Aging, Trade and Welfare in a Heckscher-Ohlin Model with Overlapping Generations*

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Abstract

This paper uses a computable $2 \times 2 \times 2$ Heckscher-Ohlin model with overlapping generations (OLG) to investigate the economic implications of unsynchronized global population aging. Interregional differences in the extent and timing of aging affect the relative abundance of factors of production in each region and lead to the emergence of Heckscher-Ohlin trade patterns. Without recourse to social security, trade liberalization in the light of unsynchronized aging patterns leads to strong distributional effects across regions *and* generations. Gains from trade are not universal, i.e. openess may be immiserizing. In the fast aging region, generations born in the run up to and the beginning of the demographic transition gain while those born in the midst of the transition and thereafter stand to incur substantial utility losses. In the short-run, international trade potentially alleviates the adverse economic effects of aging. Our results are robust over a broad range of empirically plausible parameter configurations.

Keywords: Population aging; International trade; Heckscher-Ohlin; Overlapping generations; Trade liberalization; Computable general equilibrium modeling

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1 Introduction

Most developed countries currently undergo large demographic changes with significant economic consequences. The extent and timing of predicted aging processes, however, differ substantially across countries (see Fig. 1). In an increasingly globalized world economy the presence of unsynchronized demographic patterns is likely to induce international flows of capital, labor, and goods. While this has been widely acknowledged, the various types of

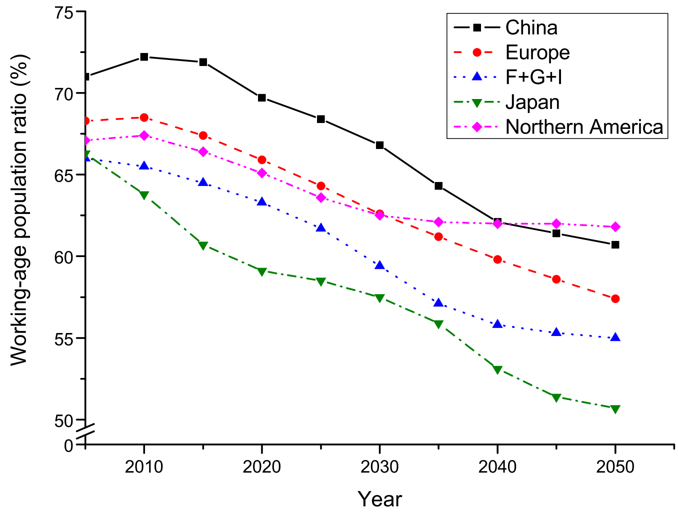

Fig. 1: Unsynchronized global aging patterns

Note: The working-age population ratio is defined as the population aged between 15-64 as a fraction of the total population. 'F+G+I' denotes the country average for France, Germany, and Italy. Source: United Nations, World Population Prospects: The 2004 Revision. Medium projection variant.

economic linkages between countries have received different levels of attention. Studies in this area almost exclusively focus on the implications of population aging for international capital flows and labor migration in one-sector models. The potential significance of trade in goods with different factor intensities has been largely neglected.

The present paper aims to fill this gap by investigating the economic effects of demographic change in a model of international trade. In particular, we address the question whether international trade in goods can function as a mechanism that alleviates the adverse effects brought about by population aging. To this end, we construct a computable version of a neoclassical dynamic two region, two factor, two commodity Heckscher-Ohlin model with overlapping generations (OLG) in the tradition of Auerbach and Kotlikoff [1987]. We assume that production technologies as well as households' preferences are identical across regions but postulate that the extent and timing of demographic shocks differ. We compare the impact on key macroeconomic and household variables under autarky and trade conditions when international goods markets are perfectly integrated. To our knowledge, this paper is the first to provide a detailed discussion of the intergenerational welfare effects of population aging in the presence of globalized goods markets. The principal contribution of the paper is to integrate a comprehensive OLG demand system into an international trade context to obtain a computable general equilibrium model that can be used to derive transitional as well as steady state effects and to evaluate welfare changes.

Our results are as follows. Under autarky, interregional differences in the extent and timing of population aging affect the relative abundance of factors of production across countries. Aging leads to a capital deepening in the economy. Given differing factor intensities across sectors this leads to differentials in commodity and factor prices under autarky. In a world with international trade, Heckscher-Ohlin trade patterns emerge. The fast aging region becomes relatively capital-abundant therefore exporting the capital-intensive commodity while the slow aging region becomes relatively labor-abundant therefore exporting the labor-intensive commodity. Without recourse to social security, trade liberalization in the light of unsynchronized aging patterns leads to strong distributional effects across generations and regions. In contrast to what would be expected from standard neoclassical trade theory, liberalizing trade in the presence of globally unsynchronized aging patterns does not unambiguously lead to welfare gains, i.e. openness may be immiserizing. Depending on the evolution of factor price differentials between trade and autarky over the lifetime of a given generation, openness of a country can either decrease or increase lifetime utility. In the fast aging region, generations born in the run up to and the beginning of the demographic transition gain while those to be born in the midst of the transition and thereafter stand to incur substantial utility losses. This is in stark contrast to what can be found in the literature. Sayan [2005] finds —in a framework that is incapable of assessing *generational* welfare changes— that the fast aging region loses overall. Furthermore, we find that during the economic transition phase there is always one region that benefits from trade at the expense of the other. In this sense, international trade can be viewed as a mechanism which potentially alleviates in the short-run the adverse economic effects that are associated with population aging. We also show that aging is a slow moving economic process extending far beyond the time horizon of the demographic transition phase. It profoundly affects the well-being of future generations born *after* the demographic transition is completed.

A substantial and growing literature has been calling attention to the economic implications of population aging in open economies. Besides empirical studies, computable OLG models have emerged as the dominant analytical tool.[1] The literature examining the impact of population dynamics on international capital flows has developed only recently. Abstracting from the presence of a public pension system, these studies (e.g. Attanasio and Violante [2005], Feroli [2003], Brooks [2003], Domeij and Floden [2004]) compare closed-economy versions of their models to two-country or multi-country world scenarios where international capital markets are perfectly integrated and overwhelmingly find that unequal demographic trends across countries put in place strong incentives to international capital mobility. For example, Feroli [2003] uses a multi-region OLG model that is calibrated to match demographic differences among the major industrialized countries over the past 50 years and finds that that demographic differences can explain not all but some of the observed long-term capital movements; Brooks [2003] finds that retirement saving by aging baby boomers will raise the supply of capital substantially above investment in both the European Union and North America, causing both regions to export large amounts of capital to Latin America and other emerging markets in the years ahead. Beyond 2010, however, baby boomers in the European Union and North America will dissave in retirement, causing both regions to become capital importers. This shift will be financed by capital flows from Latin America and other emerging markets, while Africa will remain dependent on foreign capital for the foreseeable future because of continued high population growth. Domeij and Floden [2004] empirically test to what extent international capital flows predicted by the model match historically observed current account positions and find that a small but significant fraction of international capital movements can be explained by the simulation model. A second strand of the literature analyzes the role of pension reform and associated savings patterns for international capital flows in the light of population aging; e.g., Attanasio and Violante [2005], Börsch-Supan, Ludwig, and Winter [2005], INGENUE [2001], and Roeger [2006]. These studies find that capital flows from fast aging countries to the rest of the world will initially be substantial but that trends are reversed when aging is progressed and households decumulate savings. Moreover, the status quo of public pension systems is shown to crucially determine the magnitudes of international capital movements. A number of related papers (Fehr, Jokisch, and Kotlikoff [2003], Fehr, Halder, and Jokisch [2004], and Fehr, Halder, Jokisch, and Kotlikoff [2004]) develop closed- and open economy versions of Auerbach and Kotlikoff [1987]-type OLG economies that comprise a large number of realistic features within a single framework and allow for

[1] See Bryant and McKibbin (1998, 2003) for good surveys of multi-country macroeconometric and multi-country simulation models that incorporate demographic change.

joint mobility of capital and labor across regions. Most notably, Fehr, Jokisch, and Kotlikoff [2004] study the implications of global demographic change on international labor migration. Developing a three-region OLG model, they analyze the scope for general and skill-specific immigration policy to alleviate the economic stress that is put on fiscal institutions by the projected demographic change. They find that even a significant expansion of immigration, whether across all skill groups or among particular skill groups, will do remarkably little to alter the major capital shortage, tax hikes, and reductions in real wages that can be expected along the demographic transition.

Common to all these models is the assumption of a single commodity world and disintegrated international goods markets. The treatment of population aging in models of international trade has largely been overlooked. A notable exception is the study by Sayan [2005] who uses a simple Heckscher-Ohlin framework with overlapping generations to investigate the implications of differential population dynamics for trade. Our analysis is close to Sayan [2005] but makes several advances relative to his contribution which are required to carefully study the *intergenerational welfare effects* of population aging in the context of international trade. First, we allow for endogenous labor supply decisions of households. Second, we employ a numerical solution technique that is capable of accommodating corner solutions of economic equilibrium. Both, a labor-leisure trade-off as well as the endogenous regime switching property of our numerical model are particularly important for a proper assessment of households' behavior and associated welfare effects. Third, households in our model live for more than two periods which enables us to study a more detailed aging process that better portrays the stylized nature of real-world demographic change. Fourth, we allow for a strictly positive capital depreciation rate which drives the endogenous capital accumulation behavior of the aggregate economy and thus constitutes another critical element in assessing dynamic welfare effects.

The rest of this paper is organized as follows. Section 2 sets out the theoretical framework. Section 3 formulates the dynamic general equilibrium of the model in a mixed complementarity (MCP) format whose structure is exploited by the computational solution algorithm. In section 4, we briefly sketch our calibration strategy and describe the nature of the stylized demographic shocks that are implemented to mimic globally unsynchronized aging patterns. Section 5 presents results from various simulation experiments investigating the aggregate economy and intergenerational welfare effects of aging under autarky and trade. Section 6 performs a number of sensitivity analyses to check for the robustness of our results. Section 7 summarizes the main results and concludes.

2 The model

The economy is represented by a dynamic infinite-horizon two region, two sector, two factor trade model in which capital is accumulated endogenously and in which the demand side is characterized by overlapping generations.[2] Time is discrete and extends to infinity, $t = 0, \ldots, \infty$. In each period the two regions, indexed by $r, s \in \{a, b\}$, are assumed to be identical with respect to the production structure and technologies as well as households' preferences. Regions experience, however, differential population dynamics over time. Factor intensities vary across sectors to allow for Heckscher-Ohlin trade patterns to emerge. We assume that output and factor markets are perfectly competitive and that there is no aggregate or household-specific uncertainty in the model. We abstract from any government activity and do not consider a public sector pension system.[3]

Households live over a deterministic lifespan of $a = 0, \ldots, N$ periods. Each generation, indexed by $g = 0, ..., \infty$, is described by a continuum of homogenous households. At the beginning of each year $t = g$, a household of generation g is born. Thus, at each point in time there are $N + 1$ generations alive.[4] We abstract from intracohort heterogeneity[5] as well as from intergenerational altruistic preferences and bequests. In each period over the life cycle households are endowed with units of time that they allocate between labor and leisure. Households are assumed to be forward-looking individuals that form rational point expectations (perfect foresight) over the infinite horizon. Lifetime utility of generation g, u_g, is additively separable over time and is of the constant-intertemporal-elasticity-of-substitution form (CIES). The representative agent of each generation chooses optimal consumption and leisure paths over his life cycle subject to lifetime budget and time endowment constraints. Suppressing the trivial expectations operator and the region index, we can write the optimization problem for a typical household of generation g more formally as

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[2] See, e.g., Galor [1992] for a tractable closed-economy version of this model in which agents live two periods.

[3] It can be argued, however, that the model features a world in which a fully-funded pension system is in place that implicitly operates through the (private) life-cycle savings behavior of households. The model is silent on the role played by public pension schemes that have intergenerational redistributive power such as, e.g., a Pay-as-you-go system.

[4] Our specification of the overlapping generations demand system is due to Diamond [1965]-Samuelson [1958]. One virtue of this approach vis-à-vis the Blanchard [1985] - Weil [1989] - Yaari [1965] framework is the ability to keep track of different cohorts, their savings decisions and wealth stocks over the life cycle.

[5] The terms "generation" and "household" are therefore used interchangeably.

$$\begin{aligned}
\underset{c_{t,g},\, l_{t,g}}{max} \qquad & u_g(z_{t,g}) = \sum_{t=g}^{g+N} \left(\frac{1}{1+\rho}\right)^{t-g} \frac{z_{t,g}^{1-\theta}}{1-\theta} \\
s.t. \qquad & z_{t,g} = \left(\alpha\, c_{t,g}^{\sigma} + (1-\alpha)\, l_{t,g}^{\sigma}\right)^{1/\sigma} \\
& \sum_{t=g}^{g+N} p_t^c\, c_{t,g} \leq \sum_{t=g}^{g+N} p_t^l\, \pi_{t,g}\,(\omega_{t,g} - l_{t,g}) \\
& l_{t,g} \leq \omega_{t,g} \\
& c_{t,g}\,, l_{t,g} \geq 0\,.
\end{aligned} \tag{1}$$

Here, $c_{t,g}$ and $l_{t,g}$ denote material and leisure consumption of household g at time t, respectively, which are combined to a composite (CES-) consumption good $z_{t,g}$ where $1/(1-\sigma)$ denotes the intratemporal elasticity of substitution between c and l and where α is a share parameter. ρ is a time-invariant periodic utility discount rate, and $1/\theta$ is the intertemporal elasticity of substitution. In the second constraint, p_t^c and p_t^l denote the price of consumption and the wage rate, respectively. $\pi_{t,g}$ is an index of labor productivity that varies exogenously over the life cycle and $\omega_{t,g}$ denotes the exogenous time endowment of generation g at time t. The lifetime budget constraint states that the sum of periodic consumption expenditures over the life cycle cannot exceed lifetime labor earnings. In each period, the time allocated to leisure consumption cannot be larger than the periodic time endowment. Choices for material and leisure consumption are restricted to be nonnegative.[6] The time endowment of generations evolves according to

$$\omega_{r,g+1,t+1} = (1+\gamma_{r,t})\,\omega_{r,g,t} \quad , \forall\, t \geq 1\,, \forall\, g \geq 1\,, \tag{2}$$

where $\gamma_{r,t}$ is a region-specific and time-varying exogenous population growth rate and where $\omega_{r,0,0}$ is normalized to unity. Note that there is no growth in time endowments over the life cycle. Thus, while the number of households across generations increases over time, the size of a cohort over its life cycle remains constant. We solely focus on the role of population aging in determining growth dynamics and do not consider technological change. This implies that in the steady state per capita variables are constant.[7]

[6] Note that due to the convex structure of CES-preferences the nonnegativity constraints on c and l are never binding in the optimum.

[7] It is, however, straightforward to incorporate technical change into the model so as to render the model capable of explaining historically observed positive per capita growth. Yet, simply adding another *exogenous* engine of growth does not change our results with respect to the implications of differential population dynamics.

Production structure and technologies in both regions are assumed to be identical. Production occurs within a period and technologies are stationary over time. Competitive firms in both regions have access to two linearly homogenous production technologies of the constant-elasticity-of-substitution (CES) form, indexed by $i, j \in \{1, 2\}$, that combine services of capital, $K_{t,r,i}$, and labor, $L_{t,r,i}$ to produce a homogenous sectoral output good $Y_{t,r,i}$ according to

$$Y_{t,r,i} = \left(\alpha_i \, K_{t,r,i}^{\epsilon_i} + (1 - \alpha_i) \, L_{t,r,i}^{\epsilon_i}\right)^{1/\epsilon_i} , \tag{3}$$

where α_i is a sector-specific share parameter and where $1/(1 - \epsilon_i)$ is the elasticity of substitution between input factors. In both regions, sector 1 is assumed to be relatively capital-intensive, i.e. $\alpha_1 > \alpha_2$. This assumption is central to our results and determines along with changes in the relative abundance of factors the structure of trade patterns.[8] Factors of production can be allocated between sectors within regions but are assumed to be immobile internationally. Hence, we abstract from international capital flows and labor migration. Sectoral outputs can be traded freely at no costs between regions[9] and can be combined to form a composite commodity $A_{t,r}$ according to the following time-invariant CES-aggregator:

$$A_{t,r} = \left(\alpha_A \, Y_{t,r,i}^{\epsilon_A} + (1 - \alpha_A) \, Y_{t,r,j}^{\epsilon_A}\right)^{1/\epsilon_A} , \; i \neq j \, , \tag{4}$$

where α_A is a share parameter that is identical across regions, and where $1/(1 - \epsilon_A)$ is the elasticity of substitution between inputs. The composite good can be used for domestic consumption and investment purposes. Assuming that it takes one time period to build up future capital the law of motion for the capital stock in region r is described by

$$K_{t+1,r} = (1 - \delta) \, K_{t,r} + I_{t,r} \, , \tag{5}$$

where δ and $I_{t,r}$ denote the periodic depreciation rate for capital (identical across regions) and gross investment carried out at time t in region r, respectively. We abstract from capital adjustment costs. This completes the description of the model.

[8] Note that the choice of CES production functions is sufficient to rule out factor intensity reversals for both sectors at all factor and commodity prices; see Wong [1990]. I owe this point to Emami Namini [2006].

[9] In order to avoid degenerate solutions for the base case scenario in which regions are also identical with respect to population dynamics, iceberg trade costs of a small magnitude are introduced. These apply to exports of commodity 1 (2) from region $b\,(a)$ to $a\,(b)$. Except for negligible quantitative effects, this does not influence our results.

3 Dynamic general equilibrium in a MCP format

This section formulates the dynamic general equilibrium of the model in a mixed complementarity (MCP) format.[10] The structure of equilibrium is exploited by the GAMS/MPSGE program and the solver we use to compute the transition and steady state of the infinite-horizon economy.[11] Mathiesen [1985] and Rutherford [1995,1999] showed that a general equilibrium model can be formulated and efficiently solved as a complementarity problem, i.e., given a function F: $\mathbb{R}^n \longrightarrow \mathbb{R}^n$, find $z \in \mathbb{R}^n$ such that $F(z) \geq 0$, $z \geq 0$, and $z^T F(z) = 0$. It has also been demonstrated that such an equilibrium formulation is advantageous because of its ability to address weak inequalities and complementary slackness which allow to accommodate corner solutions of economic equilibrium. We exploit this feature by studying the endogenous retirement decisions of households in response to demographic shocks.

An equilibrium in complementarity format can be posed as a system of three classes of nonlinear inequalities in three sets of economic variables: commodity and factor prices, activity levels, and income levels. The first class of equilibrium conditions are zero profit conditions which state that all production activities earn zero excess profit in equilibrium. Zero-profit condition exhibit complementary slackness with respect to the associated activity level. The second class of equilibrium conditions are market clearance conditions. They state that supply must be greater than or equal to demand for each primary factor and produced good. Market clearance conditions exhibit complementary slackness with respect to market prices. The third class of equilibrium conditions are income balance equations that state that a household's expenditure must not exceed his income. Note that complementary slackness is a feature of the equilibrium allocation even though it is not imposed as an equilibrium condition, per se [Rutherford, 1995]. This means that in equilibrium, any production activity which is operated makes zero profit and any production activity which earns a negative net return is idle. Likewise, any commodity which commands a positive price has a balance between aggregate supply and demand, and any commodity in excess supply has an equilibrium price of zero.

In order to set up a computable counterpart for the theoretical model in section 2, the infinite-horizon economy has to be approximated by an appropriate finite-dimensional complementarity problem. To this end, the dynamic equilibrium formulation comprises a fourth

[10] The expression "mixed complementarity format" reflects the fact that the equilibrium formulation incorporates mixtures of equations and inequalities.

[11] For an overview of applied general equilibrium modeling with GAMS (General Algebraic Modeling System) and the GAMS/MPSGE subsystem see e.g. Rutherford [1998,1999], and http://www.mpsge.org.

class of terminal conditions that relate the transition to the steady state. We follow here the method proposed by Lau, Pahlke, and Rutherford [2002] for approximating infinite-horizon equilibria in neoclassical growth models. In the context of an OLG model with perpetual entry and exit of households, the additional complication arises how to handle households that are alive in the initial period and that live beyond the terminal period of the numerical model. We follow here Rasmussen and Rutherford [2004] who have put forward a systematic approach for modeling overlapping generations in a complementarity format.

3.1 Zero profit conditions

Zero profit conditions are formulated using the dual of production functions and have the following form:

$$-\Pi_j(p) \geq 0\,, \quad y_j \geq 0\,, \quad \Pi_j(p)\, y_j = 0\,,$$

where $\Pi_j(p)$ denotes the unit profit function for activity y_j and p is the associated price vector. In short hand and to indicate the complementarity aspect of economic equilibrium, this can be written as:

$$-\Pi_j(p) \geq 0 \quad \perp \quad y_j \geq 0\,.$$

If not stated otherwise, each equilibrium condition holds for periods $t = 0, \ldots, T$, where T denotes the terminal period of the numerical model. Note that for some activities and markets the complementarity aspect is trivial, i.e., weak inequalities are always binding in the optimum and associated complementary variables are strictly positive. This notwithstanding, we keep the formulation of economic equilibrium as general as possible and always express equilibrium conditions in complementary slackness form. The zero profit condition for sector $Y_{t,r,i}$ is given by

$$-\left[py_{t,r,i} - \left(\alpha_i^{1/(1-\epsilon_i)}\, r_{t,r}^{\epsilon_i/(\epsilon_i-1)} + \alpha_i^{1/(1-\epsilon_i)}\, w_{t,r}^{\epsilon_i/(\epsilon_i-1)}\right)^{1-1/\epsilon_i}\right] \geq 0 \quad \perp \quad Y_{t,r,i} \geq 0\,, \tag{6}$$

where $py_{t,r,i}$ is the price for commodity $Y_{t,r,i}$, and $r_{t,r}$ and $w_{t,r}$ denote the capital rental and wage rate, respectively.[12] The zero profit condition for production of the composite good is

[12] The second term in parentheses on the left-hand side of (6) represents the unit cost function that corresponds to the production function given in (3). In general, the unit cost function that is associated with a CES production function of the type

$$f(z, x_1, ..., x_n) = z\left(\sum_{j=1}^{n} \alpha_j\, x_j^{\epsilon}\right)^{1/\epsilon},$$

where x_j, $j = 1, ..., n$, denote inputs and $z > 0$, $\alpha > 0$, and $\epsilon > 0$ are parameters, can be derived from (continued on next page)

given by

$$- \left[pa_{t,r} - \left(\alpha_A^{1/(1-\epsilon_A)} \, py_{t,r,1}^{\epsilon_A/(\epsilon_A - 1)} + \alpha_A^{1/(1-\epsilon_A)} \, py_{t,r,2}^{\epsilon_A/(\epsilon_A - 1)} \right)^{1-1/\epsilon_A} \right] \geq 0 \quad \perp \quad A_{t,r} \geq 0 \, , \quad (7)$$

where $pa_{t,r}$ denotes the price for the composite commodity $A_{t,r}$. Capital is rented to firms and adds (net of depreciation) to the capital stock in the next period. The zero profit condition for the production of capital is thus given by

$$- \left[r_{t,r} + (1 - \delta) \, pk_{t+1,r} - pk_{t,r} \right] \geq 0 \quad \perp \quad K_{t,r} \geq 0 \, , \tag{8a}$$

where $pk_{t,r}$ denotes the capital purchase price. Since the numerical model ends at time T, we introduce a new variable for the price of post-terminal capital, i.e. the price of capital in period $T+1$. The zero profit condition for capital in the terminal period is therefore given by

$$- \left[(1 - \delta) \, pkt_r - pk_{t,r} \right] \geq 0 \quad \perp \quad K_{t,r} \geq 0 \, , \; t = T \, , \tag{8b}$$

where pkt_r is the post-terminal purchase price of capital. By assumption, the composite good functions as the investment good. The zero profit condition for investment is therefore given by

$$- \left[pk_{t+1,r} - pa_{t,r} \right] \geq 0 \quad \perp \quad I_{t,r} \geq 0 \, , \tag{9a}$$

where $I_{t,r}$ denotes investment. In the terminal period condition (9a) is replaced by

$$- \left[pkt_r - pa_{t,r} \right] \geq 0 \quad \perp \quad I_{t,r} \geq 0 \, , \; t = T \, . \tag{9b}$$

Households of each generation face a leisure/labor-tradeoff. A positive amount of labor is supplied if the shadow price of time (measured in units of lifetime utility) is equal to the market wage. Households do not supply labor if their reservation wage exceeds the market price for labor. The zero profit condition for labor supply is thus given by:

$$- \left[\pi_{t,r,g} \, w_{t,r} - rw_{t,r,g} \right] \geq 0 \quad \perp \quad LS_{t,r,g} \geq 0 \, , \tag{10}$$

[12] the following cost minimization problem

$$\min_{x_1, \ldots, x_n \, \geq 0} \; \sum_{j=1}^{n} p_j \, x_j \quad s.t. \quad f(z, x_1, \ldots, x_n) \geq 1 \, ,$$

where p_j, $j = 1, \ldots, n$, are associated factor prices. Rearranging optimality conditions that arise from the corresponding Lagrangian yields the unit cost function

$$c \, (p_1, \ldots, p_n) = z^{-1} \left(\sum_{j=1}^{n} \alpha_j^{1/(1-\epsilon)} \, p_j^{\epsilon/(\epsilon - 1)} \right)^{1-1/\epsilon} \, .$$

where $rw_{t,r,g}$ and $LS_{t,r,g}$ denote the reservation wage and labor supply for generation g, respectively. The zero profit condition for the production of the composite consumption good (which is a CES aggregate of material and leisure consumption; see eq. (1)) is given by

$$-\left[pz_{t,r,g} - \left(\alpha^{1/(1-\sigma)}\, pa_{t,r}^{\sigma/(\sigma-1)} + \alpha^{1/(1-\sigma)}\, rw_{t,r,g}^{\sigma/(\sigma-1)}\right)^{1-1/\sigma}\right] \geq 0 \quad \perp \quad z_{t,r,g} \geq 0\,. \tag{11}$$

Utility is treated as a commodity demanded by the different generations which implies that the utility function is modeled as any other production activity. A problem arises here for generations that live beyond the horizon of the numerical model (henceforth called "terminal generations") because the price for the composite consumption good is only defined for $t \leq T$. We therefore introduce a price for post-terminal consumption which is denoted by $pzt_{t',r,\hat{g}}$, where $T - N < \hat{g} \leq T$ identifies terminal generations and $t' = T, \ldots, T + N$ is an index for post-terminal periods. The price for post-terminal consumption is determined endogenously by imposing an appropriate terminal condition on the numerical model that approximates the infinite-horizon economy using a steady state assumption (see section 3.3). For generations that do not live beyond the terminal period, i.e. $0 \leq g \leq T - N$, the zero profit condition for the 'production' of a unit of lifetime utility is given by

$$-\left[pu_{r,g} - \left(\sum_{t=g}^{g+N}\left(\frac{1}{(1+\rho)^{t-g}}\right)^{1/\theta} pz_{t,r,g}^{1-1/\theta}\right)^{\theta/(\theta-1)}\right] \geq 0 \quad \perp \quad U_{r,g} \geq 0\,, \tag{12a}$$

where $U_{r,g}$ and $pu_{r,g}$ denote the lifetime utility and the price of a unit of lifetime utility for generation g, respectively.[13] For generations that live beyond the model horizon the following zero profit conditions apply:

$$-\left[pu_{r,\hat{g}} - \left(\sum_{t=\hat{g}}^{T}\left(\frac{1}{(1+\rho)^{t-\hat{g}}}\right)^{1/\theta} pz_{t,r,\hat{g}}^{1-1/\theta} + \sum_{t'=\hat{g}}^{\hat{g}+N}\left(\frac{1}{(1+\rho)^{t'-\hat{g}}}\right)^{1/\theta} pzt_{t',r,\hat{g}}^{1-1/\theta}\right)^{\theta/(\theta-1)}\right] \geq 0$$

$$\perp \quad U_{r,\hat{g}} \geq 0\,. \tag{12b}$$

[13] Prior to deriving the unit cost function for the production of lifetime utility, we monotonically transform the CEIS preferences in (1) to obtain a linearly homogenous CES representation of preferences as in

$$u_g(z_{t,g}) = \left(\sum_{t=g}^{g+N}\left(\frac{1}{1+\rho}\right)^{t-g} z_{t,g}^{1-\theta}\right)^{1/(1-\theta)}.$$

This formulation of preferences has the advantage that it can be represented easily in the GAMS/MPSGE program that we use for simulation of the model. Recall that a monotonic transformation does not alter the underlying preference orderings and hence optimization yields the same demand functions.

Sectoral outputs $Y_{t,r,i}$ can be sold in the respective foreign market without incurring any trade costs. The level of exports is thus determined by the following zero profit condition for international trade

$$-\left[py_{t,s,i} - py_{t,r,i}\right] \geq 0 \quad \perp \quad Y_{t,rs,i} \geq 0\,, \quad r \neq s\,, \tag{13}$$

where $Y_{t,rs,i}$ denotes exports of commodity i from region r to region s.

3.2 Market clearance conditions

For each produced good the market clearance conditions are of the following form:

$$\xi_i \geq 0\,, \quad p_i \geq 0\,, \quad \xi_i\, p_i = 0$$

where ξ and p_i denote the net supply and price of good i, respectively. In short hand and to indicate the complementarity aspect of economic equilibrium, this can be written as

$$\xi_i \geq 0 \quad \perp \quad p_i \geq 0\,.$$

We exploit Shephard's Lemma to obtain conditional factor demands for each activity. The market clearance condition for commodity $Y_{t,r,i}$ is given by

$$Y_{t,r,i} - \frac{\partial\, c_{A_r}(py_{t,r,i}\,,\, py_{t,r,j})}{\partial\, py_{t,r,i}} A_{t,r} - Y_{t,rs,i} \geq 0 \quad \perp \quad py_{t,r,i} \geq 0\,, \quad i \neq j\,,\, r \neq s\,, \tag{14}$$

where $c_{A_r}(\cdot)$ denotes the unit cost function for the composite commodity as given by (7). The market clearance condition for the composite commodity $A_{t,r}$ is given by

$$A_{t,r} - I_{t,r} - \sum_{g=t-N}^{t} \frac{\partial\, pz_{t,r,g}}{\partial\, pa_{t,r}}\, z_{t,r,g} \geq 0 \quad \perp \quad pa_{t,r} \geq 0\,, \tag{15}$$

where $\frac{\partial\, pz_{t,r,g}}{\partial\, pa_{t,r}}$ denotes the amount of the composite commodity that is demanded by generation g at time t. The time endowment of generations is viewed as a commodity that is being traded on the market for time. The corresponding market clearance condition is given by

$$\omega_{t,r,g} - LS_{t,r,g} - \frac{\partial\, pz_{t,r,g}}{\partial\, rw_{t,r,g}}\, z_{t,r,g} \geq 0 \quad \perp \quad rw_{t,r,g} \geq 0\,, \tag{16}$$

where $\frac{\partial\, pz_{t,r,g}}{\partial\, rw_{t,r,g}}$ denotes the amount of time that goes into leisure consumption of generation g at time t. The following market clearance condition for labor determines the wage rate:

$$\sum_{g=t-N}^{t} LS_{t,r,g} - \sum_{i=1}^{2} \frac{\partial\, c_{Y_{r,i}}(r_{t,r}\,,\, w_{t,r})}{\partial\, w_{t,r}}\, Y_{t,r,i} \geq 0 \quad \perp \quad w_{t,r} \geq 0\,, \tag{17}$$

where $c_{Y_{t,i}}(\cdot)$ denotes the unit cost function for sectoral output i as given by (6). The market clearance condition for capital services is given by

$$K_{t,r} - \frac{\partial\, c_{Y_{r,i}}(r_{t,r}\,,\, w_{t,r})}{\partial\, r_{t,r}}\, Y_{t,r,i} \geq 0 \quad \perp \quad r_{t,r} \geq 0\,. \tag{18}$$

The level of capital stock is determined by investment decisions in the previous period and by the capital stock (net of depreciation) in the previous period. The market clearance condition for the capital stock for periods is therefore given by

$$(1-\delta)\, K_{t-1,r} + I_{t-1,r} - K_{t,r} \geq 0 \quad \perp \quad pk_{t,r} \geq 0\,,\, 1 \leq t < T\,. \tag{19a}$$

A particular version of (19a) applies for the first and terminal period. In the first period generations that were born $\widetilde{g} = -N, \ldots, -1$ periods prior to year zero are endowed with exogenous assets denoted by $m_{r,\widetilde{g}}$. Assets holdings by households represent claims on the domestic capital stock. The market clearance condition for the first period can therefore be written as

$$\sum_{\widetilde{g}} \frac{m_{r,\widetilde{g}}}{(1+\bar{r})} - K_{t,r} \geq 0 \quad \perp \quad pk_{t,r} \geq 0\,,\, t = 0\,, \tag{19b}$$

where $\bar{r}$ denotes the exogenous steady state interest rate used for initial calibration of the model. Dividing assets by $(1+\bar{r})$ gives the value of assets one period prior to year zero assuming that the economy is initially in a steady state. See section 4.1 for further details on the calibration procedure. In the last model period the following market clearance condition for the capital stock applies:

$$(1-\delta)\, K_{t,r} + I_{t,r} - KT_r \geq 0 \quad \perp \quad pkt_r \geq 0\,,\, t = T\,, \tag{19c}$$

where KT_r denotes the post-terminal capital stock which is selected by imposing an appropriate terminal condition on the model (see section 3.3). The market clearance condition for the composite consumption good is given by

$$z_{t,r,g} - \frac{\partial\, pu_{r,g}}{\partial\, pz_{t,r,g}}\, U_{r,g} \geq 0 \quad \perp \quad pz_{t,r,g} \geq 0\,, \tag{20}$$

where $\frac{\partial\, pu_{r,g}}{\partial\, pz_{t,r,g}}$ denotes the demand by generation g at time t. The market clearance condition for lifetime utility of generation g is given by

$$U_{r,g} - M_{r,g}/pu_{r,g} \geq 0 \quad \perp \quad pu_{r,g} \geq 0\,, \tag{21}$$

where $M_{r,g}$ denotes the income of generation g in region r as defined in (22).

3.3 Income balance and terminal conditions

In equilibrium the income of each generation equals the value of its time endowment over the life cycle:

$$M_{r,g} = \sum_{t=g}^{g+N} rw_{t,r,g}\ \omega_{t,r,g} \quad , \quad \text{if } g \neq \widetilde{g}\ \wedge\ g \neq \hat{g}\,. \tag{22a}$$

Generations that were born before the first model period, i.e., those generations that are of age $a = \mid \widetilde{g} \mid$ at year zero, receive an exogenous endowment of assets in year zero and are endowed in subsequent periods of their life as any other generation. The income definition for generation $\widetilde{g}$ is therefore given by

$$M_{r,g} = pk_{0,r}\,\frac{m_{r,\widetilde{g}}}{(1+\bar{r})} + \sum_{t=0}^{N-|\widetilde{g}|} rw_{t,r,\widetilde{g}}\ \omega_{t,r,\widetilde{g}} \quad , \quad \text{if } g = \widetilde{g}\,. \tag{22b}$$

Formulating the income balance equations for terminal generations requires addressing two additional issues that relate the transition to the steady state and ensure that there are no undesirable terminal effects brought about by the behavior of generations that live into post-terminal periods. First, in order to satisfy demands of generations living into post-terminal periods, post-terminal consumption and leisure enter as endogenous endowments. Second, terminal generations are required to leave behind an amount of assets in the terminal period. This is modeled by assuming that generations living into the post-terminal periods receive a negative endowment of capital thus preventing these generations from fully running down their assets in the last model period. The income balance equation for terminal generations is hence given by

$$M_{r,g} = \sum_{t=\hat{g}}^{T} rw_{t,r,g}\ \omega_{t,r,g} + \sum_{t'=\hat{g}}^{\hat{g}+N} pzt_{t',r,\hat{g}}\, zt_{t',r,\hat{g}} - KT_r\, pkt_r \quad , \quad \text{if } g = \hat{g}\,, \tag{22c}$$

where $zt_{t',r,\hat{g}}$ denotes post-terminal consumption and leisure. The level of post-terminal consumption and leisure for generations living into post-terminal periods is selected by a terminal constraint which guarantees that the behavior of these generations is consistent with a steady state situation after T. The level of zt is selected endogenously according to a steady-state projection which commands that the present value price of post-terminal full consumption, pzt, declines with the steady-state interest rate:

$$pz_{T,r,\hat{g}}\,(1+\bar{r})^{T-t'} = pzt_{t',r,\hat{g}} \quad \perp \quad zt_{t',r,\hat{g}} > 0\,. \tag{23}$$

Given $zt_{t',r,\hat{g}}$, the price for post-terminal consumption and leisure is then determined by the following market clearance condition:

$$zt_{t',r,\hat{g}} - \frac{\partial\, pu_{r,\hat{g}}}{\partial\, pzt_{t',r,\hat{g}}}\, U_{r,\hat{g}} \geq 0 \quad \perp \quad pzt_{t',r,\hat{g}} \geq 0\,, \tag{24}$$

where $\frac{\partial\, pu_{r,\hat{g}}}{\partial\, pzt_{t',r,\hat{g}}}$ denotes the demand by generations $\hat{g}$ at post-terminal period t'. A second terminal constraint selects the level of post-terminal capital that also appears in the income balance equation (22c). We follow here the approximation method proposed by Lau, Pahlke, and Rutherford [2002] according to which the infinite horizon economy can be decomposed into two distinct programs where one runs from $0, \ldots, T$ and the other one running from $T+1, \ldots, \infty$.[14] Both subproblems are linked through the post-terminal capital stock in period $T+1$. The level of post-terminal capital is computed endogenously by requiring that investment grows at the steady-state rate in the last model period:

$$\frac{I_{T,r}}{I_{T-1,r}} = 1 + \gamma_{T,r} \quad \perp \quad KT_r > 0\,. \tag{25}$$

Note that condition (23) already requires that a steady state is reached sufficiently ahead of T. It is therefore not feasible to select terminal investment growth endogenously by anchoring it to any other stable quantity in the model.[15]

So far, the equilibrium has been formulated for a full two-region version of the model. It is straightforward to obtain a corresponding system of equilibrium conditions for a closed economy by dropping the zero profit conditions for trade activities given by (13) and by deleting the export variables $Y_{t,rs,i}$ that appear in market clearance conditions (14).

4 Calibration and the nature of demographic shocks

The square system of equations as specified by (6)–(25) does not have an analytical solution. We therefore numerically solve the model using a mixed-complementarity solver in GAMS which exploits the specific structure of our equilibrium formulation. Besides the ability to address corner solutions of economic equilibrium, one virtue of this computational algorithm is that it does not rely on a log-linear approximation of equilibrium decision rules that solve the Euler equations. The numerical model therefore allows for discontinuities and is more reliable in analyzing large perturbations to the steady state growth path.

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[14] Note that this method for approximating the infinite horizon relies on the assumption of time-separable utility functions.

[15] This has been suggested by Lau, Pahlke, and Rutherford [2002] to circumvent the necessity of having to impose a steady state in the terminal period.

4.1 Calibration strategy

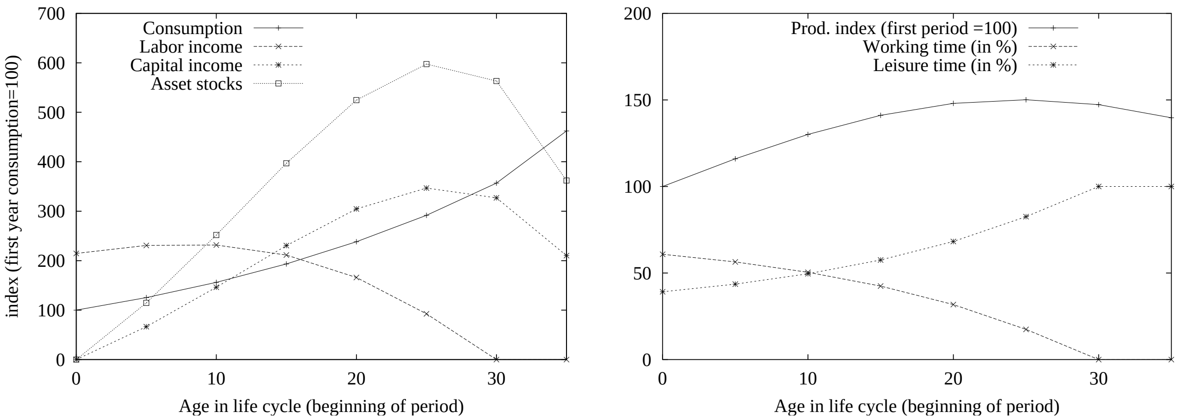
For the calibration of the model we use a synthetic database. In order to isolate the role of differential population dynamics across regions, we impose symmetry with respect to technology and preferences. We calibrate the model to a steady state situation in which population growth rates in both countries are identical over time. Table 1 shows the parametrization

Table 1: Base Case Parametrization

(i) Preferences		
Intertemporal elasticity of substitution, eq.(1)	$1/\theta$	0.5
Intratemporal elasticity of substitution, eq.(1)	$1/(1-\sigma)$	0.8
Consumption share parameter, eq.(1)	α	0.4
(ii) Technology		
Sector-specific share parameter, eq.(3)	α_1	0.7
Sector-specific share parameter, eq.(3)	α_2	0.3
Elasticity of substitution, eq.(3)	$1/(1-\epsilon_i)\,,\, i=1,2$	2
Share parameter for composite good, eq.(4)	α_A	0.5
Elasticity of substitution, eq.(4)	$1/(1-\epsilon_A)$	2
Annual capital depreciation rate, eq.(5)	δ	0.07
(iii) Steady-state quantity and price index		
Annual growth in time endowment, eq.(2)	$\gamma_{t,r}\,,\,\forall\, t, \forall\, r$	0.03
Annual interest rate, eq.(22b)	$\bar{r}$	0.05

that is chosen for the base case scenario. It follows from the first order conditions of the households' optimization problem that —in a steady state where prices decline with the constant rate $1+\bar{r}$ and where quantities grow at the constant rate $1+\gamma$— the utility discount factor is implicitly given by $\rho = \frac{1+\bar{r}}{(1+\gamma)^{-\theta}} - 1$. Labor productivity is assumed to follow a humped-shaped profile over the life cycle. We draw here on Auerbach and Kotlikoff [1987] and adapt the following specification: $\pi_{a,r,g} = exp(4.47 + 0.033\,a - 0.00067a^2)/exp(4.47)\,, \forall\, r,\, \forall\, g$. We therefore assume an identical and time-invariant labor productivity profile for all generations in both regions. Fig. 2, Panel (b), plots the labor productivity over the life cycle. In order to limit the computational burden of our simulations, we solve the model in five year intervals. Households are assumed to live for $N = 35$ years.

We use the calibration procedure for OLG models described in Rasmussen and Rutherford [2004]. The virtue of this approach is that the calibration of the OLG demand system is independent from the specific structure of the production side of the economy. The calibration to an initial steady state is carried out in two steps. The first step solves for the optimal behavior of a single reference generation taking into account the existence of aggregate consistency conditions which ensure that individual choices by households match the behavior of the overall economy. In the second step, the results from the household calibration model are used to set up the entire baseline reference path, including generations that are already alive at year zero, by extrapolating optimal decision profiles of the single reference generation using a steady-state assumption.



(a) Baselineprofiles (b) Productivity index and allocation of time

Fig. 2: Steady state profiles and productivity index

Note: Baseline profiles for a representative generation along the reference path are derived in the first calibration step using a steady state assumption [Rasmussen and Rutherford, 2004]. The household calibration model is parameterized as shown in Table 1, (i) and (iii).

The resulting income and consumption profiles as well as asset positions over the life cycle are shown in Fig. 2, Panel (a). In accordance with the life cycle theory [Modigliani and Brumberg, 1954], households arrange savings so to smooth consumption over their lifetime — as it is implied by the intertemporal Euler equation. Due to the humped-shaped productivity profile, time devoted to labor and consequently labor income is increasing in the first years and then decreases —as it is implied by the intratemporal Euler equation for leisure. Fig. 2, Panel (b) shows the allocation of leisure and working time over the life cycle. In the last five years, labor supply drops to zero with the reservation wage exceeding the market wage. Note that this is an instance of the endogenous regime-switching property of our specific equilibrium formulation, i.e., there is a corner solution for labor supply during the last five years of the life cycle. Households accumulate assets in younger to middle ages and run down their asset positions when being old; capital income evolves accordingly. Zero labor income in the last years requires to build up sufficiently large asset stocks in old age.

4.2 Region-specific population dynamics

Population aging is modeled by assuming that population growth rates, or equivalently, fertility rates, decrease over time. Shocks to the population growth rate alter the age composition of the population.[16] Unsynchronized aging patterns are modeled by assuming that the speed of decline in growth rates varies across regions.

Region a is the fast aging region and experiences a relatively fast decline in its population growth rate. Region b is the slow aging region. In the initial steady state both regions are characterized by a common annual population growth rate of $\gamma_{t,r} = 0.03, \forall\, r$. From year zero onwards population dynamics begin to diverge and both regions experience a linear decline in growth rates that occurs at a different speed. Eventually, growth rates in both regions settle to a new permanent level which is lower than in the initial steady state. This ensures that there is a well-defined steady state for the world economy and that neither region becomes negligible in terms of size as time approaches infinity. More formally, the time profile for population growth rates in the fast aging region a is given by

$$\gamma_{t,a} = \begin{cases} 0.03 & \text{if} \quad t = 0 \\ 0.03 - 0.0004\,(t - 40) & \text{if} \quad 0 < t < 20 \\ 0.02 & \text{if} \quad 20 \leq t < T\,, \end{cases} \tag{26a}$$

whereas the time profile for population growth rates in the slow aging region b is given by

$$\gamma_{t,b} = \begin{cases} 0.03 & \text{if} \quad t = 0 \\ 0.03 - 0.0002\,(t - 40) & \text{if} \quad 0 < t < 50 \\ 0.02 & \text{if} \quad 50 \leq t < T\,, \end{cases} \tag{26b}$$

where the time horizon for the numerical model is set to $T = 150$.[17] Fig. 3, Panel (a) and (b), show the time profiles of population growth rates and the impact on the impact on the working-age population ratio, respectively. In the given context, we define the working-population

[16] From an empirical point of view it might seem unsatisfactory to capture the nature of population aging by declining fertility rates only, given that the severity of demographic change in most industrialized countries is also heavily driven by falling mortality rates. The model does not feature stochastic lifetimes and uncertainty with respect to the point of death and is consequently not capable of addressing the implications of increasing life expectancy. Note, however, that in terms of the stylized impact on the age structure of the population that we aim to characterize here, it suffices to solely concentrate on decreasing fertility rates.

[17] The model reaches its new long-run equilibrium around $t = 120$.

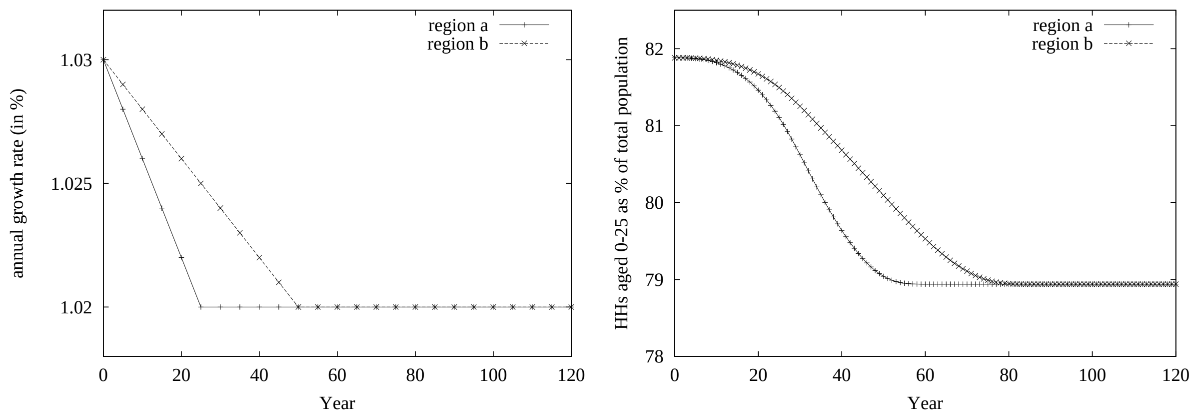
ratio as the number of households aged between 0-25 as a fraction of the total population. The temporary decline in population growth rates effectively decreases the proportion of young people in the economy. The extent of population aging is more pronounced in region a which experiences the relatively fast drop in population growth rates. Note that the demographic transition extends far beyond the point in time where fertility rates have converged: it nearly takes the full lifetime of a generation to reach a stationary age structure again.

5 Model results

This section presents the results from simulation experiments that (i) use the base case parameter configuration (Table 1) and (ii) assume that both regions experience population dynamics as specified in (26a) – (26b). We consider the following two scenarios:

1. *Autarky case:* International trade is prohibited and the world economy is composed of two regions that operate under autarky. Benchmark I: initial steady state.

2. *International trade:* International goods markets for each homogenous good $Y_{t,r,1}$ and $Y_{t,r,2}$, $r \in \{a, b\}$, are perfectly integrated. Benchmark II: autarky case.



(a) Time profile of fertility rates

(b) Impact on working-age population ratio

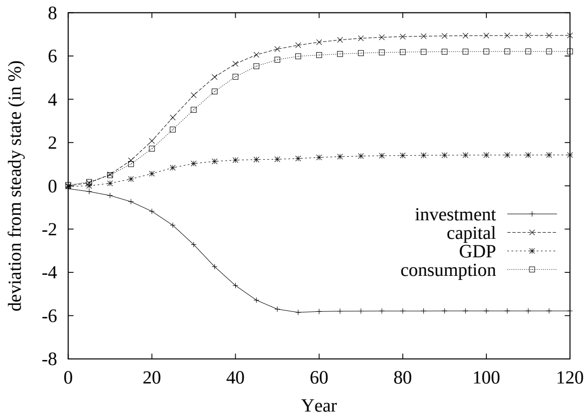
Fig. 3: Demographic shocks and population aging

Note: Panel (a) shows the series of demographic shocks as specified in eq. (26a) – (26b). Panel (b) shows the impact on the working-age population ratio. It is defined as the number of households in age groups 0-25 as a fraction of the total population. For both graphs, we interpolate quinquennial data to obtain annual observations.

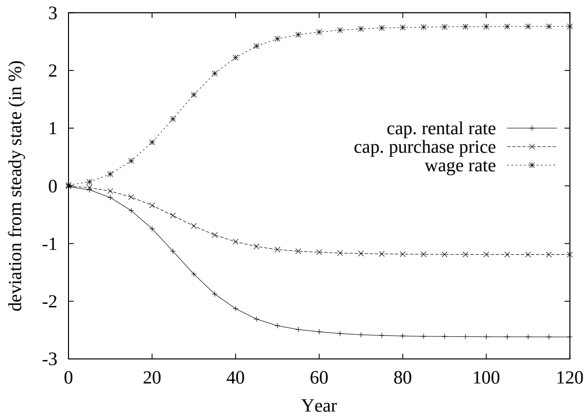
5.1 Aggregate economy effects

5.1.1 Aging under autarky

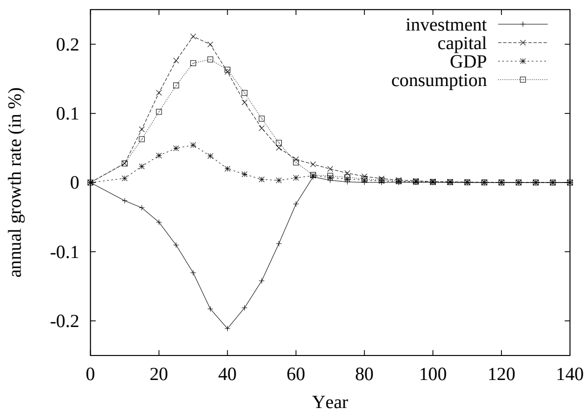
In order to develop some intuition for the economic impact of the demographic shocks, we first look at the aggregate economy effects of aging in a closed economy. Fig. 4, Panels (a) – (c), show the evolution of key macroeconomic variables for the fast aging region a under autarky. The decline in fertility rates and the associated aging of the population cause the per capita



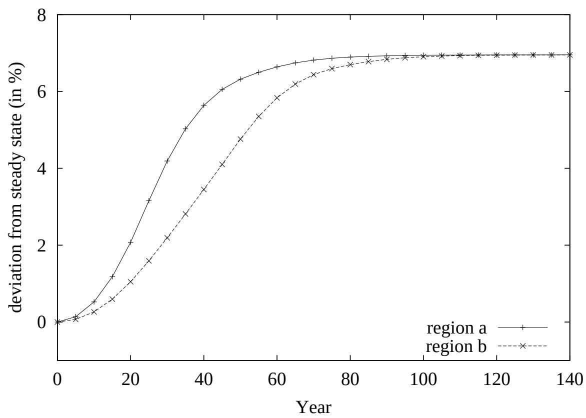
(a) Per capita variables, region a



(b) Real factor prices, region a



(c) Per capita growth rates, region a



(d) Per capita capital stock, region a and b

Fig. 4: Per capita macroeconomic variables under autarky

Note: The plots show the transition to a new long run equilibrium in response to demographic shocks as specified in eq. (26a) – (26b). Panel (a): Consumption is defined as total private sector consumption. Investment is gross investment. Gross domestic product (GDP) is defined as the quantity of the composite good A produced at a given point in time. Panel (b): Factor prices are nominal factor prices deflated by the price index for the composite good A. Panel (c): Per capita growth rates are expressed in % and refer to annual changes.

capital stock to rise continuously until it settles to a new higher long-run level. This capital deepening in the aging economy stems from the underlying life cycle savings behavior of households: older generations hold larger amounts of assets than younger generations (there is a decline at older ages but the levels of capital holdings exceed those in younger ages (Fig. 2)). Since the fraction of older people during the demographic transition rises the average capital holding, i.e. the per capita capital stock, also increases.[18] Thus, we have:

Result 1: *Aging brought about by lower fertility rates leads to a higher per capita capital stock during the transition and increases the steady state per capita capital stock.*

Investment per capita decreases in response to lower future returns on capital and because the propensity to save is lower in an economy whose population is made up of a higher fraction of older people. Lower investment means that per capita consumption rises. Linear homogeneity and monotonicity of production technologies imply that per capita GDP moves with the per capita capital stock. Since population growth is the only source of sustainable growth in the model, growth rates of per capita variables converge back to zero in the long-run (Fig. 4, Panel (c)).

A higher per capita capital stock means that the economy has become capital abundant and labor scarce relative to its initial steady state. The output of the capital-intensive sector 1 therefore expands relatively to the labor-intensive sector 2. Note that we only observe a "dampened" Rybczynski effect due to feedback effects of induced commodity price changes which cushion —but not reverse— the "pure" Rybczynski effect that would be produced if prices were fixed.[19] Fig. 5, Panel (a), plots the relative size of sector 1 over time illustrating the reallocation of resources between sectors that is brought about by aging. There is a relative contraction of the labor-intensive sector 2. Demand and supply consequently force the relative price of commodity 1 to fall (Fig. 5, Panel (b)). The fall in the relative price of commodity 1 lowers the real return to the factor used intensively in the production of that good, here capital, and hence the capital rental rate falls, and increases the real return to the other factor, here labor, and hence the wage rate rises. Changes in factor prices thus reflect the varying degree of scarcity of each primary factor. Since in each period changes in commodity prices are a weighted average of changes in factor prices, the wage rate (capital

[18] The result of a higher steady state level of per capita capital and output in response to a permanently lower population growth rate is also obtained in a Solow-Swan growth framework. In the Ramsey model, however, changes in the population growth rate have only transitional effects and long run levels of both per capita variables are unaffected. In the case of Solow-Swan, the result is driven by the assumption of an exogenous constant savings rate. In an OLG framework, the very presence of finite lifetimes of households prevents a full insulation from demographic shocks which is achieved in the Ramsey model only where agents face infinite planning horizons. For more details see, e.g., Barro and Sala-i-Martin [2004].

[19] See, e.g., Jones [1965, pp.562].

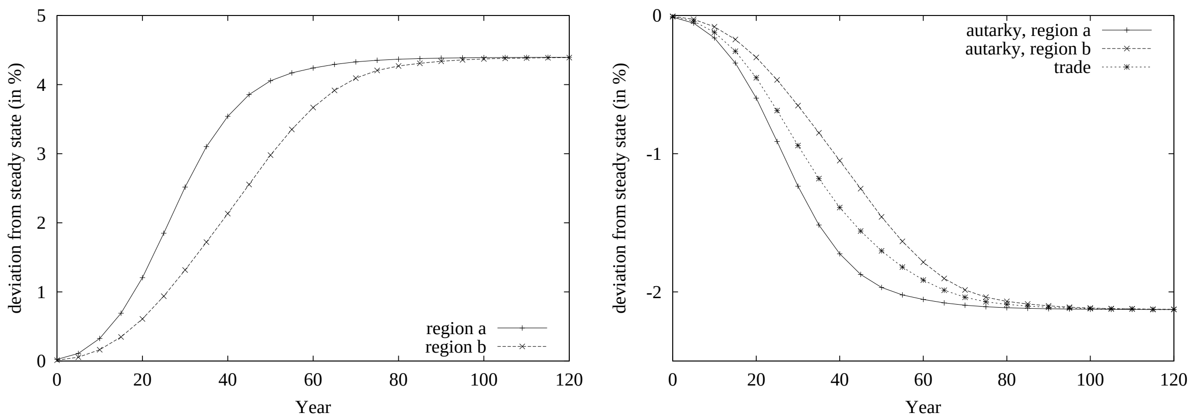
rental rate) increases (decreases) in percentage terms by more than the relative price of commodity 1. Thus, we have:

Result 2: *In a two sector economy where commodities are produced with different factor intensities, aging leads to a relative contraction (expansion) of the labor-intensive (capital-intensive) sector. Real wages (capital rental rates) increase (decrease) during the transition and are higher (lower) in the new steady state.*

It is interesting to compare the length of the demographic adjustment with the number of periods the economy needs to reach its new steady state equilibrium; compare Fig. 3, Panel (b) with, e.g., Fig. 4, Panel (c). The age structure of the population is stationary from year 80 onwards whereas the economic transition phase extends until around year 120. Hence, even if demographic shocks occur over a small number of periods —here, 25 years in the case of region a— cause lengthy economic adjustment processes.

5.1.2 Unsynchronized aging patterns and international trade

Differences in the extent and timing of aging patterns across regions lead to quantitatively distinct responses in both economies. The capital deepening in the slow aging region b is less pronounced vis-à-vis the fast aging region a as the share of older people in the economy is lower throughout the transition. Capital (labor) is more abundant (scarce) in the fast aging



(a) Relative size of sector 1 (autarky)

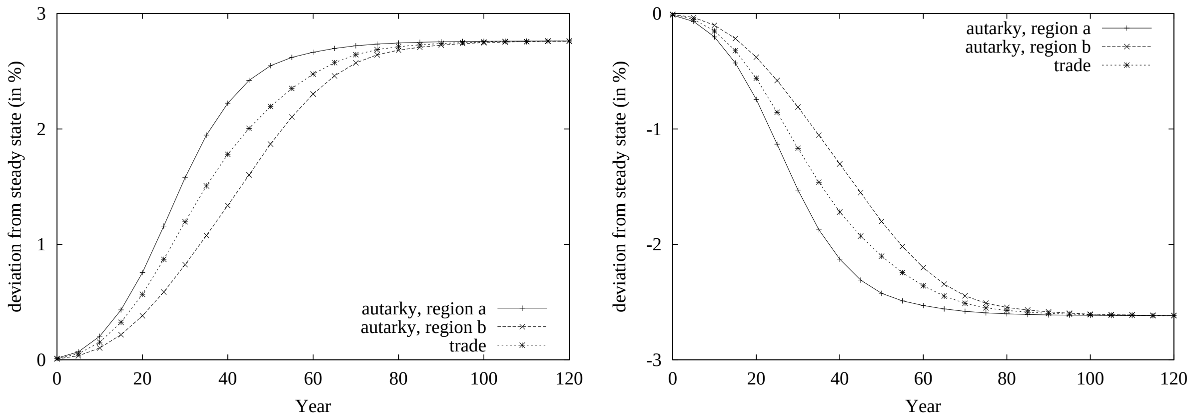
(b) Relative price of commodity 1 (autarky vs. trade)

Fig. 5: Relative size of sector 1 and relative price of commodity 1

Note: The plots show the transition to a new long run equilibrium in response to demographic shocks as specified in eq. (26a) – (26b). The relative size of sector 1 is measured as the ratio of sectoral outputs. Prices in year zero are calibrated to unity. The relative price of commodity 1 is defined as the ratio of the price for commodity 1 and 2.

region as compared to the slow aging region. Thus, the per capita capital stock in region b is lower than in region a, i.e., the slow (fast) aging region becomes relatively labor (capital) abundant (Fig. 4, Panel (d)). Consequently, the ”dampened” Rybczynski effect is weaker for region b implying a less strong relative expansion of the capital-intensive sector 1 (Fig. 5, Panel (a)). In the steady state both regions are characterized by the same per capita capital stock as differences in the age composition of populations have vanished.

Under autarky, differences in relative factor endowments cause equilibrium factor prices to diverge (Fig. 6). The wage rate (capital rental rate) in the relatively capital abundant,



(a) Real wage rate

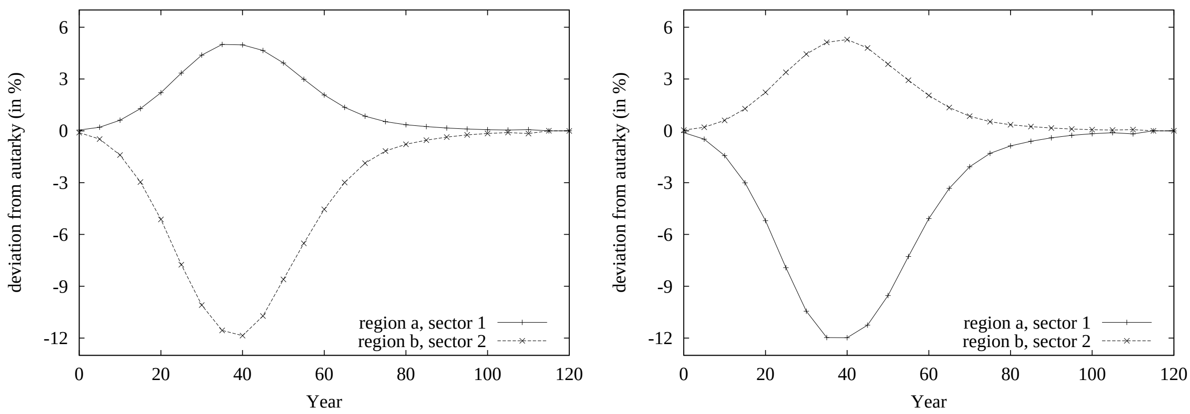
(b) Real rental rate of capital

Fig. 6: Equilibrium factor prices under autarky and trade

Note: The plots show the transition to a new long run equilibrium in response to demographic shocks as specified in eq. (26a) – (26b). Prices in year zero are calibrated to unity. Real factor prices are nominal factor prices deflated by the price index for the composite good A.

labor scarce region a is higher (lower) throughout the transition vis-à-vis the relatively capital scarce, labor abundant region b. International differentials in factor prices reach their maximum when differences in the per capita capital stock peak and gradually diminish as population dynamics converge. Despite identical production technologies in both regions, differentials in factor prices imply different costs of production which in turn translate into relative price differences of produced commodities under autarky (Fig. 5, Panel (b)). The fast aging region a has a cost advantage in producing the capital-intensive commodity 1 relative to the labor-intensive commodity 2 —vice versa for region b. Hence, the relative price of commodity 1 in region a stays below the one in region b as long as differences in relative factor endowments across regions are sustained.

Divergence of relative commodity prices under autarky create incentives for trade. If international trade is liberalized, each region will export the good where it has a cost advantage. Exploiting the higher relative price of commodity 1 in region b, the fast aging, capital abundant region a consequently exports the capital-intensive commodity 1, whereas the slow aging, labor abundant region b exports the labor-intensive commodity 2. Each region specializes in the production of the good which it can produce more efficiently thereby even more increasing productivity of the factor used intensively in this sector. Comparing trade with autarky, capital is more productive in sector 1 in region a and less productive in sector 2 in region b, and vice versa for the productivity of labor (Fig. 7, Panel (a) and (b)). Free trade



(a) Capital productivity (b) Labor productivity

FIG. 7: FACTOR PRODUCTIVITY BY SECTORS (TRADE RELATIVE TO AUTARKY)

Note: The plots show the percentage changes in factor productivity by sectors comparing trade with autarky. Productivity is measured as the average product, i.e. sectoral output divided by the quantity of the respective factor used as an input in production.

establishes a common relative price across regions where the autarky prices provide lower and upper bounds (Fig. 5, Panel (b)). Trade in goods is a perfect substitute for trade in factors and therefore leads to the equalization of factor prices (Fig. 6).[20] Thus, we have:

Result 3: *Capital (labor) is more abundant (scarce) in the fast aging region vis-à-vis the slow aging region. Thus, in a $2\times2\times2$ economy, the fast (slow) aging region exports the capital-intensive (labor-intensive) commodity during the transition.*

Although immobile internationally, capital is shifted between regions to the extent that it is embedded in traded goods. By exporting the capital-intensive commodity during the transition, the capital abundant region a is thus characterized by a lower capital stock per capita under trade as compared to autarky, whereas per capita capital in the labor abundant

[20] In our model, all conditions for factor price equalization are met. See, e.g., Feenstra [2004].

region b, which exports the labor-intensive good, is higher under trade as compared to autarky (not shown).

5.2 Aging and trade: Household behavior and intergenerational welfare effects

This section takes a more disaggregated view and analyzes the economic consequences of unsynchronized aging patterns on a household level if international trade is liberalized. It turns out that impacts on household behavior in region a and b are reversed to each other. For most part of the analysis, it therefore suffices to focus on either region; we concentrate on the fast aging region a. Fig. 8, Panel (a), shows the welfare change for each generation measured as the percentage change in Hicksian equivalent variation (EV) on a full lifetime basis comparing trade with autarky. In contrast to standard neoclassical trade theory, trade liberalization yields ambiguous results with respect to welfare effects, i.e. openness may be immiserizing. From Fig. 8, Panel (a), it can be seen that:

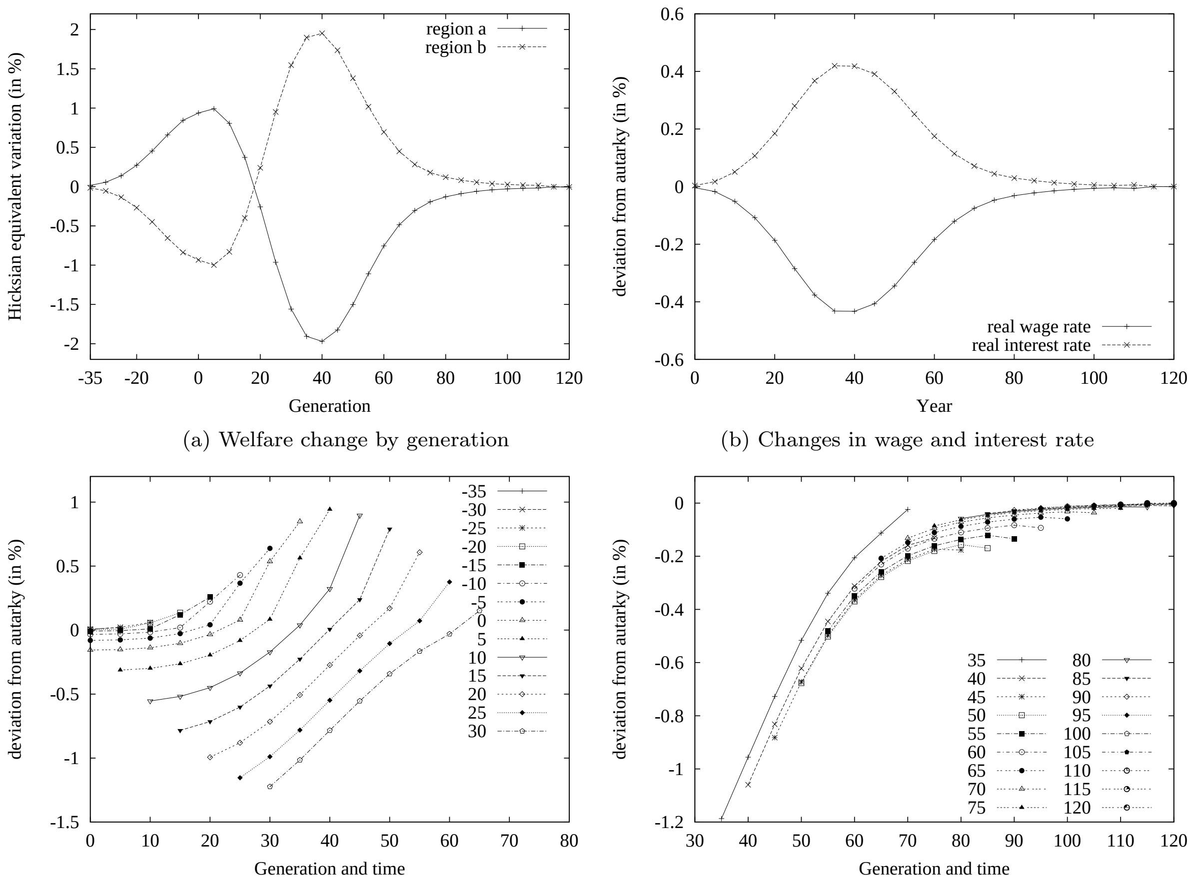
Result 4: *Gains from trade in the presence of globally unsynchronized aging patterns are not universal, i.e. there are in both regions generations that gain and lose. In the fast aging region, only generations that are born in the run-up to and in the beginning of the demographic transition gain while all others lose.*

Why are there winners and losers? What explains the marked differences in sign and magnitude of welfare changes? The answer lies in the different extent of *real* factor price changes that each generation experiences over its lifetime. As evident from the previous analysis, trade leads to factor price equalization and thereby increases (decreases) the real capital rental rate (real wage rate) in the fast aging region a as compared to autarky. Fig. 8, Panel (b), plots the gap between the trade and autarky real wage and interest rate. Now consider, for example, generation 0 which lives until year 35, and generation 40 that lives until year 75. Comparing the extent of changes in wage rates for a given age of both generations, it is easy to see that generation 40 undergoes larger decreases than generation 0. This is particularly true for earlier periods of the life cycle when labor productivity and consequently hours worked are high. A similar picture applies for changes in real interest rates. Here, generation 0 experiences relatively large increases in old age when accumulated assets and capital income are high whereas generation 40 benefits from modest increases in old age only.

Household consumption

Changes in real factors prices alter the consumption and labor supply choices of households and consequently impact on lifetime utility. To see why some generations win and others lose, look first at changes in lifetime consumption profiles of generations (Fig. 8, Panel (c)-(d)). The observed change in periodic consumption —focussing on households in the fast aging region a— is a composite of the following three effects:

1. For a given level of savings higher interest rates make households wealthier. Thus,



(a) Welfare change by generation

(b) Changes in wage and interest rate

(c) Changes in periodic cons. ($g = -35, \ldots, 30$)

(d) Changes in periodic cons. ($g = 35, \ldots, 120$)

Fig. 8: Welfare and consumption profiles by generation (trade vs. autarky)

Note: The plots show compare trade with autarky under population dynamics as specified in (26a) – (26b). All changes are denoted in %. Panel (a): Welfare is computed on a lifetime basis and is measured in Hicksian equivalent variation (EV). Panel (b): Plots show region a. The real interest rate is defined as the real capital rental rate plus depreciation. Panel (c) and (d): Change in periodic material consumption $c_{t,g}$ over the respective lifetime for generations $g = -35, \ldots, 30$ and $g = 35, \ldots, 120$ in region a, respectively.

consumption rises (*wealth effect*).

2. Changes in interest rates affect the relative price of consumption between different periods in time. Higher interest rates imply that consumption today becomes more expensive vis-à-vis consumption tomorrow. Households therefore allocate consumption towards later periods of their life (*substitution effect*).

3. For a given level of labor supply lower wage rates imply that households are less wealthy. Thus, consumption decreases (*wage effect*).

For generations born prior to year zero, we see that old-age consumption is slightly increased due to a positive income and a intertemporal reallocation effect which shifts consumption towards later periods of the life cycle. Consumption in earlier periods is almost unchanged because the drop in wage rates and hence negative wage effects are relatively small. For subsequent generations $g = 0, \ldots, 30$, i.e. moving along the x-axis from the left to the right in Panel (c), losses in wage income become increasingly substantial and lead to larger and larger reductions of consumption in earlier periods of the life cycle. At the same time, the interest rate gap widens which strengthens the positive income and intertemporal substitution effect causing increases in old-age consumption. However, as the extent of real factor price changes over the lifetime of a generation becomes less favorable, losses in wage income cannot be compensated for by the positive income effect and hence there is a downward adjustment in consumption for all periods of the life cycle for generations $g \geq 35$ (Panel (d)). From year 40 onwards, the trade-autarky differentials in factor prices begin to diminish and households born during this period experience smaller and smaller decreases in wage rates over their lifetime. Hence, moving across generations, the size of reductions in young-age consumption diminishes.

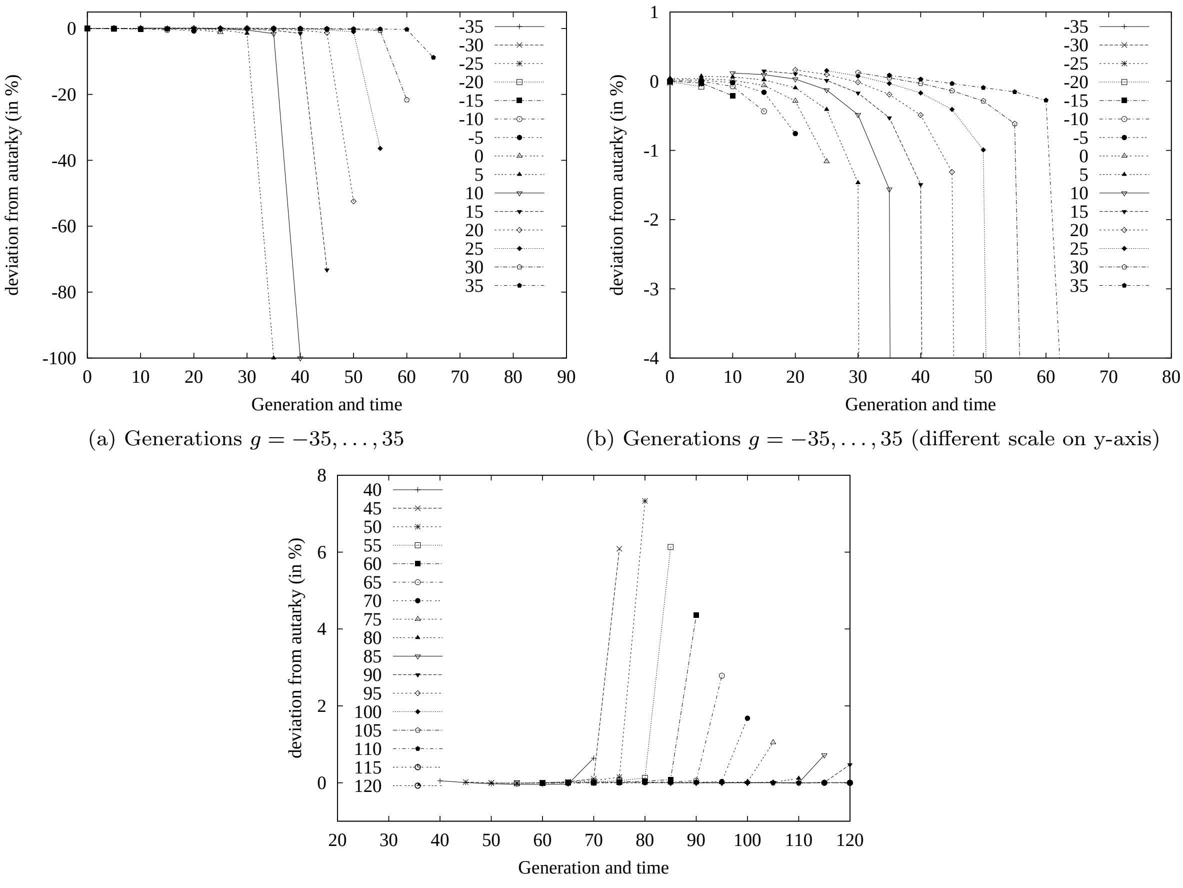
Labor supply and endogenous retirement

Fig. 9, Panel (a)-(c), shows the resulting impact on labor supply profiles of generations. The observed change in periodic labor supply —focussing on households in the fast aging region a— is a composite of the following two effects:

1. For a given level of savings, higher interest rates imply that households have become more wealthy; because there is disutility from working this creates a tendency for hours worked to fall (*wealth effect*).

2. Hours worked respond directly to changes in the real wage (*price effect*). The intuitive case is that labor supply is increasing in the wage rate. In principle, however, this effect may work in both directions and the labor supply schedule may also exhibit a backward-bending part, i.e. leisure consumption is a Giffen good.

Generations born prior to the demographic shocks, i.e. $g = -35, \ldots, -5$, leave labor supply largely unchanged as real wages in early stages of the life cycle when labor productivity is high are almost unchanged. At the same time, they benefit in older ages from higher interest rates on their savings and therefore adjust labor supply downwards in later periods of their life cycle. As wage and interest rate gaps between trade and autarky broadens, decreases in labor supply become increasingly substantial for generations $g \leq 10$ when moving, both, over



(a) Generations $g = -35, \ldots, 35$

(b) Generations $g = -35, \ldots, 35$ (different scale on y-axis)

(c) Generations $g = 40, \ldots, 120$

FIG. 9: CHANGES IN PERIODIC LABOR SUPPLY OVER LIFE CYCLE (TRADE VS. AUTARKY)

Note: The plots show the percentage change in periodic labor supply for households in region a comparing trade with autarky under population dynamics as specified in (26a) – (26b).

the life cycle of a given generation as well as across generations. There are huge drops in labor supply for generations 5 and 10 that belong —looking at the respective welfare changes in Fig. 8, Panel (a)— to the biggest winners of trade liberalization. Albeit decreases in real wages during younger ages, the positive wealth effect is strong enough to make these households significantly better off. In their case, the wealth effect stemming from higher interest rates is sufficiently strong to facilitate early retirement (as compared to autarky), i.e., there is an endogenous regime change in labor supply for generation 5 and 10 in year 35 and 40, respectively. Thus, in a fully-funded pension system international trade in the presence of globally unsynchronized aging patterns potentially alters the optimal retirement age. For subsequent generations $g \geq 15$, factor price changes over the respective lifetime become increasingly less favorable implying that households decrease their periodic labor supply by less than preceding generations. Generations $g \geq 40$ even have to increase labor supply in the second last period of their life cycle, i.e. in their last period before retirement, to generate sufficiently high labor income (Fig. 9, Panel (c)).

Intergenerational welfare effects

As evident from the foregoing discussion, liberalizing trade in the presence of globally unsynchronized aging patterns has very distinct *quantitative and qualitative* effects across generations and regions. Changes in consumption and labor supply profiles explain the observed realized welfare gains and losses of generations. In the fast aging region, generations born in the run up to and the beginning of the demographic transition gain: lifetime utility of generations $g = -35, \ldots, 15$ in the fast aging region increases (Fig. 8, Panel (a)) due to (*i*) positive wealth effects stemming from higher interest rates and (*ii*) increased leisure consumption that (*iii*) overcompensate losses in wage income. This is in stark contrast to what can be found in the literature. Sayan [2005] finds —in a framework that is incapable of assessing *generational* welfare changes— that the fast aging region loses overall. Generations born in the midst of the transition and thereafter ($g > 15$) stand to incur substantial utility losses. As the age structure of populations in both regions converges, the adverse welfare effects for newly born generations diminish. A reverse situation in terms of welfare changes applies to the slow aging region. Here, currently old generations lose while all subsequent generations win. However, without imposing a social welfare function that involves a weighting of current versus future generations it is not possible to assess the welfare impact of trade liberalization on a country level. We only focus on the generational welfare implications.

On grounds of the previous analysis, a set of further results can be established. It obvious that:

Result 5: *Aging is a slow moving economic process extending far beyond the time horizon of the demographic transition phase. It profoundly affects the well-being of future generations born after the demographic transition is completed.*

Furthermore, as it is well-known, international trade leads to a redistribution of world income. If goods markets are integrated, differences in the extent and timing of aging patterns lead to international flows in goods and therefore impact on the well-being of nations. Fig. 8, Panel (a), reveals that welfare changes across regions for a given generation are diametrically opposed. Without loss of generality, i.e. by applying an arbitrary but identical social welfare function to both regions, it can thus be argued that gains from trade for both regions over the entire model horizon sum up to zero. This implies, however, that during the transition either of the regions temporarily benefits from trade at the expense of the other. Thus, we have:

Result 6: *In the presence of globally unsynchronized aging patterns, international trade provides a mechanism which alleviates in the short-run the adverse economic effects of aging for either region at the expense of the other.*

6 Sensitivity analysis

This section explores the robustness of our results with respect to key parameters of the model. Besides a piecemeal analysis, it also checks for the sensitivity with respect to various parameter configurations. We confine the analysis to empirically plausible parameter values and take standard values from the literature as in, e.g., Backus, Kehoe, and Kydland [1992] and Ambler, Cardia, and Zimmermann [2002]. In order to account for the uncertainty of parameter estimates we consider a range of values for each of the following six key parameters: intertemporal elasticity of substitution parameter $\theta = [0.5; 4]$, intratemporal elasticity of substitution parameter $\sigma = [0.5; 2]$, consumption share $\alpha = [0.3; 1]$, elasticity of substitution between capital and labor $\epsilon_i = [0.5; 4]$, elasticity of substitution for composite good $\epsilon_A = [0.5; 4]$, capital depreciation rate $\delta = [0; 1]$.

Closed economy: household and production side parameters

Table 2 shows the sensitivity of the steady state per capita capital stock for the closed economy case (fast aging region) for different configurations of θ, ϵ_i, ϵ_A, and α. The corresponding

Table 2: Sensitivity of steady state per capita capital stock (closed economy)

endogenous labor supply ($\alpha^* = 0.4 < 1$)

	ϵ_i, eq.(3)				ϵ_A, eq.(4)			
θ	0.5	1	2^*	4	0.5	1	2^*	4
0.5	0.50 (-6.03)	1.57 (-5.03)	2.92 (-3.77)	4.27 (-2.50)	2.61 (-4.06)	2.72 (-3.96)	2.92 (-3.77)	3.27 (-3.44)
1	3.08 (-3.63)	4.14 (-2.63)	5.08 (-1.75)	5.80 (-1.08)	4.88 (-1.93)	4.95 (-1.87)	5.08 (-1.75)	5.28 (-1.56)
2^*	6.25 (-0.66)	6.66 (-0.28)	6.95 (–)	6.98 (0.03)	6.89 (-0.05)	6.92 (-0.03)	6.95 (–)	7.01 (0.05)
3	8.68 (1.61)	8.20 (1.17)	7.72 (0.72)	7.36 (0.38)	7.82 (0.814)	7.79 (0.78)	7.72 (0.72)	7.62 (0.62)
4	10.71 (3.51)	9.41 (2.30)	8.36 (1.31)	7.66 (0.66)	8.56 (1.50)	8.49 (1.44)	8.36 (1.31)	8.15 (1.12)

exogenous labor supply ($\alpha = 1$)

	ϵ_i, eq.(3)				ϵ_A, eq.(4)			
θ	0.5	1	2^*	4	0.5	1	2^*	4
0.5	4.49 (3.97)	5.26 (3.64)	6.11 (3.11)	6.85 (2.48)	5.92 (3.24)	6.00 (3.20)	6.11 (3.11)	6.31 (2.95)
1	6.53 (3.36)	7.02 (2.78)	7.43 (2.24)	7.69 (1.80)	7.35 (2.36)	7.38 (2.32)	7.43 (2.24)	7.51 (2.12)
2^*	8.94 (2.53)	8.63 (1.85)	8.39 (1.34)	8.23 (1.16)	8.43 (1.44)	8.42 (1.40)	8.39 (1.34)	8.34 (1.25)

Note: Sensitivity of steady state per capita capital stock for the closed economy case (fast aging region) for different configurations of θ, ϵ_i, ϵ_A, and α. Figures denote percentage changes from initial steady state. In the top panel ($\alpha = 0.4$), figures in parentheses denote the percentage deviation from the base case level. In the bottom panel ($\alpha = 1$), figures in parentheses denote the percentage deviation from the respective value in the upper panel. An asterisk denotes base case parameter values.

transition paths are shown in Appendix A, Fig. A–1 and Fig. A–2.

For all parameter configurations the decline in fertility rates leads to an increase in the long-run per capita capital stock. In nearly all cases the transition is monotonic. Magnitudes of change, however, differ substantially ranging from 0.5% to around 10%. Ceteris paribus, a higher intertemporal elasticity of consumption (moving up a column in Table 2) leads to a smaller increase in the per capita capital stock. A higher value of θ means that households are more responsive to price changes and tolerate larger fluctuations in periodic consumption. Following the drop in interest rates, the reduction in households' savings is hence the stronger, the higher is $1/\theta$. Therefore, the fall in investment per capita is increasing in $1/\theta$ and the increase in the short-run as well as steady state per capita capital stock is the smaller, the higher the intertemporal elasticity of substitution.

This also holds if labor is supplied inelastically ($\alpha = 1$). Furthermore, note that for all parameter configurations the increase in the steady state per capita capital stock is larger in the case of exogenous labor supply as compared to $\alpha < 1$. Figures in parentheses in the lower panel refer to percentage changes from the respective value in the upper panel. If $\alpha = 1$, households lack the opportunity to react to factor price changes via adjustments in hours worked. Consumption smoothing therefore has to be accomplished by transferring a larger fraction of wealth to the future. Facing lower future interest rates, this implies that the reduction in savings and in per capita investment under $\alpha = 1$ is smaller than under $\alpha < 1$. This means that more capital is accumulated and thus the steady state per capita capital stock is higher. In fact, the increase in the per capita capital stock is the higher, the closer α is to unity (not shown).

Varying the intertemporal elasticity of substitution between consumption and leisure has very minor effects on time pathes of key variables (not shown). Not surprisingly, there is a strong sensitivity of the steady state per capita capital stock with respect to the capital depreciation rate δ. The larger δ, the smaller is the increase in the capital stock (Fig. A–1).

Sensitivity of gains from trade

Since for all parameter configurations the transitional as well as steady state per capita capital stock under autarky increases, factor prices also move in the same directions under all scenarios. The qualitative welfare implications of unsynchronized aging patterns in the presence of integrated goods markets are therefore unchanged. Table 3 shows the sensitivity of gains from trade for the biggest winner and loser generations for different configurations of θ, ϵ_i, ϵ_A, and α. As an example, Fig. A–3 in the Appendix shows the sensitivity of welfare changes with respect to θ for *all* generations given that either $\{\epsilon_i = 2, \alpha = 0.4\}$ or $\{\epsilon_i = 2, \alpha = 1\}$. It can be seen that the qualitative results are preserved: gains from trade are not universal, i.e., there are winners and losers in both countries. Also note that for all parameter configurations, generations 10 and 40 stay the biggest winner and loser generation, respectively. Table 3 therefore focusses on these generations. Under autarky, high values for the intertemporal elasticity of consumption mean that the different extent of aging across regions creates relatively large differences with respect to the per capita capital stock and factor prices. Trade leads to factor price equalization and therefore differentials

Table 3: Sensitivity of gains from trade (selected generations)

endogenous labor supply ($\alpha^* = 0.4 < 1$)

		ϵ_i, eq.(3)				ϵ_A, eq.(4)			
Gen.	θ	0.5	1	2*	4	0.5	1	2*	4
$g = 10$	0.5	0.76	0.72	0.61	0.45	0.64	0.63	0.61	0.57
$g = 40$		-2.03	-1.78	-1.43	-1.02	-1.51	-1.48	-1.43	-1.33
$g = 10$	1	1.37	1.15	0.84	0.54	0.91	0.89	0.84	0.76
$g = 40$		-2.98	-2.49	-1.83	-1.19	-1.98	-1.93	-1.83	-1.67
$g = 10$	2*	1.50	1.34	0.99	0.61	1.07	1.04	0.99	0.89
$g = 40$		-3.45	-2.72	-1.98	-1.32	-2.13	-2.07	-1.98	-1.83
$g = 10$	3	2.09	1.54	1.01	0.62	1.13	1.09	1.01	0.92
$g = 40$		-3.86	-3.04	-2.11	-1.30	-2.31	-2.24	-2.11	-1.89
$g = 10$	4	2.40	1.70	1.08	0.64	1.21	1.16	1.08	1.03
$g = 40$		-4.18	-3.24	-2.20	-1.33	-2.42	-2.34	-2.20	-1.98

exogenous labor supply ($\alpha = 1$)

		ϵ_i, eq.(3)				ϵ_A, eq.(4)			
Gen.	θ	0.5	1	2*	4	0.5	1	2*	4
$g = 10$	0.5	1.51	1.18	0.82	0.51	0.89	0.87	0.82	0.73
$g = 40$		-2.91	-2.34	-1.67	-1.06	-1.82	-1.77	-1.67	-1.50
$g = 10$	1	2.15	1.54	0.96	0.62	1.09	1.05	0.96	0.86
$g = 40$		-4.08	-3.04	-1.98	-1.22	-2.21	-2.14	-1.98	-1.77
$g = 10$	2*	2.76	1.82	1.08	0.63	1.29	1.17	1.08	0.97
$g = 40$		-5.06	-3.56	-2.21	-1.26	-2.51	-2.38	-2.21	-1.05

Note: Sensitivity of gains from trade in % for biggest winner ($g = 10$) and loser ($g = 40$) generation for different configurations of θ, ϵ_i, ϵ_A, and α. An asterisk denotes base case parameter values.

between autarky and trade prices are the larger, the higher is $1/\theta$. Recalling the discussion from section 5.2, this in turn explains why generational welfare gains and losses increase with $1/\theta$. Similarly, higher values for production elasticities induce a smaller extent of factor price changes under autarky, thereby leading to smaller autarky-trade differentials. Welfare changes are thus the smaller, the higher is either ϵ_i or ϵ_A. As it has been argued above, inelastic labor supply leads to a higher accumulation of capital per capita vis-à-vis endogenous labor supply. Therefore, factor price differentials between trade and autarky are the larger, the closer α is to unity, and hence the larger are welfare gains and losses.

Multiple equilibria

A major concern of our sensitivity analysis was to check for the potential existence of multiple equilibria. Galor [1992] pointed out the possibility of global indeterminacy of the perfect-foresight equilibrium in the two-sector neoclassical OLG model. In principle, this contingency cannot be ruled out by specific structure and assumptions of our model. At first sight, the problem potentially exacerbates because we consider a high-dimensional version of Galor's model in which there are two countries and in which households live for more than two periods. The non-existence of a closed form solution makes it impossible to analytically characterize the dynamical system and establish conditions for uniqueness. A priori it seems reasonable to expect that in our framework indeterminacy will be an issue, too. Results from the previous sensitivity analysis, however, suggest that changes in key parameters of the model do not lead to qualitative reversals. Moreover, varying parameter values alters equilibrium outcomes of endogenous variables in a 'monotonic way'. We interpret these findings as negative evidence for the existence of multiple equilibria.

7 Conclusions

This paper investigated the economic implications of unsynchronized global demographic patterns in a world which is characterized by perfectly integrated goods markets. The ongoing discussion about the international dimension of population aging has largely concentrated on one-sector models thereby neglecting the role played by trade in goods with different factor intensities.

We aim to fill this gap by constructing a computable two region, two factor, two commodity Heckscher-Ohlin model with overlapping generations in the tradition of Auerbach and Kotlikoff [1987]. Our analysis assumes that regions are identical with respect to production technologies and household preferences. We introduce demographic shocks that differ in size and timing across regions thereby capturing in a stylized manner observed real-world population aging. Interregional differences in aging patterns affect the relative abundance of factors of production across regions and lead to differentials in commodity and factor prices under autarky. During the transition, the relatively fast aging region is characterized by a higher capital-labor ratio vis-à-vis the slow aging region. In a world where international trade is liberalized, Heckscher-Ohlin trade patterns emerge.

The main focus of this contribution is to analyze the intergenerational welfare effects of globally unsynchronized aging patterns in the presence of integrated goods markets. As yet, this aspect has not been addressed in the literature. Without recourse to social security, we find that trade liberalization in the light of unsynchronized aging patterns leads to strong distributional effects across generations and regions. Surprisingly, liberalizing trade in the presence of globally unsynchronized aging patterns does not unambiguously lead to welfare gains, i.e. openness may be immiserizing. Depending on the evolution of factor price differentials between trade and autarky over the lifetime of a given generation, openness of a country can either decrease or increase lifetime utility. This is in contrast to what can be found in the literature. Sayan [2005] finds —in a framework that is incapable of assessing *generational* welfare changes— that the fast aging region loses overall. We also find that during the economic transition phase there is always one region that benefits from trade at the expense of the other. In this sense, international trade can be viewed as a mechanism which potentially alleviates in the short-run the adverse economic effects that are associated with population aging. We also argue that aging is a slow moving economic process extending far beyond the time horizon of the demographic transition phase. It profoundly affects the well-being of future generations born *after* the demographic transition is completed.

The results obtained are subject to several caveats that may also serve as suggestions for future research. First, by focussing on the goods market channel as the only linkage between countries, the critical assumption is made that factors of production are internationally immobile. Integrating cross-border migration and international capital flows into the model would add an additional mechanism to arbitrage away regional differences in observed population dynamics. Other things being equal, this would most likely reduce the magnitudes of adjustment responses of economies including a lower level of interregional goods flows and less pronounced changes in households' welfare. Second, the model abstracts from any government activity and in particular does not consider a public sector pension system. As pointed out in section 1, recent studies emphasize the role of a public pension system in determining households' savings behavior. Our model takes an extreme standpoint on this issue by assuming the presence of a fully-funded pension system which implicitly operates through private life-cycle savings decisions of households. Third, in reality there are numerous obstacles which prevent international goods markets from being fully integrated. As long as barriers to trade exert a symmetric effect on both regions, the qualitative implications of our analysis should be preserved. However, cross-country differences in the active set of trade policy instruments are likely to result in an asymmetric distribution of gains and losses from trade. Fourth, a departure from the perfect neoclassical world is clearly needed to enhance the policy relevance of the simulation experiments. Whether the inclusion of those factors would materially alter our conclusions is a question for future research.

Appendix

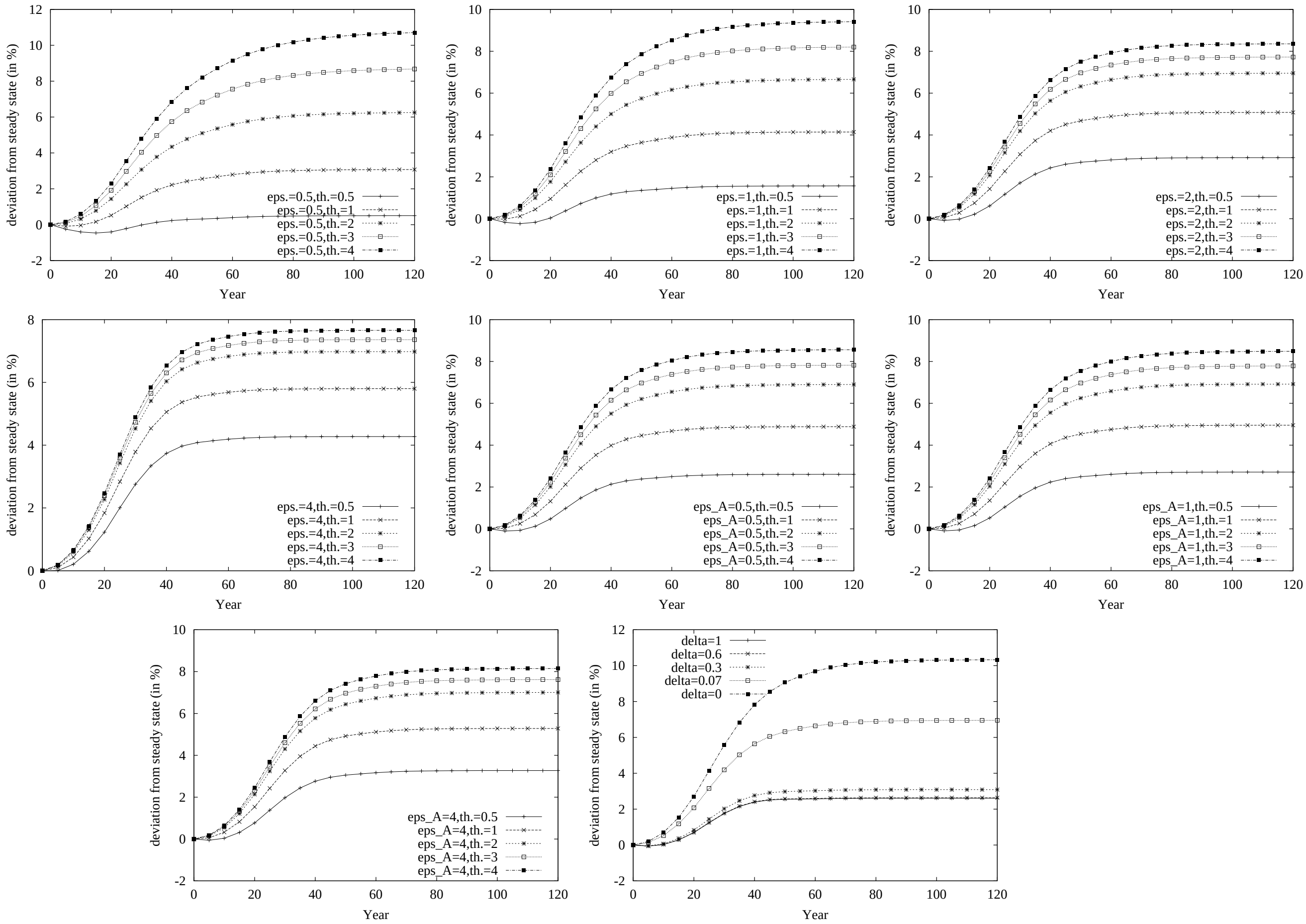


Fig. A–1: Sensitivity of per capita capital stock (closed economy)

Note: Sensitivity of per capita capital for different configurations of ϵ_i and θ for the closed economy case (fast aging region) assuming that households face a labor-leisure tradeoff ($\alpha = 0.4 < 1$). Graph in bottom right corner shows sensitivity with respect to δ.

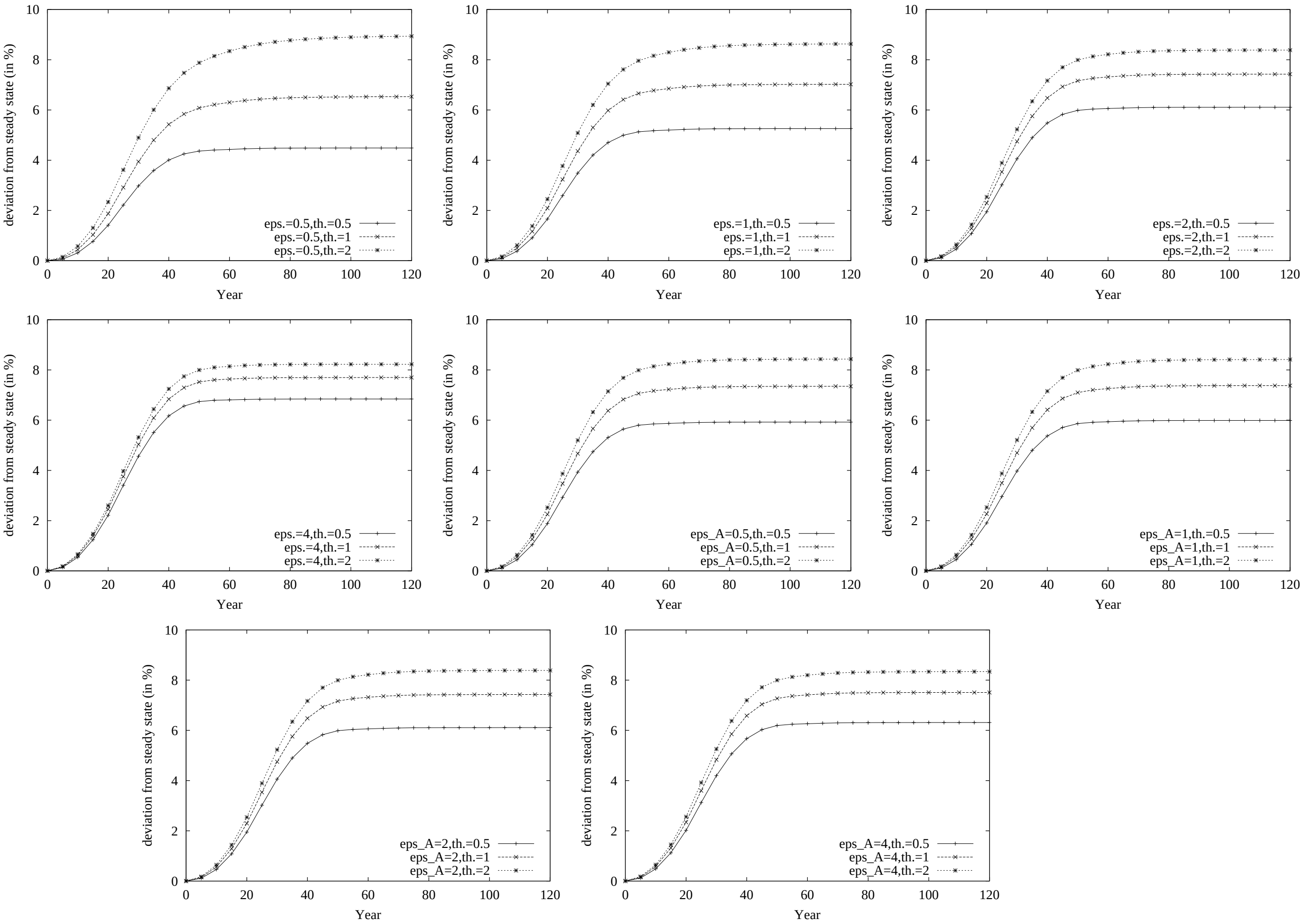
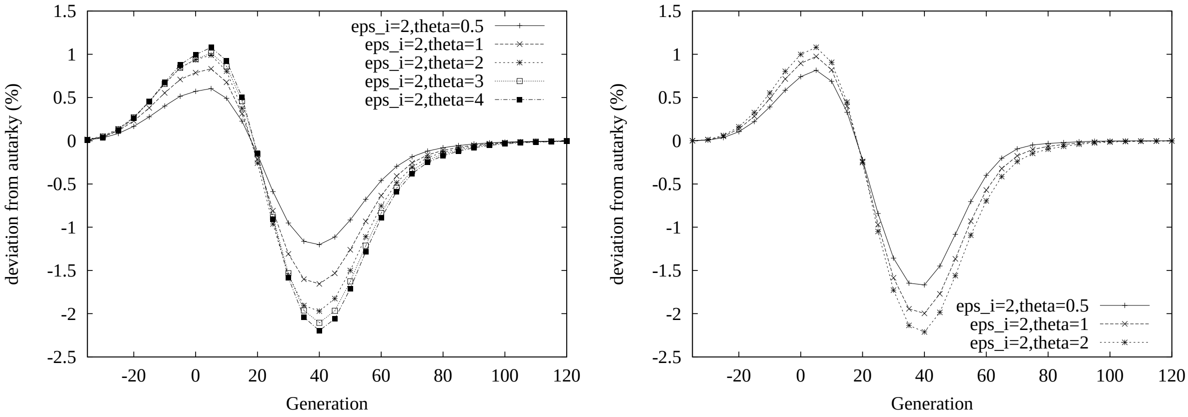


Fig. A–2: Sensitivity of per capita capital stock (closed economy)

Note: Sensitivity of per capita capital for different configurations of ϵ_i and θ the closed economy case (fast aging region) assuming that household supply labor inelastically ($\alpha = 1$).



(a) Endogenous labor supply

(b) Exogenous labor supply

FIG. A–3: SENSITIVITY OF GAINS FROM TRADE

Note: Sensitivity of gains from trade with respect to θ. Panel (a): $\alpha = 0.4$. Panel (b): $\alpha = 1$.

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