive payoff maximizers. It is given $x_2^{rel} = (\frac{r}{n}V, \cdots, \frac{r}{n}V)$.

In comparison the unique pure strategy Nash equilibrium with absolute payoff maximizers is given by $x_2^{abs} = (\frac{n-1}{n^2}rV, \cdots, \frac{n-1}{n^2}rV)$

Therefore the incentive of the relative maximizer to reach the second round is smaller, because they have a higher effort in the second stage and thus a smaller payoff.

Proposition 1:
The individual expenditures and aggregate rent-dissipation in the equilibrium with relative maximizers of the second stage are always higher than in the Nash equilibrium with absolute maximizers. $\frac{r}{n}V > \frac{r}{n}V - \frac{r}{n^2}V$

In a private good contest for V, as in the second stage of our model, the contest with relative maximizers always leads to a higher expenditure than the one with absolute maximizer.[4] The Nash equilibrium in the case with absolute preferences occurs at a solution, that yields a positive value of the first derivative of the relative maximization function. A marginal increase of expenditures beyond the optimal absolute Nash level increases relative payoff. First a marginal increase of expenditure has second-order negative effects on a players own payoff because the

[3]For $r > 1$ it is a mess to prove it, but see for it Hehenkamp, Leininger und Possajennikov (2004).

[4]see Leininger (2002) and Hehenkamp, Leininger and Possajennikov (2004)

first derivative of his payoff function is zero at the absolut Nash level. And second a marginal increase has first-order negative effect on the other players payoff because cross derivatives in the absolute Nash equilibrium are always negative. As a consequence the difference between own and others payoff increases.

Definition 1:
An equilibrium x^*_{rel} displays overdissipation (full dissipation, underdissipation) if and only if $\sum_{i=1}^n x_i^* > (=, <) V$, respectively.

The technology parameter r determines the dissipation rate.[5]

Proposition 2:
Aggregate rent dissipation in x_2^{rel} of the second stage is independent of the number of contestants in the rent-seeking tournament.
$n \cdot \frac{r}{n} V = rV$

r represents the efficiency of the given contest technology. For $r < 1$ we find an underdissipation of the rent, for $r = 1$ full dissipation and for $r > 1$ overdissipation. Assuming $r = 1$ we have full rent dissipation. Therefore there would not be an incentive for the relative rent-seeker to make a positive bid in the first period. Hence we assume $r < 1$ in our analysis.

We have to distinguish between material payoff and utility. Material payoff determines the material success of players in absolute values. Utility represents the preferences of the individuum. In the case of individuals with absolute preferences those two are identical. But in a model with rent-seekers with relative preferences they differ. The utility is the difference between own material payoff and average payoff of the others. Relative-seekers try to maximize their utility, but what they get in the end of the tournament is the material payoff. To calculate the benefit of entering the second round we have to use the material payoff. For it we substitute the optimal effort $(\frac{r}{n}V)$ in Eq.(11) and find that the individuals valuation of the benefit of entering the second round is $\frac{1-r}{n}V$ for a member of group X. The value of entering the second round is lower for a member of a larger group, because the prize is diluted by the number of members. Hence in the first round each member of group X solves the rent seeking maximization for a reduced rent:

[5]See for this result Hehenkamp, Leininger and Possajennikov (2004).

$V_n^{rel} = \frac{1-r}{n}V$ for group X and $V_m^{rel} = \frac{1-r}{m}V$ for group Y.

4.2 The first stage

Group sizes n of group X and m of group Y are given in this stage.

The payoff of a relative maximizing individuum i of group X: $\Pi_i^{rel} = \Pi_i - \bar{\Pi}$

where is $\bar{\Pi}$ the population mean payoff

$$\begin{aligned}\bar{\Pi} &= \frac{\sum_{k=1}^{n}\Pi_k + \sum_{j=1}^{m}\Pi_j}{n+m} \\ &= \frac{\frac{\sum_{k=1}^{n}x_{1k}^r}{\sum_{k=1}^{n}x_{1k}^r+\sum_{j=1}^{m}y_{1j}^r}nV_n^{rel} - \sum_{k=1}^{n}x_{1k} + \frac{\sum_{j=1}^{m}y_{1j}^r}{\sum_{k=1}^{n}x_{1k}^r+\sum_{j=1}^{m}y_{1j}^r}mV_m^{rel} - \sum_{j=1}^{m}y_{1j}}{n+m}\end{aligned}$$

and p_X the success function of group X

$$p_X = \frac{\sum_{k=1}^{n}x_{1k}^r}{\sum_{k=1}^{n}x_{1k}^r + \sum_{j=1}^{m}y_{1j}^r}$$

With these we get

$$\begin{aligned}\Pi_i^{rel} &= \frac{\sum x_{1k}^r}{\sum x_{1k}^r + \sum y_{1j}^r}V_n - x_{1i} \\ &-\frac{1}{n+m}\left(\frac{\sum x_{1k}^r}{\sum x_{1k}^r + \sum y_{1j}^r}nV_n^{rel} - \sum x_{1k} + \frac{\sum y_{1j}^r}{\sum x_{1k}^r + \sum y_{1j}^r}mV_m^{rel} - \sum y_{1j}\right)\end{aligned} \quad (13)$$

Calculating the first order condition

$$\begin{aligned}\frac{\partial\Pi_i^{rel}}{\partial x_i} &= \frac{rx_{1i}^{r-1}(\sum x_{1k}^r + \sum y_{1j}^r) - (\sum x_{1k}^r)rx_{1i}^{r-1}}{(\sum x_{1k}^r + \sum y_{1j}^r)^2}V_n^{rel} - 1 - \\ &\frac{1}{n+m}\left(\frac{rx_{1i}^{r-1}(\sum x_{1k}^r + \sum y_{1j}^r) - (\sum x_{1k}^r)rx_{1i}^{r-1}}{(\sum x_{1k}^r + \sum y_{1j}^r)^2}nV_n^{rel} - 1 - \frac{(\sum y_{1j}^r)rx_{1i}^{r-1}}{(\sum x_{1k}^r + \sum y_{1j}^r)^2}mV_m^{rel}\right) \\ &\overset{!}{=} 0\end{aligned}$$

Following our assumption that members in each group are identical, we can write that $\sum x_{1k}^r = nx_1^r$ and $\sum y_{1j}^r = my_1^r$. Substituting this in the equation, we yield

$$\frac{rx_1^{r-1}(nx_1^r + my_1^r) - nx_1^r r x_1^{r-1}}{(nx_1^r + my_1^r)^2} V_n^{rel} - 1$$
$$-\frac{1}{n+m}\left(\frac{rx_1^{r-1}(nx_1^r + my_1^r) - nx_1^r r x_1^{r-1}}{(nx_1^r + my_1^r)^2} nV_n^{rel} - 1 + \frac{-my_1^r r x_1^{r-1}}{(nx_1^r + my_1^r)^2} mV_m^{rel}\right) = 0$$

Equivalent for a player of group Y:

$$\frac{ry_1^{r-1}(nx_1^r + my_1^r) - my_1^r r y_1^{r-1}}{(nx_1^r + my_1^r)^2} V_m^{rel} - 1$$
$$-\frac{1}{n+m}\left(\frac{ry_1^{r-1}(nx_1^r + my_1^r) - my_1^r r y_1^{r-1}}{(nx_1^r + my_1^r)^2} mV_m^{rel} - 1 + \frac{-nx_1^r r y_1^{r-1}}{(nx_1^r + my_1^r)^2} nV_n^{rel}\right) = 0$$

After some manipulations and equate both equations we get

$$m^2 y_1^{rel} = n^2 x_1^{rel} \tag{14}$$

Substituting this in the above equations we yield

$$\begin{aligned} x_1^{rel} &= \frac{1}{n+m-1} \frac{rm^2(\frac{m^2}{n^2})^r (V_n^{rel} + V_m^{rel})}{(n(\frac{m^2}{n^2})^r + m)^2} \\ &= \frac{1}{n+m-1} \frac{rm(\frac{m^2}{n^2})^r \frac{n+m}{n}(1-r)V}{(n(\frac{m^2}{n^2})^r + m)^2} \\ &= \frac{1}{n+m-1} \frac{m^{2r-1} n^{2r-3}(r-r^2)V}{(m^{2r-1} + n^{2r-1})^2} \end{aligned} \tag{15}$$

In comparison the optimal effort in the first stage with absolute maximizers is

$$\begin{aligned} x_1^{abs} &= \frac{rmV_n(\frac{mV_n^{rel}}{nV_m^{rel}})^r}{(n(\frac{mV_n^{rel}}{nV_m^{rel}})^r + m)^2} \\ &= -\frac{rmV(\frac{m^3(nr-n-r)}{n^3(mr-m-r)})^r (nr-n-r)^2}{(n^2(n\frac{m^3(nr-n-r)}{n^3(mr-m-r)})^r + m} \end{aligned}$$

Proposition 3:
Total expenditure in the equilibrium with relative preferences of the first stage are always smaller than those in the Nash equilibrium with absolute maximizers.

Proposition 4:
In the equilibrium of the first stage with relative preferences it holds true that

- individual expenditures decrease with increasing size of a contestants own group
- individual expenditures decrease with increasing rival group size

4.3 The overall relative two-stage contest

If we take a look on the contest of the welfare perspective, the main interest is focused on the aggregate expenditures of the player in the whole contest. Therefore we adding the individual efforts with their corresponding appearance probabilities

$$x_1^{rel} = \frac{1}{n+m-1} \frac{rm(\frac{m^2}{n^2})^r \frac{n+m}{n}(1-r)V}{(n(\frac{m^2}{n^2})^r + m)^2}$$

$$y_1^{rel} = \frac{1}{n+m-1} \frac{rn(\frac{n^2}{m^2})^r \frac{n+m}{m}(1-r)V}{(m(\frac{n^2}{m^2})^r + n)^2}$$

$$x_2^{rel} = \frac{r}{n}V$$

$$y_2^{rel} = \frac{r}{m}V$$

$$\begin{aligned} totaleffort &= nx_1^{rel} + my_1^{rel} + \frac{nx_1^{rel}}{nx_1^{rel} + my_1^{rel}} nx_2^{rel} + \frac{my_1^{rel}}{nx_1^{rel} + my_1^{rel}} my_2^{rel} \\ &= nx_1^{rel} + my_1^{rel} + rV \end{aligned}$$

In the case of players with relative preferences and decreasing returns to scale technology ($r < 1$) there is always underdissipation of the rent.[6] With increasing

[6] see Hehenkamp,Leininger and Possajennikov (2004)

number of players the aggregate expenditures converge to rV from above, but still it never reaches rV with a finite number of players.

5 Comparison

5.1 Relative vs. absolute maximizer in two-stage contests

From the above analysis we see in the first stage that the aggregate expenditure in equilibrium with relative preferences is always lower or equal than in absolute Nash equilibrium. This result is driven by the different valuation to enter the second round of the tournament. If we compare the benefit of entering tht second stage of absolute maximizer ($\frac{1-r}{n}V + \frac{r}{n^2}V$) with those of relative-seekers ($\frac{1-r}{n}V$), we find that absolute player have a higher valuation of the rent in the first stage. Exactly, for them the valuation of second round entering is $\frac{r}{n^2}V$ higher than for rent-seekers with relative preferences. This creates an asymmetry in behavior. In the second stage the effect is vice versa, the aggregate expenditure in equilibrium with relative preferences is always higher than in the Nash equilibrium with absolute-seekers. The effects of both stages counteract in their direction, but the effect of the second stage is stronger and dominates the first-stage effect.

Proposition 5:
The total expenditure in equilibrium of the overall two-stage contest with relative preferences is higher than the overall expenditure in the contest with absolute maximizers.

In a public good group contest the incentive to increase own expenditure is counteracted by the free-rider problem. An increase above the Nash level still advances ones own position in relation to members of the other group, but it puts oneself also at a disadvantage in relation to own group members. They can free-ride on the additional spiteful effort. Free-riding is in a relative-seeker context sensible spiteful behavior against members of the own group. It increases ex post the payoff of members of the other group, which brings along a decrease in ones own relative payoff, but it increases ones own payoff by even more.[7]

[7] see for this problem Leininger (2002)

5.2 Comparison with one-stage contests

An alternative to a two-stage contest like one explained above is in the most cases an individual one-stage contest. All members of both groups are pooled together and rentseek in a one-stage Tullock contest for the rent V or for a part of the rent. To compare these two different structures of a contest, we calculate the effort of the palyer in the one-stage contest; once for the case with absolute payoff-maximizers and once for the case with relative maximizers.

One-stage contest with Tullock probability function for $(n+m)$ player:

$$x^*_{abs} = \frac{n+m-1}{(n+m)^2} rV \tag{16}$$

Thus the dissipation rate is given by $\delta_{abs} = \frac{n+m-1}{(n+m)} r$.

$$x^*_{rel} = \frac{r}{(n+m)} V \tag{17}$$

Thus the dissipation rate is given by $\delta_{rel} = r$.

After comparing the results of a one-stage contest with those of a two-stage contest, we find the following:

Proposition 6:

- Total expenditure with absolute maximizers of a one-stage contest is always higher or equal than those in a two-stage contest.
- Total expenditure with relative maximizers of a one-stage contest is always lower or equal than those in a two-stage contest.

To find an optimal contest structure the principal has to consider this phenomenom. If he thinks his agents have absolute preferences, he should prefer an one-stage contest to get a higher dissipation rate. And if he thinks his agents are relative-seekers, he should favour the two-stage contest.

6 Political implications

The German political federalism system is organized like our model. Political decisions are made on two hierachical stages. First the overall, countrywide federation-government decides about the divide of money, for example the GNP pie. Afterwards on a second stage there is the division of the pie on a local regions stage. Exactly these two-stage approach is analysed in the above part of the paper. In the first stage there are two (or more) groups, the different federal state, competing against each other in a contest organized by the federation government. In the second stage the individuals of the winning province start a intra-contest for the rent. The individuals of the second stage can also be interest groups composed of many individual agents like cities, single political parties or lobbyists, but we abstract in this paper from the problem of enforcing cooperation within such a group.[8]

An important function of the government is to distribute rents, parts of the GNP pie, through regulations. Abstractly seen, there are two different endogenous formation of jurisdiction for a two region country thinkable. On the one hand there is the system of federalism. A federation is a union comprising a number of partially self-governing regions united by a central government. Thus sovereignty is constitutionally divided between a central governing authority and constituent political units like states or regions. On the other hand there is the system of an unitary state. Both regions are combined into a single jurisdiction, a unification. We assume, that the unification does not have any (technological)efficiency gains or losses concerning the GNP. Thus the rent is in both jurisdictions the same.

This model of federalism, in which the federal government allocates GNP shares to two regions, in which separate contests then determine individual shares, and unification, in which the whole GNP pie is directly divided between the individuals, is analysed by Wärneryd (1998) in his paper over the endogenous formation of jurisdictions. Under the assumption that the political process is a costly fight to acquire shares of the GNP cake he found out that less resources are spent in aggregate on appropriative activities under a hierarchical system of federalism than in a unified jurisdiction with a single central government. Therefore federal-

[8]see Olson (1965) and Nitzan (1991) for this problem

ism ameliorates competition for GNP shares, because the rate of rent dissipation is strictly lower than under unification.

Our result under the assumption of relative preferences of the player contrasts with Wärneryd (1998), who showed that it is more efficient for a principal respective the government to induce unification, because then the rent dissipation rate is higher. In our model with relative-seekers and decreasing returns to scale technology the federalism system induces a strictly higher rate of rent dissipation than the unified jurisdiction. Thus the unification does not always increase competition in the sense that the global rate of dissipation increases such as Wärneryd (1998) stated.

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A Appendix

A.1 SOC for the second stage maximization of the contest with absolute preferences

We found for our model with absolute payoff maximizers a unique Nash optimum. It is given by $x_2^{abs} = \frac{n-1}{n^2} rV$. This is our only interior solution candidate. To check it it is really a maximum we calculate the second order condition.

$$\begin{aligned}\frac{\partial\partial\Pi_{2i}^{abs}}{\partial\partial x_{2i}} &= \frac{(r(r-1)x_{2i}^{r-2}\sum x_{2l}^{r} + r^2 x_{2i}^{2r-2} - x_{2i}^{2r-2}(2r-1)r)V}{(\sum x_{2l}^{r})^2} \\ &\quad - \frac{2(rx_{2i}^{r-1}\sum x_{2l}^{r} - x_{2i}^{2r-1}r)Vx_{2i}^{r-1}r}{(\sum x_{2l}^{r})^3} < 0\end{aligned}$$

Simplification under the assumption that all player are identical lead to

$$\frac{Vr(rn^2 - n^2 - 3rn + n + 2r)}{n^3 x^2} < 0$$

this holds in the symmetric solution $x = \frac{n-1}{n^2} rV$ if $\frac{n(rn-n-2r)}{Vr(n-1)} < 0 \iff rn - 2r - n < 0$

Thus the SOC for local maximization holds for $r < \frac{n}{n-2}$.

Also note that SOC implies global concavity of the payoff function, if $r \leq 1$ holds: The bracketed term in the numerator is always negative, the one in the denominator always positive. Hence the local optimum is a global one for all $r \leq 1$.

As mentioned before, to hold the analysis interesting, we concentrate on the case of decreasing returns to scale technology, which means $r < 1$. Therefore the SOC for local maximazation is always fullfilled: $r < 1 < \frac{n}{n-2}$