The Stability of the Banking Sector and Credit Default Swaps

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Abstract

This paper considers credit default swaps (CDS) in which both, the protection buyer and the protection seller is a bank. These CDS improve the diversification of the banks' risk which encourages the banks to invest more into an illiquid, risky credit portfolio and less into a relatively liquid, safe asset. On the one hand, this bank behavior implies a higher expected utility of their depositors, but on the other hand, it reduces the stability of the banking sector in a macroeconomic downturn, and it may reduce this stability in a macroeconomic upturn.

JEL classification: G10, G21

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1 Introduction

Since the end of the 1990s the use of credit derivatives, which allow for the transfer of credit risks, has increased substantially. According to the British Bankers' Association (2006), the size of the global credit derivatives market increased from 586 billion US dollars in the year 1999 to 20,207 billion US dollars in 2006, and it is expected that it will have expanded to 33,120 billion US dollars in the year 2008. The major market participants are banks, hedge funds, and insurance companies. Although the market share of hedge funds has increased substantially during the last six years, banks still constitute the majority of market participation in both buying and selling credit protection.[1] The mostly traded product in the credit derivatives market is the credit default swap (CDS).[2] CDS are bilateral contracts where the risk seller pays a fixed periodic fee to the risk buyer in exchange for a payment contingent on a credit event such as the default on a reference asset (compensation payment).

It is especially the rapid growth of the credit derivatives market which has evoked an ongoing debate on the consequences of the use of credit derivatives for financial stability. In the literature, there is no unambiguous answer to the question of whether these derivatives have a positive or negative effect on financial stability. The Deutsche Bundesbank (2004) argues that on the one hand, developed and liquid credit risk transfer markets would allow for a broader diversification and more efficient price-setting which would improve the allocation of credit risk and therefore would foster financial stability. However, on the other hand, there would be risks involved in credit risk transfer which could have a negative impact on financial stability. These risks would result from: ineffective safeguards and inaccurate ex-ante assessments of risk/return profiles, a high concentration of intermediary services on

[1] According to the British Bankers' Association (2006), in the year 2006 the banks' market share of buying credit protection was 59 %, of selling credit protection 44 %. The corresponding numbers for hedge funds were 28 % and 31 %. In the year 2000, the market share of banks was 81 % and 63 % respectively, that of hedge funds 3 % and 5 %.

[2] The major products in the credit derivative market are single-name credit default swaps, index trades, and synthetic collateralized debt obligations. According to British Bankers' Association (2006), the market share of single-name credit default swaps was 33 %, of full index trades 30 %, of tranched index trades 8 %, and of synthetic collateralized debt obligations 16 % in the year 2006.

only a small number of market participants, the possibility of using regulatory arbitrage, asymmetric information, and from the interaction between credit risk transfer markets and other financial markets. Wagner (2006) argues that new credit derivative instruments would improve the banks' ability to sell their loans making them less vulnerable to liquidity shocks. However, this again might encourage banks to take on new risks because a higher liquidity of loans enables them to liquidate them more easily in a crisis. This effect would offset the initial positive impact on financial stability. Wagner and Marsh (2006), on the other hand, argue that especially the transfer of credit risk from banks to non-banks would be beneficial for financial stability as it would allow for the shedding of aggregate risk which would otherwise remain within the relatively more fragile banking sector. However, Allen and Carletti (2006) show that the transfer between the banking sector and the insurance sector can lead to damaging contagion of systemic risk from the insurance to the banking sector as the credit risk transfer induces insurance companies to hold the same assets as banks. If there is a crisis in the insurance sector, insurance companies will have to sell these assets forcing down the price which implies the possibility of contagion of systemic risk to the banking sector since banks use these assets to hedge their idiosyncratic liquidity risk.

This short survey shows that there cannot be a general answer to the question of whether credit derivatives raise or lower financial stability, but that it is necessary to analyze specific aspects of this complex issue. This paper focuses on credit risk transfer within the banking sector and on CDS, i.e. CDS in which both the protection buyer and the protection seller is a bank are considered. The main result is that on the one hand, these CDS increase the expected utility of the banks' depositors but that on the other hand, they reduce the stability of the banking sector in a macroeconomic downturn, and that they may reduce this stability in a macroeconomic upturn. The crucial point is that this credit risk transfer induces the banks to reduce their investments into a safe, relatively liquid asset and to increase their investments into a risky, illiquid credit portfolio.

The basic story: We consider four banks. Each of them seeks to maximize the expected utility of its risk averse depositors. Each bank has to decide on how much of its deposits it invests, inter alia, into a risky, illiquid credit portfolio and how much it invests into a relatively liquid, safe asset, where the credit portfolio has a higher expected return than the safe asset. The risks of the credit portfolios of the four banks are independent from each other so that there is scope for diversification. The conclusion of CDS contracts allows to make use of the diversification possibility. The thereby reduced credit risk implies an increase in the depositors' expected marginal utility from the credit portfolio so that the banks, which seek to maximize the expected utility of their risk averse depositors, increase their investment into the credit portfolio and reduce their investment into the safe asset. The conclusion of the CDS contracts and the resulting investment shifting increases the depositors' expected utility, but in a macroeconomic downturn this bank behavior reduces the stability of the banking sector and in a macroeconomic upturn it may reduce this stability. The intuition is that in a macroeconomic downturn, with a corresponding high number of credit defaults, the investment shifting to the credit portfolio implies a decrease in the banks' assets and therefore in their ability to absorb an asset shock. Furthermore, the investment shifting to the illiquid credit portfolio impairs the banks' ability to absorb a liquidity shock which increases the danger of contagion. In a macroeconomic upturn, the number of credit defaults is relatively low and the investment shifting to the credit portfolio implies an increase in the banks' assets fostering financial stability. However, also in favorable macroeconomic circumstances the ability of the banking sector to absorb a liquidity shock may be impaired as a result of the investment shifting due to the illiquidity of the credit portfolio.

The remainder of this paper is structured as follows. Section 2 presents the model without CDS. In section 3, we insert CDS into the model and analyze how these contracts influence the banks' optimal investment decision, the expected utility of their depositors, and the banks' ability to absorb shocks which determines the stability of the banking sector. Section 4 briefly summarizes the paper.

2 Model

2.1 Technology and Liquidity Preferences

There are three dates $t = 0, 1, 2$. We consider a continuum of ex ante identical consumers. Each consumer has one unit of a consumption good which serves as a numéraire. This good can not only be consumed but also be invested in assets to produce future consumption. There are three types of assets: a short-term, safe asset; a long-term, safe asset; and a long-term, risky asset. The short-term, safe asset is represented by a storage technology: one unit of the consumption good at date t produces one unit of the consumption good at date $t+1$. The two long-term assets can only be invested at date 0. One unit of the consumption good invested in the long-term, safe asset produces $R > 1$ units of the good at date 2 or $0 < r < 1$ units at date 1. Consequently, the long-term, safe asset is not completely illiquid: It can be liquidated at date 1 but only at a loss. The investment of one unit of the consumption good into the long-term, risky asset, a credit portfolio, yields a random return K at date 2. With probability α the investment succeeds and $K = H > R$, with probability $1 - \alpha$ the investment fails and $K = L < 1$. We assume that $E(K) > R$. The uncertainty is resolved at date 1.

The consumers are risk-averse and have the usual Diamond-Dybvig preferences: with probability ω they are early consumers who only value consumption at date 1, with probability $1 - \omega$ they are late consumers who only value consumption at date 2. Their utility of consumption is represented by the function

$$U(c) = \ln c. \tag{1}$$

The uncertainty is restricted to the level of the individual consumers. At the aggregate level, we assume that the law of large numbers applies so that with probability one a fraction ω of all consumers are early consumers and a fraction $1 - \omega$ are late consumers. The uncertainty about the individual consumption preferences generates demand for liquidity and a role for banks which have a comparative advantage of providing this liquidity. Therefore, at date 0, each consumer deposits his consumption good in a bank.The deposit contract allows the depositor to withdraw either c_1

	A	B	C	D
S_1	ω_H	ω_L	ω_H	ω_L
S_2	ω_L	ω_H	ω_L	ω_H

Table 1: Fractions of Early Consumers

units of the consumption good at date 1 or c_2 units at date 2. The units c_1 and c_2 specified in the contract depend on the bank's investment decision. The bank has to choose how to split its deposits investing x units in the short-term asset, y units in the long-term, safe asset, and u units in the risky credit portfolio. We assume that banks compete to raise deposits so that the contract a bank offers maximizes its depositors' expected utility.

2.2 Interbank Deposit Market

Using the approach of modelling contagion introduced by Allen and Gale (2000), we consider four ex ante identical regions, labelled A, B, C, and D and assume that the behavior of the banks in each region can be captured by a representative bank. Each of the four representative banks has a continuum of measure one of depositors. The probability ω of being an early consumer, which is equal to the fraction of a representative bank's depositors who demand withdrawal at date 1, is a random variable. There are two possible values of ω, a high value (ω_H) and a low value (ω_L), where $0 < \omega_L < \omega_H < 1$. The probability for each value to be realized is the same. As Allen and Gale (2000), we define two states S_1 and S_2. If the state S_1 occurs, the banks A and C face a high demand for liquidity at date 1, but the banks B and D face a low demand and vice versa if the state S_2 occurs (see table 1). Consequently, each of the four banks faces an uncertain demand for liquidity, but there is no aggregate uncertainty. This allows for insurance via an interbank deposit market. As in Allen and Gale (2000, section IV.B), we assume that each representative bank holds deposits with only one adjacent representative bank:[3] bank A holds deposits with bank B, bank B with bank C, bank C with bank D and bank D with bank A. The units of the interbank deposits correspond to $\omega_H - \gamma$, where γ denotes the average fraction of early deposits which is given by $\gamma = (\omega_H + \omega_L)/2$. All uncertainty is resolved at date 1. Then each consumer learns whether he is an early or late consumer which reveals the actual fraction of early consumers for each bank. Then those banks with liquidity surpluses provide liquidity for banks with liquidity shortages.

2.3 Optimal Investment

Each representative bank chooses how to split its deposits, i.e. it has to choose x, y, and u, to maximize the depositor's expected utility. It must consider that a fraction of γ becomes early consumers and of $(1-\gamma)$ late consumers. The bank uses the proceeds of the short-term asset to provide the early consumers with a level of consumption

$$c_1 = \frac{x}{\gamma}. \tag{2}$$

The proceeds of the long-term assets are used to provide the late consumers with a level of consumption

$$c_2 = \begin{cases} \frac{yR+uL}{1-\gamma} =: c_{2,L} & \text{if} \quad K = L \\ \frac{yR+uH}{1-\gamma} =: c_{2,H} & \text{if} \quad K = H. \end{cases} \tag{3}$$

Consequently, each bank has to solve the optimization problem (note that c_1 depends on x and c_2 on y and u)

$$E[U] = \gamma \ln c_1 + (1-\gamma)\left[\alpha \ln c_{2,H} + (1-\alpha)\ln c_{2,L}\right] =: f(x,y,u) \to \max \tag{4}$$

[3] Since each representative bank holds deposits with only one adjacent bank, we assume that the interbank deposit market is incomplete. As Allen and Gale (2000, section IV.B.) we motivate this incompleteness with transaction and information costs which prevent banks from acquiring claims on banks in the other regions.

s.t.

$$x + y + u = 1, \quad x, y, u \geq 0, \quad \text{and} \quad c_1 \leq c_{2,L}. \tag{5}$$

The first term on the right hand side of the objective function represents the utility of the early consumers. The second term represents the expectation of the uncertain utility of the late consumers. It is uncertain since it depends on the outcome of the investment into the risky credit portfolio. The first constraint is the bank's budget constraint at date 0. The second constraint, the non-negativity constraint, implies that beside the deposits, the bank cannot take on more liabilities. The third constraint is the incentive compatibility constraint meaning that the bank's investment decision must ensure that a bank run is avoided, i.e. even in case it turns out at date 1 that the bad outcome of the credit portfolio will be realized at date 2, it must still be (weakly) optimal for the late consumers to withdraw at date 2. This constraint is due to an asymmetric information problem. The banks cannot distinguish between early and late consumers. Consequently, if $c_1 > c_{2,L}$, the late consumers will be better off withdrawing at date 1 which implies that all consumers pretend to be early consumers and demand withdrawal at date 1, i.e. a bank run occurs.[4]

If the inequality constraints are neglected, we obtain the following solution of the optimization problem:

$$x^* = \gamma, \tag{6}$$

$$y^* = \frac{(HR(1-\alpha) + LR\alpha - HL)(1-\gamma)}{(H-R)(R-L)}, \tag{7}$$

and

$$u^* = \frac{R(L(1-\alpha) + H\alpha - R)(1-\gamma)}{(H-R)(R-L)} \tag{8}$$

[4]In a situation without asymmetric information banks can distinguish between early and late depositors. Then, there is no incentive compatibility constraint, i.e. the banks can offer a contract in which $c_{2,L} < c_1 < c_{2,H}$ without causing a bank run if the bad state L is realized.

which implies that

$$c_1^* = 1, \tag{9}$$

and

$$c_2^* = \begin{cases} \frac{(H-L)R(1-\alpha)}{H-R} & \text{if} \quad K = L \\ \frac{(H-L)R\alpha}{R-L} & \text{if} \quad K = H. \end{cases} \tag{10}$$

Now we investigate, under which circumstances the inequality constraints are satisfied by this solution. The non-negativity constraints for x and u are automatically satisfied because of $x^* = \gamma > 0$ and $u^* = \frac{R(E(K)-R)(1-\gamma)}{(H-R)(R-L)} > 0$.

The constraint $y^* \geq 0$ is only satisfied if

$$1 - \alpha \geq \frac{L(H-R)}{R(H-L)}, \tag{11}$$

and the incentive compatibility constraint is only satisfied if

$$1 - \alpha \geq \frac{H-R}{R(H-L)}. \tag{12}$$

In what follows, we assume that

$$1 - \alpha > \frac{H-R}{R(H-L)}. \tag{13}$$

Due to this assumption, it holds that $c_1^* < c_{2,L}^*$ and $y^* > 0$, i.e. we assume that the expected return on the credit portfolio compared to the return on the safe asset is not that high that without asymmetric information the banks would offer a deposit contract in which $c_{2,L}^* < c_1^*$ (see footnote 4) and it is not that high either that the bank prefers to invest its long-term deposits totally into the credit portfolio. Consequently, all inequality constraints are satisfied, none of them is binding, the solution given by (6) to (8) solves the optimization problem, and c_1^* and c_2^* are the return payments from the bank to the consumer specified in the deposit contract.

Note that the promised return payment in the second period is state-dependent, i.e. as in Hellwig (1994) the depositors bear some risk.[5]

As in Allen and Gale (2000) a bank is assumed to be bankrupt if it cannot meet the demand of its depositors by liquidating all its assets. This will be the case if at date 1 a shock occurs implying that $c_2 < c_1^*$, for example. Then all depositors pretend to be early consumers, and all of them demand withdrawal at date 1. For satisfying this demand, which is equal to 1, the bank has to liquidate all its assets at date 1. The problem is that the liquidation value of its total assets at date 1 is $x^* + y^* r < 1$.[6] Consequently, the bank is not able to meet the demands of its depositors by liquidating all its assets and goes bankrupt. Note that the definition of bankruptcy implies that if the shock leads to $c_1^* \leq c_2 < c_{2,L}^*$, the bank will not be bankrupt although it will not be able to pay the promised units c_2^* of consumption to the late consumers. However, we assume that as long as $c_1^* \leq c_2$, it is (weakly) optimal for the late consumers to withdraw at date 2, i.e. the bank does not have to liquidate all its assets at date 1, the bank is not bankrupt.

2.4 Stability of the Banking Sector

The stability of the banking sector is reflected by its ability to absorb shocks. We assume that at date 1, there is an idiosyncratic asset shock: There is negative

[5] Furthermore, it is worth mentioning that in our model c_1^* is equal to 1, contrary to Diamond and Dybvig (1983) in which the return payment specified in the deposit contract at date 1 is strictly larger than 1. In the Diamond-Dybvig model, this result motivates the role for banks as $c_1^* > 1$ is Pareto optimal, and this optimum cannot be achieved by a market economy, in which $c_1 = 1$, but by implementing a financial intermediary. However, for obtaining the result that a financial intermediary can solve the problem of an inefficient market outcome, a stronger condition than just risk averse depositors is necessary, namely that $c \mapsto cU'(c)$ is decreasing which is interpreted as that the intertemporal elasticity of substitution must be larger than 1 (Freixas and Rochet, 1997, p. 47/48). The condition that $c \mapsto cU'(c)$ is decreasing is equivalent to $-cU''(c)/U'(c) > 1$. This condition is not satisfied in our model in which $U(c) = \ln c$, so that $-cU''(c)/U'(c) = 1$. Consequently, one could argue that in our model, there is no role for banks since the optimal allocation can also be achieved by the market. However, we assume, without explicitly modelling this issue, that banks have a comparative advantage of providing the liquidity as they can monitor credits at lower costs, for example.

[6] At date 1, only the short-term asset and the long-term, safe asset can be liquidated. The credit portfolio is assumed to be totally illiquid so that its liquidation value at date 1 is equal to zero. Consequently, the liquidation value of a bank's total assets at date 1 is $x^* + y^* r$. This is less than 1 since $x^* + y^* + u^* = 1$, $u^* > 0$, and $r < 1$.

information about one representative bank's future asset returns: the realization of that bank's credit portfolio will only be $(L - s)$ instead of L, where $s > 0$ is a shock that is assigned zero probability at date 0. This shock implies that c_2 will be smaller than expected for the bad state L, i.e. $c_2 < c^*_{2,L}$. How large can this shock be without inducing c_2 to be smaller than c^*_1, i.e. without inducing a bank run? The condition is

$$c^*_1 = \frac{x^*}{\gamma} \leq \frac{y^* R + u^*(L - s)}{(1 - \gamma)} = c_2. \tag{14}$$

Solving (14) for s one obtains for the asset buffer of that bank:

$$B^{As} = \frac{y^* R + u^* L - x^* \frac{1-\gamma}{\gamma}}{u^*}. \tag{15}$$

If $s > B^{As}$, late consumers will be better off withdrawing at date 1, i.e. the bank faces a run. The resulting bankruptcy of this bank will lead to a banking crisis if it triggers the bankruptcy of other banks (contagion). Let us assume that Bank A is subject to an asset shock s and that $s > B^{As}$ so that A goes bankrupt. In this case, the value of bank D's deposits in bank A decreases, i.e. they are worth less than the deposits of bank C in bank D,[7] so that bank D faces a loss when cross holdings of deposits are liquidated. Whether bank D can absorb this liquidity shock depends on the amount of the long-term, safe asset that bank D can liquidate at date 1 without causing c_2 to become smaller than c^*_1. Consequently, the condition for a run not to occur is

$$c^*_1 = \frac{x^*}{\gamma} \leq \frac{yR + u^* K}{1 - \gamma} = c_2 \tag{16}$$

which implies that the bank must keep at least $\frac{x^* \frac{1-\gamma}{\gamma} - u^* K}{R}$ units of the long-term, safe asset to prevent a run. Consequently, the amount of the long-term, safe asset which can be liquidated at date 1 is

$$y^{crit} = y^* - \frac{x^* \frac{1-\gamma}{\gamma} - u^* K}{R}. \tag{17}$$

[7] As Allen and Gale (2000) we assume that if a bank goes bankrupt, all its assets are liquidated at date 1, and the proceeds of the liquidation are split pro rata among all depositors including banks.

Therefore, the bank's liquidity buffer is given by

$$B^{Li} = \min\left[ry^{crit}, ry^{*}\right]. \tag{18}$$

At date 1, one unit of the long-term, safe asset is worth r. The bank can liquidate y^{crit} units of this asset without causing a run. The bank actually holds y^{*} units of this asset. Obviously, the minimum of both determines bank D's liquidity buffer. If $y^{crit} > y^{*}$, the bank could liquidate more assets without causing a run, the problem is that it has no more *liquid* assets. If $y^{crit} < y^{*}$, the bank had more assets to liquidate, the problem is that it cannot liquidate them without causing a bank run. Consequently, a bank's liquidity buffer is either limited by its total assets, then ry^{crit} is the relevant term, or by its stock of liquid assets, then ry^{*} determines its liquidity buffer. Note also that due to assumption (13) both y^{crit} and y^{*} are strictly positive.

If the loss of bank D's deposit in bank A is larger than B^{Li}, also bank D is bankrupt. There is a spillover effect from bank A to bank D, there is contagion. If bank D is bankrupt, bank C's deposits in bank D will become worth less, so that bank C will face the same liquidity problem as bank D. If the loss in its deposits is higher than the liquidity buffer given by equation (18), also bank C will go bankrupt.[8]

Summing up, the stability of the banking sector depends on the asset buffer given by equation (15) which determines whether a single bank can absorb an idiosyncratic asset shock and by the liquidity buffer given by equation (18) which determines in how far banks can absorb the resulting liquidity shock. The liquidity buffer is the decisive buffer against contagion.

[8] Note that in Allen and Gale (2000), all banks will go bankrupt as soon as bank D goes bankrupt. In our model, this is not necessarily the case but it depends on whether bank D's credit portfolio realizes H or L. If H is realized, i.e. if the credits succeed, and nevertheless bank D is not able to liquidate enough assets to absorb the liquidity shock, no bank can liquidate enough assets, independently of the performance of its credit portfolio, i.e. all other banks go bankrupt, too. However, if L is realized the other banks can but they do not need to go bankrupt.

3 Introduction of Credit Default Swaps

3.1 Modelling Credit Default Swaps

We assume that the risks of the banks' credit portfolios are perfectly correlated within a region but that they are independent between regions. Consequently, there is scope for diversification, but due to frictions which imply that the banks can conclude credit contracts only in their own region, banks cannot make use of this possibility of diversification. However, credit default swaps (CDS) offer a possibility to overcome the frictions. Each of the four representative banks A, B, C, and D concludes two CDS. In one CDS it is a risk taker in the other it is a risk shedder as illustrated by figure 1. Let us consider bank A, for example. On the one hand, it concludes a CDS with bank B as a risk shedder, i.e. bank B undertakes to make a compensation payment to bank A if a predefined credit event occurs. On the other hand, bank A also concludes a CDS with bank D in which it is the risk taker. The

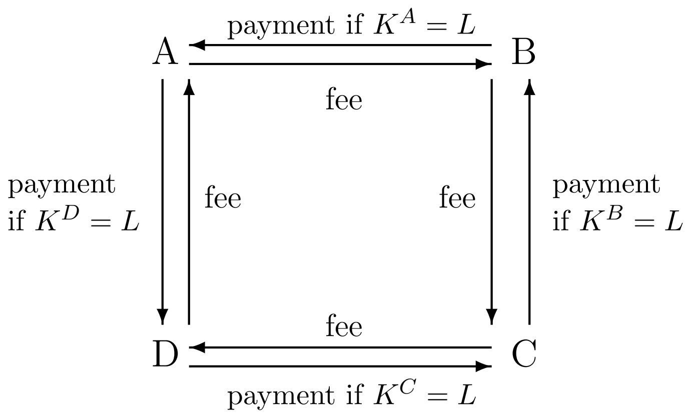

Figure 1: Credit Default Swaps

CDS contracts are specified as follows: If a bank is the risk shedder, it has to pay a fee to the risk taker at date 0, and in return, it receives a payment ug at date 2, where u are the units invested into the credit portfolio and

$$g = \begin{cases} \frac{1-L}{2} = l & \text{if} \quad K = L \\ 0 & \text{if} \quad K = H, \end{cases} \tag{19}$$

i.e. each bank insures half of its possible loss and takes half of the possible loss of another bank. This reveals that the only motive for concluding a CDS is diversification. That only half of a possible loss is insured is owed to moral hazard problems. Since all banks are assumed to be ex ante identical, the fee as well as the compensation payment are the same for all banks, i.e. at date 0 the CDS is balance sheet neutral since each bank receives and pays a fee of the same amount. Furthermore, we assume that if a bank is bankrupt, it will not fulfil its obligations from the CDS, i.e. it makes no compensation payment independently of the outcome of the credit portfolio of that bank it has concluded a CDS with as a risk taker ($g = 0$ for $K = H$ and for $K = L$). Moreover, we assume that if a bank has to be liquidated at date 1, it does not receive any payment from the CDS it has concluded as a risk shedder. A compensation payment has to be paid only at date 2.

3.2 Optimal Investment

At date 1, there are no payments due to the CDS. Consequently, also with CDS c_1 is given by equation (2). However, at date 2 the level of consumption does not only depend on the performance of the credit portfolio of the depositor's own bank as without CDS, but also on the performance of that bank its bank has concluded a CDS with as a risk taker: For a depositor of bank D, for example, the consumption at date 2 depends on the realization of K^D and on K^C so that four possible states can be defined:

LL: the credit portfolio of bank D realizes L, that of bank C realizes L,

LH: the credit portfolio of bank D realizes L, that of bank C realizes H,

HL: the credit portfolio of bank D realizes H, that of bank C realizes L,

HH: the credit portfolio of bank D realizes H, that of bank C realizes H.

These four states can be defined analogously for all banks. Consequently, the level of consumption at date 2 is given by

$$c_2 = \begin{cases} \frac{yR+uL}{1-\gamma} & =: \quad c_{2,LL} \quad \text{in the state} \quad LL \\ \frac{yR+uL+ul}{1-\gamma} & =: \quad c_{2,LH} \quad \text{in the state} \quad LH \\ \frac{yR+uH-ul}{1-\gamma} & =: \quad c_{2,HL} \quad \text{in the state} \quad HL \\ \frac{yR+uH}{1-\gamma} & =: \quad c_{2,HH} \quad \text{in the state} \quad HH. \end{cases} \tag{20}$$

Equation (20) shows that if the credit portfolios of both banks (the depositor's own bank and the bank its bank has concluded a CDS with as a risk taker) perform badly, compensation payments have no influence on c_2. The reason is that the bank receives a compensation payment but it also has to make one. Consequently, the net effect will be zero. Obviously, compensation payments play no role in case both banks perform well either. Only in case the performance of the two portfolios is different, compensation payments have to be considered.

Consequently, with CDS a bank's optimization problem becomes

$$\begin{aligned} E[U] = \gamma \ln c_1 + (1-\gamma)\Big\{&(1-\alpha)^2 \ln c_{2,LL} + (1-\alpha)\alpha \ln c_{2,LH} \\ &+ \alpha(1-\alpha)\ln c_{2,HL} + \alpha^2 \ln c_{2,HH}\Big\} =: g(x,y,u) \rightarrow \max \end{aligned} \tag{21}$$

s.t. the constraints given by (5).

Again, we first neglect the inequality constraints. The necessary and sufficient (since g is concave and the constraint is linear) optimality conditions for an optimal solution $(\tilde{x}, \tilde{y}, \tilde{u})$ of (21) s.t. the equality constraint $x + y + u = 1$ are

$$\frac{\gamma}{\tilde{x}} = \lambda, \tag{22}$$

$$\left[\frac{(1-\alpha)^2}{\tilde{c}_{2,LL}} + \frac{(1-\alpha)\alpha}{\tilde{c}_{2,LH}} + \frac{(1-\alpha)\alpha}{\tilde{c}_{2,HL}} + \frac{\alpha^2}{\tilde{c}_{2,HH}}\right] R = \lambda, \tag{23}$$

$$\frac{(1-\alpha)^2}{\tilde{c}_{2,LL}} L + \frac{(1-\alpha)\alpha}{\tilde{c}_{2,LH}}(L+l) + \frac{(1-\alpha)\alpha}{\tilde{c}_{2,HL}}(H-l) + \frac{\alpha^2}{\tilde{c}_{2,HH}} H = \lambda, \tag{24}$$

together with $\tilde{x} + \tilde{u} + \tilde{y} = 1$, where $\tilde{c}_2 = c_2(\tilde{u}, \tilde{y})$ and where λ is the Lagrange multiplier. These optimality conditions imply $\tilde{x} = \gamma$.

Proof: See proof I in the appendix.

Unfortunately, we are not able to solve the optimality conditions to get the optimal values $\tilde{u}$ and $\tilde{y}$. However, we are able to show that $\tilde{u} > u^*$ and that $\tilde{y} < y^*$, i.e. that the introduction of CDS implies that the banks reduce their investment into the safe asset and increase their investment into the credit portfolio. For showing this, we eliminate y by using the equality constraint in a first step. Then, we obtain a new objective function $h(x, u) := g(x, 1 - x - u, u)$, where h depends on c_1 and c_2 in the same way as g in (21), but the consumption c_2 now depends on x and u in the following way

$$\begin{aligned} c_{2,LL} &= \frac{(1-x)R + u(L-R)}{1-\gamma}, \\ c_{2,LH} &= \frac{(1-x)R + u(L+l-R)}{1-\gamma}, \\ c_{2,HL} &= \frac{(1-x)R + u(H-l-R)}{1-\gamma}, \\ c_{2,HH} &= \frac{(1-x)R + u(H-R)}{1-\gamma}. \end{aligned} \tag{25}$$

We compute the partial derivative $\partial h / \partial u$ at the point (x^*, u^*), where x^* and u^* are the optimal x and u without CDS. Using the fact that x^* and u^* satisfy (6) and (8), we obtain

$$\begin{aligned} \frac{\partial h}{\partial u}(x^*, u^*) = (1-\alpha)\alpha \Bigg[(L-R) \left(\frac{1}{c^*_{2,LH}} - \frac{1}{c^*_{2,LL}} \right) + l \left(\frac{1}{c^*_{2,LH}} - \frac{1}{c^*_{2,HL}} \right) \\ + (H-R) \left(\frac{1}{c^*_{2,HL}} - \frac{1}{c^*_{2,HH}} \right) \Bigg] > 0, \end{aligned} \tag{26}$$

where c_2^* denotes the level of consumption according to (25) at the point (x^*, u^*). Since we already know that for the maximizer $(\tilde{x}, \tilde{u})$ of h it holds that $\tilde{x} = x^* = \gamma$,

(26) implies that $\tilde{u} > u^*$.[9] This in turn means that $\tilde{y} < y^*$.

By the preceding considerations we easily see that the solution $(\tilde{x}, \tilde{y}, \tilde{u})$ of the optimization problem with neglected inequality constraints already satisfies the non-negativity constraints for x and u. But we cannot exclude that this solution violates the incentive compatibility constraint or even the weaker non-negativity constraint for y. Even though (13) holds, so that without CDS the incentive compatibility constraint is not binding, it can become binding with CDS because of $\tilde{c}_{2,LL} < c^*_{2,L}$. Nevertheless, analyzing the Karush-Kuhn-Tucker (KKT) conditions for the optimization problem with inequality constraints, we can proof that the optimal solution $(\bar{x}, \bar{y}, \bar{u})$ for (21) including all constraints (5) still has the property $\bar{u} > u^*$ and $\bar{y} < y^*$. It might happen that $\bar{x} < x^*$, but this happens only if the incentive compatibility constraint becomes binding.

Proof: See proof II in the appendix.

This leads us to

> *Result 1: The CDS imply an investment shifting: The banks reduce their investment into the long-term, safe asset and increase their investment into the risky credit portfolio. The short-term investment remains constant or is reduced. But the latter case occurs only if the incentive compatibility constraint becomes binding.*

The reason for the investment shifting ($\bar{u} > u^*$, $\bar{y} < y^*$, $\bar{x} \leq x^*$) is as follows. The CDS allow the banks to make use of diversification possibilities. The resulting reduced credit risk implies an increase in the risk averse depositors' expected marginal utility from the credit portfolio $\partial E[U]/\partial u$ and a decrease in their expected marginal utility from the long-term, safe asset $\partial E[U]/\partial y$. In case the incentive compatibility constraint is not binding, optimality requires the expected marginal utility from all three assets to be the same, i.e. $\partial E[U]/\partial x = \partial E[U]/\partial y = \partial E[U]/\partial u$ (see the first order conditions given by (22) to (24)). This implies that the banks' optimal

[9] The strict concavity of h with respect to u implies in general that $h(x^*, u) - h(x^*, u^*) < (u - u^*)\frac{\partial h}{\partial u}(x^*, u^*)$ for $u \neq u^*$, hence $h(x^*, u) < h(x^*, u^*)$ if $u < u^*$.

reaction to the change in the expected marginal utilities is to expand their investment into the risky credit portfolio (u increases) and to reduce their investment into the long-term, safe asset (y decreases) until $\partial E[U]/\partial x = \partial E[U]/\partial y = \partial E[U]/\partial u$. However, the incentive compatibility constraint may prevent a sufficient investment shifting from the safe asset to the credit portfolio. Holding x constant in that case, would imply that $\partial E[U]/\partial y < \partial E[U]/\partial x < \partial E[U]/\partial u$. Consequently, a further adjustment of the expected marginal utilities requires a reduction in x, since this implies a decrease in c_1 which allows for a further investment shifting from the long-term, safe asset to the risky credit portfolio without violating the incentive constraint $c_1 \leq c_{2,L}$. Note that the reduction in x only implies a further adjustment of the expected marginal utilities but a total adjustment as in the case in which the constraint is not binding is not achieved (see the KKT conditions given in the appendix).

The investment shifting from the long-term, safe asset to the risky credit portfolio implies an on average higher consumption in the second period since $E(K) > R$. This leads us to

> *Result 2: The conclusion of the CDS contracts, which implies an improved diversification of risk, together with the investment shifting, which implies an on average higher consumption, leads to an increase in the consumers' expected utility.*

Proof: See proof III in the appendix.

3.3 Stability of the Banking Sector

For analyzing the consequences of the CDS contracts for the stability of the banking sector, we look at their influence on the banks' asset and liquidity buffer.

Looking at their influence on the **asset buffer** first, we have to define two further states for each bank what we do exemplarily for bank A:

$[L-s]L$: bank A is hit by a shock, its credit portfolio realizes $(L-s)$, that of bank D realizes L,

$[L-s]H$: bank A is hit by a shock, its credit portfolio realizes $(L-s)$, that of bank D realizes H.

These two states can be defined analogously for all banks. Then, analogously to the case without CDS, one obtains for a bank's asset buffer

$$B^{As} = \begin{cases} \frac{\bar{y}R+\bar{u}L-\bar{x}\frac{1-\gamma}{\gamma}}{\bar{u}} & \text{in the state} \quad [L-s]L \\ \frac{\bar{y}R+\bar{u}L-\bar{x}\frac{1-\gamma}{\gamma}}{\bar{u}} + l & \text{in the state} \quad [L-s]H. \end{cases} \tag{27}$$

Comparing the asset buffer with and without CDS (equations (15) and (27)) reveals that in the state $[L-s]L$ the CDS unambiguously reduce the asset buffer, while in the state $[L-s]H$ their effect on the buffer is ambiguous.

The reason for the reduction in the state $[L-s]L$ is the investment shifting from the long-term, safe asset to the risky credit portfolio (reduction in y and increase in u). We have shown that, as long as the introduction of the CDS does not imply that the incentive compatibility constraint becomes binding, the short-term investment remains constant ($\bar{x} = x^* = \gamma$). In this case the investment shifting reduces the asset buffer because the return on the safe asset is higher than on the badly performing credit portfolio ($R > L$). However, if the incentive compatibility constraint becomes binding, the short-term investment may be reduced, i.e. the introduction of the CDS may imply a decrease in x ($\bar{x} < x^* = \gamma$). Nevertheless, the effect of the CDS on the asset buffer is unambiguously negative also in this case: If the constraint becomes binding we have $\bar{c}_1 = \bar{c}_{2,LL}$. This means that in the worst expected case LL, the payment to the late consumers is the same as to the early consumers. Consequently, in this case the asset buffer reduces to zero. However, without CDS there is always a positive asset buffer since (13) implies that even in the worst expected case L, the payment to the late consumers is strictly larger than to the early consumers. Therefore, also if the CDS imply that the incentive compatibility constraint becomes binding so that there is a reduction in x, their effect on the asset buffer in the state $[L-s], L$ is still negative.

In the state $[L-s]H$, the effect of the CDS on the asset buffer is ambiguous because the compensation payment l the bank receives may overcompensate the negative effect resulting from the investment shifting. If the state $[L-s]L$ is realized, the compensation payment will not influence the asset buffer because on the one hand the bank receives a payment but on the other hand it has to make the same payment to the bank it has concluded a CDS with as a risk taker so that the net effect will be zero. This leads us to

Result 3: The CDS will imply a decrease in a bank's asset buffer if the credit portfolio of that bank it has concluded a CDS contract with as a risk taker fails. If this credit portfolio succeeds, the influence of the CDS on the bank's asset buffer will be ambiguous.

A bank's **liquidity buffer**, i.e. the buffer which determines a bank's ability to absorb a liquidity shock, is given by

$$B^{Li} = \max\left\{\min\left[r\bar{y}^{crit}, r\bar{y}\right], 0\right\}, \tag{28}$$

where

$$\bar{y}^{crit} = \begin{cases} \bar{y} - \frac{\bar{x}\frac{(1-\gamma)}{\gamma} - \bar{u}(L-l)}{R} & \text{in the state} \quad LL \\ \bar{y} - \frac{\bar{x}\frac{(1-\gamma)}{\gamma} - \bar{u}L}{R} & \text{in the state} \quad LH \\ \bar{y} - \frac{\bar{x}\frac{(1-\gamma)}{\gamma} - \bar{u}H}{R} & \text{in the state} \quad HH \\ \bar{y} - \frac{\bar{x}\frac{(1-\gamma)}{\gamma} - \bar{u}(H-l)}{R} & \text{in the state} \quad HL. \end{cases} \tag{29}$$

As without CDS, the buffer is either limited by the bank's total assets, then $r\bar{y}^{crit}$ is the relevant term, or by the stock of its liquid assets, then the term $r\bar{y}$ determines the liquidity buffer (see page 12 for details). If the latter applies, the CDS will unambiguously reduce the bank's liquidity buffer due to the $y-$reducing investment shifting.

If the bank's total assets limit the buffer, the buffer will be determined by the performance of the bank's own credit portfolio as well as by the performance of the credit portfolio of that bank it has concluded a CDS with as a risk taker: Let us assume that the asset buffer of bank A has been too small to absorb the shock s so that bank A is bankrupt. Then, bank D's liquidity buffer determines whether there is contagion, and this liquidity buffer is determined by the performance of bank D's credit portfolio and of bank C's credit portfolio. In the following, we describe exemplarily the influence of the CDS contracts on bank D's liquidity buffer in the four possible states LL, LH, HH, and HL by comparing the liquidity buffers given by the equations (18) and (28).

First, we consider the case in which the incentive compatibility constraint does not become binding, i.e. the case in which x remains constant ($\bar{x} = x^* = \gamma$). If both portfolios perform badly (LL), the liquidity buffer with CDS is smaller than without CDS. The reason is twofold. Firstly, in this state the investment shifting has a negative impact on the buffer because the return on the safe asset is higher than on the credit portfolio ($R > L$). Secondly, bank D has to make a compensation payment to bank C, but due to the bankruptcy of bank A, bank D does not receive a compensation payment although its own portfolio realizes L (see assumptions on page 14). The investment shifting also implies that in case only bank D's portfolio performs badly (LH), the liquidity buffer with CDS is smaller. If bank D's portfolio performs well, the investment shifting has a positive effect on the bank's total assets and therefore on this liquidity buffer because the return on the safe asset is smaller than on the credit portfolio ($R < H$). Consequently, if also bank C's portfolio performs well (HH), bank D's liquidity buffer is higher with CDS. However, if bank C's portfolio performs badly (HL), the positive effect from the investment shifting may be overcompensated by the compensation payment bank D has to make to bank C.

Now let us consider what happens to this liquidity buffer if the introduction of the CDS implies that the incentive compatibility constraint becomes binding, meaning that a reduction in x may occur. Although this reduction has a positive effect on this buffer (the reduction in x reduces the claims of the early consumers implying that a higher portion of the long-term asset can be liquidated without causing a run), the influence of the CDS on this buffer qualitatively remains the same: In the states LL and LH, the effect is unambiguously negative since in these two states the buffer reduces to zero. In the state LL, the buffer reduces to zero since if the incentive compatibility constraint becomes binding, $\bar{c}_{2,LL} = \bar{c}_1$. In the state LH, the buffer reduces to zero although $\bar{c}_{2,LH} > \bar{c}_{2,LL} = \bar{c}_1$ because due to the bankruptcy of the risk taker, the bank does not receive a compensation payment so that $\bar{c}_{2,LH} > c_{2,LH} = \bar{c}_{2,LL} = \bar{c}_1$. In the state HL, the effect of the CDS on the buffer is ambiguous. There are two positive effects, one resulting from the investment shifting from the long-term, safe asset to the risky asset and the other resulting from the reduction in x, but these effects can be overcompensated by the negative effect due to the compensation payment the bank has to make. In the state HH, the bank does not have to make a compensation payment, so that in this state, the effect of the CDS on the buffer is unambiguously positive.

This leads us to

> *Result 4: In case a bank's own credit portfolio performs badly (states LL and LH), the CDS will reduce the bank's liquidity buffer. If its credit portfolio performs well (states HL and HH), the effect of the CDS on its liquidity buffer will be ambiguous.*

The effects of the CDS on a bank's asset buffer and liquidity buffer are summarized in table 2.

State	$[L-s]L$	$[L-s]H$	LL	LH	HL	HH
Asset buffer	↓	↓ ↑				
Liquidity buffer limited by total assets			↓	↓	↓ ↑	↑
Liquidity buffer limited by liquid assets			↓	↓	↓	↓

Table 2: Effects of CDS on a Bank's Asset Buffer and Liquidity Buffer

What general conclusions can be drawn from these results for the stability of an economy's banking sector? Let us assume that a cyclical downturn is involved with a high fraction of badly performing credits and a cyclical upturn with a high fraction of well performing credits.[10] Then, our results imply that in an economic downturn CDS reduce the stability of the banking sector since they imply a decrease in the

[10] For a possibility of explicitly modelling this aspect, see Pennacchi (2006).

banks' asset and liquidity buffer. In an economic upturn, the effect of CDS on the banking sector's stability is ambiguous.

4 Summary

The aim of this paper is to analyze the consequences of CDS contracts in which both the protection buyer and the protection seller is a bank for the stability of the banking sector. In our study, the stability of the banking sector is determined by two buffers. The asset buffer determines a bank's ability to absorb an asset shock, the liquidity buffer its ability to absorb a liquidity shock. The latter is decisive for the danger of contagion. Concerning the liquidity buffer, we distinguish between two cases: either the buffer is limited by a bank's liquid assets or by a bank's total assets. If the buffer is limited by a bank's total assets, the bank has sufficient liquid assets to meet a liquidity shock, but it cannot liquidate them without causing a bank run, i.e. the return payments to the depositors who want to withdraw later would become too small. If the buffer is limited by a bank's liquid assets, the bank could liquidate more assets without causing a bank run, but it has no more liquid assets. The main idea of this paper is to analyze in how far the introduction of CDS influences the asset and the liquidity buffer to study their impact on the stability of the banking sector.

The crucial point in our analysis is that the CDS induce the banks to increase their investments into a risky, illiquid credit portfolio and to reduce their investments into a safe, relatively liquid asset. The reason for this investment shifting is that the CDS improve the diversification of the banks' credit risk. The resulting lower risk for the depositors allows the banks, which seek to maximize the expected utility of their risk averse depositors, to invest more into the risky credit portfolio which has a higher expected return than the safe asset.

This bank behavior (the conclusion of CDS contracts and the investment shifting) implies on the one hand an increase in the depositors' expected utility. On the other hand, it reduces the stability of the banking sector in a macroeconomic downturn, and it may reduce this stability in a macroeconomic upturn: The investment shifting from the relatively liquid asset to the illiquid credit portfolio unambiguously reduces the bank's *liquidity buffer* if it is limited by the banks' liquid assets. If the liquidity buffer is limited by the banks' total assets, the impact of the CDS on this buffer depends on the macroeconomic circumstances. In a macroeconomic downturn with a corresponding high number of credit defaults, the credit portfolio is worth less than the safe asset so that the investment shifting implies lower total assets and therefore a reduction in this buffer. However, in a macroeconomic upturn which is characterized by a low number of credit defaults, the credit portfolio is worth more than the safe asset so that the investment shifting implies higher total assets and therefore a higher buffer. The *asset buffer*, which determines in how far a bank can absorb an idiosyncratic asset shock as a sudden decrease in the value of its credit portfolio, will unambiguously decrease in a macroeconomic downturn if CDS are introduced and it may decrease in a macroeconomic upturn. The reason is that the investment shifting implies a decrease in the bank's total assets since due to the shock, the credit portfolio is worth less than the safe asset, independently of the macroeconomic circumstances. However, in a macroeconomic upturn, this negative effect on the buffer may be overcompensated by the compensation payment from the CDS contract the bank receives so that the overall effect is ambiguous. In a macroeconomic downturn the bank receives a compensation payment too, but it also has to make such a payment itself, so that the net effect of the compensation payments is zero.

Consequently, on the one hand, the CDS imply a higher expected utility of the depositors, but on the other hand they lead to a higher systemic risk. In a macroeconomic downturn, they reduce the stability of the banking sector, and in a macroeconomic upturn they may reduce this stability.

In our analysis, we have focused on CDS in which both, the protection buyer as well as the protection seller, is a bank. Currently, banks still constitute the majority of both. However, especially as protection sellers also insurance companies play an important role in the markets for credit risk transfer, and the fraction of hedge funds as protection sellers and protection buyers increases rapidly (British Bankers' Association, 2006). The consequences for financial stability of credit risk transfer between the banking sector and the insurance sector has been analyzed by Allen and Gale (2006) and in Allen and Carletti (2006), for example. The consequences for financial stability of credit risk transfer in which one contract party is a hedge funds should be an interesting topic for future research.

Appendix

Proof I:
It has to be shown that $\tilde{x} = \gamma$. Multiplying both sides of equation (22) with $\tilde{x}$, of equation (23) with $\tilde{y}$, and of equation (24) with $\tilde{u}$, we obtain

$$\begin{aligned}\gamma &+ \frac{(1-\alpha)^2}{\tilde{c}_{2,LL}}\tilde{y}R + \frac{(1-\alpha)\alpha}{\tilde{c}_{2,LH}}\tilde{y}R + \frac{(1-\alpha)\alpha}{\tilde{c}_{2,HL}}\tilde{y}R + \frac{\alpha^2}{\tilde{c}_{2,HH}}\tilde{y}R + \frac{(1-\alpha)^2}{\tilde{c}_{2,LL}}\tilde{u}L \\ &+ \frac{(1-\alpha)\alpha}{\tilde{c}_{2,LH}}\tilde{u}(L+l) + \frac{(1-\alpha)\alpha}{\tilde{c}_{2,HL}}\tilde{u}(H-l) + \frac{\alpha^2}{\tilde{c}_{2,HH}}\tilde{u}H = \lambda(\tilde{x}+\tilde{y}+\tilde{u}) = \lambda.\end{aligned}$$

This implies that

$$\gamma + (1-\alpha)^2(1-\gamma) + 2(1-\alpha)\alpha(1-\gamma) + \alpha^2(1-\gamma) = \lambda.$$

Consequently, $\lambda = 1$ implying that $\tilde{x} = \gamma$ according to equation (22). ■

Proof II:
The KKT conditions for a maximizer $(\bar{x}, \bar{y}, \bar{u})$ of (21) s.t. all constraints (5), being necessary and sufficient since the objective g is concave and all constraints are linear (for details about KKT conditions see e.g. Sun and Yuan (2006), Theorem 8.2.7, Corollary 8.2.9 and Theorem 8.2.11), are the following

$$\frac{\gamma}{\bar{x}} + \mu_x - \frac{\nu}{\gamma} = \lambda, \tag{30}$$

$$\left[\frac{(1-\alpha)^2}{\bar{c}_{2,LL}} + \frac{(1-\alpha)\alpha}{\bar{c}_{2,LH}} + \frac{(1-\alpha)\alpha}{\bar{c}_{2,HL}} + \frac{\alpha^2}{\bar{c}_{2,HH}}\right]R + \mu_y + \frac{\nu R}{1-\gamma} = \lambda, \tag{31}$$

$$\begin{aligned}&\frac{(1-\alpha)^2}{\bar{c}_{2,LL}}L + \frac{(1-\alpha)\alpha}{\bar{c}_{2,LH}}(L+l) + \frac{(1-\alpha)\alpha}{\bar{c}_{2,HL}}(H-l) + \frac{\alpha^2}{\bar{c}_{2,HH}}H \\ &\qquad + \mu_u + \frac{\nu L}{1-\gamma} = \lambda,\end{aligned} \tag{32}$$

$$\mu_x, \mu_y, \mu_u, \nu \geq 0, \tag{33}$$

$$\mu_x \bar{x} = \mu_y \bar{y} = \mu_u \bar{u} = \nu \left(\frac{R\bar{y} + L\bar{u}}{1-\gamma} - \frac{\bar{x}}{\gamma} \right) = 0, \tag{34}$$

$$\bar{x} + \bar{y} + \bar{u} = 1, \quad \bar{x}, \bar{y}, \bar{u} \geq 0, \quad \frac{R\bar{y} + L\bar{u}}{1-\gamma} - \frac{\bar{x}}{\gamma} \geq 0. \tag{35}$$

Multiplying both sides of equation (30) with $\bar{x}$, of equation (31) with $\bar{y}$, and of equation (32) with $\bar{u}$, adding the 3 equations and regarding (34) and $\bar{x} + \bar{y} + \bar{u} = 1$ we obtain again $\lambda = 1$.

If we assume that $\bar{x} > \gamma$, then (30) implies $\mu_x > \frac{\nu}{\gamma} \geq 0$, i.e. $\bar{x} = 0$ due to (34), a contradiction. Consequently it must hold $\bar{x} \leq \gamma$. Moreover, if $\bar{x} < \gamma$ then (30) implies $\frac{\nu}{\gamma} > \mu_x \geq 0$, hence $\frac{R\bar{y}+L\bar{u}}{1-\gamma} - \frac{\bar{x}}{\gamma} = 0$, i.e. the incentive compatibility constraint must be binding.

It remains to show that $\bar{y} < y^*$, since then we also have

$$\bar{u} = 1 - \bar{x} - \bar{y} > 1 - x^* - y^* = u^*.$$

We first consider the case that the incentive compatibility constraint becomes binding, i.e. that it holds

$$\frac{R\bar{y} + L\bar{u}}{1-\gamma} = \frac{\bar{x}}{\gamma}.$$

After inserting $\bar{u} = 1 - \bar{x} - \bar{y}$ into this equation and some rearrangement we obtain

$$(R - L)\bar{y} = \left(\frac{1-\gamma}{\gamma} + L \right) \bar{x} - L.$$

On the other hand, due to (13) we obtained

$$\frac{Ry^* + Lu^*}{1-\gamma} > \frac{x^*}{\gamma}$$

implying

$$(R - L)y^* > \left(\frac{1-\gamma}{\gamma} + L \right) x^* - L.$$

Because of $\bar{x} \leq x^*$ we can conclude

$$(R-L)\bar{y} = \left(\frac{1-\gamma}{\gamma} + L\right)\bar{x} - L \leq \left(\frac{1-\gamma}{\gamma} + L\right)x^* - L < (R-L)y^*,$$

i.e. $\bar{y} < y^*$.

If the incentive compatibility constraint does not become binding then we have $\bar{x} = x^* = \gamma$ and we can argue that it must hold $\bar{u} > u^*$ and $\bar{y} < y^*$ in the same way like we did it for $\tilde{u}$ and $\tilde{y}$ in the case where the inequality constraints have been neglected. ■

Proof III:

It has to be shown that

$$\max g(x, y, u) > \max f(x, y, u).$$

Let (x^*, y^*, u^*) be the solution of the optimization problem without CDS, i.e. the maximizer of f and $c^*_{2,LL}$, $c^*_{2,LH}$, $c^*_{2,HL}$ and $c^*_{2,LL}$ the consumption at *this* point with CDS. Then, we have

$$g(x^*, y^*, u^*) - f(x^*, y^*, u^*) = (1-\gamma)\alpha(1-\alpha)[\ln c^*_{2,LH} + \ln c^*_{2,HL} - \ln c^*_{2,LL} - \ln c^*_{2,HH}].$$

Furthermore, we have

$$c^*_{2,LL} + \frac{u^* l}{1-\gamma} = c^*_{2,LH} < c^*_{2,HH} = c^*_{2,HL} + \frac{u^* l}{1-\gamma}.$$

Setting $a := \frac{u^* l}{1-\gamma}$, we obtain

$$g(x^*, y^*, u^*) - f(x^*, y^*, u^*) = (1-\gamma)\alpha(1-\alpha)\Big\{ \left[\ln(c^*_{2,LL} + a) - \ln c^*_{2,LL}\right] - \left[\ln(c^*_{2,HL} + a) - \ln c^*_{2,HL}\right] \Big\} > 0$$

by the strict concavity of ln. Consequently,

$$\max g(x, y, u) \geq g(x^*, y^*, u^*) > f(x^*, y^*, u^*) = \max f(x, y, u)$$

what had to be shown. ■

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