Environmental amenities and public open space in an efficient urban spatial structure

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Abstract. Environmental amenities from public open space (e.g., parks or open squares) constitute one of the most important components of urban quality of life. In this paper, we consider an urban economics model with environmental amenities stemming from public open space, which (i) needs space in rivalry to residential use and (ii) has positive spatial spillover-effects. For a large average spillover length, the efficient urban spatial structure is characterized by an inner area, completely used as residential space, and a greenbelt, which is completely used as public open space. Population density is maximal at an intermediate distance between the city center and the greenbelt.

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1 Introduction

Environmental amenities from public open space (e.g., parks or open squares) constitute one of the most important components of urban quality of life. Accordingly, they are of great concern for the urban public. For rapidly growing megacities in many developing countries as well as for advancing “urban sprawl” in developed countries, the provision of open space one of the most immediate environmental issues. In the USA, for example, hundreds of referenda in recent years (between 1998 and 2003) raised an amount of over 21 billion US $ for open space conservation in jurisdictions at the state, county, and local levels (Kotchen and Powers 2006:373-374). Figures like this raise the question how to provide urban public spaces in an efficient way.

This question has found increased interest in the recent literature of urban and environmental economics. Yet, a comprehensive answer has not been derived. From a theoretical perspective, public open space is one part of an overall efficient urban spatial structure. Two issues make it difficult to determine such an allocation: (i) the provision of public open space is in rivalry to other uses of urban space. (ii) Environmental amenities from public open space benefit not only the inhabitants of the respective neighborhood itself, but also the inhabitants of adjacent neighborhoods. In other words, there are positive externalities (‘spillovers’) of public space provision in one neighborhood to other neighborhoods.

As environmental amenities are public goods, their spatial allocation affects the spatial allocation of households in a residential equilibrium. Wu and Plantinga 2003 and Wu 2006 study this question in a modeling analysis. They analyze, how different open spaces affect the spatial distribution of households, when there are positive spillovers of environmental amenities. Among others, they consider parks, greenbelts, or corridors of different size and at different locations and show that open space can change the urban landscape considerably. Open space provided far from the urban center can even be so attractive to residents that a sub-center forms close to the open space.

How public space is allocated, has an important impact on urban land rents: they will rise in the areas which benefit from environmental amenities Irwin (2002), Anderson and West (2006). This reflects the urban inhabitants' preferences for public open space. When the demand for environmental amenities is different for population groups with different income, the allocation of these amenities can completely re-shape the spatial pattern of where in a city different income groups choose to live (Brueckner et al. 1999).

As argued above, the provision of public space is in rivalry to other uses of urban space, simply because it consumes space. Thus, the scarcity of space, reflected by the land-rent, has to be taken into account when an optimal allocation is to be determined. There are only few models that take these interrelations into account. Yang and Fujita (1983) an urban economics model with public open space. They derive propositions about the optimal allocation of public open space when there are no spillovers, i.e., when environmental amenities from public open space accrue only in the own neighborhood. When urban residents have identical Cobb-Douglas-type preferences, it is optimal to distribute open space uniformly: all neighborhoods should have the same amount of open space. In a numerical example of a linear city with five discrete neighborhoods, Yang and Fujita (1983) show that this result does not longer hold, when there are positive spillover effects. Rather, they conclude that relatively more open space should be supplied in the city center. Recently, Tajibaeva et al. 2006 analyzed a similar model in a two-dimensional model of a city with discrete neighborhoods. In numerical simulations, they show how different allocations of population and open space arise under different assumptions on commuting costs and spillover lengths. However, also their analysis is restricted to examples with very few (5×5) such neighborhoods.

Following a similar modeling approach, we determine the efficient urban spatial structure in an urban economics model with environmental amenities from public open space. The model takes into account that public open space precludes the use of that space for residential purposes and that there are positive spillovers of public open space. We show that the optimal density of public open space increases from the city center towards the periphery (in contrast to the finding of Yang and Fujita 1983).[1] As environmental amenities attract residents, the maximum population

[1] In the special case without spillover effects, we reproduce their result that the optimal density of public space is constant across the city.

density is not in the city center (as in models without environmental amenities), but further outwards. When the spillover-length is high, the outer areas of the city are completely used as open space, while in the center, it is optimal to provide no open space at all. That is, having a greenbelt is the efficient urban spatial structure when the spillover length is large. When, on the other hand, the spillover length is small, a homogenous distribution of public spaces is optimal.

The paper is structured as follows: the model is laid out in Section 2. In Section 3, we derive analytical results on the efficient urban spatial structure and illustrate these results with numerical examples. The final section, as always, concludes.

2 The model

The analysis is based on a general spatial equilibrium model of a monocentric city. The model comprises three goods (an aggregate consumption commodity, lot size, and environmental amenities, which are a public good) and a continuum of identical households. The city is closed, i.e., there is no migration over the borders of the city, and features a central business district (CBD) at the origin. Production activities do not need space and are concentrated at the CBD.

2.1 Households

There is a continuum of $\hat{N} > 0$ identical households living in the city, who have identical preferences on private consumption of goods (amount c), lot size s, and environmental amenities Q. Each household chooses c, s and the residence location (z, ϕ) such as to maximize utility $u(c, s, Q)$. In the numerical calculations, we will use the specification (similar to Wu and Plantinga 2003)

$$u(c, s, Q) = c^{\alpha} \, s^{1-\alpha} \, Q^{\gamma}, \tag{1}$$

where $\alpha, \gamma \in (0, 1)$, i.e. $u(c, s, Q)$ is increasing and concave in the consumption of goods c, lot size s, and environmental amenities Q.

2.2 Environmental amenities

Public open space contributes to environmental amenities. The highest contribution stems from the open space at the household's place of residence, but there are also positive spillovers from open space at other locations. The closer the the open space is to the household's address, the higher is its contribution to environmental amenities. Specifically, given the density $q(z)$ of public space, environmental amenities $Q(z)$ are given by[2]

$$Q(z) = \int_0^Z q(z')\,\Phi(z,z')\,z'\,dz', \tag{2}$$

where

$$\Phi(z,z') = \int_0^{2\pi} \left[\max\left\{0, 1 - \delta\,\epsilon\,\sqrt{z^2 + z'^2 - 2\,z\,z'\,\cos(\phi)}\right\}\right]^{\frac{1}{\epsilon}} d\phi, \tag{3}$$

with $\epsilon \leq 1$ and $\delta \geq 0$. $1/\delta$ can be interpreted as an average spillover-distance. For $\epsilon = 0$, $\Phi(z,z') = \int \exp\left[-\delta\,\sqrt{z^2 + z'^2 - 2\,z\,z'\,\cos(\phi)}\right]\,d\phi$.

The function $\Phi(z,z')$ determines the strength of the spillovers, i.e., the contribution of a unit of public open space at the distance z' from the CBD to environmental amenities at a distance z from the CBD.

2.3 Definition of optimal allocation

The optimal allocation is found by allocating land for residential use and open space and consumption of goods, and distributing households in the city such as to achieve a Pareto optimum.

The distribution of households is described by the function $N(A(z))$ which is the population living in the area $A(z) = \pi\,z^2$. Population density $n(z)$ at z is given by $n(z) = N'(A)$. Lot sizes $s(z)$, density $q(z)$ of open space and consumption of goods $c(z)$ are variables which can be chosen independently at each location $z \in [0, Z]$, and are assumed to be continuous in space. Thus, an allocation is a

[2] We model the spillovers of public open space as Yang and Fujita (1983, equation 23). Similar spatial spill-overs are also considered in Lucas (2001) as well as Lucas and Rossi-Hansberg (2002).

collection of functions (s, q, c) on $[0, Z]$ and N on the circular plane with radius Z that describe lot sizes, the density of open space, consumption of goods and population distribution. To be feasible, an allocation must satisfy the non-negativity constraints $s(z), q(z), c(z), N(\pi z^2) \geq 0$ for all z, and fulfill the constraints given in below.

The spatial structure of the city is shaped by commuting costs which increase with the distance between the household's place of residence and the CBD. Commuting every working day from the place of residence to the CBD takes $t_c(z)$ units of time per year, where $t_c(z)$ increases monotonically in the distance commuted, $t_c'(z) > 0$. A household living in the immediate neighborhood of the CBD has no commuting time, $t_c(0) = 0$. Apart from the opportunity costs of commuting which are due to foregone wage income, no further commuting costs are considered.

Both environmental amenities and commuting costs are determined by spatial distances. In the optimum, these distances are determined by the allocation of land for residential use and open space. Assuming that one unit of residential space requires one unit of land, land constraint requires

$$n(z)\, s(z) + q(z) = 1 \quad \text{for all} \quad z \in [0, Z]. \tag{4}$$

Each individual household is endowed with one unit of time, i.e. the gross time being available for working and commuting is $\hat{N}$. (Leisure is ignored for reasons of simplicity.) Formally,

$$L = \hat{N} - \int_0^Z n(z)\, t_c(z)\, z\, dz = \int_0^Z n(z)\, (1 - t_c(z))\, z\, dz. \tag{5}$$

Finally, we have the constraint that aggregate consumption must not exceed (in the optimum, equal), total production:

$$F(L) = \int_0^Z n(z)\, c(z)\, z\, dz. \tag{6}$$

3 Analysis and Results

The planner's problem is choose a feasible allocation $s(z), q(z), c(z), N(\pi z^2)$ in order to maximize the level $\bar{u}$ of utility given that all individuals enjoy this level of utility, i.e.,

$$u(c(z), s(z), Q(z)) = \bar{u} \quad \text{for all} \quad z \in [0, Z], \tag{7}$$

and given the constraints (4), (5) and (6). The Lagrangian for this problem is

$$\begin{aligned}
\mathcal{L} = & \int_0^Z \lambda(z)\, n(z)\, \left[u(c(z), s(z), Q(z)) - \bar{u}\right] z\, dz \qquad (8) \\
& + \int_0^Z \psi(z) \left[-Q(z) + \int_0^Z q(z')\, \Phi(z, z')\, z'\, dz' \right] z\, dz \\
& + \int_0^Z \Big[\rho(z)\, \left[1 - n(z)\, s(z) - q(z)\right] + \nu(z)\, q(z) + \eta(z)\, \left[1 - q(z)\right] \Big] z\, dz \\
& + \omega \left[-L + \int_0^Z n(z)\, \left[1 - t_c(z)\right] z\, dz \right] \\
& + \mu \left[F(L) - \int_0^Z n(z)\, c(z)\, z\, dz \right]
\end{aligned}$$

The first order conditions with respect to L, $c(z)$, $s(z)$, $q(z)$, $Q(z)$ and $N(\pi\, z^2)$ are (Appendix **??**)

$$\frac{\partial \mathcal{L}}{\partial L} = 0 \qquad \mu\, F'(L) = \omega \tag{9}$$

$$\frac{\partial \mathcal{L}}{\partial c(z)} = 0 \qquad \lambda(z)\, u_c(z) = \mu \tag{10}$$

$$\frac{\partial \mathcal{L}}{\partial s(z)} = 0 \qquad \lambda(z)\, u_s(z) = \rho(z) \tag{11}$$

$$\frac{\partial \mathcal{L}}{\partial q(z)} = 0 \qquad \int_0^Z \psi(z')\, \Phi(z, z')\, z'\, dz' = \rho(z) + \nu(z) - \eta(z) \tag{12}$$

with $\nu(z) > 0$ for $q(z) = 0$, $\nu(z) = 0$ for $q > 0$, $\eta(z) > 0$ for $q(z) = 1$ and $\eta(z) = 0$ for $q(z) < 1$.

$$\frac{\partial \mathcal{L}}{\partial Q(z)} = 0 \qquad \lambda(z)\, n(z)\, u_Q(z) = \psi(z) \tag{13}$$

$$\frac{\partial \mathcal{L}}{\partial N(\pi\, z^2)} = 0 \qquad \lambda(z)\, u_Q(z)\, Q'(z) = \rho'(z)\, s(z) + \omega\, t_c'(z) \tag{14}$$

Rearranging these conditions, we obtain the following conditions for the efficient spatial structure of the city.

Proposition 1

The efficient urban spatial structure is determined by the conditions

$$\frac{u_s(z)}{u_c(z)} = \frac{\rho(z)}{\mu} \tag{15}$$

$$\int_0^Z n(z')\, \frac{u_Q(z')}{u_c(z')}\, \Phi(z,z')\, z'\, dz' = \frac{\rho(z)+\nu(z)-\eta(z)}{\mu} \tag{16}$$

with $\nu(z) > 0$ *for* $q(z) = 0$, $\nu(z) = 0$ *for* $q > 0$, $\eta(z) > 0$ *for* $q(z) = 1$ *and* $\eta(z) = 0$ *for* $q(z) < 1$.

$$-\frac{\rho'(z)}{\mu}\, s(z) + \frac{u_Q(z)}{u_c(z)}\, Q'(z) = F'(L)\, t_c'(z) \tag{17}$$

These conditions have straightforward economic interpretations. Condition (15) is the condition for the efficient allocation of private goods, that the marginal rate of substitution between the consumption of commodities and residential space equals the ratio of shadow prices. Condition (16) determines the efficient allocation of the environmental amenities, which are a public good. It is a modified version of the Lindahl-Samuelson condition, which states that for the efficient supply of pure public goods, the ratio of shadowprices equals the marginal rate of substitution between the public and the private good, aggregated over all users of the public good. The good under consideration here, environmental amenities, is not a pure public good, but obtains its character as a public good from the positive spillovers. Accordingly, the aggregation over the users of environmental amenities is weighted by the strength of the spillovers, given by the term $\Phi(z, z')$.

The shadowprice for residential space, $\rho(z)$, differs across locations in the city, because different places of residence differ with respect to the distance from the CBD and the endowment with public open space. Condition (17) determines, how exactly the shadowprice of residential space varies with the distance from the CBD. On the right hand side of this condition, we have the costs of a marginal relocation further outwards from the CBD: the time available for working decreases by an amount $t_c'(z)$, thus, marginal costs $F'(L)\, t_c'(z)$ in terms of foregone production output occur. On the right hand side, we have the net benefits of such a relocation. They comprise two parts: the fist term is a decrease (or, potentially, increase) in the shadowprice of residential space, the second term is the additional marginal benefit (or loss) from higher (or lower) environmental amenities further away from the CBD. The sign of both terms can only be determined by solving the system of equations for the whole spatial allocation.

The following lemma goes a step towards the solution of the model.

Lemma 1

Consumption of goods is given by

$$c(z) = C_1 - \alpha\, F'(L)\, t_c(z) \tag{18}$$

where C_1 is a constant of integration which is determined by the condition for the goods market equilibrium. The optimal spatial allocation of environmental amenities is given by

$$\int_0^Z \frac{1-q(z')}{Q(z')}\, c(z')^{\frac{1}{1-\alpha}}\, Q(z')^{\frac{\gamma}{1-\alpha}}\, \Phi(z,z')\, z'\, dz' = \frac{1-\alpha}{\gamma}\, c(z)^{\frac{1}{1-\alpha}}\, Q(z)^{\frac{\gamma}{1-\alpha}} \tag{19}$$

Proof: see Appendix **??**.

Unambiguously, however, the optimal density of public open space increases with the distance from the CBD.

Proposition 2

For finite δ, the optimal density of public open space increases with z, $q'(z) \geq 0$.

Proof: see Appendix **??**.

The result of Proposition 2 does not imply that environmental amenities also increase with the distance from the CBD, because, on average, the city center is particularly close to all other locations in the city. Residents near the city center thus have a particularly high benefit in terms of environmental amenities from the positive spillovers of public open space elsewhere in the city.

Proposition 2 holds for the case of finite average spillover length $1/\delta$. As an interesting benchmark case, we consider $\delta \to \infty$, i.e., that open space is a purely local good. In this case, $\Phi(z,z') = 1$ for $z = z'$ and $\Phi(z,z') = 0$ for $z \neq z'$. Environmental amenities thus $Q(z)$ at z are equal to open space at z, i.e., $Q(z) = q(z)\, z\, dz$. Under this condition, the optimal density of public open space is (cf. Condition (19) for $\delta \to \infty$)

$$q(z) = \frac{\gamma}{1-\alpha+\gamma}, \tag{20}$$

i.e. public open space is the same all across the city. (This reproduces a result from Yang and Fujita (1983), when there are no spillovers.)

In the other extreme case, $\delta \to 0$, open space is a public good, and $\Phi(z, z') = 1$ for all z, z'. Environmental amenities are the equal throughout the city, i.e., $Q(z) = Q$ for all $z \in [0, Z]$ with

$$Q = \int_0^Z q(z')\, z'\, dz'. \tag{21}$$

Under this condition, condition (19) is

$$\int_0^Z [1 - q(z')]\, c(z')^{\frac{1}{1-\alpha}}\, z'\, dz' = \frac{1-\alpha}{\gamma}\, c(z)^{\frac{1}{1-\alpha}} \int_0^Z q(z')\, z'\, dz' \tag{22}$$

The right hand side of this condition is monotone decreasing with z; the left hand side is independent of z. Hence, a $\hat{z}$ must exist, such that for all $z < \hat{z}$ the left hand side is smaller than the right hand side for all $z > \hat{z}$ the left hand side is larger than the right hand side. Thus, $\hat{z}$ is determined by

$$\int_0^{\hat{z}} c(z')^{\frac{1}{1-\alpha}}\, z'\, dz' = \frac{1-\alpha}{\gamma}\, c(\hat{z})^{\frac{1}{1-\alpha}}\, (Z - \hat{z}). \tag{23}$$

This proves the last part of the following proposition

Proposition 3

There exist a $\delta > 0$, $\delta_2 > \delta_1$ and $\delta_3 > \delta_2$, such that

1. *for all $\delta > \delta_1$, a $\overline{z} < Z$ exists, such that $q(z) = 1$ for all $z > \overline{z}$.*

2. *for all $\delta > \delta_2$, a $\underline{z} > 0$ exists, such that $q(z) = 0$ for all $z < \underline{z}$.*

3. *for all $\delta > \delta_3$, a $\hat{z} > 0$ exists, such that $q(z) = 0$ for all $z < \hat{z}$ and $q(z) = 1$ for all $z > \hat{z}$.*

Proof: see Appendix **??**.

For a sufficiently large average spillover length $1/\delta$, there will be a circular disc close to the CBD which is exclusively used for residential purposed. Residents in this area purely benefit from the positive spillovers of public open spaces in other areas of the city. In the outer areas of the city, there is a 'greenbelt', i.e., a belt of space which is completely used as public open space.

Figure 1 shows the density of open space, environmental amenities, and population density as a function of the distance z of location from the CBD. Three different values of average spillover length are assumed. The solid lines depict the case of a very large δ, i.e., a very short spillover length $1/\delta$, we are almost in the case without spillovers. The optimal density of public open space is constant and so are environmental amenities. Population density is at maximum near the CBD. For $\delta = 1.75$, depicted with broken lines, we have a purely residential area in the city center and a green belt further outwards. Environmental amenities reach a maximum close to the border of the green belt. As the environmental from the green belt provide an incentive to move away from the CBD, the maximum population density is at an intermediate distance from the CBD. In the case of a very large spillover length, $\delta = 0.75$, depicted with dotted lines, there is almost no area in the city with a mixed land use of both residential areas and public open space. Rather, there is a clear border between an inner area purely used for residential purposes and an outer area which completely is used as public open space. The population maximum is close to this border right besides the green belt.

4 Conclusions

In this paper, we have considered an urban economics model with environmental amenities stemming from public open space. Open space has two characteristics: (i) it needs space and (ii) it has positive spillover-effects to other locations of the city.

In the presence of such spillovers, the efficient urban spatial structure is characterized by a density of public open space that increases from the CBD to the periphery. For a low average spillover length, there is a mixed use of the major part

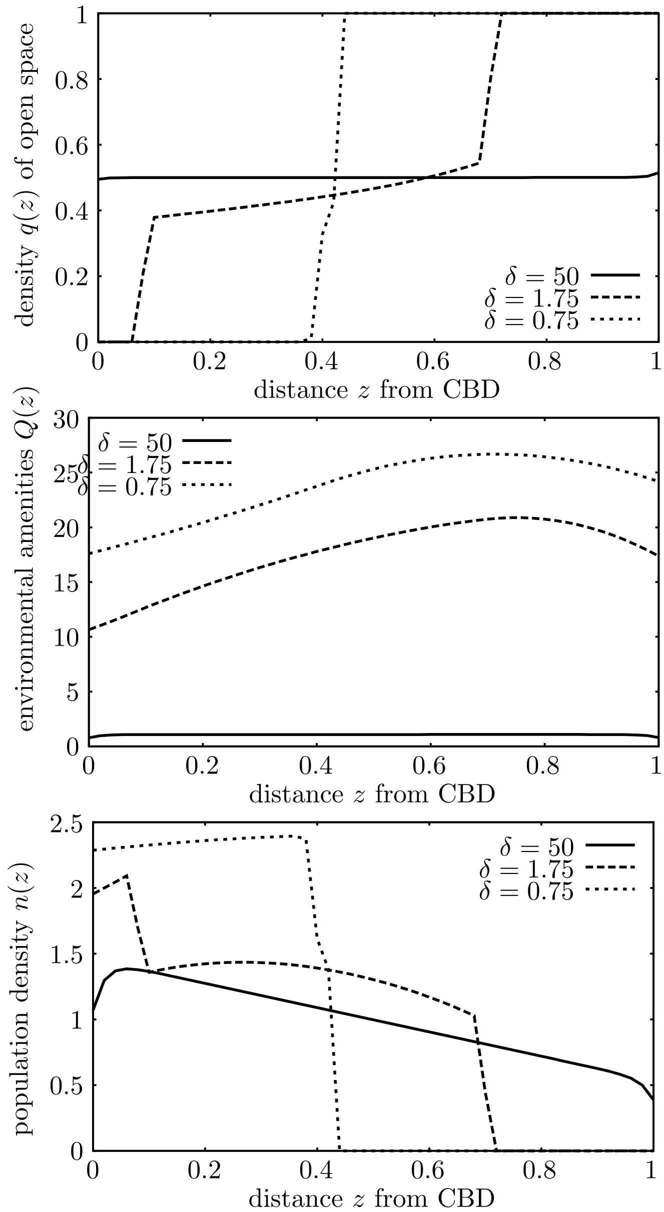

Figure 1: The density of open space, environmental amenities, and population density as a function of the distance z of location from the CBD for different values of δ, and $\tau = 1$, $\alpha = 0.5$, $\gamma = 0.5$; space is divided into 50 units.

of the city, such that most neighborhoods have both, residential areas and public open space. When the average spillover length is large enough, the efficient urban spatial structure is characterized by an inner area, completely used as residential space, and a greenbelt, an outer area, which is completely used as public open space. For a very large spillover length, there is a sharp border between the purely residential area and the greenbelt. Population density is maximal not in the city center (as predicted by urban economic models without environmental amenities), but at an intermediate distance between the city center and the greenbelt.

References

Anderson, S. T. and S. E. West (2006), 'Open space, residential property values, and spatial context', Regional Science and Urban Economics 773–789.

Brueckner, J. K., J.-F. Thisse and Y. Zenou (1999), 'Why is central Paris rich and downtown Detroit poor? An amenity-based theory', European Economic Review 43: 91–107.

Irwin, E. G. (2002), 'The effects of open space on residential property values', Land Economics 78(4): 465–480.

Kotchen, M. J. and S. M. Powers (2006), 'Explaining the appearance and success of voter referenda for open-space conservation', Journal of Environmental Economics and Management 52: 373–390.

Lucas, R. E. (2001), 'Externalities and cities', Review of Economic Dynamics 4: 245–274.

Lucas, R. E. and E. Rossi-Hansberg (2002), 'On the internal structure of cities', Econometrica 70(4): 1445–1476.

Marshall, E. (2004), 'Open-space amenities, interacting agents, and equilibrium landscape structure', Land Economics 80(2): 272–293.

Quaas, M. F. (forthcoming), 'Pollution-reducing Infrastructure and Urban Environmental Policy', *Environment and Development Economics* http://www.ufz.de/data/Quaas_EDE3958.pdf.

Tajibaeva, L., R. Haight and S. Polasky (2006), 'A discrete space urban model with environmental amenities', Working Paper, Department of Applied Economics, University of Minnesota, USA.

Wu, J. (2006), 'Environmental amenities, urban sprawl, and community characteristics', Journal of Environmental Economics and Management 52: 527–547.

Wu, J. and A. J. Plantinga (2003), 'The influence of public open space on urban spatial structure', Journal of Environmental Economics and Management 46: 288–309.

Yang, C. and M. Fujita (1983), 'Urban spatial structure with open space', Environment and Planning A 15: 67–84.