Scope of governmental policies and agglomeration

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Abstract

This paper analyzes within a regional growth model the impact of alternative regional scopes of governmental policies on the spatial distribution of economic activity. Production inputs are mobile and immobile labor, mobile private capital and a publicly provided input that may be congested at an aggregate and a relative level. Private capital is accumulated and includes the regional knowledge stock. The public input, e. g. motorways or basic knowledge, is supraregionally productive, thus there is diffusion of the productivity effect of governmental expenditure. It is shown that not only the access to the regional inputs but especially the characteristics of the public inputs affect agglomeration forces and hence the localization of economic activity. For equilibrium agglomeration, we demonstrate that concentration is too high due to the negative capital externality which arises from congestion in the public input.

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1 Introduction

Does the establishment of a university in the capital city indeed reinforce agglomeration? Does the expansion of the road networks in fact contribute to a balancing between core and periphery? With this contribution we show that the impact of governmental activities on the agglomeration process is complex: The type of governmental input and the scope of governmental policies together determine whether agglomeration is tightened or slowed down. Particularly the type of congestion turns out to be an important consideration in assessing the effect of public expenditures on agglomeration.

The existence of externalities along the entire innovation processes is undoubted thus legitimating governmental intervention (see e. g. Aghion and Howitt (1998) or Grossman and Helpman (1991) for an overview). While until now most analyses focus on subsidizing knowledge creation directly or indirectly (e. g. via subsidies of final products in case of innovative intermediates) at present also the diffusion process becomes increasingly important in economic and political debates. These debates also involve regional facets since as consequence of the globalization process not only national but also the interregional competition has increased significantly. Governments support the innovation efforts in their regions in order to survive in the international competition. Hence efficient innovation and correspondingly an efficient policy accompanying the entire innovation process is required not only at national or supranational but also at regional levels. Aside from private activity also governmental decisions are of major importance thus requiring a sophisticated treatment of the impact of governmental activity in the context of regional innovation processes.

This paper analyzes within a regional growth model the impact of alternative types of governmental policies and the scope of those policies on the spatial distribution of economic activity as measured by the regional output ratio. Following the seminal work of Barro (1990) governmental activity includes the provision of a public production input (e. g. basic research, research establishments or the provision of regional infrastructure) that is complementary to the private production inputs (see also e. g. Glomm and Ravikumar (1994b, 1994a) or Barro and Sala-I-Martin (2004) for an overview). Private capital may be accumulated by the individuals and includes the stock of knowledge of the region. Since capital is mobile knowledge diffusion may occur in the context of factor migration.

The local governments may either provide a pure public good or an input that is subject to absolute and relative congestion (see Eicher and Turnovsky (2000) for the classification of public inputs or e. g. Turnovsky (2000) for an overview of the congestion literature). We analyze two regions where private firms have access to identical production technologies. Production inputs are mobile and immobile labor, mobile private capital as well as public capital that – depending on its characteristics – may or may not be mobile. Mobile factors always move to the region having the highest marginal productivities and factor movements cease when these productivities are equilibrated. There are two types of externalities in the context of regional public inputs: One that arises in the context of the diffusion process (in the following denoted as diffusion externality) and a congestion externality:

- Diffusion externality: Most types of regional public inputs are supraregionally productive. If one region's government provides e. g. basic research, this induces a productivity shift in all surrounding regions. However, if one region's government builds a harbor, the firms of all regions will benefit from the services. Hence, there is a diffusion of the productive impact of governmental activity. This diffusion is usually not taken into account within the governments' decision about the level of the public input.
- Congestion externality: Another feature of congested inputs is that they involve a capital externality in that individuals do not realize the (negative) impact of their own accumulation on the individual availability (and hence productivity) of the public input of the others. Then private capital is over–accumulated and the public input is relatively congested. Additionally also absolute congestion may arise in the sense that congestion increases with the absolute size of the economy. Usually this absolute congestion externality is assumed to be negative. We extend the existing literature and also allow for positive values. Then the externality is due to positive capital spillovers and may be interpreted in the sense of Romer (1986).

Both externalities are crucially influenced by the decisions of the local governments on the type of public input that is provided. Consequently, not only the very provision of a public input but especially the type of the public input has a major impact on knowledge diffusion, capital and labor mobility and hence the localization of economic activity. It is shown that while relative congestion does not have structural effects concerning the number and the stability characteristics of the equilibrium capital ratio, the level of absolute congestion does very well.

To formalize concentration or dispersion the model includes convergence and divergence forces. The latter (former) arise if the mobile factors face local increasing (decreasing) returns. Divergence is reinforced whenever the public input is at least to some degree congested. Multiple equilibria may arise. They are also affected by the relative sizes of the regions – as measured by the number of local firms –, the relative factor endowments with immobile labor as well as by the exogenous and endogenous regional capital ratios. If divergence forces dominate concentrated steady states (thus reflecting agglomeration in either region and contraction of the other region) arise whereas in the context of balanced steady states both regions finally grow at the same constant rate. Due to the externalities the market equilibria turn out to be suboptimal. If as consequence of the congestion externality private capital return exceeds social capital return concentration is too high in an agglomeration equilibrium . To illustrate the main results numerical simulations are carried out.

The paper is organized as follows: After presenting the analytical framework the investment decision is discussed in section 3. The role of factor migration is discussed in Section 4. Then the impact of convergence and divergence forces for the model's dynamics are studied. Multiple equilibria and hence agglomerations are illustrated in the context of numerical simulations in Section 6 while Section 7 focusses on efficiency aspects. The paper closes with a short summary.

2 The analytical framework

2.1 Firms

Firms in both regions $\iota = 0, 1$ produce the homogenous good, Y_ι, by the same Cobb–Douglas technology. The inputs used in each region are mobile labor, M_ι, immobile labor, L_ι and private capital, K_ι. Furthermore, output depends upon regional access to a global public input that is measured by an index, D_ι. The production function for a representative firm in region ι is given by

$$Y_\iota = L_\iota^\lambda M_\iota^\mu K_\iota^\alpha D_\iota^\gamma, \quad \lambda \geq 0, \mu \geq 0, \quad 1 \geq \alpha \geq 0, 1 \geq \gamma \geq 0, \quad \alpha + \mu + \lambda = 1 \tag{1}$$

It has constant returns to scale in the three private inputs, thus guaranteeing competitive pricing at the factor markets, and increasing returns in all inputs. The global public input, D_ι, includes the regional public inputs, $G_{s\iota}$, that are separately provided by both regions. The firm's access to the other region's public input may be limited as parameterized by $0 < \beta < 1$, and we assume

$$D_1 = G_{s1} + \beta G_{s2} \tag{2a}$$

$$D_2 = G_{s2} + \beta G_{s1} \tag{2b}$$

The parameter β may also be interpreted as a measure for the spatial scope of any governmental policy. If $\beta = 0$, firms in each region only benefit from the public input provided by the government of their own region whereas $\beta > 0$ implies that firms in one region have also (possibly limited) access to the other region's public input.

The following illustration may serve as example: Consider two countries that both provide a road network as public input. As long as these networks are not connected the spatial scope of governmental policy is restricted to the own region, $\beta = 0$. Firms in region ι only benefit from their own region's roads. But if now e. g. ferries, connecting roads, tunnels or bridges are established, the road network in region 1 may be also used by firms of region 2. Formally, β increases up to $\beta = 1$ that reflects the other polar case in which firms in both regions have access to the entire public inputs provided in both regions. Then the global public input covers both road networks.[1]

The modeling of the governmental input is adopted from Eicher and Turnovsky (2000) and the congestion function includes absolute and relative congestion that are denoted by ε_A and ε_R

$$G_{s\iota} = G_\iota K_\iota^{\varepsilon_R} \bar{K}_\iota^{\varepsilon_A - \varepsilon_R}, \quad 0 \leq \varepsilon_R \leq 1 \tag{3}$$

Thereby parameter $\bar{K}_\iota$ denotes the aggregate stock of private capital in region ι and G_ι denotes the aggregate flow of government expenditure. The congestion function incorporates the potential for the public good to be associated with alternative types and also alternative degrees of congestion. Following Borcherding and Deacon (1972) and Bergstrom and Goodman (1973) there is vast empirical literature that examines the 'publicness' of governmental expenditures. Nevertheless, in contrast to Eicher and Turnovsky (2000) we do not restrict the sign of the degree of absolute congestion, ε_A, but we allow for positive and negative externalities at the aggregate level.[2] The public services can be classified into the following three categories.

(i) If $\varepsilon_A = \varepsilon_R = 0$, then government services constitute a pure public good in the sense of Samuelson (1954) and $G_{s\iota} = G_\iota$. The public input is available equally to each individual within region ι independent of the usage of others.[3] The situation $\varepsilon_R = 0$ may be interpreted as a government providing a pure public good, e. g. basic research. Its usage by one firm does not affect possible usages of the others. The same is true for the usage of the public input by firms from other regions.

(ii) Pure relative congestion arises if $\varepsilon_R > 0, \varepsilon_A = 0$: This reflects situations in which the level of the public input available to the individual is tied to this individual's usage of capital. Values $\varepsilon_R = 0$ correspond to a nonrival pure public input whereas $\varepsilon_R = 1$ reflects a situation of proportional relative congestion. Accordingly, the cases $0 < \varepsilon_R < 1$ correspond to situations of partial relative congestion, in the sense that given the individual stock of

[1] Note that both limiting cases, $\beta = 0$ and $\beta = 1$, characterize an extreme and unrealistic word but may be well useful as benchmark cases.

[2] As will be shown below this is of major importance for the resulting equilibria.

[3] Since only few examples of such pure public goods exist, this case should be treated primarily as benchmark.

capital, government spending can increase at slower rate than does $\bar{K}_\iota$ and still provide a fixed level of services to the firm. An example for $\varepsilon_R \leq 1$ could be e. g. the provision of a road network. In extreme it is proportionally congested and each of the N_ι individuals within region ι may use $1/N_\iota$ parts of the entire public input, G_ι, for production.[4]

(iii) Pure (negative) absolute congestion given that $\varepsilon_A < 0, \varepsilon_R = 0$: Absolute congestion represents a situation in which congestion is directly proportional to the aggregate level of capital in the considered region and individual action does not affect the extent of the services derived from the public input. Examples include local fire departments where congestion increases with the absolute size of the economy.

(iv) Pure positive capital externality given that $\varepsilon_A < 0, \varepsilon_R = 0$: This reflects the opposite case where positive effects of capital accumulation arise and individuals benefit from accumulation of the others. The positive effects increase with the absolute size of the economy. This externality can be interpreted in the sense of Romer (1986) and an example could be the outcomes of research centers that are financed by non–distortionary taxes.[5]

For the production technology in (1) to allow for endogenous growth in both regions, an additional constraint has to be imposed, namely $\alpha + \gamma(1 + \varepsilon_A) = 1$. This ensures global constant returns to private capital, the accumulable factor.[6] As will be shown below this knife edge assumption will be of major importance for the characteristics of divergence and convergence.

From (1), (2) and (3) output of the individual firms in both regions is given by

$$Y_1 = L_1^\lambda M_1^\mu K_1^\alpha (G_1 K_1^{\varepsilon_R} \bar{K}_1^{\varepsilon_A - \varepsilon_R} + \beta G_2 K_2^{\varepsilon_R} \bar{K}_2^{\varepsilon_A - \varepsilon_R})^\gamma \tag{4a}$$

$$Y_2 = L_2^\lambda M_2^\mu K_2^\alpha (G_2 K_2^{\varepsilon_R} \bar{K}_2^{\varepsilon_A - \varepsilon_R} + \beta G_1 K_1^{\varepsilon_R} \bar{K}_1^{\varepsilon_A - \varepsilon_R})^\gamma \tag{4b}$$

Private (average) capital productivities in both regions are evolve according to

$$\frac{Y_1}{K_1} = L_1^\lambda M_1^\mu \left(1 + \frac{\beta}{\tilde{g}_s}\right)^\gamma \left(\frac{G_1}{K_1}\right)^\gamma N_1^{\gamma(\varepsilon_A - \varepsilon_R)} \tag{5a}$$

$$\frac{Y_2}{K_2} = L_2^\lambda M_2^\mu (1 + \beta \tilde{g}_s)^\gamma \left(\frac{G_2}{K_2}\right)^\gamma N_2^{\gamma(\varepsilon_A - \varepsilon_R)} \tag{5b}$$

[4]As Turnovsky (1996, p. 364) argues, the case $\varepsilon_R > 1$ describes a situation where congestion is so great that the public input must grow faster than the economy in order for the level of services provided to the individual firm remain constant. This case is unlikely at the aggregate level, but may well be plausible for local public goods (see also Edwards (1990)). A local public good could be a harbor that is provided by the regional government. Nevertheless it also may be used by individuals coming from outside the region.

[5]Note that the positive spillovers in the model of Romer (1986) or Lucas (1988) do not exactly correspond to the framework of this model since there the spillovers arise independent of governmental activity.

[6]This interdependence between the parameters implies an adjustment of the values of α or γ whenever a change in absolute congestion, ε_A, is analyzed. Besides, together with $0 \leq \gamma \leq 1$ the condition also ensures that the marginal product of private capital in case of $\varepsilon_A < 0$ is positive.

Thereby the following variables are defined: $g \equiv G_1/G_2$, $k \equiv K_1/K_2$, $\overline{k} = \overline{K}_1/\overline{K}_2$ and $g_s \equiv G_{s1}/G_{s2} = gk^{\varepsilon_R}\overline{k}^{\varepsilon_A - \varepsilon_R}$. The variable $\tilde{g}_s = gk^{\varepsilon_A}n^{\varepsilon_A - \varepsilon_R}$ already utilizes the equilibrium condition at the capital market, $\overline{K}_\iota = N_\iota K_\iota$. Average productivities depend on the distribution of capital and governmental activity across regions, as incorporated in g_s, the ratio G_ι/K_ι as well as on the scale of the regions and congestion within the regions, $N_\iota^{-\gamma(\varepsilon_A - \varepsilon_R)}$.

2.2 Households

Households are either immobile or mobile workers. Immobile workers receive wages denoted by $w(t)$, while mobile workers receive wages denoted by $m(t)$. Mobile workers do not face any relocation cost and choose the location offering the highest value of $m(t)$. Since perfect competition at the factor markets is assumed, wages for mobile labor are highest where the private marginal productivity of mobile labor is highest.

The infinitely lived households possess identical isoelastic preferences and the representative household maximizes lifetime utility out of consumption, $C(t)$, according to

$$U = \int_0^\infty \frac{\sigma}{\sigma - 1} C(t)^{\frac{\sigma-1}{\sigma}} e^{-\rho t} dt \tag{6}$$

The subjective discount rate is denoted by ρ and σ is the elasticity of intertemporal substitution. Households save by accumulating a risk free asset. The asset value equals the value of the stock of capital at any point in time and hence the asset value of the two regions equals $V(t) \equiv q_1(t)K_1(t) + q_2(t)K_2(t)$, where q_ι denotes the stock price of capital installed in region ι.

Mobile and immobile workers earn labor income as well as capital income out of investment in both regions. Their total income evolves according to

$$\dot{V}_w(t) = w(t)L(t) + (r(t) - \delta)V(t) - C(t) \tag{7a}$$

$$\dot{V}_m(t) = m(t)M(t) + (r(t) - \delta)V(t) - C(t) \tag{7b}$$

with $r(t)$ denoting the interest rate that equals marginal productivity of capital and δ as constant depreciation rate of private capital. To fully describe the optimization problem, the transversality condition

$$\lim_{t - \infty} K(t)\xi(t) = 0 \tag{8}$$

has to be met where ξ denotes the shadow value of capital. Maximizing (6) subject to the accumulation constraint (7) leads to the well known growth rate of consumption as[7]

$$\frac{\dot{C}}{C} = \sigma(r - \delta - \rho) \tag{9}$$

Due to constant returns of capital the growth rates of consumption and capital coincide.

[7]In what follows time indices will be suppressed.

3 Investment decision and dynamic equilibria

Individuals in the two regions are able to hold capital in region 1 or 2. No–arbitrage applies if both yield identical rates of private return

$$(r+\delta)q_\iota = \dot{q}_\iota + \frac{\partial Y_\iota}{\partial K_\iota} \tag{10}$$

As long as $q_\iota = 1$, private investors are willing to invest in region ι.[8] Then $\dot{q}_\iota = 0$ and according to (10) the interest rate equals the net marginal product of capital, $r = \partial Y_\iota/\partial K_\iota - \delta$. Investment is positive, $I_\iota > 0$. If instead $q_\iota < 1$, no investment would take place and $I_\iota = 0$. Since individuals only invest in the region with the higher return, positive investment in both regions is only feasible if marginal capital productivities coincide. Then both capital stocks grow at the same rate and the capital ratio, k, is constant.

The following conditions are complementary and must be fulfilled for sustained investment in region ι

$$I_\iota \geq 0, \quad q_\iota \leq 1, \quad I_\iota(1-q_\iota) = 0 \tag{11}$$

and the goods market equilibrium condition is given by

$$Y_1 + Y_2 = \dot{K}_1 + \dot{K}_2 + \delta(K_1+K_2) + G_1 + G_2 + C \tag{12}$$

It is now possible to derive alternative types of equilibria.[9]

- *Balanced steady states* arise in case of a constant capital ratio, k. Starting point is an initial equilibrium in which capital productivities in both regions are equalized, $\partial Y_1/\partial K_1 = \partial Y_2/\partial K_2$. Then ongoing positive investment in both regions arises and both regions grow according to (9) with r being derived from (4).

- *Concentrated steady states* are the consequence of an initial disequilibrium, $\partial Y_1/\partial K_1 \neq \partial Y_2/\partial K_2$ and a capital ratio k tending either to zero or to infinity. In the latter case, mobile labor, capital and eventually also output are concentrated in region 1 and a core–periphery structure results in which region 1 is the core and region 2 becomes the periphery. Together with (5a) the marginal product of capital in region 1 then results as

$$\frac{\partial Y_1}{\partial K_1} = \frac{\alpha(\tilde{g}_s+\beta)+\gamma\varepsilon_R\tilde{g}_s}{\tilde{g}_s+\beta} L_1^\lambda M_1^\mu \left(1+\frac{\beta}{\tilde{g}_s}\right)^\gamma \left(\frac{G_1}{K_1}\right)^\gamma N_1^{\gamma(\varepsilon_A-\varepsilon_R)} \tag{13}$$

[8] If $q_\iota > 1$, investment would be infinite, hence it is sufficient to deal with $q_\iota = 1$.

[9] As will be shown below it is also possible to derive alternative numbers of equilibria, each of them being assigned to a certain type of equilibria.

which replaces the interest rate in (9). Note that $\tilde{g}_s$ includes the capital ratio, k, so that increasing the spatial distribution of capital affects the marginal product of capital in region 1 and hence influences further investment there. If k continuously increases, again a constant growth rate results but now only in region 1. Then a core–periphery structure arises: Eventually all mobile factors agglomerate there, only immobile labor and the firms remains in region 2.[10]

- *Transitions to balanced steady states*: Starting point of the argumentation is distribution of capital such that $R \neq 1$. Then future investment will only take place in regions yielding higher returns whereas the capital stock in the other region shrinks with the rate of depreciation, δ. The transition may be described by the following differential equations

$$\dot{K}_1 = Y_1 + Y_2 - \delta K_1 - C - (G_1 + G_2) \tag{14a}$$

$$\dot{K}_2 = -\delta K_2 \tag{14b}$$

$$\frac{\dot{C}}{C} = \sigma\left((\alpha + \gamma\varepsilon_R)\frac{\partial Y_1}{\partial K_1} - \delta - \rho\right) \tag{14c}$$

$$\dot{q}_2 = \left(\frac{\partial Y_1}{\partial K_1} - \delta\right) q_2 - \frac{\partial Y_2}{\partial K_2} \tag{14d}$$

which hold as long as $q_2 < 1$. (14a) is the goods market equilibrium condition, (14b) is due to exclusive investment in region 1, (14c) describes the Keynes–Ramsey rule, and (14d) is the equilibrium condition of the asset market. Note that in (14a) capital accumulation in region 1 is negatively affected by governmental policy in region 2 as it is assumed that the provision of $G_1 + G_2$ is realized out of global income $Y_1 + Y_2$. The regional decision about the governmental input is described in section 7.

4 Labor migration and capital productivities

Labor migration A migration equilibrium requires that mobile labor is distributed such that wages of mobile workers, $m(t)$, are equalized across regions at any time. Due to perfect mobility of mobile workers the migration equilibrium is given when marginal productivities of M_ι in both regions coincide, hence

$$\frac{\partial Y_1}{\partial M_1} = \frac{\partial Y_2}{\partial M_2} \quad \Rightarrow \frac{Y_1}{Y_2} = \frac{M_1}{M_2} \tag{15}$$

[10]However, firms in region 2 continue to produce though the capital stock declines with the constant depreciation rate δ, since the capital stock K_2 will never vanish completely.

Perfect mobility thus implies that the output ratio equals the ratio of mobile labor. Utilizing (4) and denoting $l \equiv L_1/L_2$, the output ratio of both regions can be written as

$$\frac{Y_1}{Y_2} = \left[l^{\lambda} k^{\alpha} \left(\frac{g_s + \beta}{1 + \beta g_s} \right)^{\gamma} \right]^{\frac{1}{1-\mu}} \tag{16}$$

Lower case letters reflect the distribution of the respective parameter across the two regions. In this context only the relative sizes and not their absolute levels gain importance. For given elasticities and given l, the distribution of mobile labor across regions only depends on the distribution of private capital, k, governmental activity as incorporated in g_s, and the spatial scope of governmental activity, β.[11]

Capital productivities Households may use income either consumtive or investive. The homogenous good is freely traded between the regions without transportation costs. Adjustment costs do not arise so that one unit of the good may be transformed into one unit of installed capital. We assume that installed capital cannot be relocated once being nailed to the ground in one region. Capital will be invested in the region with the higher marginal capital productivity. Denote

$$R \equiv \frac{\partial Y_1}{\partial K_1} / \frac{\partial Y_2}{\partial K_2} \tag{17}$$

If capital productivities are equalized (i. e. $R = 1$) the distribution of capital is stationary and *balanced steady states* arise. In this case investment in both regions is positive and both capital stocks grow at the same rate. In case of productivity disparities (i. e. $R \neq 1$) the prevailing capital ratio is not stationary but *transitions with k increasing (if $R > 1$) or decreasing (if $R < 1$) over time* will take place. Since we assumed that capital is immobile once it has been nailed down, a transition with increasing k implies that there is only investment in region 1 and no investment in region 2. The capital stock in region 2 then declines with the depreciation rate.

5 Convergence and divergence

For studying the model's dynamics, one has to see how productivities of private capital in both regions depend on the distribution of capital across the regions and governmental activity, given that mobile labor is distributed such that wages are equalized across regions any time. To do so the ratio R may be derived from the specified production functions (4) together with (15) and (16). It may be rewritten as function of the capital ratio, k, as well as of the ratio of governmental activity, g, as incorporated in $\tilde{g}_s$ and

$$R = \left[l^{\lambda} k^{\mu-(1-\alpha)} \left(\frac{\tilde{g}_s + \beta}{1 + \beta \tilde{g}_s} \right)^{\mu-(1-\gamma)} \right]^{\frac{1}{1-\mu}} \cdot \left(\frac{\alpha(\tilde{g}_s + \beta) + \gamma \varepsilon_R \tilde{g}_s)}{\alpha(1 + \beta \tilde{g}_s) + \gamma \varepsilon_R} \right) \tag{18}$$

[11]Note the the variable g_s includes scale effects in the equilibrium.

Since we focus on a growing economy, we assume that the governments in both regions set the aggregate expenditure levels, G_ι, as a constant fraction of aggregate output, K_ι, namely[12]

$$G_\iota = \theta_\iota \bar{K_\iota} \tag{19}$$

so that an expansion in government expenditure is parameterized by an increase in the capital share, θ_ι. Then

$$\tilde{g}_s = \theta k^{1+\varepsilon_A} n^{1+\varepsilon_A-\varepsilon_R}, \quad \theta \equiv \frac{\theta_1}{\theta_2} \tag{20}$$

Taking logarithms from (18)

$$i(k) \equiv (\mu-(1-\alpha))\ln k + (\mu-(1-\gamma))\ln\left(\frac{\tilde{g}_s+\beta}{1+\beta\tilde{g}_s}\right) + (1-\mu)\ln\left(\frac{\alpha(\tilde{g}_s+\beta)+\gamma\varepsilon_R\tilde{g}_s)}{\alpha(1+\beta\tilde{g}_s)+\gamma\varepsilon_R}\right) \tag{21}$$

may be derived and

$$R \gtreqless 1 \quad \Longleftrightarrow \quad i(k) \gtreqless -\lambda \ln l \tag{22}$$

A balanced steady state is attained at point(s) k^* solving $i(k^*) = -\lambda \ln l$. Then $R = 1$ and the marginal capital productivities are equalized across both regions; both regions grow at constant rates and thus the capital ratio stays constant.[13] The term $-\lambda \ln l$ reflects a threshold value. It is independent of the capital ratio, k and decreases in l and λ. In case of symmetric distribution of immobile labor, $l = 1$, the term vanishes.

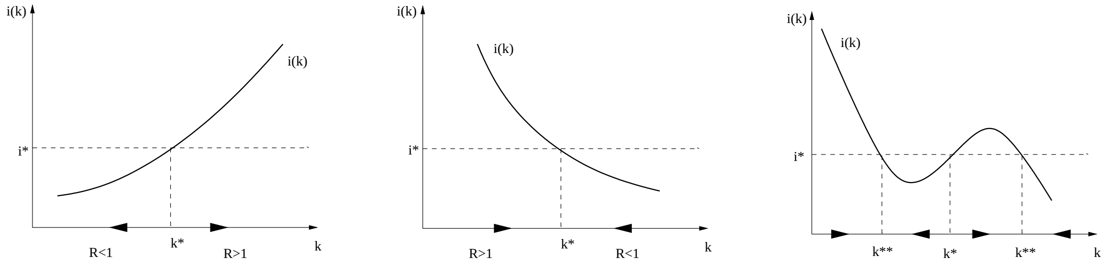
Basically it is possible to attain either no equilibrium, one unique equilibrium or three (and hence multiple) equilibria. The equilibria may show different stability characteristics: they may be stable or unstable. Stable equilibria arise whenever capital ratios outside the equilibrium strive towards the equilibrium k^*. If the capital ratio departs continuously from the equilibrium, the underlying equilibrium is unstable.

The stability implications are indicated by the arrows within Figures 1(a) – 1(c). As will be discussed below, different stability characteristics arise in case of multiple equilibria, two of them being stable and an unstable one.

Intuitively, if starting from a steady state, k^*, an increase of k generates a capital productivity advantage in region 1, this region becomes even more attractive for investment thus inducing further increases of k. Hence the economy departs from the steady state and the regions diverge. Formally, this situation arises whenever function $i(k)$ is positively sloped. If on the contrary in the equilibrium the function $i(k)$ is negatively sloped, an increase

[12]The derived results are equivalent to assuming $G_\iota = \theta Y_\iota$ but the formulation in (19) keeps the formal analysis much simpler.

[13]Note that it is anyhow possible that then both regions diverge with respect to their absolute levels of output, governmental input or private capital.



(a) $f'(k) > 0$: divergence (b) $f'(k) < 0$: convergence (c) $f'(k) \gtrless 0$: multiple eq.

Figure 1: Convergence, divergence and multiple equilibria

in k reduces the ratio of capital productivities and $R < 1$, thus giving rise to a productivity advantage in region 2. The ratio k declines and converges to its original steady state value.[14]

Convergence and divergence forces In the following we discuss issues of convergence and divergence in the context of function $i(k)$. Taking the first derivative yields

$$\frac{\partial i(k)}{\partial k} = (\mu - (1-\alpha))\frac{1}{k} \quad + \tag{23a}$$

$$(\mu - (1-\gamma)) \quad (1+\varepsilon_A)\frac{\tilde{g}_s}{k} \quad \frac{(1-\beta)(1+\beta)}{(\tilde{g}_s+\beta)(1+\beta\tilde{g}_s)} \quad + \tag{23b}$$

$$(1-\mu) \quad (1+\varepsilon_A)\frac{\tilde{g}_s}{k} \quad \frac{(\alpha(1-\beta)+\gamma\varepsilon_R)(\alpha(1+\beta)+\gamma\varepsilon_R)}{[\alpha(\tilde{g}_s+\beta)+\gamma\varepsilon_R\tilde{g}_s][\alpha(1+\beta\tilde{g}_s)+\gamma\varepsilon_R]} \tag{23c}$$

The first term (see (23a)) only includes convergences forces. This is due to the condition of competitive prices at the factor markets which effectuates unequivocally that $\mu - (1 - \alpha) < 0$. Note that this condition is equivalent to $\mu + \alpha < 1$ and hence reflects the fact of decreasing returns to mobile labor and private capital as long as the impact of capital within D_ι is neglected.

The analysis of (23b) is a bit more complex: The term decreases with rising β and vanishes if $\beta = 1$. Hence the arising forces are the stronger the less the regional interdependencies are. For all feasible parameter constellations and given that $\beta < 1$, the term is positive (negative) whenever $(\mu - (1-\gamma))(1+\varepsilon_A)$ is positive (negative). Together with the condition for endogenous growth, it is possible to derive a value denoted $\overline{\varepsilon}_A$ that satisfies $\mu+\gamma-1 \gtrless 0$ whenever $\varepsilon_A \lesseqgtr \overline{\varepsilon}_A \equiv \frac{\mu-\alpha}{1-\mu}$. Hence the sign of the entire term depends upon the interdependency between $\mu+\gamma-1$ and $1+\varepsilon_A$.

Note that due to the definition of $\overline{\varepsilon}_A$, parameter values $\varepsilon_A > 0$ ($\varepsilon_A < 0$) require $\mu > \alpha$ ($\mu < \alpha$). Depending upon the prevailing level of ε_A it is possible to distinguish four cases (see Table 1):

[14]Note that convergence in this context implies that the balanced steady state is stable whereas instable equilibria induce divergence.

Table 1: Convergence and divergence forces in (23b) if $\beta < 1$

	-1 < $\varepsilon_A < 0$	$\varepsilon_A > 0$
$\mu < \alpha$	divergence	indeterminate
$\mu > \alpha$	indeterminate	convergence

Divergence forces arise if $-1 < \varepsilon_A < \frac{\mu-\alpha}{1-\mu}$. Basically, this condition may be fulfilled for positive or negative values of ε_A. Convergence forces exist if $\varepsilon_A > \frac{\mu-\alpha}{1-\mu} > -1$, and again the corresponding level of ε_A may be positive or negative depending upon the relationship between μ and α.

The third term (23c) is positive if $\mu < 1$ and $\varepsilon_A > -1$, which both reflect sensible parameter constellations, and hence includes unequivocal divergence forces. The term decreases with rising β. If $\beta = 0$ the term reduces to $1/\tilde{g}_s$ for all degrees of congestion. If $\beta = 1$ and $\varepsilon_R = 0$ the term vanishes.

To sum up the convergence/divergence implications of (23) one finds convergence forces in (23a), either convergence or divergence forces in (23b) depending upon the concrete parameter constellations and divergence forces in (23c).

Symmetry It is possible to evaluate the sign of (23) for given capital ratios and then to make clear whether the equilibrium is stable or unstable. Multiple equilibria arise if the sign changes with increasing k. Provided that $\tilde{g}_s = l = 1$, (23) simplifies to

$$\frac{\partial i(k)}{\partial k} = \frac{1}{k}\left[\underbrace{\mu+\alpha-1}_{force1} - \underbrace{(\mu-1)(1+\varepsilon_A)\frac{2\beta\gamma\varepsilon_R}{(1+\beta)(\alpha(1+\beta)+\gamma\varepsilon_R)} + (1+\varepsilon_A)\gamma\frac{1-\beta}{1+\beta}}_{force2}\right] \tag{24}$$

and consequently

$$\frac{\partial i(k)}{\partial k} \gtrless 0 \quad \Longleftrightarrow \quad force1 \gtrless force2 \tag{25}$$

As already argued, competitive factor markets ensure that $force1$ is always negative thus inducing convergence. If however $force2$ is even smaller, the sign of (24) is negative. It is exactly this combination that cause multiple equilibria. Summarizing this finding leads to the following statement: Multiple equilibria arise if

$$0 > \mu+\alpha-1 > (\mu-1)(1+\varepsilon_A)\frac{2\beta\gamma\varepsilon_R}{(1+\beta)(\alpha(1+\beta)+\gamma\varepsilon_R)} - (1+\varepsilon_A)\gamma\frac{1-\beta}{1+\beta} \tag{26}$$

Numerical simulations within the next Section will help to enlighten these complex interdependencies.

6 Agglomeration: Concept and simulations

As argued before it is possible to derive parameter constellations that end up in an unique equilibrium capital ratio that is stable (unstable) if convergence (divergence) forces dominate. The equilibrium arises whenever $i(k)$ as given by (23) equals the threshold value $-\lambda \ln l$.[15] Increasing l or λ shifts this value downwards for all k. Due to the interaction of the convergence – divergence forces it is also possible that the economy ends up in a situation with multiple equilibria, one of them being unstable and two stable ones.[16] In case of symmetry, i. e. if $\tilde{g}_s = l = 1$, the unstable equilibrium results at a capital ratio equal to $k = 1$. Then the regions also parallel with respect to their capital stocks. Compared to this unstable equilibrium agglomerations arise whenever k departs from equal distribution with region 1 being the core (periphery) whenever $k^{**} > 1$ ($k^{**} < 1$).

To generalize, stable equilibria are called agglomerations with high concentration of mobile factors either in region 1 or region 2. The corresponding equilibrium capital ratio is then either high or low while the unstable equilibrium with an intermediate capital ratio lies in between the two stable ones. Changes in the threshold value $-\lambda \ln l$ may have consequences not only for the arising level of the equilibria but also for their number and their stability characteristics. As becomes apparent also within (26) several factors affect the existence, number and stability characteristics of the equilibria. To enlighten the complex interdependencies, numerical simulations will be carried out for the impact of μ, n and β.

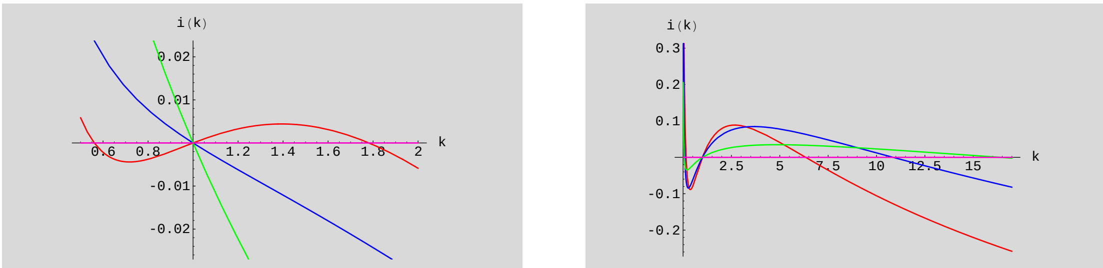
Simulations The nonlinear function represents $i(k)$ from (21) whereas the magenta function represents the threshold value $-\lambda \ln l$ which equals the abscissa since we assume equal distribution of immobile labor, i. e. $l = 1$. Other parameter values are as indicated below Figures 2 – 4. Since changes in the level of relative congestion, ε_R, only have level effects but do not affect the structure of the equilibrium we suppose intermediate relative congestion, $\varepsilon_R = 0.5$ and refrain from a detailed discussion. The non–linear functions represent alternative levels of absolute congestion, ε_A, and again the colors imply levels of absolute congestion as indicated below the Figures. Equilibria arise at intersections between the non–linear functions and the abscissa. Note that within all Figures (at least) one equilibrium at $k = 1$ arises. This is the consequence of the assumed parameters.

Productivity of mobile labor, μ, and ε_A In the context of Figure 2 it is possible to analyze the implications of the partial production elasticity of mobile labor, μ, on the stability characteristics of the model. It turns out that then the level of ε_A is of major importance for the equilibria. From Figure 2(a) it becomes apparent that for low levels, $\mu = 0.2 < \alpha$,

[15] Graphical illustrations for unique equilibria are given in Figures 1(a) and 1(b) where the threshold value is denoted by i^*.

[16] See Figure 1(c) where the stable equilibria are denoted by k^{**} and the unstable equilibrium is given by k^*.

alternative numbers of equilibria arise depending upon the prevailing ε_A. If ε_A is relatively high (blue and green functions), an unique stable equilibrium in $k = 1$ arise. If ε_A decreases sufficiently eventually $force1 > force2$ and multiple equilibria occur. Then the initially stable symmetric capital distribution, $k = 1$, becomes unstable and two stable equilibria with k being smaller or larger than 1 appear. Consequently the level of absolute congestion affects the curvature of $i(k)$. The intuition for this result is as follows: For relatively high values ε_A the convergence forces dominate for all capital ratios, k. Then absolute congestion includes a positive externality that arises from the governmental input.



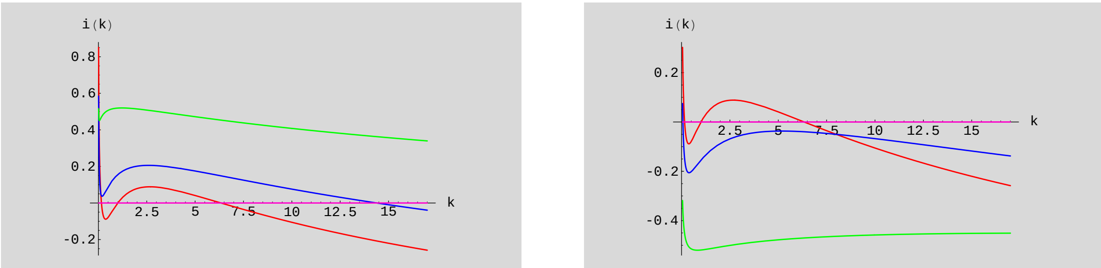
(a) $\mu = 0.2$ (b) $\mu = 0.4$

Figure 2: equilibria arising from $i(k)$ in (21) for alternative levels of μ and ε_A
parameters: $\alpha = 0.25$, $\beta = 0.5$, $\varepsilon_R = 0.5$, $\theta = 1$, $l = 1$, $n = 1$
functions $i(k)$ for alternative values of absolute congestion: red: $\varepsilon_A = 0.5$, blue: $\varepsilon_A = 0$, green: $\varepsilon_A = -0.5$; threshold value $-\lambda \ln l$ for $l = 1$ in magenta

If however ε_A falls short of $\bar{\varepsilon}_A$ then either convergence or divergence dominate depending upon the prevailing capital ratio, k. Now $k = 1$ represents a situation in which divergence forces dominate, $i(k)$ is upward sloping and the equilibrium is unstable. The other equilibria however are stable and imply either agglomeration in region 1 (high level of k^{**}) or in region 2 (low level of k^{**}).

Figure 2(a) also has another implication: Note that multiple equilibria only arise for certain levels of l that are close to $l = 1$. Strong deviations from the uniform distribution induces upward or downward shifts of the threshold level function. It is then possible that finally only one stable equilibrium exists with agglomeration in region 1 and hence $k^* > 1$ if this region has relatively more immobile labor (high l) or agglomeration in region 2 and $k^* < 1$ given that region 2 is relatively better endowed with immobile labor (small l). Note that this results independent of the value of ε_A and thus the shape of all three functions $i(k)$ remains unchanged.

Figure 2(b) assumes the same parameter constellations, except that now $\mu = 0.4 > \alpha$. Then for given equal distribution of immobile labor, $l = 1$, three equilibria arise for all levels of absolute congestion and $k^* = 1$ is always unstable. The level of ε_A affects the distance to the unstable equilibrium and it becomes apparent that stable equilibria are the closer to $k^* = 1$, the higher the prevailing level of ε_A.



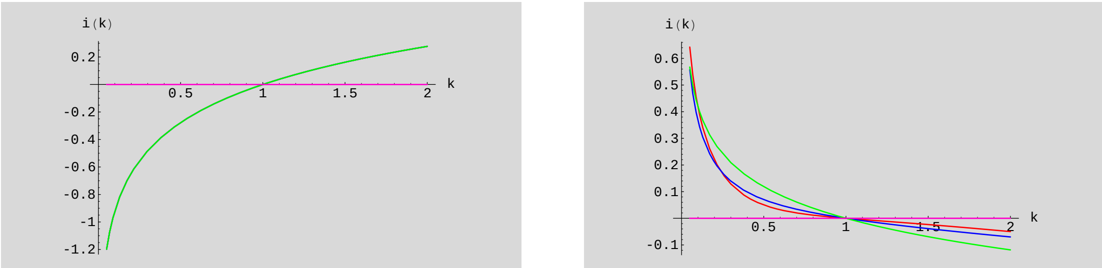
(a) $n = 0.5$ (b) $n = 2$

Figure 3: $i(k)$ in (21) for alternative levels of n
parameters: $\alpha = 0.25$, $\beta = 0.5$, $\varepsilon_R = 0.5$, $\theta = 1$, $\mu = 0.4$
functions $i(k)$ for alternative values of absolute congestion: red: $\varepsilon_A = 0.5$, blue: $\varepsilon_A = 0$, green: $\varepsilon_A = -0.5$; threshold value $-\lambda \ln l$ for $l = 1$ in magenta

Figure 3 focuses on the impact of the number of firms that are settled in one region. The analysis is carried out for $\mu = 0.4$ but the structural results hold for all other values of μ. In Figure 3(a) the number of firms in region 2 is twice as much as in region 1, $n = 0.5$, while Figure 3(b) represents the opposite case.[17] Again, whether or not multiple equilibria arise depends upon the regional distribution of immobile labor, l, and the prevailing degree of absolute congestion, ε_A. In Figure 3(a) again $\varepsilon_A = 0.5$ (red function) implies three equilibria with $k^* = 1$ being unstable, while less absolute congestion (blue and green function) end up with one stable equilibrium distribution, k^*, that increases with a reduction of ε_A. Consequently negative absolute congestion strengthens capital accumulation in region 1 even if the number of firms located there is only half of those in region 2. Figure3(b) however just may be interpreted as being a mirror of the situation just illustrated. The equilibrium value k^* now decreases with a reduction in absolute congestion and the intuition for this result may be drawn in analogy.

Implication of the scope β Figure 4 analyzes the spatial dimension within the model. It includes the benchmark cases of no interaction ($\beta = 0$) and complete interaction ($\beta =$

[17] Note that equal distribution of the number of firms across regions, $n = 1$, for the given parameters can be seen in Figure 2(b).



(a) $\beta = 0$ (b) $\beta = 1$

Figure 4: $i(k)$ in (21) for alternative levels of β
parameters: $\alpha = 0.25$, $\varepsilon_R = 0.5$, $\theta = 1$, $\mu = 0.4$, $n = 1$
functions $i(k)$ for alternative values of absolute congestion: red: $\varepsilon_A = 0.5$, blue: $\varepsilon_A = 0$, green: $\varepsilon_A = -0.5$; threshold value $-\lambda \ln l$ for $l = 1$ in magenta

1). Again the intermediate case for the parameter constellations in Figure 4 is given by Figure 2(b) which represents $\beta = 0.5$. In case of no spatial interaction ($\beta = 0$) the function $i(k)$ is identical for all levels of absolute congestion and the three functions (red – blue – green) now coincide and are given by the green function in Figure 4(a). It becomes apparent that one unstable equilibrium arises and the equilibrium capital ratio, k^*, decreases with a rise in l. The more immobile labor is settled in region 1 (higher l) the lower is the optimal capital ratio. There is no interaction between the regions and the diffusion externality does not appear.

If in contrast the spatial interaction is perfect ($\beta = 1$) then an unique equilibrium appears that now is stable. Deviations from $k^* = 1$ do not sustain. The externalities associated with the local governmental inputs penetrate the other region and the mechanisms described before arise. Note that absolute congestion does not affect $k^* = 1$ but this is only given as long as $l = 1$. In case of unequal distribution of immobile labor (shifts of the magenta function) the arising capital ratio is actually affected by ε_A.

7 Efficiency

In order to judge about different agglomeration scenarios it is necessary to compare them with the social optimal situation: Is agglomeration all bad? Which is the optimal degree of concentration? And is equilibrium concentration suboptimally high or low?

The efficient solution internalizes congestion externalities and diffusion externalities. On the one hand side, individuals neglect their influence on aggregate capital, hence they overestimate the individually available amount of the congested governmental input: there is a negative externality of capital accumulation. On the other hand side, regional governments usually neglect the productivity impact of governmental activity on the other region: there is a positive externality of governmental activity. We start with the consideration of the congestion externality. In order to evaluate the socially optimal degree of concentration, we have to take into account that private investment increases aggregate capital and hence reduces the individually available amount of the public input. If firms enlarge their truck fleet (private investment) the motorways get more crowded and there is less infrastructure applicable for each firm. The congestion function then amounts to

$$G_{sl} = \theta_l N_l^{1+\varepsilon_A-\varepsilon_R} K_l^{1+\varepsilon_A} \tag{27}$$

Hence, the optimal relation between the physical capital stocks, k, is found by maximizing the sum of income of both regions $F = Y_1 + Y_2$ with respect to k. Aggregate capital is given by $K = K_1 + K_2$, hence physical capital in region 1 amounts to $K_1 = kK_2$ and capital in region 2 is given by $K - kK_2$

$$\begin{aligned}\frac{\partial F}{\partial k} =&(F_{K_1} - F_{K_2})K_2 \\ =&\frac{Y_1}{k(g_s+\beta)}\left(\alpha(g_s+\beta)+\gamma(1+\varepsilon_A)(g_s-\beta k)\right) \\ &-\frac{Y_2}{1+\beta g_s}\left(\alpha(1+\beta g_s)\gamma(1+\varepsilon_A)\left(1-\frac{\beta g_s}{k}\right)\right)\end{aligned} \tag{28}$$

This leads to socially optimal capital accumulation determined by

$$\frac{Y_1}{Y_2 k}\frac{1+\beta g_s}{g_s+\beta}\frac{\alpha(g_s+\beta)+\gamma(1+\varepsilon_A)(1-\beta k)}{\alpha(1+\beta g_s)+\gamma(1+\varepsilon_A)\left(1-\frac{\beta g_s}{k}\right)} - 1 \gtrless 0 \tag{29}$$

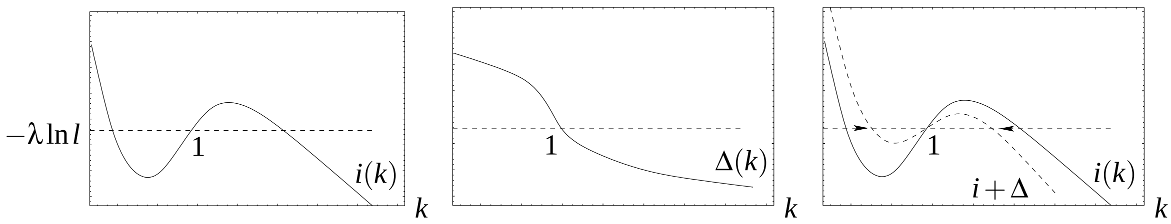
$$\iff \quad i(k) + \Delta(k) \gtrless -\lambda \ln l \tag{30}$$

with $i(k)$ as given in equation (21) and $\Delta(k)$ defined as

$$\Delta(k) = (1-\mu)\left(\ln\left(\frac{\alpha(1+\beta g_s)+\gamma\varepsilon_R}{\alpha(g_s+\beta)+\gamma\varepsilon_R g_s}\right) + \ln\left(\frac{\alpha(g_s+\beta)+\gamma(1+\varepsilon_A)(1-\beta k)}{\alpha(1+\beta g_s)+\gamma(1+\varepsilon_A)\left(1-\frac{\beta g_s}{k}\right)}\right)\right) \tag{31}$$

$\Delta(k)$ reflects the negative externality of capital accumulation and adjusts the ratio of private capital returns to the socially relevant relation. Δ decreases in k and goes through zero for the symmetric case. Furthermore, Δ is bounded from above with $\overline{\Delta} = \ln(\alpha + \gamma\varepsilon_R)$ and from below with $-\overline{\Delta}$. Therefore, the dynamics of the optimal capital relation are delivered according to figure 5.

The fact that private investment increases aggregate capital and therefore reduces the availability of the public input alters the ratio between the capital returns in the two regions. Figure 5(c) shows that agglomeration is socially optimal. Nevertheless, concentration is suboptimally high. Since individuals overestimate private capital return, they react too sensitive with respect to a regional difference in capital return. As a consequence, the degree of concentration is suboptimally high in market equilibrium.



(a) Equilibrium agglomeration. (b) Congestion externality. (c) Optimal agglomeration.

Figure 5: Equilibrium versus optimal dynamics.

The remaining point refers to the diffusion externality: Public inputs like harbors are supraregionally productive. If region 1 increases the provision of public inputs, the productivity in both regions rises. Within the optimal choice of governmental expenditures, G_i, this impact has to be properly considered. The optimal relation θ of regional public inputs ratios is found by maximizing aggregate income $F = Y_1 + Y_2$ with respect to θ and taking into account that $G_1 = \theta k n G_2$ and $G_2 = G - \theta k n G_2$ apply. The resulting condition for optimal governmental activity is

$$\begin{aligned} \frac{\partial F}{\partial \theta} =& F_{G_1} \frac{\partial G_1}{\partial \theta} + F_{G_2} \frac{\partial G_2}{\partial \theta} \overset{!}{=} 0 \\ \iff \quad & \frac{Y_1}{Y_2} \frac{1+\beta g_s(\theta)}{g_s(\theta)+\beta} = \frac{1-\beta k^{\varepsilon_A} n^{\varepsilon_A - \varepsilon_R}}{k^{\varepsilon_A} n^{\varepsilon_A - \varepsilon_R} - \beta} \end{aligned} \tag{32}$$

Using equation (16) to replace Y_1/Y_2 yields

$$\left(\frac{g_s(\theta^*)+\beta}{1+\beta g_s(\theta^*)} \right)^{\frac{\gamma+\mu-1}{1-\mu}} = \frac{1-\beta k^{\varepsilon_A} n^{\varepsilon_A - \varepsilon_R}}{k^{\varepsilon_A} n^{\varepsilon_A - \varepsilon_R} - \beta} (l^{\lambda} k^{\alpha})^{\frac{1}{\mu-1}} \tag{33}$$

Within the equilibrium analysis given in the last section, the ratio of governmental activity, θ, was assumed to be arbitrarily set. Nevertheless, a regional government would decide about the amount of governmental activity, G_i, by equating marginal costs and benefits. As the homogenous good may be changed 1:1 into governmental expenditures, marginal costs of an increase in G_i are 1. Marginal benefits result from increased productivity. It is self–evident to assume that regional governments are only concerned about the productivity in their own region. They disregard the diffusion of public inputs and hence the diffusion externality. Usually, a regional government will only provide a harbor, if the productivity gain in its own government is sufficient to warrant the harbor. The regional government will not take into account that due to the harbor other regions will experience increased productivity.

Hence, both regions equate marginal benefits and marginal costs of governmental activity according to

$$Y_{1G_1} \stackrel{!}{=} 1 \quad \text{and} \quad Y_{2G_2} \stackrel{!}{=} 1 \quad \Longrightarrow \quad Y_{1G_1} \stackrel{!}{=} Y_{2G_2}$$
$$\Longleftrightarrow \quad \frac{Y_1}{Y_2}\frac{1+\beta g_s(\theta)}{g_s(\theta)+\beta} = \frac{1}{k^{\varepsilon_A} n^{\varepsilon_A-\varepsilon_R}} \tag{34}$$

Replacing again Y_1/Y_2 by (16) leads to

$$\left(\frac{g_s(\tilde{\theta})+\beta}{1+\beta g_s(\tilde{\theta})}\right)^{\frac{\gamma+\mu-1}{1-\mu}} = \frac{1}{k^{\varepsilon_A} n^{\varepsilon_A-\varepsilon_R}}(l^{\lambda}k^{\alpha})^{\frac{1}{\mu-1}} \tag{35}$$

Comparing the optimal ratio of governmental activity, θ^*, and the corresponding equilibrium value, $\tilde{\theta}$, in the symmetric case yields $\theta^* = \tilde{\theta}$. The relative impact of the positive diffusion externality is of the same magnitude in each region. Hence, the relation between governmental expenditures is unaffected. Nevertheless, the level of governmental expenditures is suboptimally low.[18] Applying this result to figure 5 demonstrates that selfish governmental behavior has no impact on the degree of agglomeration compared with the optimal governmental activity.

8 Conclusions

In this paper we analyzed the impact of governmental activity on agglomeration. Regional public inputs usually are supraregionally productive. E. g. basic research or infrastructure which are provided in one region, induce a productivity shift in all surrounding regions. There is a diffusion of the productive impact of governmental activity. Since this diffusion is usually disregarded within the governments' decision about the level of the public input, it causes a diffusion externality. The supraregional impact of governmental activity depends mainly on the characteristics of the public input, predominantly concerning congestion aspects. A pure public good, e. g. basic research, exhibits a stronger diffusion externality than inner–city infrastructure. Consequently, the type of the regional public input has a major impact on relative capital accumulation and hence the localization of economic activity.

We show that due to convergence–divergence forces the economy may end up in a situation with multiple equilibria, one unstable and two stable ones called agglomerations. The agglomerations are affected by the size of the regions (amount of immobile labor and number of firms) as well as on the degrees of relative and absolute congestion of the public input and on the diffusion intensity. For high productivity of mobile labor, the divergence forces ceteris paribus dominate around the symmetric distribution of physical

[18] This is easily seen as the direct marginal returns, Y_{iG_i}, are lower than the socal returns, F_{G_i}.

capital and induce equilibrium agglomeration. The degree of absolute congestion displays the positive or negative spillover effects of (mobile) capital. Hence, with an increase in absolute congestion, negative externalities gain importance and the capital distribution in the equilibrium agglomerations is more unequal. Anyway, optimal agglomeration internalizes the congestion externality as well as the diffusion externality and therefore is less concentrated than agglomeration in market equilibrium.

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