Sectoral Heterogeneity, Resource Depletion, and Directed Technical Change: Theory and Policy

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January 2007

Abstract

We analyze an economy in which the sectors are heterogenous with respect to the intensity of resource use, the productivity of R&D, and specialization gains. The long-term dynamics of the economy are characterized by the essential use of a non-renewable natural resource and two types of research allowing for directed technical change. We first study the balanced growth path and determine the impact of heterogeneity on the stability conditions. Then we focus on two different types of policies that aim at abetting sustainable development. According to the nature of the problem, we look at the impact of actors which are especially interested in the long run: pension funds. We show that a disproportionate investment in stocks of specific firms has no impact on economic growth, whereas development is influenced when supporting sector-specific knowledge creation.

Keywords: sustainable development, directed technical change, overlapping generations, heterogeneity, pension funds

JEL Classification: O4 (economic growth), Q01 (sustainable development), Q3 (non-renewable resources), G23 (pension funds)

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1 Introduction

In the course of the last decades, considerable attention has been devoted to the analysis of long-run implications of resource scarcity. It has been shown in the framework of various endogenous growth analyses[1] that growth might be compatible with the essential use of non-renewable resources. Yet these studies focus on economies in which the sectoral heterogeneity is not considered. Scholz/Ziemes (1999) e.g. employ a Romer (1990) framework with symmetric product differentiation. As final goods production is modeled assuming a unitary elasticity of substitution among inputs, the production shares of the individual varieties are given.

In this paper we model an economy that comprises two final goods sectors – a *modern* and a *traditional* sector. The two sectors differ according to the intensity of natural resource use and the productivity gains which arise from diversification. It is furthermore assumed that modern and traditional research differ in the effort it takes to conduct research. Which sector is more or less productive in research and in which higher gains of specialization arise is not necessarily undisputed and can be subject to discussion, yet the following assumptions seem to be intuitively appealing: Production in the modern sector is assumed to be less resource intensive and enjoys higher gains of specialization. On the other hand, research activities take more effort in the modern sector. The intention of this paper is not to justify these specific assumptions, but rather to show the implications that sectoral heterogeneity in general exerts on short- and long-run growth and the direction of development.

In our model, the production shares of modern and traditional goods are determined endogenously as we assume a CES-type production technology in the final goods sector. Economic policy might therefore not only affect aggregate economic growth and the rate of resource depletion in general, but might also influence the sector shares of modern and traditional goods production as well as the direction of technical change. The paper adds to the literature in two main respects. First, we show the existence of a balanced growth path and the stability conditions of the system near the long-term path. The stability analysis turns out to be substantially different from the case with homogenous sectors used in recent literature. Second, we study implications of two types of policies that are carried out by a pension fund. The pension fund's aim to assure old-age consumption is close to support sustainability, meaning that later generations enjoy a level of welfare which equals or exceeds the welfare of the current generation. In our model, abetting sustainable development becomes manifest in two investment rules for pension funds. On the one hand, pension funds invest relatively more in shares of the modern sector

[1]See e.g. Schou/Ziemes (1999), Groth/Schou (2001), Schou (2001, 2002) as well as Grimaud/Rougé (2003, 2005)

compared to private investors. On the other hand, pension funds promote the modern sector by providing basic knowledge to modern firms which decreases costs of modern goods. This might take the form of subsidizing basic research or giving (cheap) credit to the modern sector. An example for this kind of activity is found in Switzerland, where pension fund money can be used for real estate investment and low energy technologies, that is more advanced technologies, get better credit conditions than normal housing.

There are several reasons why pension funds are increasingly driven to include these types of rules in their business and investment strategies. First, the large pooling of savings for long-term investments suggests to take basic economic, environmental and social problems into account because these affect long-term capital return. Second, the growing size and market shares enables pension funds to exert a noticeable impact on firms' activities, the more so as financial intermediators have to take part in the monitoring and control of corporations. Third, as pension are of high political interest, a large variety of stakeholders is affected by their activities. Fourth, corporate responsibility may in certain cases appear as an appropriate response to governmental regulation or a good anticipation of future regulation by the government.

We adopt an OLG framework in which the young generation saves for the retirement age. Savings are in the form of bonds, two types of innovations and resource stock. Pensions guarantee a statutory minimum consumption of the old generation in terms of their previous consumption. The quantity and the direction of long-term investments decide on changes in natural resource abundance and increase of knowledge stocks which are crucial for welfare growth. In both sectors, positive externalities emerge from research raising the public stock of knowledge.

As a basic result it will turn out from the model that the two policy types have very different impacts on the overall economy. Whereas the investment rule for the stock market has no impact on economic dynamics, the provision of additional knowledge to the modern sector is efficient. The economic intuition behind this result is that the pension funds' endeavors on stock markets are countered by neutral investors while there is no comparable counter-effect in the case of credit markets.

The paper is related to various strands in literature. Regarding the dynamic behavior of the modeled economy, intergenerational transfers and long-run investment within a dynamic OLG framework were already studied in Hammond (1975) and Kotlikoff et al. (1988). Contributions on the impact of natural resource use on development in continuous time approaches without intergenerational aspects are Bovenberg and Smulders (1995) and Stokey (1998). Papers that deal with environmental and resource aspects in a discrete time framework include Howarth and Norgaard (1992), John and Pecchenino (1994) and Marini and Scaramozzino (1995). The relationship between social security and long run investments, e.g. in the environmental or in the education sector, has been studied by Rangel (2003) who finds that social security plays a crucial role in sustaining investments favouring future generations. Attanasio and Rohwedder (2003) find mixed effects of pension funds savings on overall savings for the case of the UK.

The modelling of the OLG setting and the inclusion of non-renewable resources in our approach draws on the contributions of Quang and Vousden (2002) and Agnani, Gutierrez, and Iza (2003), respectively. Technology assumptions are based on Romer (1990); directed technical change is related to Acemoglu (2002) and Smulders and de Nooij (2003). The impact of natural resource use in this kind of framework is treated in Bretschger (2003). Pittel (2002) provides a broad survey on the impact of the natural environment on economic growth and Bretschger and Pittel (2005) derive first results on the long-term impact of pension funds in a model without directed technical change and with a smaller set of pension funds activities.

Finally, with respect to the theory on corporate governance and the stakeholder approach, Tirole (2001) enumerates the various control and management problems in the principal agent context. La Porta et al. (2000) emphasize that corporate governance is also a set of mechanisms through which outside investors protect themselves against expropriation by managers and controlling shareholders. Several sources and contributions present interesting empirical evidence. That the share of total savings managed by pension funds has reached respectable dimensions becomes evident in Social Investment Forum (2003), Eurosif (2003) and Swiss Federal Statistical Office (2004). La Porta et al. (2000) show empirical evidence for the impact of legal corporate governance arrangements on financing structures of firms. In a broad study Smith (1996) finds that a majority of firms targeted by the large and well-known Californian pension fund CalPERS adopt proposed changes. Faccio and Lasfer (2000) conclude that pension funds have large incentives to monitor companies in which they hold large stakes but are, according to empirical results for the UK, not very effective monitors. Del Guercio and Hawkins (1999) study pension funds behavior and find a large variety of activism objectives, tactics, and the impact on target firms; the major motive is found to be value maximization. Prevost and Rao (2000) derive that firms receiving proposals of pension funds the first time experience a temporary decrease in shareholder wealth while firms targeted repeatedly are faced with longer-lasting negative effects.

The remainder of the paper is organized as follows. Section 2 describes the model in detail. The dynamics of the model are then analyzed in section 3 and 4 with section 3 focusing on characteristics of the balanced growth path and section 4 deriving the short-run dynamics of the model for the pure market economy. Section 5 deals with the effects of economic policies striving at a higher share of modern goods production. Finally, section 6 concludes.

2 The Model

The general model structure is as follows (see Figure 1): Horizontally differentiated products are produced in the modern as well as in the traditional sector. Blueprints for new products are developed by two distinct sectoral research activities and then sold to monopolistic producers in each sector. In each sector, the produced goods are combined to a homogeneous final good. Besides natural resources, labor constitutes the second primary input which is employed in the research sectors as well as in sectoral production.

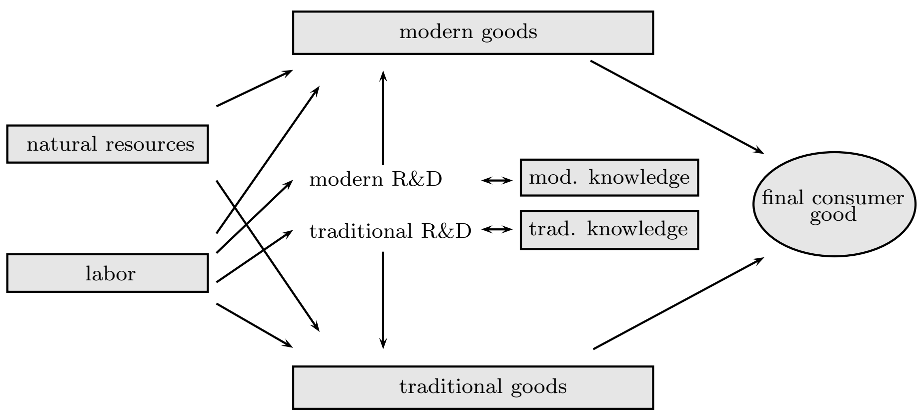

Figure 1: Production sectors

With respect to consumers we consider an overlapping generations economy with individuals maximizing utility over their two-period life. Each generation consists of a continuum of consumers who in the first period work, consume and save. In the second period they retire and live of their savings. In the this and the following section, savings are conducted voluntarily by consumers while in section 4 we additionally introduce forced savings via contributions to pension funds. Savings are either in the form of investment in natural resources or in R&D. In the second period of life, consumers receive the returns from investment in R&D and from the sale of their natural resources to either firms or the next generation.

2.1 Production

At every point in time t, a homogeneous final good C is assembled from aggregates of modern and traditional goods, $\tilde{X}$ and $\tilde{Z}$, according to the following CES-production function

$$C_t = \left(\tilde{X}_t^{\frac{\nu-1}{\nu}} + \tilde{Z}_t^{\frac{\nu-1}{\nu}} \right)^{\frac{\nu}{\nu-1}}, \qquad \nu > 0,\, \nu \neq 1 \tag{1}$$

where ν denotes the elasticity of substitution between $\tilde{X}$ and $\tilde{Z}$. The market for the final good is competitive and in every period, C_t is consumed by the two currently living generations, i.e. $C_t = C_{1t} + C_{2t}$ with C_1 being consumption when young and C_2 consumption when old.

The two aggregates $\tilde{X}$ and $\tilde{Y}$ each consist of a continuum of horizontally differentiated goods, x_{i_t}, $i \in [l_{t-1}, l_t]$, and z_{j_t}, $j \in [m_{t-1}, m_t]$, where l and m denote the number of varieties in the respective sectors at time t and $t-1$. Gains from specialization arise, i.e. the larger the variety of goods, the more productive the aggregate:[2]

$$\tilde{X}_t = \left(\int_{l_{t-1}}^{l_t} x_{i_t}^{\beta} \, di \right)^{1/\beta} \qquad \text{and} \qquad \tilde{Z}_t = \left(\int_{m_{t-1}}^{m_t} z_{j_t}^{\gamma} \, dj \right)^{1/\gamma} . \tag{2}$$

As already indicated, it assumed that modern goods give rise to higher gains from specialisation than traditional goods ($\beta < \gamma$). Competition in modern and traditional production is assumed to be monopolistic. Each type of good is produced by only one firm that had to acquire the according patent or blueprint for the design first. For simplicity we assume that blueprints are used for one period only and then become outdated. This assumption does not alter the qualitative results.

Modern as well as traditional goods are produced from labor L and non-renewable resources R by the following Cobb-Douglas production technologies:

$$x_{i_t} = (L_{x_i t})^{\alpha} (R_{x_i t})^{1-\alpha} \qquad \text{and} \qquad z_{j_t} = \left(L_{z_j t}\right)^{\delta} \left(R_{z_j t}\right)^{1-\delta} \tag{3}$$

where L_k and R_k, $k = x_i, z_j$, denote the input of labor and resources in the production of x_i and z_j. The production technologies in the modern and traditional sector are assumed to differ with respect to their resource intensity with the modern sector being able to employ natural resources more effectively ($\alpha > \delta$).

Blueprints for new types of goods are generated in two separate modern and traditional R&D sectors. The only rival input to research is labor, yet production also profits from cumulated past research activities which gives rise to positive sector specific spill-overs. Production is linear in labor and accumulated research experience which is equal to the number of blueprints generated in the past, l_t and m_t:

$$l_{t+1} - l_t = \frac{L_{lt}}{a_l} l_t \qquad \text{and} \qquad m_{t+1} - m_t = \frac{L_{m_t}}{a_m} m_t \tag{4}$$

with L_l and L_m being the inputs of labor in the two sectors. a_l and a_m are productivity parameters for the respective sector where it is assumed that research in the modern sector requires relatively more labor input ($a_l > a_m$).

[2] In contrast to the productivity adjusted aggregates, $\tilde{X}$ and $\tilde{Z}$, we denote aggregate physical amounts of x_i and z_i by $X = \int_{l_{t-1}}^{l_t} x_{i_t} di$ and $Z = \int_{m_{t-1}}^{m_t} z_{j_t} dj$. The prices for $\tilde{X}$ and $\tilde{Z}$ are $p_{\tilde{X}}$ and $p_{\tilde{Z}}$. p_{x_t} and p_{z_t} on the other hand denote the sector specific prices for individual goods in the symmetric equilibrium.

Research activities are financed by consumers' investments, such that

$$p_{C_t} S^c_{l_t} = w_t L_{l_t} \qquad \text{and} \qquad p_{C_t} S^c_{m_t} = w_t L_{l_t} \tag{5}$$

where S^c_n with $n = l, m$ denotes consumers' investment in the respective sectors and p_C is the price index of final goods C.

Natural resources are assumed to be exhaustible. The complete resource stock H is owned by consumers who can sell the right to extract part of the stock R_t to x- and z-producers. Resources are for simplicity assumed to be extracted at no cost. Equilibrium on the resource markets is given for $R_t = R_{X_t} + R_{Z_t}$ with R_{k_t}, $k = X, Z$ being the sectoral employment of resources, ($R_{X_t} = \int_{l_{t-1}}^{l_t} R_{x_i} di$) and $R_{Z_t} = \int_{m_{t-1}}^{m_t} R_{z_j} dj$. At any point in time we have:

$$H_t = H_{t+1} + R_{t+1}. \tag{6}$$

The relation between the resource stock at time t and the amount of extracted resources is determined what we label the extraction rate $\tau_t = \frac{R_t}{H_t}$.

We normalize the population size to unity, such that the labor market equilibrium requires

$$L_{X_t} + L_{Z_t} + L_{l_t} + L_{m_t} = 1 \tag{7}$$

where L_{k_t}, $k = X, Z$ are the sectoral employments of labor in goods production, $L_{X_t} = \int_{l_{t-1}}^{l_t} L_{x_i} di$ and $L_{Z_t} = \int_{m_{t-1}}^{m_t} L_{z_j} dj$.

2.2 Consumers

The representative consumer works in the first period of his life, receives wage income and allocates this income between consumption and savings. In the second period, the consumer is retired and lives on the returns to his first-period savings.

Three types of savings are considered: First, consumers can invest in modern or traditional R&D. As consumers are indifferent between investment in the modern and traditional sector, S^c_l and S^c_m, and capital markets are assumed to be perfect, the returns to investment in both sectors are equalized in equilibrium. In their second period of life consumers are paid back their original investment plus the interest. Secondly, consumers can buy natural resource stocks from the preceding generation, keep them until retirement and then sell them to the following generation or to the x- and z- producing firms.

Consumers supply labor inelastically, such that the following budget constraints apply in the two periods:

$$p_{C_t} C_{1t} + p_{H_t} H_t + p_{C_t}(S^c_{l_t} + S^c_{m_t}) = w_t \tag{8}$$

$$p_{C_{t+1}} C_{2t+1} = (1 + r_{t+1}) p_{C_t}(S^c_{l_t} + S^c_{m_t}) + p_{R_{t+1}} R_{t+1} + p_{H_{t+1}} H_{t+1} \tag{9}$$

where C_{1t} and C_{2t+1} denote first- and second-period consumption.

In equilibrium the prices for resources sold by the consumers to firms, p_R, and to the next generation, p_H, are equalized, as they are perfect substitutes to the consumer. As resources are also a perfect substitute to investment in R&D, their price has to increase at the same rate in equilibrium, i.e. the interest rate, leading to the familiar Hotelling rule:

$$p_{R_{t+1}} - p_{R_t} = r_{t+1}. \tag{10}$$

The representative consumer maximizes lifetime utility U from consumption in the working and retirement period:

$$U_t = \ln C_{1t} + \frac{1}{1+\rho} \ln C_{2t+1} \tag{11}$$

where ρ denotes the individual discount rate. Utility maximization is subject to the two budget constraints, (8) and (9), plus the law of motion of the resource stock (6).

As it proves to facilitate calculations considerably without loss of generality, we choose consumption expenditure ($p_{C_t} C_t = 1$) to be the numeraire of the system.

3 Characteristics of the balanced growth path

It can be shown that a long-run equilibrium in our economy exists in which not only expenditure shares are constant, but also output grows at a constant rate and the labor allocation between sectors is stationary.

Inspection of (1) shows that balanced growth exists only if production of the modern and traditional aggregates grows at the same constant rate, i.e.[3]

$$g_{\tilde{X}} = g_{\tilde{Z}}. \tag{12}$$

As it can be shown that profit maximization in the final goods sector results in

$$\frac{\tilde{X}_t}{\tilde{Z}_t} = \left(\frac{p_{\tilde{X}_t}}{p_{\tilde{Z}_t}}\right)^{-\nu} \quad \Leftrightarrow \quad \frac{p_{\tilde{X}_t}\tilde{X}_t}{p_{\tilde{Z}_t}\tilde{Z}_t} = \left(\frac{\tilde{X}_t}{\tilde{Z}_t}\right)^{-\frac{1-\nu}{\nu}} = \frac{\phi_t}{1-\phi_t}, \tag{13}$$

identical growth rates of $\tilde{X}$ and $\tilde{Z}$ also imply that the sectoral expenditure shares ϕ_t and $1-\phi_t$ have to be constant over time.

The equilibrium labor shares can be derived from the the first-order conditions of profit maximization , equilibrium profits in the monopolistic sectors and the consumers'

[3]Throughout this paper $g_{k_t} = \frac{k_{t+1}}{k_t}$ is referred to as the growth rate of a variable. Variables and growth rates that don't carry time indices refer to values along the balanced growth path (BGP).

optimization problem (see Appendix A). The input of labor in traditional research along the BGP is given by

$$L_m = \frac{1-\gamma}{E} \qquad \text{with} \qquad E = (1-\gamma) + (1-\beta)\tilde{\phi} + (1+r)(\alpha\beta\tilde{\phi} + \gamma\delta) \tag{14}$$

where $\tilde{\phi}$ denotes the relative sector share $\frac{\phi}{1-\phi}$. The remaining labor shares can now be expressed in terms of L_m:

$$L_l = \tilde{\phi}\frac{1-\beta}{1-\gamma}L_m \tag{15}$$

$$L_X = \tilde{\phi}\frac{\alpha\beta}{1-\gamma}(1+r)L_m \tag{16}$$

$$L_Z = \frac{\delta\gamma}{1-\gamma}(1+r)L_m. \tag{17}$$

The relative input of labor in the respective sectors is determined by their relative productivity, but of course also by the relative importance of modern versus traditional goods production as represented by $\tilde{\phi}$. Furthermore, the interest rate affects the allocation of labor between research and goods production. This results from the fact that labor employed in goods production today also incurs profits today while labor employed in R&D today only incurs profits in the next period when the generated blueprints are used in the production of intermediates. Consequently, firms discount these profits at the interest rate (see also Appendix A).

The wage rate for which the labor market clears

$$w = \left(\delta\gamma + \frac{1-\gamma}{1+r}\right) + \tilde{\phi}\left[(\alpha\beta - \delta\gamma) + \frac{\gamma-\beta}{1+r}\right] \tag{18}$$

can be determined from the first-order condition for labor in the modern goods sector, (47), and the equilibrium labor shares, (14) and (16). As (18) shows, the effect of a relative increase in modern production, $\tilde{\phi}$, on the income of labor is not clear from the outset, given that $\alpha > \delta$ and $\beta < \gamma$. This ambiguity is due to the effect that an increase in $\tilde{\phi}$ exerts on labor demand in goods production. On the one hand labor demand increases due to the higher labor intensity in modern production, but on the other hand labor demand may fall due to the higher gains from specialization in the x-sector. With respect to the labor demand in research, an increase in $\tilde{\phi}$ always induces a net increase for our parameter specification. As the gains from specialization are higher in the x-sector, a higher share of modern goods production increases equilibrium profits in the modern sector more than it lowers profits in the traditional sector. Due to this increased profitability, demand for modern blueprints increases more than the demand for traditional blueprints decreases and overall labor demand in research rises.

Depending on the net effect on aggregate labor demand in production and research, the equilibrium wage may rise or fall following an increase in $\tilde{\phi}$.

With respect to households investment in modern and traditional research it follows from the equilibrium profits in monopolistic production and the no-arbitrage conditions for the patent markets that relative investment, S_l/S_m, is driven by relative profits in the modern and traditional sectors, Π_X/Π_Z:

$$\frac{S_l}{S_m} = \frac{\Pi_X}{\Pi_Z} = \tilde{\phi}\frac{1-\beta}{1-\gamma}. \tag{19}$$

The ratio depends on the relative sector share and the relative gains of specialization, but is independent of the interest rate as households are indifferent between investment in the modern or traditional sector.

The condition that along the BGP aggregate production in both sectors has to grow at the same rate carries important implications for equilibrium R&D. Considering that intermediate firms are symmetric in each sector such that $x_{i_t} = x_t$ and $z_{i_t} = z_t$ holds, aggregate production can be expressed as

$$\tilde{X}_t = (l_t - l_{t-1})^{\frac{1-\beta}{\beta}} L_{X_t}^{\alpha} R_{X_t}^{1-\alpha} \qquad \text{and} \qquad \tilde{Z}_t = (m_t - m_{t-1})^{\frac{1-\gamma}{\gamma}} L_{Z_t}^{\delta} R_{Z_t}^{1-\delta}. \tag{20}$$

Along the BGP labor shares are constant which also implies constant growth of research in the two sectors. Furthermore it can be shown from (57) and the Hotelling rule that the time path of resource extraction is given by

$$R_{k_{t+1}} = \left(\frac{1-\tilde{\phi}_t}{1-\tilde{\phi}_{t+1}}\frac{1}{1+r_{t+1}}\right) R_{k_t}, \qquad\qquad k = X, Z. \tag{21}$$

Taking into account that for balanced growth to be feasible, sector shares and the interest rate have to be constant over time this implies that the equilibrium growth rate of resources is given by $g_Z = (1+r)^{-1}$, i.e. the invers of the growth rate of resource prices.

Consequently, we get the following condition for aggregate production in the two sectors to grow at the same rate

$$\frac{g_l^{\frac{1-\beta}{\beta}}}{g_m^{\frac{1-\gamma}{\gamma}}} = (1+r)^{\alpha-\delta}. \tag{22}$$

This condition states that for balanced growth to be feasible the difference in resource intensity between the sectors has to be compensated by research. Given our assumption that the modern sector is less resource intensive ($\alpha < \delta$), it is less hurt by the declining input of natural resources over time. Consequently, for growth in the traditional sector to keep up with growth in the modern sector, traditional research has to compensate for the higher drag on growth from resources. As we assumed the gains from specialization to be higher in the modern sector ($\beta < \gamma$), this condition directly implies that the growth rate of traditional R&D is higher in equilibrium than the growth rate of modern R&D ($g_m > g_l$). Whether or not this also implies that more labor is employed in m-research than in l-research, depends on the productivity of research.

While the aggregate amounts in both sectors grow at the same rate in equilibrium, the amounts produced of each variety in either sector decrease over time. The quantities develop according to $g_x = (1+r)^{\alpha-1} < 1$ and $g_x = (1+r)^{\delta-1} < 1$ where the reduction in the produced amounts is due to the decreasing input of natural resources. Since the traditional sector is more resource dependent than the modern sector, z falls faster than x. It can be shown that, as economic intuition suggests, the rising scarcity of resources induces the price ratio of x- to z-products to follow a time path that is inverse to the development of quantities. Yet, in the aggregate, the upward pressure on individual variety prices that follows from the Hotelling rule is compensated by the increase in new product varieties.

With respect to consumption it follows straightforwardly from (1) and (12) that consumption growth along the balanced path is given by

$$g_C = g_l^{\frac{1-\beta}{\beta}} (1+r)^{1-\alpha} = g_m^{\frac{1-\gamma}{\gamma}} (1+r)^{1-\delta}. \tag{23}$$

As was to be expected, research growth affects growth positively while the scarcity of resources exerts a negative effect.

4 Transitional dynamics and stability

In order to analyze the transitional dynamics of the economy, we express the dynamic system in terms of variables which are constant along the BGP. We reduce the system to three first-order difference equations which are functions of the extraction rate, τ, the relative factor share, $\tilde{\phi}$, and a composite stock variable

$$l_{R_t} = \frac{l_t^{\frac{1-\beta}{\beta}}}{m_t^{\frac{1-\gamma}{\gamma}}} H_t^{\delta-\alpha}. \tag{24}$$

The latter captures the already discussed prerequisite for a BGP that the knowledge weighted resource use in the two sectors has to be constant in equilibrium. Due to the constant extraction rate along the BGP this prerequisite can also be expressed by the constancy of the ratio of the three stock variables of this economy: modern and traditional knowledge on the one hand and the resource stock on the other hand.

Expressions for the dynamics of $\tilde{\phi}$, τ and l_{R_t} are derived in Appendix B. We get the following system of first-order difference equations in implicit form:

$$\begin{aligned}
\tilde{\phi}_t^{\frac{1}{\nu-1}-\frac{1-\beta}{\beta}} &= f(\tilde{\phi}_{t+1}, \tilde{\phi}_t, \tau_t, l_{R_t}) && (25)\\
\tau_{t+1} &= g(\tilde{\phi}_{t+1}, \tilde{\phi}_t, \tau_t) && (26)\\
l_{R_{t+1}} &= h(\tilde{\phi}_{t+1}, \tilde{\phi}_t, \tau_t, l_{R_t}). && (27)
\end{aligned}$$

To check for the stability of the system, we derive the Jacobian of (25) to (27) in the proximity of the steady state[4]

$$D = \begin{pmatrix} \frac{\partial \tilde{\phi}_{t+1}}{\partial \tilde{\phi}_t} & \frac{\partial \tilde{\phi}_{t+1}}{\partial \tau_t} & \frac{\partial \tilde{\phi}_{t+1}}{\partial l_{R_t}} \\ \frac{\partial l_{R_{t+1}}}{\partial \tilde{\phi}_t} & \frac{\partial l_{R_{t+1}}}{\partial \tau_t} & \frac{\partial l_{R_{t+1}}}{\partial l_{R_t}} \\ \frac{\partial \tau_{t+1}}{\partial \tilde{\phi}_t} & \frac{\partial \tau_{t+1}}{\partial \tau_t} & \frac{\partial \tau_{t+1}}{\partial l_{R_t}} \end{pmatrix}_{\tilde{\phi},\tau,l_R}$$

As a result it turns out that, due to the models' complexity, the first-order difference system cannot be expressed explicitly. Thus it is not feasible to conduct an analytical proof in this case. Instead, we use a Taylor expansion to derive the elements of D and use numerical estimations for varying calibrations of the model. Then, the system gives rise to two eigenvalues outside the unit circle and one within. Following the Balanchard/Kahn conditions this implies that, in order to have a unique and stable trajectory, the system should contain one predetermined variable whose initial value is known while the initial values of the remaining two variables can be chosen freely. Considering the underlying model structure, where the initial values of H_0, l_0 and m_0 are given, it is actually true that the initial values of ϕ_t and τ_t can be chosen freely.

To conclude, we establish a unique steady state and demonstrate stability with the help of numerical methods. To get additional insights it proves instructive to take a look at a simplified version of the model. Let us specifically consider the following case with reduced heterogeneity:

$$\alpha = \delta, \qquad \beta = \gamma, \qquad a_l \neq a_m. \tag{28}$$

Alternatively we could also regard the two cases in which the sectors either differ only with respect to the resource intensities or gains from specialization. Yet, the chosen specification has the advantage of allowing to derive local stability ranges that depend explicitly on the parameters of the model. First note that the expression for the relative sector share, (65), reduces for $\alpha = \delta$ and $\beta = \gamma$ to

$$\tilde{\phi}_{t+1} = \left(\frac{l_{t+1} - l_t}{m_{t+1} - m_t}\right)^{\frac{1-\beta}{\beta}(\nu-1)}, \tag{29}$$

[4]Appendix C shows that a unique steady state exists for the dynamic system under consideration.

showing that $\tilde{\phi}$ is predetermined by the available number of patents at each point in time. Employing the first-order conditions for the optimal labor input in R&D in the two sectors, $L_{l_t} = \tilde{\phi}_t L_{m_t}$, and the production functions of R&D, (4), then gives

$$\tilde{\phi}_{t+1} = \left(\frac{a_l}{a_m}\frac{l_t}{m_t}\right)^{\frac{\frac{1-\beta}{\beta}}{\frac{1}{\nu-1}-\frac{1-\beta}{\beta}}}. \tag{30}$$

It can already be seen that the relative sectoral share is now solely determined by the production conditions in the research sector. At each point in time it is predetermined by the ratio of the patent stocks in the two sectors and the respective research productivity. Since no differences exist with respect to resource intensity and gains of specialization, these cannot give rise to a deviation from a symmetric distribution of sector shares. In the previous model version, in which research had to compensate for differences in resource intensities and specialization gains, the ratio of the knowledge stocks in the two sectors changed over time even in the long-run equilibrium. Now $\frac{l_t}{m_t}$ is constant along the BGP.

(67) reduces to

$$l_{m_{t+1}} = \left(\frac{\tilde{\phi}_{t+1}\frac{L_{m_t}}{a_l}+1}{\frac{L_{m_t}}{a_m}+1}\right) l_{m_t} \tag{31}$$

with $l_{m_t} = \frac{l_t}{m_t}$. The labor share in traditional research can after substitution of (30) and (63) be expressed as

$$L_{m_t} = \left(1+\left(\frac{a_l}{a_m}\frac{l_t}{m_t}\right)^{\frac{\frac{1-\beta}{\beta}(\nu-1)}{1-\frac{1-\beta}{\beta}(\nu-1)}}\right)^{-1}\left(1+(1+\rho)\frac{\alpha\tau_t}{\alpha\tau_t-(2+\rho)(1-\alpha)}\right)^{-1}. \tag{32}$$

Using (32), $l_{m_{t+1}}$ can be written as a function of l_{m_t} and τ_t only. Combining (59) and (63) furthermore gives a first-order difference equation for the extraction rate:

$$\tau_{t+1} = \left(\frac{(1+\rho)(1-\beta)}{\alpha\beta\tau_t-(2+\rho)(1-\alpha)\beta}-1\right)^{-1} \tag{33}$$

which is independent of the relative sector share $\tilde{\phi}$.

From (31) the steady state value of $\tilde{\phi}$ can be derived: Taking into account that along the BGP $l_{m_{t+1}} = l_{m_t}$ has to hold, we get

$$\tilde{\phi} = \frac{a_l}{a_m}. \tag{34}$$

Given that the modern sector is less efficient with respect to research ($a_l > a_m$) this implies $\tilde{\phi} > 1$, i.e. the sectoral share of the modern sector exceeds the share of the traditional sector. Inserting $\tilde{\phi}$ into (30) gives the steady state ratio of the two knowledge stocks

$$\left(\frac{l_t}{m_t}\right) = \left(\frac{a_l}{a_m}\right)^{\frac{\frac{1}{\nu-1}-\frac{1-\beta}{\beta}}{\frac{1-\beta}{\beta}}-1} \tag{35}$$

which additionally depends on the elasticity of substitution between modern and traditional goods and the gains from specialization. Inserting this result into (31) shows that along the BGP $g_m = g_l$ holds for this simplified case.

To solve for the equilibrium extraction rate, we set $\tau_{t+1} = \tau_t$ which gives a second-order polynomial. Given that the parametrization gives rise to an interior solution ($\tau < 1$), it can again be shown that the polynomial has one positive and one negative root. Consequently we get a unique $\tau = r$.

Let us now consider the stability of the system. From (30), (31) and (33) we get two first-order difference equations

$$l_{mt+1} = g(l_{m_t}, \tau_t) \tag{36}$$

$$\tau_{t+1} = h(\tau_t) \tag{37}$$

whose linearization around the steady state gives the following Jacobian:

$$D_2 = \begin{pmatrix} \frac{\partial l_{m_{t+1}}}{\partial l_{m_t}} & \frac{\partial l_{m_{t+1}}}{\partial \tau_t} \\ 0 & \frac{\partial \tau_{t+1}}{\partial \tau_t} \end{pmatrix}_{\tau, l_m}$$

The two eigenvalues of this simple system are $\left.\frac{\partial l_{m_{t+1}}}{\partial l_{m_t}}\right|_{\tau,l_m}$ and $\left.\frac{\partial \tau_{t+1}}{\partial \tau_t}\right|_{\tau,l_m}$. It can be shown that

$$\left.\frac{\partial \tau_{t+1}}{\partial \tau_t}\right|_{\tau} = (1+\tau)^2 \frac{\alpha\beta}{(1-\beta)(1+\rho)} \tag{38}$$

always exceeds unity. For the second eigenvalue we get

$$\left.\frac{\partial l_{m_{t+1}}}{\partial l_{m_t}}\right|_{\tau,l_m} = 1 + \frac{\frac{1-\beta}{\beta}}{\frac{1}{\nu-1} - \frac{1-\beta}{\beta}} \frac{\frac{L_m}{a_m}}{\frac{L_m}{a_m}+1}, \tag{39}$$

such that whether or not the eigenvalue is below or above unity depends crucially on the substitution elasticity ν. The eigenvalue lies inside the unit circle if

- $\nu < 1$ and
- $\nu > \nu^*$ with $\nu^* = \frac{\frac{1+\beta}{a_m E_t}+2}{\left(\frac{1}{a_m E_t}+2\right)(1-\beta)}$.

For the dynamic system to give rise to a unique stable saddle path, given that one eigenvalue exceeds unity while the absolute value of the other is smaller than one, the initial value of one variable should be predetermined while the other can be chosen freely. In this case $l_{m_0} = \frac{l_0}{m_0}$ is predetermined, while τ_0 can be chosen freely.

The stability of the system thus depends upon the elasticity of substitution in the CES-production function for final output. The result that for $\nu < 1$ the system is saddle-path stable corresponds to recent literature but is shown to hold in the presence of an essential non-renewable resource. Note that for $1 < \nu < \nu^*$ the system is unstable while for values of $\nu > \nu^*$ it is again stable.

5 Policy analysis

As has already been shown, the CES production technology in final output implies that in our model the expenditure shares of modern and traditional goods, ϕ and $1 - \phi$, are determined endogenously. This would be of no particular interest if the modern and traditional sector were identical. Yet, as the two sectors differ not only with respect to technologies in research and specialization gains, but also with respect to resource intensity, a policy maker might strive to influence these shares in favor of the cleaner modern sector. We have already seen that in a long-run equilibrium the aggregate amounts of $\tilde{X}$ and $\tilde{Z}$ have to grow at the same rate. Consequently, growth in both sectors would profit equally from growth enhancing policies in the long run. Yet the ratio of the level of production in the two sectors might change due to economic policy. As the two sectors differ with respect to their resource intensity, a change in the allocation of expenditure shares also affects the timing and pricing of resource extraction.

In the following we consider two different types of policies that could be conducted in order to affect the direction of growth in this economy. First we consider a disproportionate increase in the share of investment going to the modern sector. Let us assume, for example, that a pension fund that has the statutory obligation to invest more than the equilibrium market share in the modern sector.

In a second step we alternatively assume that the policy maker wants to support production in the modern sector by investing in activities that generate public knowledge which is specific to the modern sector and results in an increase in productivity in this sector. These types of activities can also be depicted by implementing a pension fund. In this case, the pension fund is more flexible in its investment strategy.

Let us consider the following modified model set-up which is depicted in Figure 2: The pension fund in our economy has to assure for a minimum standard of living of the consumers in their retirement period. To be able to pursue this task, the pension fund collects a share τ_t of the young consumer's wage income w_t. In investing the collected revenues, the pension fund has to follow certain rules. These rules may take two forms. The pension fund can invest the collected revenues specifically in modern R&D and/or it can use part of the raised contributions to improve productivity in the modern sector. The productivity improvement can, for example, result from investing in the public provision of sector specific infrastructure or fundamental productive knowledge. Alternatively the pension fund may directly provide subsidized credits to firms in the modern sector. These credits have to be used for investment in productivity improvements that are made available to all firms in the sector. If it is mandatory that this is done without compensation, this of course entails a subsidy rate of unity – as we assume here. The difference between the market and subsidized credit rate is paid out of the revenues the pension fund collects.

The pension paid to the consumer is defined in terms of expenditures for first-period consumption $p_{C_t}C_{1t}$. The share ξ' of $p_{C_t}C_{1t}$ to which the pension P_{t+1} has to amount is politically determined.

As the pension fund uses at least part of the collected revenues to invest in research, (5) changes accordingly:

$$p_{C_t}(S^c_{l_t} + S^{pf}_{l_t}) = w_t L_{l_t} \qquad \text{and} \qquad p_{C_t}(S^c_{m_t} + S^{pf}_{m_t}) = w_t L_{l_t}, \tag{40}$$

where S^{pf}_n, $n = l, m$, denotes the pension fund's investment in modern and traditional research.

If the pension fund uses a share μ of the collected revenues for public, productivity enhancing investment, this investment has also to be financed from the contributions of the consumers in their working period. Consequently the total amount of contributions of the young is given by

$$\tau_t w_t = (\xi + \mu) p_{C_t} C_{1_t}, \qquad 0 < \xi, \mu < 1,\ \xi + \mu < 1 \tag{41}$$

where $\xi = \xi'(1 + r_{t+1})^{-1}$. μ is also assumed to be exogenously determined by a political process. If the pension fund invests in public knowledge κ, this investment affects productivity in the production of modern goods positively, such that x_{i_t} in (3) can be rewritten as

$$x_{i_t} = \kappa_t \left(L_{x_{i_t}}\right)^{\alpha} \left(R_{x_{i_t}}\right)^{1-\alpha}. \tag{42}$$

Investments that are undertaken in $t-1$ are assumed to translate one-to-one into public knowledge in t, i.e. $\kappa_t = \mu p_{C_{t-1}} C_{1t-1}$.

5.1 Investment in modern R&D

Let us first take a look at the effects of an disproportional investment in modern R&D. Starting from a given equilibrium allocation of aggregate savings and taking the allocation of consumer savings to be constant of a moment, the adoption of such a rule

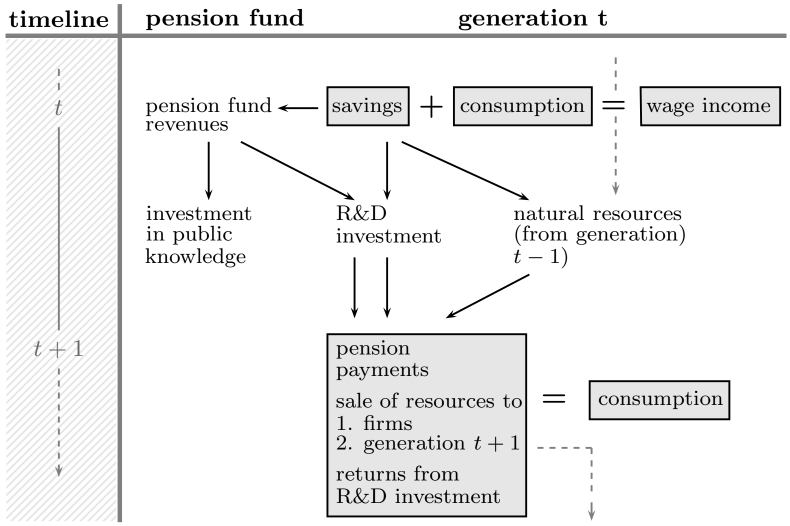

Figure 2: Timeline of consumers' and pension fund's activities

drives up aggregate investment in the modern sector at the expense of research in the traditional sector. As a consequence the number of blueprints for modern goods rises compared to the number of traditional blueprints.

Now consider equilibrium profits attainable from producing x- and z-goods from the generated blueprints. The ratio of sectoral profits is given by

$$\frac{\Pi_{X_t}}{\Pi_{Z_t}} = \frac{(1+\pi_{x_t})}{(1+\pi_{z_t})} \frac{S^{tot}_{l_{t-1}}}{S^{tot}_{m_{t-1}}} \tag{43}$$

where π_i, $i = x, z$, denotes the rate of return to investment in the modern and traditional sectors and S_i^{tot} is the aggregate investment of consumers and the pension fund in each sector. Sectoral profits are used to pay off the investment in R&D of the previous period. Arbitrage leads on the one hand to the intertemporal equalization of profits and investment costs and, on the other hand, to an equalization of the returns to investment in the two sectors ($\pi_{i_t} = r$). Consequently it follows from profit maximization that the profit ratio is constant in equilibrium:

$$\frac{\Pi_{X_t}}{\Pi_{Z_t}} = \tilde{\phi}_t \frac{1-\beta}{1-\gamma}. \tag{44}$$

Combining (43) and (44) shows that changing the savings relation in favor of the modern sector would have to come at the expense of the return to investment in this sector. Due to instantaneous arbitrage processes, the returns of investment are however equalized. Consumers adjust their investment portfolio by withdrawing funds from the modern sector and investing them in traditional research, such that the equilibrium savings allocation remains unchanged. We have to emphasize that this happens despite of assuming a CES-technology which normally gives rise to directed technological change.

So, as long as the pension fund's investment does not crowd out private investment completely, investing more than the equilibrium share in modern research, has neither an effect on overall nor on sectoral investment.[5] This result can be generalized to the extent that in equilibrium only the aggregate investment shares in x- and z-research are determined. Consumers' and pension fund's shares of the sectoral investment remain undetermined as they constitute perfect substitutes.

5.2 Investment in public knowledge

The second option we consider for the pension fund to abet sustainable development is to invest in public knowledge dedicated at raising the productivity of modern goods production.

From the first-order conditions of utility and profit maximization, the budget constraints and sectoral profits we are able to solve for the unique equilibrium expenditure share of modern goods

$$\phi = [1 - \frac{a\frac{1-\beta}{1+r} - \alpha\beta + \frac{1-\alpha}{\alpha}\frac{1+a}{r}}{a\frac{1-\gamma}{1+r} - \delta\gamma + \frac{1-\delta}{\delta}\frac{1+a}{r}}]^{-1} \tag{45}$$

where $a = (1+\rho)(1+\mu)$ denotes the ratio between the true costs of first-period consumption ($(1+\mu)p_{C_t}C_{1_t}$) and interest bearing savings. The term $1+\mu$ in a reflects the distortion of the consumers savings decision. This distortion is due to the consumer assumed inability to internalize the effect of an additional unit of first-period consumption on the contributions to the pension fund. If $\mu = 0$, this inability does not matter, as pension fund savings and private savings are perfect substitutes. If, however, part of the contributions are not paid back with interest in the second period, but rather used for public investment ($\mu > 0$), the assumed inability drives a wedge between the expected return to savings and the true return. The consumer's distorted savings decision is based on the assumption that the return to savings is equal to $1+r$. Yet, if pension funds use part of the collected contribution for public investment, the true return is given by $\frac{1+r}{1+\mu}$ and the income in the second period is lower than expected. If the consumer internalized the feedback effect of his decisions on contributions, this negative income effect would be compensated by an increase in savings that results from an intertemporal substitution effect: The link between first-period consumption and pension fund contributions implies a de facto increase in the opportunity costs of first-period consumption

[5] As Bretschger/Pittel 2005 show, this neutrality result would not hold if, e.g., consumers had a preference for own investment. But even then, the pension fund's increased investment in x-research would only raise overall investment, but would not alter the allocation of investment between sectors.

(decrease in the opportunity costs of saving). If the consumer internalized this effect, he would adjust the optimal ratio between consumption and savings accordingly. Without the internalization, it follows from the first-order conditions of utility maximization that consumption expenditures in the two periods are again allocated following the standard Ramsey-rule. If, however, the consumer internalized the effect, the optimal ratio would rather be given by $\frac{1+r}{1+\rho}(1+\mu)$, i.e. the consumer would increase savings. As we assumed a unitary intertemporal elasticity of substitution (see 11), this substitution effect would just offset the income effect, such that $a = (1+\rho)$.

From (45) it follows that whether or not an increase in public knowledge investment, i.e. a rise in μ, has a positive effect on ϕ depends on production technologies as well as on the gains from specialization. On the one hand an increase in μ always results in an increase of a in (45). But, this increase only translates into a rise of ϕ if

$$\beta\left(\alpha - (1-\alpha)\frac{1}{r}\right) > \gamma\left(\delta - (1-\delta)\frac{1}{r}\right). \tag{46}$$

The economic intuition behind this condition is as follows: The negative income effect associated with an increase of μ – and therefore a – induces consumers to reduce consumption as well as savings. The implied reduction in factor demands leads to a downward pressure on factor prices which in turn induces an offsetting increase in factor demands until demand and supply are equalized again. The magnitude of the increase in factor demands depends on the labor and resource elasticities and the respective gains from specialization. As these differ across sectors, the adjustment results in a reallocation of factors and thereby a change in sector expenditure shares.

Assuming that labor in x-production is more productive than labor in the z-sector ($\alpha > \delta$, while taking $\beta = \gamma$ for the moment) implies that the increase in labor demand in the modern sector is higher than in the traditional sector, such that in the new equilibrium more labor is allocated to x- relative to z-production. As labor and resources are optimally employed in fixed relations (see (47)), this reallocation of labor is met by a reallocation of resources in the same direction. The higher overall factor input in the modern sector implies an increase in the sector's expenditure share. The positive effect on the factor share is enforced by the lower resource dependency of the modern sector (second term in brackets). An increase in ϕ reallocates factor income shares away form resources and towards labor. As the modern sector is less resource intensive, it suffers less from this decrease.

While the higher labor productivity in the x-sector induces a positive reaction of ϕ to an increase of μ, this effect is at least partially offset as the gains from specialization in the modern sector are higher than in the traditional sector ($\beta < \gamma$). Higher gains from specialization imply a lower price elasticity of demand for the individual x- or z-product, such that increasing production induces a relatively stronger decline in prices and therefore profits. This diminishes the incentive to reallocate labor to a sector in which specialization gains are high, in this case the modern sector.

Summing up, a pension fund that is interested in increasing the sectoral share of modern production should only increase μ, if the positive effects of the higher labor productivity ($\alpha > \delta$) are not outweighed by the negative effects of a higher degree of monopoly power ($\beta < \gamma$).

6 Conclusions

The paper at hand analyzes the short- and long-run consequences of sectoral heterogeneity. We consider differences between sectors with respect to the intensity of resource use, specialization gains and research productivity. It is shown that, when sectors differ in resource intensity, research activities are crucial for balanced growth to be feasible. Not only has research to overcome the general drag on growth that arises from the rising scarcity of resources. Resource intensive sectors can in the long-run only stay competitive if they succeed to conduct faster research growth. Similarly, lower productivity gains from specialization also need to be compensated by research for a sector to hold its market share. Consequently, along the BGP research growth is higher in sectors that are less efficient with respect to resources and specialization. We furthermore show that due to the sectoral heterogeneity, the stability of the system depends not only – as well known from the literature – on the scope for substitution between the sectoral outputs, but also on, e.g., the ability of the sectors to gain from a more diversified product range.

In a second part of the paper we analyze the consequences of two types of policy aiming at abetting the share of modern production in the economy. To carry out these policies, we introduce a pension fund into the system. It is shown that disproportional investment in the modern sector has no effect on the aggregate economy. This result holds even under a CES production technology in the final goods sector and two different types of research which usually bring about directed technical change, that is a redirection of research and growth after asymmetrical shocks in the different sectors. As a second result we derive that policy can actively and effectively promote the modern sector and aggregate growth by providing additional public knowledge to modern goods production. The fundamental difference between the two policy instruments lies in the asymmetry regarding the reaction of market participants. While in the first case the biased investment on stock markets is offset by investment changes of neutral investors there are no similar reactions when providing public knowledge. As a consequence, an increasing share of modern goods investment does not alter the direct capital return for the inner solution whereas the provision of public goods results in additional costs for savings in pension funds. But under favorable circumstances there are also benefits, of course, because the modern goods development entails relatively more knowledge spillovers and is therefore better for aggregate growth.

The present research can be extended in at least two dimensions. First, the asymmetry between the two instruments would be different and eventually smaller when assuming the research investments in the modern and the traditional sector to be incomplete substitutes. This could be modeled in terms of different risks in the two sectors. Second, the costs and benefits of knowledge creation by pension funds calls for deriving a social planner solution to determine the optimum amount of subsidies. These issues are left for future research.

7 Appendix

A. Derivation of equilibrium labor shares

To derive the equilibrium labor shares we first have to solve for the consumer's optimal allocation of consumption between the working and retirement period as well as for the profit maximizing allocation of labor and resources between modern and traditional research and production.

From the maximization of profits in the monopolistic production of modern and traditional goods we get the first-order conditions for the individual products. Considering that $x_i = x$ and $z_j = z$ in equilibrium, aggregating over all varieties gives

$$\alpha\beta \frac{\phi_t}{L_{X_t}} = w_t \qquad \text{and} \qquad \delta\gamma \frac{(1-\phi_t)}{L_{Z_t}} = w_t. \tag{47}$$

From (47) and the production functions for x and z, (3), the equilibrium profits in modern and traditional goods production can be derived:

$$\Pi_{x_t} = \phi_t(1-\beta) \qquad \text{and} \qquad \Pi_{z_t} = (1-\phi_t)(1-\gamma). \tag{48}$$

Profits from period t are used to repay the research investment of the previous period. As patents are worthless after one period, the no-arbitrage conditions for the patent markets read

$$\Pi_{x_{t+1}} = (1+r_{t+1})p_{C_t}S_{l_t} \qquad \text{and} \qquad \Pi_{z_{t+1}} = (1+r_{t+1})p_{C_t}S_{m_t}. \tag{49}$$

Since consumers' savings are invested in R&D and research firms operate at zero profits

$$p_{C_t}S_{i_t} = w_t L_{i_t}, \qquad i = l, m \tag{50}$$

has to hold in equilibrium.

We can now derive conditions for the equilibrium allocation of labor: From (47) – (50) we get

$$L_{l_t} = \frac{1-\beta}{1-\gamma}\tilde{\phi}_{t+1} L_{m_t} \tag{51}$$

$$L_{X_t} = \frac{\alpha\beta}{1-\gamma}\tilde{\phi}_t \frac{1+\tilde{\phi}_{t+1}}{1+\tilde{\phi}_t}(1+r_{t+1})L_{m_t} \tag{52}$$

$$L_{Z_t} = \frac{\delta\gamma}{1-\gamma}\frac{1+\tilde{\phi}_{t+1}}{1+\tilde{\phi}_t}(1+r_{t+1})L_{m_t}. \tag{53}$$

where $\tilde{\phi}_t = \frac{\phi_t}{1-\phi_t}$ and therefore $\frac{1-\phi_t}{1-\phi_{t+1}} = \frac{1-\tilde{\phi}_{t+1}}{1-\tilde{\phi}_t}$. Combining (51) - (53) with the equilibrium condition for the labor market, (7), gives the equilibrium input of labor in traditional research:

$$L_{m_t} = \frac{1-\gamma}{E_t} \tag{54}$$

with $$E_t = \frac{1-\gamma}{(1-\gamma) + (1-\beta)\tilde{\phi}_{t+1} + (1+r_{t+1})\frac{1+\tilde{\phi}_{t+1}}{1+\tilde{\phi}_t}(\alpha\beta\tilde{\phi}_t + \gamma\delta)} \tag{55}$$

such that (51) to (53) can be rewritten as

$$L_{l_t} = \frac{(1-\beta)\tilde{\phi}_{t+1}}{E_t}, \quad L_{X_t} = \frac{\alpha\beta(1+r_{t+1})\frac{1+\tilde{\phi}_{t+1}}{1+\tilde{\phi}_t}\tilde{\phi}_t}{E_t}, \quad \text{and} \quad L_{Z_t} = \frac{\gamma\delta(1+r_{t+1})\frac{1+\tilde{\phi}_{t+1}}{1+\tilde{\phi}_t}}{E_t}. \tag{56}$$

Considering that along the BGP labor shares as well as the interest rate and sector shares are constant, we get (14) to (17).

B. Derivation of the system of first-order difference equations

An expression for the dynamics of the extraction rate can be obtained from the optimal extraction of resources and consumer optimization. From $R_t = R_{Z_t} + R_{X_t}$, the first-order conditions for resource use in monopolistic production

$$(1-\alpha)\beta\frac{\phi_t}{R_{X_t}} = p_{R_t} \quad \text{and} \quad (1-\delta)\gamma\frac{(1-\phi_t)}{R_{Z_t}} = p_{R_t} \tag{57}$$

and the Hotelling rule (10) we get for the growth rate of resource extraction

$$g_{R_t} = \frac{1}{1+r_{t+1}}\frac{1+\tilde{\phi}_t}{1+\tilde{\phi}_{t+1}}\frac{(1-\alpha)\beta\tilde{\phi}_{t+1} + (1-\delta)\gamma}{(1-\alpha)\beta\tilde{\phi}_t + (1-\delta)\gamma}. \tag{58}$$

Making use of the definition of the extraction rate $\tau_t = \frac{R_t}{H_t}$ which implies $\frac{H_{t+1}}{H_t} = \frac{1}{1+\tau_{t+1}}$ we can write the interest factor as a function of τ_t and production shares only

$$1 + r_{t+1} = \tau_t\frac{1+\tau_{t+1}}{\tau_{t+1}}\frac{1+\tilde{\phi}_t}{1+\tilde{\phi}_{t+1}}\frac{(1-\alpha)\beta\tilde{\phi}_{t+1} + (1-\delta)\gamma}{(1-\alpha)\beta\tilde{\phi}_t + (1-\delta)\gamma} \tag{59}$$

On the consumer side we get the standard Ramsey rule

$$p_{C_{t+1}}C_{2t+1} = \frac{1+r_{t+1}}{1+\rho}p_{C_t}C_{1t}. \tag{60}$$

from maximizing utility (11) with respect to C_{1_t}, $C_{2_{t+1}}$ and S_t subject to (6), (8), (9). Substituting (6) into the budget constraints, (8) and (9) gives

$$p_{C_t}C_{1t} = (w_t(1 - L_{l_t} - L_{m_t}) - p_{R_t}H_t) \tag{61}$$

$$p_{C_{t+1}}C_{2t+1} = (1+r_{t+1})(w_t(L_{l_t} + L_{m_t}) + p_{R_t}H_t). \tag{62}$$

Using (47) and $H_t = \frac{1}{\tau_t} R_t$ we can rewrite the above expressions in labor shares only. The combination of (60) with (61) and (62) then gives after inserting (54) and (56):

$$1 + r_{t+1} = \frac{(1+\rho)[(1-\beta)\tilde{\phi}_{t+1} + (1-\gamma)]}{[\alpha\beta\tilde{\phi}_t + \delta\gamma] - \frac{2+\rho}{\tau_t}[(1-\alpha)\beta\tilde{\phi}_t + (1-\delta)\gamma]} \frac{1+\tilde{\phi}_t}{1+\tilde{\phi}_{t+1}}. \tag{63}$$

From (59) and (63) we finally get the dynamics of the extraction rate as a function of the relative production share

$$\tau_{t+1} = \left[\frac{(1+\rho)((1-\alpha)\beta\tilde{\phi}_t + (1-\delta)\gamma)}{\tau_t(\alpha\beta\tilde{\phi}_t + \delta\gamma) - (2+\rho)((1-\alpha)\beta\tilde{\phi}_t + (1-\delta)\gamma)} \right. \tag{64}$$
$$\left. \frac{((1-\beta)\tilde{\phi}_{t+1} + (1-\gamma))}{((1-\alpha)\beta\tilde{\phi}_{t+1} + (1-\delta)\gamma)} - 1\right]^{-1}.$$

To derive the dynamics of $\tilde{\phi}$, substitute (20) into (13) which gives

$$\tilde{\phi}_t^{\frac{\nu}{\nu-1}} = \frac{(l_t - l_{t-1})^{\frac{1-\beta}{\beta}}}{(m_t - m_{t-1})^{\frac{1-\gamma}{\gamma}}} \frac{L_{X_t}^{\alpha}}{L_{Z_t}^{\delta}} \frac{R_{X_t}^{1-\alpha}}{R_{Z_t}^{1-\delta}}. \tag{65}$$

By making use of (21), (47) and (54) as well as (56), (**??**) and (63) the dynamics of $\tilde{\phi}_{t+1}$ can be expressed in the form of the following implicit first-order difference equation

$$\tilde{\phi}_{t+1}^{\frac{1}{v-1} - \frac{1-\beta}{\beta}} = F l_{R_t} (1 - \gamma + (1-\beta)\phi_{t+1})^{\alpha - \delta + \frac{\gamma-\beta}{\gamma\beta}} \cdot \left(\frac{\gamma(\delta(1+\tau_t) - 1) + \beta(\alpha(1+\tau_t) - 1)}{\tau_t(\alpha\beta\tilde{\phi}_t + \gamma\delta) - (2+\rho)((1-\alpha)\beta\phi_t + (1-\delta)\gamma)}\right)^{\frac{\beta-\gamma}{\gamma\beta}}$$
$$\cdot \left(-(1+\rho)((1-\beta)\tilde{\phi}_{t+1} + 1 - \gamma) \right. \tag{66}$$
$$\left. + \frac{[\tau_t(\alpha\beta\tilde{\phi}_t + \gamma\delta) - (2+\rho)((1-\alpha)\beta\phi_t + (1-\delta)\gamma)](\tilde{\phi}_t\beta + \gamma)}{(1-\alpha)\beta\phi_t + (1-\delta)\gamma}\right)^{\delta-\alpha}.$$

where $F = \frac{\beta}{\gamma} \frac{(1-\beta)^{\frac{1-\beta}{\beta}}}{(1-\gamma)^{\frac{1-\gamma}{\gamma}}} \left(\frac{\alpha}{\delta}\right)^{\alpha} \frac{(1-\alpha)^{1-\alpha}}{(1-\delta)^{1-\delta}} \frac{a_m^{\frac{1-\gamma}{\gamma}}}{a_l^{\frac{1-\beta}{\beta}}} \frac{(1+\rho)^{2(\alpha-\delta)}}{(2+\rho)^{\frac{\gamma-\beta}{\gamma\beta} + \alpha - \delta}}$.

Finally, to complete the description of the system dynamics we can express $\tilde{l}_{R_{t+1}}$ in terms of $\tilde{\phi}_{t+1}$, $\tilde{\phi}_t$ and τ_t:

$$l_{R_{t+1}} = \frac{\left(\tilde{\phi}_t \frac{1-\beta}{1-\gamma} \frac{A_t}{a_l} + 1\right)^{\frac{1-\beta}{\beta}}}{\left(\frac{A_t}{a_m} + 1\right)^{\frac{1-\gamma}{\gamma}}} \frac{1}{1+\tau_t} l_{R_t}. \tag{67}$$

So the complete dynamics of the system can in more general form be denoted by (25) to (27).

C. Balanced growth path

To show that a BGP exists consider the following: Along the BGP $\tau = r$ holds, such that

$$r^2 \quad + \quad r\left[1-(2+\rho)\frac{(1-\alpha)\beta\tilde{\phi}+(1-\delta)\gamma}{\alpha\beta\tilde{\phi}+\delta\gamma}-(1+\rho)\frac{((1-\beta)\tilde{\phi}+(1-\gamma))}{\alpha\beta\tilde{\phi}+\delta\gamma}\right]$$
$$-(2+\rho)\frac{(1-\alpha)\beta\tilde{\phi}+(1-\delta)\gamma}{\alpha\beta\tilde{\phi}+\delta\gamma}=0.$$

(68) gives the equilibrium interest rate, r as a function of $\tilde{\phi}$.[6] From (68) we can then derive the equilibrium relative sector share, $\tilde{\phi}$, which is implicitly determined by

$$1+r(\tilde{\phi})=\left[\frac{\left(\tilde{\phi}\frac{1-\beta}{a_l E(\tilde{\phi})}+1\right)^{\frac{1-\beta}{\beta}}}{\left(\frac{1-\gamma}{a_m E(\tilde{\phi})}+1\right)^{\frac{1-\gamma}{\gamma}}}\right]^{\frac{1}{\delta-\alpha}} \tag{68}$$

It can be shown that the RHS of (68) is increasing in $\tilde{\phi}$ as $\frac{\partial E}{\partial\tilde{\phi}}>0$ and $\beta<\gamma$.[7] The LHS of (68) is a decreasing function of $\tilde{\phi}$. Given a suitable parametrization of the model the unique intersection of the RHS and LHS function determines the equilibrium $\tilde{\phi}$. Note that the equilibrium $\tilde{\phi}$ and consequently also the sector shares are independent of the elasticity of substitution between the two sectors, ν. Finally

$$\tilde{l}_R=\left[(\tilde{\phi})^{\frac{1}{\nu-1}-\frac{1-\beta}{\beta}}F^{-1}E(\tilde{\phi})^{\frac{\gamma-\beta}{\beta\gamma}-\alpha+\delta}\right](1+r(\tilde{\phi}))^{\frac{1}{\alpha-\delta}} \tag{69}$$

determines the equilibrium value $\tilde{l_R}$.

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[6] As $(2+\rho)\frac{(1-\alpha)\beta\tilde{\phi}+(1-\delta)\gamma}{\alpha\beta\tilde{\phi}+\delta\gamma}>0$ we always get, independently of the equilibrium value of $\tilde{\phi}$, a unique equilibrium r.

[7] $\frac{\partial E}{\partial\tilde{\phi}}>0$ can be obtained from taking the derivative of

$$E=\frac{\frac{2+\rho}{r}[(1-\alpha)\beta\tilde{\phi}+(1-\delta)\gamma]+\rho(\alpha\beta\tilde{\phi}+\gamma\delta)}{\frac{2+\rho}{r}[(1-\alpha)\beta\tilde{\phi}+(1-\delta)\gamma]-\ \ (\alpha\beta\tilde{\phi}+\delta\gamma)}\left((1-\gamma)+(1-\beta)\tilde{\phi}\right)$$

with respect to $\tilde{\phi}$ and considering $\alpha>\delta$.

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