Fundamental Stock Price with the Consumption CAPM: An International Comparison

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Abstract

What is the fundamental value of a stock? Do stock prices deviate from this fundamental value? If yes, do they go back to their fundamental value? This paper proposes to answer these three questions by using a stock valuation model based on the Consumption-Capital Asset Pricing Model (C-CAPM). I first show how to use the C-CAPM to get a fundamental price that can be empirically estimated. This fundamental price is a function of expected future dividends and future consumption, of their future conditional variance and covariance and of agents' risk aversion. Secondly, I estimate the C-CAPM fundamental price for the United States, the United Kingdom, Japan and Switzerland for a period going from 1965 through to 2006. I identify several periods in which stock prices deviate significantly from their fundamental value. Thirdly, I show that after a shock, the gap between the price and its fundamental value decreases with time, which suggests that stock prices go back to the C-CAPM fundamental value. Finally, I show that forecasts using the C-CAPM fundamental price are more accurate than forecasts based on the observed price only. Additionally, I show that the C-CAPM forecasts systematically outperform forecasts made with other fundamental models (e.g. models based on the price-to-dividend ratios). This result holds for any forecasts horizon in the United States and in the United Kingdom.

Keywords: Fundamental stock price, Consumption CAPM, Out-of-sample forecasts.

JEL-Classifications: D53, E44, G12.

1 Introduction

What is a stock really worth? Do stock prices deviate from this fundamental (or fair) value? And if they deviate, do they eventually go back to their fundamental value? Such questions have been prominent topics for decades in the finance profession. Many economic agents are interested in their answers: shareholders, who are comparing investment alternatives, traders, who are looking for speculation opportunities, or central bankers, who try to identify stock market imbalances of which unwinding could have an impact on the economy. In this paper, I study these three questions with a stock valuation model based on the Consumption-Capital Asset Pricing Model (C-CAPM) (Lucas 1978 and Breeden 1979). I first show how to use the no-arbitrage condition of the C-CAPM to express the fundamental stock price as a function of expected dividends and consumption, as well as of their covariance. I then estimate the fundamental stock price (and deviations from it) for four countries: the United States, the United Kingdom, Japan and Switzerland. Finally, I assess the out-of-sample forecast accuracy of the C-CAPM fundamental model. I find that forecasts based on the C-CAPM fundamental price significantly outperform forecasts based on the observed price or on indicators that are known to have some predictive power (e.g. the price-to-dividend ratio). Both long term and short term forecasts are improved by using the C-CAPM fundamental price.

Not surprisingly given the interest that fundamental stock prices arouse, academics or practitioners have proposed several models to estimate them. A large majority of them are based on the discounted cash flow model (or net present value model), which states that the fundamental stock price is equal to the sum of the discounted expected payoffs of the stock.[1] This type of model requires a forecast of future cash flows generated by the stock, along with an appropriate discount rate. The most basic

[1] Lee (1998), Dupuis and Tessier (2003), Zong, Darrat and Anderson (2003) and Borio and Lowe (2002) are some of the few exceptions to the net present value model (although the former three indirectly build their method on it). They all measure the fundamental price by separating the permanent component of stock prices from their temporary and non-fundamental component.

model in this category is the Gordon growth model (Gordon and Shapiro 1956, Gordon 1962). In this model, stocks payoffs are equal to dividends and future discount rates, as well as the future dividend growth rate are constant. Many authors have refined this model.[2] The first line of innovation is to use dividend forecasts that are more realistic than a constant growth rate. For example, Shiller (1981 and 2005) uses ex-post realized dividends. Kaplan and Ruback (1995), Becchetti and Mattesini (2005) or Bagella, Becchetti and Adriani (2005) use a two-stage model, in which short-term forecasts are given by analysts and long-term forecasts are determined by the historical growth rate. A similar three-stage model is proposed by Panigirtzoglou and Scammell (2002). Yao (1997) separates increasing dividends from decreasing dividends. The second line of developments concentrates on the definition of future cash-flow. Ang and Liu (2001), Vuolteenaho (2002) or Dong and Hirshleifer (2005) use earnings instead of dividends; Black, Fraser and Groenewold (2003a,b) use profits. Cohen, Polk and Vuolteenaho (2003) or Pástor and Veronesi (2006) chose the market-to-book value ratio instead of the price-dividend ratio. The third line of improvement concerns the econometric methodology with the use of panel studies rather than the single times series (Lee, Myers and Swaminathan 1999, Becchetti and Adriani 2004 or Gentry, Jones and Mayer 2004). Lastly, some authors have studied the net present value fundamental model in a general equilibrium framework (Black, Fraser and Groenewold 2003a,b and Kinley 2004). In opposition to the dividends dynamic, the dynamic of the discount factor has received little attention. In general, the discount factor is constant and estimated by the CAPM. Campbell and Shiller (1987 and 1988a,b) have filled this gap by modelling the dynamic of both dividends and interest rates with a VAR model. They use the estimated joint dynamic to get a proxy of agents' expectations. Their VAR approach is the starting point of impressive literature (which not necessarily devoted to the estimation of fundamental values).

[2] An exhaustive survey of the literature on fundamental prices is beyond the scope of this paper. Only a selective list of the main innovations is presented here.

The fundamental model presented in this paper is in the spirit of the VAR fundamental model developed by Campbell and Shiller (CS hereafter). Like the models cited previously, it is based on the discounted cash-flow model, but it differs from them by using a stochastic discount factor (SDF) based on the no-arbitrage condition of the C-CAPM (Lucas 1978 and Breeden 1979). In most of the papers cited above, the SDF is given by the traditional CAPM. I use the C-CAPM for two main reasons: firstly, this model is based on sound economic arguments explaining consumption and investment decisions of a representative agent in a general model of production economy (see Breeden 1979 or Cox, Ingersoll and Ross 1985). Secondly, the model links consumption to asset prices. It is therefore well adapted to study the relation between the real economy and financial markets. These characteristics have made the C-CAPM one of the cornerstone of asset pricing (see e.g. Cochrane 2000). To summarize, in the C-CAPM, a rational representative agent splits her income between consumption and savings in a risky asset in order to maximize the utility of both her present and future consumption. The no-arbitrage equation states that the utility lost in investing one unit of consumption in a stock today must be equal to the expected utility of the additional future consumption obtained with the stock's payoff. With this condition, the stock price is equal to expected future payoff of the stock discounted with the intertemporal marginal rate of substitution of the representative agent. This rate is a function of the marginal utility of present and future consumption and thus, indirectly, it is a function of present and future consumption.[3] Consequently, the fundamental asset price is a function of the expected present value of future dividends and future consumption. The fundamentals variables (or fundamentals) are thus dividends and consumption instead of dividends and discount rates as in CAPM based models. To my knowledge, only Campbell and Shiller (1988a) and Lund and Engsted (1996, LE hereafter) have used the C-CAPM to compute a fundamental stock price.[4]

[3]The intertemporal marginal rate of substitution also depends on the form of the utility function and thus on the risk aversion of the representative agent.

[4]Shiller (2005) also presents a fundamental price based on the C-CAPM. However, his computation

The model presented here differs from CS and LE in several ways: firstly, I assume that the agents adapt their forecasts to structural changes in the fundamentals' dynamic. As shown in section 4, consumption growth rates are characterized by a structural decrease in each country of our sample. At the same time, as documented by Lettau and van Nieuwerburg (2005) for the United States, we observe a structural increase of the PD ratio in the each country of our sample. These two observations are in-line with the prediction of the C-CAPM. Indeed, if the agents integrate the structural decrease in consumption growth in their expectations, then they should expect a smaller consumption growth rate in the long term, which implies a higher intertemporal marginal rate of substitution for them. This higher intertemporal marginal rate of substitution makes future dividends worth more for the agents and thus yields an appreciation of stock prices.

Secondly, the fundamental price developed in this paper is based on a second-order Taylor approximation of the no-arbitrage condition. CS and LE stop at the first-order approximation. With an additional order of approximation the fundamental price becomes a function of the first and second moments of dividends and consumption, including their covariance. I estimate these second moments with a multivariate GARCH model. The advantage of this approach is to capture the impact of *time-varying covariances* on the fundamental price. This is new in the context of fundamental stock prices. Note that the second-order approximation presented in this paper is not restricted to the C-CAPM framework and can be used to compute the fundamental price derived from any other SDF model.

Thirdly, in each step of the computation of the fundamental price for time t, I have been particularly careful to use only the information available at that time. Thus, I put myself in the same position as an investor, who computes the fundamental price at time t. As a result, the fundamental price estimated here is a true *ex-ante* price. CS and

is based on ex-post dividends and consumption growth rate and the coefficient of relative risk aversion is not estimated but arbitrarily set equal to 3.

LE compute an *ex-post* price by using the whole sample to estimate the fundamentals' dynamic.

Finally, I assess the out-of-sample accuracy of forecasts based on the fundamental price and compare it with other simpler fundamental models. The ability of simple fundamental models to forecast future prices in the long term is now well documented. Campbell and Shiller (1998 and 2001) or Rapach and Wohar (2005), for example, show that, price-dividends (PD) ratios can help to forecasts stock price movements (in-sample) for horizons of 6 up to 10 years. Recently, Rapach and Wohar (2006) bring evidence that this result also holds out-of-sample. One of the main results of this paper is to show that the C-CAPM fundamental price is able to improve out-of-sample forecasts *even for horizons shorter than 6 years*. In fact, in both the United States and the United Kingdom, the information contained in the C-CAPM fundamental price helps to improve the forecasts for all horizons. The improvement is particularly spectacular for the United Kingdom, where the fundamental model performs at least 20% better[5] than the random walk with drift for all horizons longer than 3 months! No other simpler fundamental model tested in this paper is able to systematically outperform the forecasts of the C-CAPM model. In addition, I show that, in the United States and in the United Kingdom, the accuracy of the C-CAPM fundamental model increases when the price is far from its empirical value. This suggests that the tendency of the price to move back toward its fundamental value is stronger when the gap is wide. For Switzerland, the C-CAPM fundamental model improves the forecasts for horizons longer than 5 years and for shorter horizons if the gap between the fundamental and the observed price is small. In Japan, the C-CAPM fundamental model fails to improve forecasts in horizons shorter than 6 years.

The fact that the C-CAPM fundamental price is able to give out-of-sample forecasts is a sign that there is a link between the link between the market price and the funda-

[5]The accuracy is measured in terms of mean absolute error.

mental price. This forecast ability is observable in three out of four countries, Japan being the exception. I give an additional piece of evidence of this link by showing that the gap between the price and its fundamental value is mean-reverting in the long term in all countries.

The paper is structured as follow: Section 2 presents the fundamental price equation and the transformations that are necessary to estimate it. Section 3 describes the data. Section 4 documents the link between the structural evolution of consumption and its impact on the PD ratio. Section 5 describes the econometric methodology used to estimate the different coefficients of the fundamental equation. Section 6 presents the fundamental prices and the dynamic of the gap between the price and its fundamental value. Section 7 assesses the forecasting performance of the fundamental model. Section 8 concludes.

2 The Fundamental Stock Price Equation

The equation for the fundamental stock price is based on the no-arbitrage equation derived from the C-CAPM (Section 2.1). From this equation, I express the fundamental price as the present value of expected future fundamentals, which are the dividends and the expected marginal utility of consumption (Section 2.2). Section 2.3 shows how to linearize the fundamental equation and to express the fundamental price as a linear function of expected fundamentals. Section 2.4 explains how to compute the expectations about fundamentals. Finally, Section 2.5 combines these elements to give the final equation for the fundamental price.

2.1 The C-CAPM no arbitrage equation

As mentioned in the introduction, the first step for computing the fundamental price of stocks is to chose one model, from which the fundamental equation will be derived.

In this paper, I define the fundamental price as follows

Definition 1 *At time t, the fundamental price of an asset is the equilibrium price resulting from the optimal choice made by a rational representative agent, who allocates her income between consumption and savings (in the asset) in order to maximize the utility of her present and expected future consumption.*

This definition corresponds to the maximization problem at the center of the C-CAPM (Lucas 1978 and Breeden 1979). The Euler equation given by the first order condition of this maximization problem is

$$P_t = E_t \left[M_{t+1} \left(P_{t+1} + D_{t+1} \right) \right] \tag{1}$$

with

$$M_{t+1} = \beta \frac{U' \left(C_{t+1} \right)}{U' \left(C_t \right)} \tag{2}$$

where P_t is the stock price at time t, D_{t+1} is the dividend paid by the stock at the end of period t, β is the subjective discount factor of the representative agent and $U' \left(C_t \right)$ is the marginal utility of consumption C_t in period t. M_{t+1} is called the stochastic discount factor (SDF). This equation is a no arbitrage equation, which states that the utility lost by reducing consumption of one unit in period t and investing it in the stock is equal to the discounted and expected increase in utility obtained from the extra payoff at time $t+1$.

To be able to compute SDF, I assume that

Assumption 1 *The representative agent has a power utility function*

With a power utility function, the SDF is

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\gamma} \tag{3}$$

where γ is the coefficient of relative risk aversion of the representative agent.

Of course, many other utility functions are possible (e.g. exponential utility, habit formation, prospect utility, etc...). In particular, utility functions with time-varying risk aversion have attracted a lot of attention in the recent literature.[6] Their success is mainly due to their ability to capture cyclical variation in the SDF. However, the goal of this paper is to estimate the long term fundamental value of stock prices rather than explaining their short term variations. In that sense, as mentioned by Campbell and Viceira (2002, p. 25), the constant relative risk aversion implied by power utility is "inherently attractive and is required to explain the stability of financial variables in the face of secular economic growth". However, if there is a need to use another utility function, the methodology proposed here is easily adaptable, by replacing the SDF with the adequate expression.[7] The only requirement is that the utility function should give a SDF which is a linear function of observable variables. Habit formation functions or loss aversion functions, for example, have a SDF that fits into this framework (cf. Monnin 2007).

2.2 Fundamental present value equation

By forward iteration of the future price in the no arbitrage equation (1), the fundamental price can be expressed as[8]

$$P_t = E_t \sum_{i=1}^{\infty} \left(\prod_{j=1}^{i} M_{t+j} \right) D_{t+i} \tag{4}$$

[6]For example, habit formation models initiated by Constantinides (1990) or Campbell and Cochrane (1999) have been used extensively to empirically study time-varying risk aversion.

[7]Most asset pricing models can be expressed in the SDF model. They only differ by the form taken by M_{t+1} (cf. Cochrane 2001).

[8]Formally, the transversality condition $\lim_{i\to\infty} E_t \left[\left(\prod_{j=1}^{i} M_{t+j} \right) P_{t+i} \right] = 0$ is imposed to get equation (4). This condition rules out bubbles in the infinite horizon.

This equation simply tells that the fundamental asset price in period t is equal to the *expected present value of future dividends* paid by the asset. The discount factor M_{t+i} used to compute the present value is a function of the expected marginal utilities of future consumption. Thus, the two fundamental variables driving the fundamental asset price are dividends and consumption.

Since prices and dividends are not stationary, it is convenient, for empirical purposes, to express the present value in equation (4) in terms of stationary variables. For that, as suggested by Cochrane (1992), divide both sides by dividends to get

$$PD_t = E_t \sum_{i=1}^{\infty} \prod_{j=1}^{i} M_{t+j} \gamma_{t+j} \tag{5}$$

where $PD_t = P_t/D_t$ is the price dividend ratio (PD ratio) at time t and $\gamma_t = D_t/D_{t-1}$ is the gross growth rate of dividends between t and $t-1$.

2.3 Linearization

The right hand side of equation (5) is clearly non linear, which is not convenient for empirical estimations. To cope with this problem, it is possible to linearize the fundamental price by taking the logarithm of equation (5) and by using a second order Taylor expansion of the right hand side of this equation around its mean. This yields (see proof in Appendix A.1)

$$pd_t = E_t\left(pd_t^*\right) + \frac{1}{2} V_t\left(pd_t^*\right) + R_t \tag{6}$$

where $pd_t = \ln PD_t$ and $pd_t^* = \ln PD_t^*$ with $PD_t^* = \sum_{i=1}^{\infty} \prod_{j=1}^{i} M_{t+j}\gamma_{t+j}$ being the PD ratio with all future fundamentals known with certainty (or, in other terms, the PD ratio with perfect forecasts). R_t is the remainder of the Taylor expansion, which is a function of third and higher expected moments of pd_t^*.

Note that, by definition, PD_t^* is equal to

$$PD_t^* = M_{t+1}\gamma_{t+1}\left(1 + PD_{t+1}^*\right) \tag{7}$$

Taking the logarithm of this equation yields

$$pd_t^* = m_{t+1} + \Delta d_{t+1} + \ln\left(1 + PD_{t+1}^*\right) \tag{8}$$

where $m_{t+1} = \ln M_{t+1}$, $\Delta d_{t+1} = d_{t+1} - d_t$ and $d_t = \ln D_t$. This expression can be linearized with a first-order Taylor approximation. As shown by Campbell, Lo and MacKinlay (1997), the last term of equation (8) can be approximated by (cf. Appendix A.2)

$$\ln\left(1 + PD_{t+1}^*\right) \simeq \kappa + \rho pd_{t+1}^* \tag{9}$$

where $\rho = 1/\left(1 + \exp\left(-\overline{pd}_t\right)\right)$ and $\kappa = -\ln\rho - (1-\rho)\ln(1/\rho - 1)$ are both linearization coefficients and $\overline{pd}_t$ is the average log PD ratio observed until time t.[9] Substituting this approximation in equation (8) gives

$$pd_t^* \simeq \kappa + m_{t+1} + \Delta d_{t+1} + \rho pd_{t+1}^* \tag{10}$$

Finally, by substituting pd_{t+1}^* forward, we get the following linear approximation of pd_t^*

$$pd_t^* \simeq \sum_{i=1}^{\infty} \rho^{i-1}\left(\kappa + m_{t+i} + \Delta d_{t+i}\right) \tag{11}$$

[9] Note that the linearization coefficient ρ is time varying since it changes each time that the observed average log PD ratio $\overline{pd}_t$ changes. To keep notation simple, I did not use a subscript for the time with ρ, but the reader should keep in mind that ρ is reestimated at each period to compute the fundamental price. The reestimation of ρ is necessary to get a fundamental price that is computed only on the data observable at time t.

If we now use the approximation (11) in equation (6), we get that

$$pd_t = E_t\left(\sum_{i=1}^{\infty}\rho^{i-1}\left(m_{t+i}+\Delta d_{t+i}\right)\right)+\frac{1}{2}V_t\left(\sum_{i=1}^{\infty}\rho^{i-1}\left(m_{t+i}+\Delta d_{t+i}\right)\right)+c_t' \tag{12}$$

where $c_t' = R_t + \kappa/(1-\rho)$. The next step is to replace the log SDF m_{t+i} by its definition given in equation (3)[10] and to express the fundamental log PD ratio in vector terms to simplify notation. For that, let us define the vector $x_t = \begin{bmatrix}\Delta c_t & \Delta d_t\end{bmatrix}'$ which collects all the fundamentals . Using this notation, we can rewrite equation (12) as

$$pd_t = \sum_{i=1}^{\infty}\rho^{i-1}g'E_t\left(x_{t+i}\right)+\frac{1}{2}V_t\left(\sum_{i=1}^{\infty}\rho^{i-1}g'x_{t+i}\right)+c_t \tag{13}$$

where $g' = \begin{bmatrix}-\gamma & 1\end{bmatrix}$ and $c_t = R_t + (\kappa + \ln\beta)/(1-\rho)$. Developing the conditional variance in the second term of the right hand side of this equation yields

$$pd_t = \sum_{i=1}^{\infty}\rho^{i-1}g'E_t\left(x_{t+i}\right)+\frac{1}{2}\sum_{i=1}^{\infty}\sum_{j=1}^{\infty}\rho^{i+j-2}g'E_t\left(\Lambda_{t+i,t+j}\right)g+c_t \tag{14}$$

where $\Lambda_{t+i,t+j} = \varepsilon_{t,t+i}\varepsilon_{t,t+j}'$ and $\varepsilon_{t,t+i} = x_{t+i} - E_t\left(x_{t+i}\right)$ is the error of a forecast made at time t about the fundamental x_{t+i}. The expression $E_t\left(\Lambda_{t+i,t+j}\right)$ is the difference of the expected conditional covariance *at time* t between the $t+i$ and $t+j$ forecast errors.[11] Equation (14) simply states that the fundamental log PD ratio is a linear function of expected fundamentals (dividend growth and consumption growth) and of their expected conditional covariances and autocovariances.

[10] Note that $m_{t+1} = \ln M_{t+1} = \ln\beta - \gamma\Delta c_{t+1}$ where $\Delta c_{t+1} = c_{t+1} - c_t$ with $c_t = \ln C_t$.
[11] The term covariance refers both to variances and covariances.

2.4 Expectations about future fundamentals

As stated in equation (14), the fundamental log P/D ratio is a function of the representative agent's expectations about fundamentals and their conditional covariance. Therefore, to estimate the fundamental price, we have to specify how does the representative agent forms her expectation. According to Definition 1, the representative agent is rational, which implies, by definition, that she will use all the relevant information available at time t to make her forecasts. At time t, the relevant information set is constituted of all present and past fundamentals x_t and of all present and past variables, which have some forecasting power for the fundamentals. The latter variables are collected in the $(p \times 1)$ vector z_t. Note that z_t can include the observed log PD ratio if this one helps to predict future fundamentals. Additionally, we assume that

Assumption 2 *The representative agent forms her expectations about future fundamentals in two steps. In the first step, she estimates the dynamic of the variables in her information set with a VAR model, in which the conditional covariance is modeled with a multivariate GARCH. In the second step, she uses the estimated VAR M-GARCH to forecast future fundamentals and their conditional covariance.*

The VAR part of the model is used to forecast the first part of the right hand side of equation (14) (future fundamentals). The GARCH part estimates the dynamic of the conditional covariance, which is then used to forecast the second part of the right hand side of equation (14) (future conditional variance). Concretely, assumption 2 implies that the representative agent uses the following model to make her forecasts about future fundamentals:

$$y_t = A_0 + A_1 y_{t-1} + ... + A_j y_{t-j} + \varepsilon_t \tag{15}$$

$$\varepsilon_t \sim N(0, H_t) \tag{16}$$

where $y_t = \begin{bmatrix} x_t & z_t \end{bmatrix}'$ is a vector collecting present and past observations (until lag j), A_i are matrices of coefficients[12] estimated at time t and ε_t is an error term, which is normally distributed with a time-varying covariance matrix H_t. Assumption 2 also specifies that the covariance matrix H_t is modelled as a multivariate GARCH. The multivariate GARCH is a generalization of the univariate GARCH and it estimates time-varying covariances in addition to time-varying variances. A general formulation of the multivariate GARCH is the vech model of Bollerslev, Engle and Wooldridge (1988), which has the following specification

$$h_t = K + Bh_{t-1} + Ce_{t-1} \tag{17}$$

$$e_t = h_t + u_t \tag{18}$$

where h_t =vech$\{H_t\}$, e_t =vech$\{\varepsilon_t \varepsilon_t'\}$ where the vech operator converts the lower triangle of a symmetric matrix into a vector. The matrix B and C are matrices of coefficients[13] and u_t is an error term which is normally distributed with a constant covariance matrix Ω. In this model, the covariance matrix is a linear function of its last past values and of last past residuals.[14] For more clarity, it is useful to express this VAR-GARCH model with its companion form

$$\mathbf{y}_t = \mathbf{A}\mathbf{y}_{t-1} + \mathbf{e}_t \tag{19}$$

$$\mathbf{h}_t = \mathbf{B}\mathbf{h}_{t-1} + \mathbf{u}_t \tag{20}$$

[12]The coefficient matrices are time-varying since a rational agent update her estimation with each new observation. I did not use any subscript for the time to simplify the notation. The reader should however bear in mind that a different VAR-GARCH is estimated for each period using only the information available at that time.

[13]The coefficient matrices are time-varying (cf. footnote 12).

[14]In the original vech model, the correlation matrix can be a function of more than one lag (and of other exogeneous variables). However, for simplicity and as it will be formally expressed in assumption 5, I restrict the model to one lag (with no exogeneous variables).

where

$$\mathbf{y}_t = \begin{bmatrix} y_t & y_{t-1} & ... & y_{t-j+1} & 1 \end{bmatrix}' \qquad \mathbf{h}_t = \begin{bmatrix} h_t & e_t & 1 \end{bmatrix}'$$

$$\mathbf{e}_t = \begin{bmatrix} \varepsilon_t & 0 & ... & 0 \end{bmatrix}' \qquad \mathbf{u}_t = \begin{bmatrix} 0 & u_t & 0 \end{bmatrix}'$$

$$\mathbf{A} = \begin{bmatrix} A_1 & \cdots & A_{j-1} & A_j & A_0 \\ 1 & \cdots & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & \cdots & 1 & 0 & 0 \\ 0 & \cdots & 0 & 0 & 1 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} B & C & K \\ B & C & K \\ 0 & 0 & 1 \end{bmatrix}$$

If the representative agent uses this VAR-GARCH model to make her forecasts, her expectation about $\mathbf{y}_{t+i}$ and H_{t+i} will be

$$E_t\left(\mathbf{y}_{t+i}\right) = \mathbf{A}^i \mathbf{y}_t \tag{21}$$

$$E_t\left(H_{t+i}\right) = \text{vech}^{-1}\left\{\mathbf{B}^i \mathbf{h}_t\right\} \tag{22}$$

where the operator vech^{-1} converts a vector into the lower triangle of a symmetric matrix.

Finally, before using these expectations in the fundamental equation (14), it is useful to define $\mathbf{H}_{t+i} = \mathbf{e}_{t+i}\mathbf{e}'_{t+i}$. We have that

$$\begin{aligned} E_t\left(\mathbf{H}_{t+i}\right) &= E_t\left(\mathbf{e}_{t+i}\mathbf{e}'_{t+i}\right) = \begin{bmatrix} E_t\left(H_{t+i}\right) & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix} \\ &= \mathbf{q}\text{vech}^{-1}\left\{\mathbf{B}^i \mathbf{h}_t\right\}\mathbf{q}' \end{aligned} \tag{23}$$

where $\mathbf{q}' = \begin{bmatrix} \mathbf{I}_{2+p} & \mathbf{0}_{(2+p)\times(2+p)(j-1)} \end{bmatrix}$.

2.5 Fundamental PD ratio

Once the expectations have been defined, it is possible to derive the fundamental log PD ratio as a function of the estimated VAR-GARCH and of the observable variables. For that, let us rewrite equation (14) with the new notation:

$$pd_t = \sum_{i=1}^{\infty} \rho^{i-1} \mathbf{g}' E_t \left(\mathbf{y}_{t+i}\right) + \frac{1}{2} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \rho^{i+j-2} \mathbf{g}' E_t \left(\mathbf{\Lambda}_{t+i,t+j}\right) \mathbf{g} + c \qquad (24)$$

where $\mathbf{g}' = \begin{bmatrix} g & \mathbf{0}_{(2+p)(j-1)+p} \end{bmatrix}$ and $\mathbf{\Lambda}_{t+i,t+j} = \mathbf{e}_{t,t+i}\mathbf{e}'_{t,t+j}$ with $\mathbf{e}_{t,t+i} = \begin{bmatrix} \varepsilon_{t,t+i} & 0 & ... & 0 \end{bmatrix}'$. The vector $\mathbf{g}'$ is a row vector which selects x_t in $\mathbf{y}_t$ and multiplies it by g.[15]

Let us first consider the second part of the right hand side of equation (24). Using the fact that $\mathbf{y}_{t+1} = \mathbf{A}_t\mathbf{y}_t + \mathbf{e}_{t+1}$, we can express the error made at time t about the vector $\mathbf{y}_{t+i}$ as a function of one-period shocks

$$\mathbf{e}_{t,t+i} = \sum_{k=1}^{i} \mathbf{A}^{i-k} \mathbf{e}_{t+k} \qquad (25)$$

Plugging that into $\mathbf{\Lambda}_{t+i,t+j}$ yields

$$\mathbf{\Lambda}_{t+i,t+j} = \sum_{k=1}^{i} \sum_{l=1}^{j} \mathbf{A}_t^{i-k} E_t \left(\mathbf{e}_{t+k}\mathbf{e}'_{t+l}\right) \left(\mathbf{A}'_t\right)^{j-l} \qquad (26)$$

Using this expression and the fact that the error terms are not autocorrelated,[16] we

[15] Note that, given assumption 2, expected third and higher moments are constant. Therefore, the remainder R_t of the Taylor expansion, and thus c_t, are also constant.

[16] Cf. equation (16).

can rewrite equation (24) as

$$pd_t = \sum_{i=1}^{\infty} \rho^{i-1} \mathbf{g}' E_t \left(\mathbf{y}_{t+i} \right) + \frac{1}{2} \sum_{i=1}^{\infty} \rho^{i-1} \mathbf{g}' \left(\mathbf{I} - \rho \mathbf{A} \right)^{-1} E_t \left(\mathbf{H}_{t+i} \right) \left(\mathbf{I} - \rho \mathbf{A}' \right)^{-1} \mathbf{g} + c \quad (27)$$

We can use the expectation derived in equations (21) and (23) into our fundamental equation to get

$$pd_t = \sum_{i=1}^{\infty} \rho^{i-1} \mathbf{g}' \mathbf{A}^i \mathbf{y}_t + \frac{1}{2} \sum_{i=1}^{\infty} \rho^{i-1} \mathbf{g}' \left(\mathbf{I} - \rho \mathbf{A} \right)^{-1} \mathbf{q} \text{vech}^{-1} \left\{ \mathbf{B}^i \mathbf{h}_t \right\} \mathbf{q}' \left(\mathbf{I} - \rho \mathbf{A}' \right)^{-1} \mathbf{g} + c \quad (28)$$

Finally, using the fact that the vech^{-1} operator is has the following properties

$$a \text{vech}^{-1} (x) = \text{vech}^{-1} (ax) \quad (29)$$

$$\text{vech}^{-1} (x) + \text{vech}^{-1} (y) = \text{vech}^{-1} (x + y) \quad (30)$$

we get the last fundamental equation

$$pd_t = \mathbf{g}' \left(\mathbf{I} - \rho \mathbf{A} \right)^{-1} \mathbf{A} \mathbf{y}_t + \frac{1}{2} \mathbf{g}' \left(\mathbf{I} - \rho \mathbf{A} \right)^{-1} \mathbf{q} \text{vech}^{-1} \left\{ \left(\mathbf{I} - \rho \mathbf{B} \right)^{-1} \mathbf{B} \mathbf{h}_t \right\} \mathbf{q}' \left(\mathbf{I} - \rho \mathbf{A}' \right)^{-1} \mathbf{g} + c \quad (31)$$

Equation (31) expresses the fundamental log PD ratio as a function of 1) the observable variables in $\mathbf{y}_t$, 2) the estimated covariance matrix in $\mathbf{h}_t$, 3) the estimated VAR-GARCH dynamic in $\mathbf{A}$ and $\mathbf{B}$, 4) the coefficient of relative risk aversion γ in $\mathbf{g}$ and 5) the linearization parameter ρ. The next sections present the empirical estimation of this fundamental equation.

3 Data

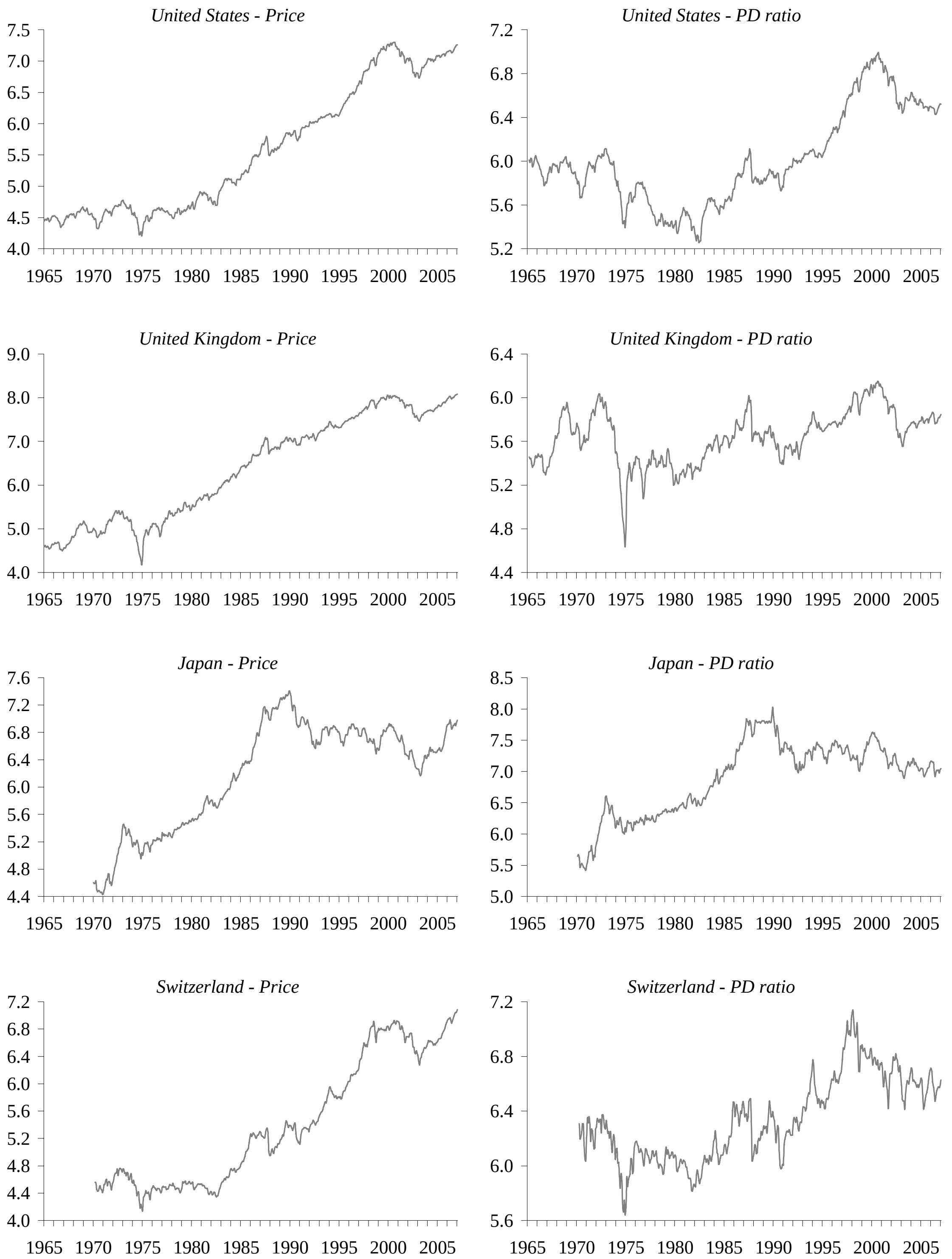
The next assumption concerns the set of variables y_t that is used to forecast future fundamentals (i.e. future consumption and dividends).

Assumption 3 *The representative agent uses past and present fundamentals (i.e. consumption and dividends) to forecasts future fundamentals.*

Following assumption 3, the data set contains stock prices, dividends and consumption data for four countries: Unites States, United Kingdom, Japan and Switzerland. All series are monthly. Stock prices are measured by the S&P 500 index for United States, the FTSE All shares index for Untied Kingdom and the MSCI indexes for Japan and Switzerland. Monthly data are obtained by taking the monthly average of daily prices. Dividends are computed with dividend yield data given with each stock price indexes. Consumption is measured by personal consumption expenditures in the United States and by the households' consumption expenditure for the other countries. Consumption data for the United Kingdom, Japan and Switzerland are quarterly. They have been converted to monthly data by using Eviews cubic spline conversion method.[17] The samples go from January 1965 through to January 2007 for the United States and the United Kingdom, from January 1970 through to January 2007 for Japan and from March 1970 through to January 2007 for Switzerland. All data stem from Datastream, except consumption data for Japan and Switzerland which stem from the IMF database and the Swiss National Bank database respectively. Figure 1 displays the log stock price indexes and the log PD ratios. Figure 2 shows the log consumption and the log dividends.

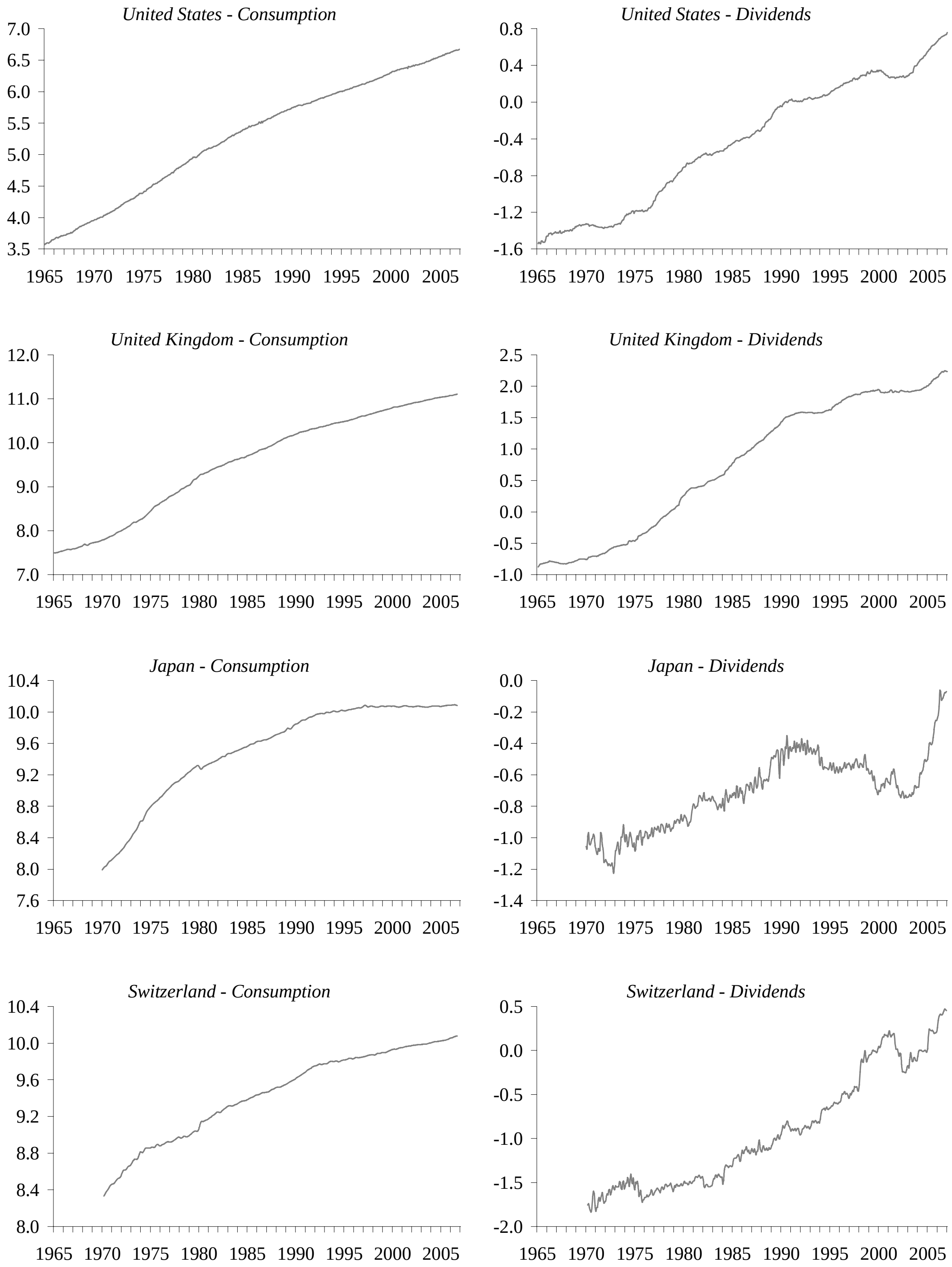
[17] This method assigns each value in the low frequency series to the last high frequency observation associated with the low frequency period, then places all intermediate points on a natural cubic spline connecting all the points.

Figure 1: Stock Prices and PD Ratios



Price indexes: S&P 500 for the Untied States, FTSE All shares for the United Kingdom, MSCI for Japan and Switzerland. Dividends are computed with the dividend yield associated with each of these indexes. Prices are nominal. Both variables are expressed in logarithm. Source: Datastream.

Figure 2: Consumption and Dividends



Consumption: personal consumption expenditures for the United States and households' final consumption for the United Kingdom, Japan and Switzerland. Dividends are computed with the dividend yield associated with each stock price index. Both variables are nominal and expressed in logarithm. Source: Datastream, IMF and Swiss national bank.

Table 1: Structural Breaks in Consumption Growth Rates

	1 break vs. 0 break	2 breaks vs. 1 break	3 breaks vs. 2 breaks	Date(s)
United Stated	30.4246**	4.3361	4.7397	1985.09
United Kingdom	110.0182**	101.7920**	61.2723**	1971.04, 1980.03, 1990.06
Japan	209.0538**	73.0209**	7.4241	1977.07, 1992.04
Switzerland	71.8520**	32.6817**	1.4754	1975.10, 1992.07

* (**) denotes the rejection of the null hypothesis of k breaks for the alternative hypothesis of k+1 breaks at a 5% (1%) confidence level. The critical value for the test are given by Bai and Perron (2003b) for a trimming parameter of 0.15. The number of break dates is equal to the minimum breaks for which the null hypothesis is not rejected. The hypothesis of three vs. four breaks is not rejected for the United Kingdom, which suggests a number of three breaks in this series (to save space, the result of this test does not appear in the table).

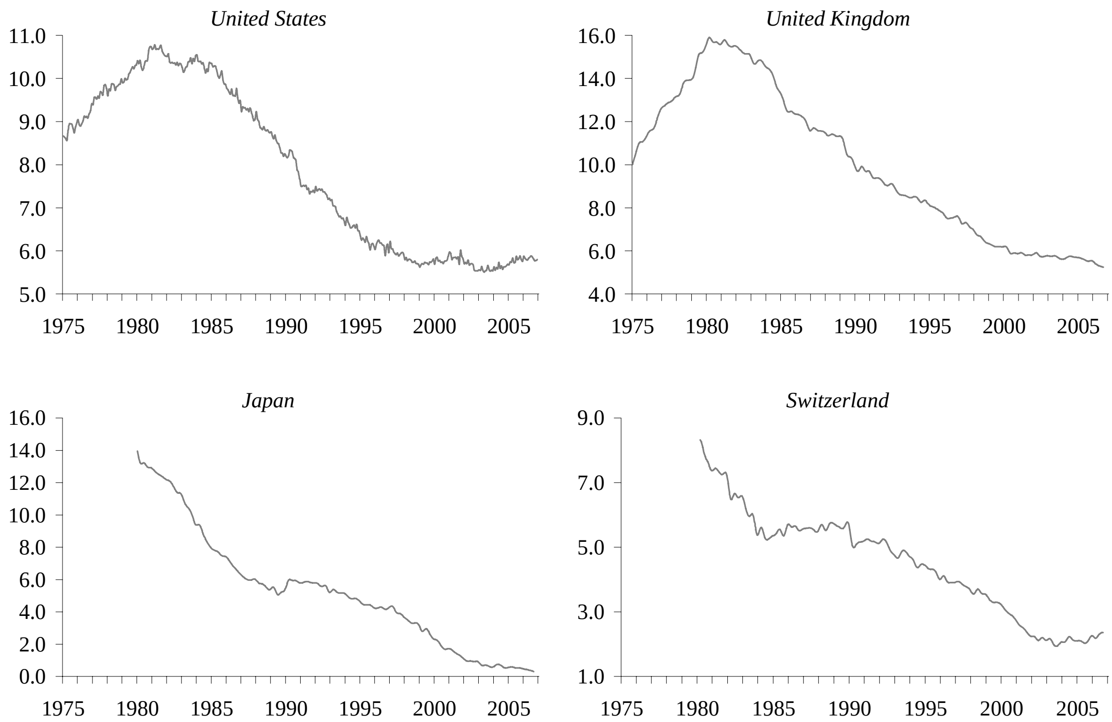
4 Evolution of long-term consumption growth

A first glance at the left-hand side panels of Figure 2 suggests that the consumption growth rate has decreased between 1965 and 2007 in all countries. Since the fundamental price is a function of the expected consumption growth, I investigate in more details the impact of this evolution on agents' expectations before moving on to the empirical estimation of the fundamental price.

Figure 3 presents the 10-year moving average of the consumption growth rate. In each country, we observe an important and continuous decrease after 1980. Structural break tests confirm this decline. Table 1 gives the results of the sup$F$ structural break test developed by Bai and Perron (1998 and 2003a) for the consumption growth rate in the different countries. This test assesses the probability of structural breaks in the average growth rate and estimate their most probable date. The Bai and Perron's sup$F$ statistics clearly detects some structural breaks in the consumption growth rate for each country (one for the United States, three for the United Kingdom and two for Japan and Switzerland). After each break, the average consumption growth rate decreases.[18] All these results point toward a decrease of long term consumption growth rates during the observed period.

[18]The only exception is an increase after the first break (April 1971) in the United Kingdom. The complete results of Bai-Perron tests (in particular the estimated jumps for average consumption growth rates) are available on request.

Figure 3: Evolution of the Consumption Growth Rates

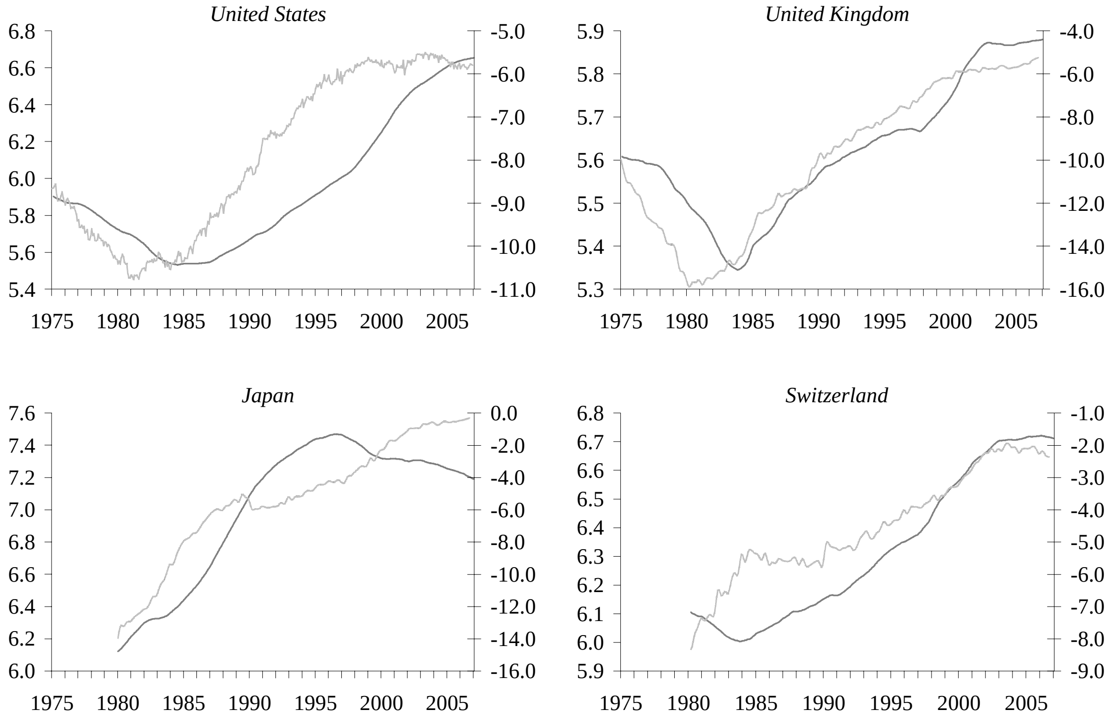


Each panel gives the 10 – year moving average of the consumption growth rate. The growth rate is expressed in annualized percentage.

A structural decrease in the consumption growth rate has an indirect impact on agents' stock valuation. Indeed, if a rational agent observes a structural modification of the consumption growth, she will adapt her expectations to this new dynamic. Since the fundamental PD ratio is a function of expected consumption growth rates, a realignment of the forecasts with the new long term value of the consumption growth rate will directly affect the fundamental PD ratio. Thus, a structural decrease in consumption growth rate leads to a structural increase of the PD ratio. In economic terms, if an agent observes a permanent decrease in her consumption growth rate, then her marginal rate of intertemporal substitution for consumption[19] increase, which makes future dividends more valuable to her and yields an appreciation of the asset

[19] The marginal rate of intertemporal substitution for consumption corresponds to the ratio between tomorrow and today marginal utility of consumption, as stated in equation (2).

Figure 4: Link Between PD Ratios and (minus) Consumption Growth Rates



The black line is the 10 – year moving average of the PD ratio expressed in logarithm (left axis). The grey line is (minus) the 10 – year moving average of the consumption growth rate, expressed in annualized terms (right axis).

price.

Figure 4 illustrates the negative link between the consumption growth rate and asset prices. It clearly shows that the 10-year moving average of the PD ratio has increased in each country during the observation period, following the decrease in consumption growth rate (or the increase of the negative consumption growth rate given by the grey line in Figure 4). Structural break tests also spot significant increases in the PD ratio.[20] This is in line with the predictions of the fundamental model for decreasing

[20]The results of the structural break tests for PD ratios are not presented here. They are available on request. They show that a structural break in consumption growth is followed by several breaks in the PD ratio average. The delay between the break in consumption growth and those in the PD ratio average is rather long (e.g. up to 10 years after the 1985.09 break for the United States). However, a Bai-Perron test with a rolling window shows that the hypothesis of no break in consumption growth would be rejected for the first time in 1992.02 at a 10% confidence level and in 1995.01 at a 1% confidence level. These results suggest that it take time for the agents to notice a structural break and that they might take it gradually into account depending on the confidence they have in their

consumption growth rates. It suggests that agents realize (immediately or after some delay) that the long term consumption growth rate has changed and thus adapt their asset valuation to the new growth rate. To take this phenomenon into account, I use the following hypothesis

Assumption 4 *The representative agent bases her forecasts about future consumption growth rates on the average consumption growth rate observed in the last five years.*

By taking the average growth rate over the last five years instead of the average over the whole period, the agent get an estimate of the long term growth rate which is currently driving consumption. Assumption 4 does not mean that the agent abandons the use the VAR-MGARCH model, but rather that she uses it to forecasts deviations from the long term average.

5 Econometric methodology

To be able to compute the fundamental price given by equation (31), we must know **A**, **B**, ρ and c. The next two sections explain how to estimate these coefficients. Note that each estimation is made with the sample observable at time t, such that the sample used reflects exactly the information set of the representative agent at that time.

5.1 Estimation of the VAR-GARCH model

The first step of the computation of the fundamental stock price is to estimate the matrices **A** and **B** of the VAR-GARCH model. This requires the simultaneous estimation of equations (15) and (17). For the estimation of the GARCH dynamic to remain computationally feasible, it is necessary to restrict the number of coefficients in **A** and **B**. For that, I make the following assumption:

estimations. This is in line with the succession of structural breaks observed in the PD ratio and the delay between the breaks in consumption growth and those in the PD ratio. Note that Lettau and van Nieuwerburg (2005) find results similar to mine for breaks in PD ratio mean for the United States.

Assumption 5 *Each element of the conditional covariance matrix depends only on its own last value and last residuals.*

This assumption means that the GARCH dynamic is a BEKK model (Engle and Kroner 1995) with one lag and no exogenous variable. The BEKK model can be estimated by the traditional maximization of the log likelihood function as explained in Hamilton (1994, p. 670).

I estimated a VAR-GARCH model for each period using only the data available at that time. With this procedure, I placed myself exactly in the same situation as investor living at time t. This yields a truly "out-of-sample" estimation of the fundamental price, i.e. based only on *ex-ante* data. In each period I used 6 lags in the VAR part of the model. In addition to the matrices $\mathbf{A}$ and $\mathbf{B}$, the estimation of the VAR-GARCH also gives an estimation of the conditional variance-covariance vector $\mathbf{h}_t$.

5.2 Estimation of the relative risk aversion coefficient

After the estimation of the matrices $\mathbf{A}$ and $\mathbf{B}$, the only unknown remaining in the fundamental equation (31) is the relative risk aversion coefficient γ appearing in $\mathbf{g}$. Note that, without using the matrix notation, equation (31) is equivalent to

$$pd_t = -\gamma q_{1,t} + q_{2,t} + \frac{1}{2}\gamma^2 q_{3,t} - \gamma q_{4,t} + \frac{1}{2} q_{5,t} + c \tag{32}$$

where $q_{1,t}$ and $q_{2,t}$ are the first and second elements of $(\mathbf{I} - \rho\mathbf{A})^{-1}\mathbf{A}\mathbf{y}_t$ for $\mathbf{A}$ and ρ estimated with the sample available at time t, and $q_{3,t}$, $q_{4,t}$ and $q_{5,t}$ are the (1,1)-th, (1,2)-th and (2,2)-th element of

$$(\mathbf{I} - \rho\mathbf{A})^{-1}\mathbf{q}vech^{-1}\left\{(\mathbf{I} - \rho\mathbf{B})^{-1}\mathbf{B}\mathbf{h}_t\right\}\mathbf{q}'(\mathbf{I} - \rho\mathbf{A}')^{-1} \tag{33}$$

for $\mathbf{A}$, $\mathbf{B}$, ρ and $\mathbf{h}_t$ estimated with the sample available at time t. The parameters γ and c can then be estimated, for each period, by estimating equation (32) with OLS and a sample containing only the variables $q_{\cdot,t-k}$ (for all k between 0 and $t-1$) available at this time. The estimated coefficients of relative risk aversion lie, depending on the period, between 0 and 4 for the United States, between 0 and 6 for the United Kingdom, between 0 and 0.6 for Japan and between 0 and 0.4 for Switzerland. This is in the range of the admissible values given by Mehra and Prescott (1986).

6 Fundamental stock prices

This section presents the estimated fundamental stock price for the different countries and then studies the dynamic of the gap between the fundamental and the observed prices.

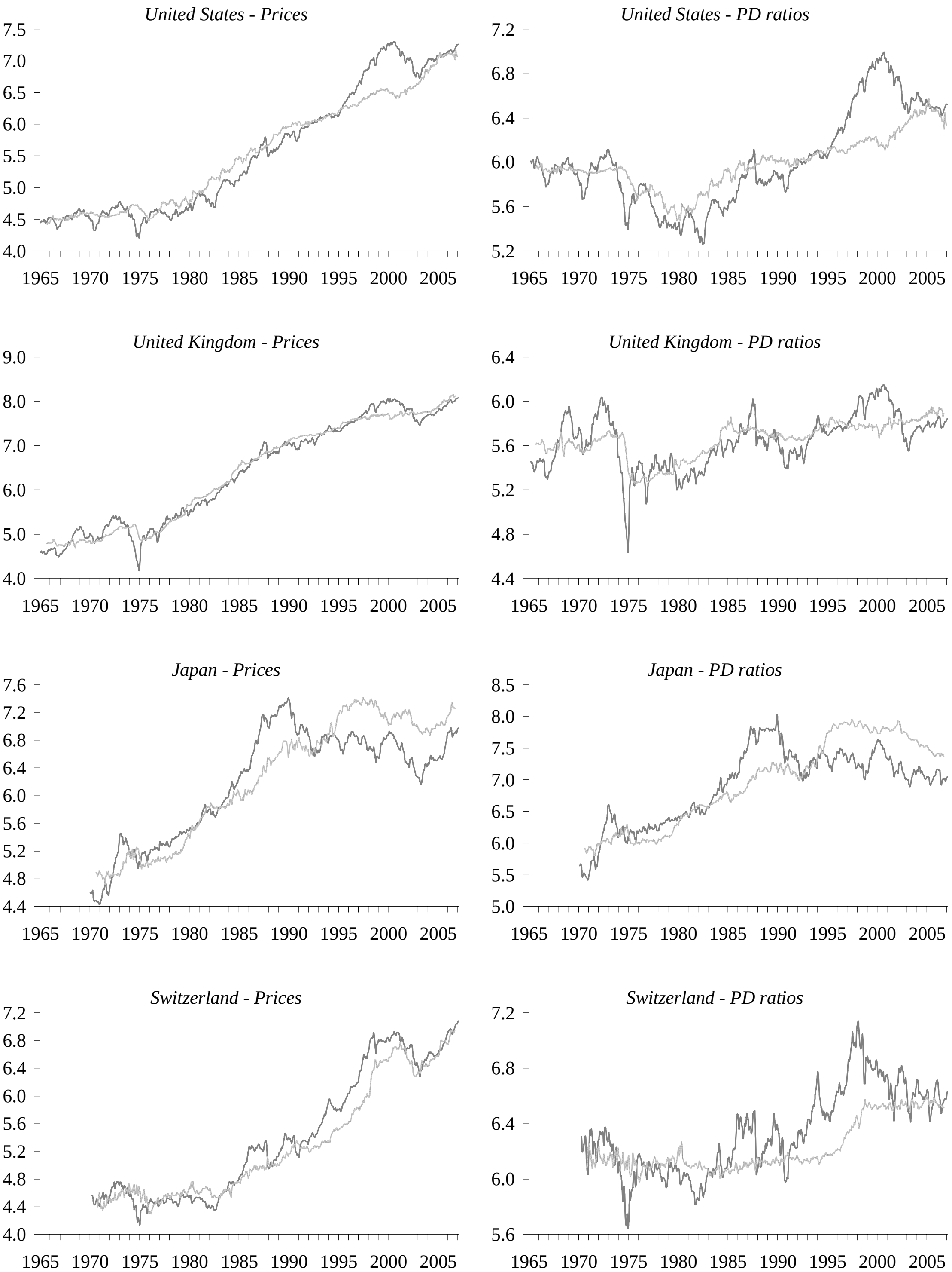
6.1 Empirical results

The fundamental log PD ratio can be computed with equation (31) for parameters estimated as explained in section 5. The fundamental price can then be recovered from the fundamental log PD ratio by adding d_t. Figure 5 shows the estimated fundamental price and the observed price (left panels) and the estimated fundamental and observed PD ratios (right panels). The gap between the observed and the fundamental prices is presented in Figure 6.

In each country, we observe that the price can diverge significantly and for long periods from its fundamental value. The American stock market is characterized by a huge gap at the end of the nineties, which is associated with the Internet bubble. Before that, and according to the C-CAPM model, the stock prices were most of the time undervalued, with some short episodes of overvaluation (e.g. in 1974 or in 1987).[21] The stock market in the United Kingdom follows a similar pattern, with a

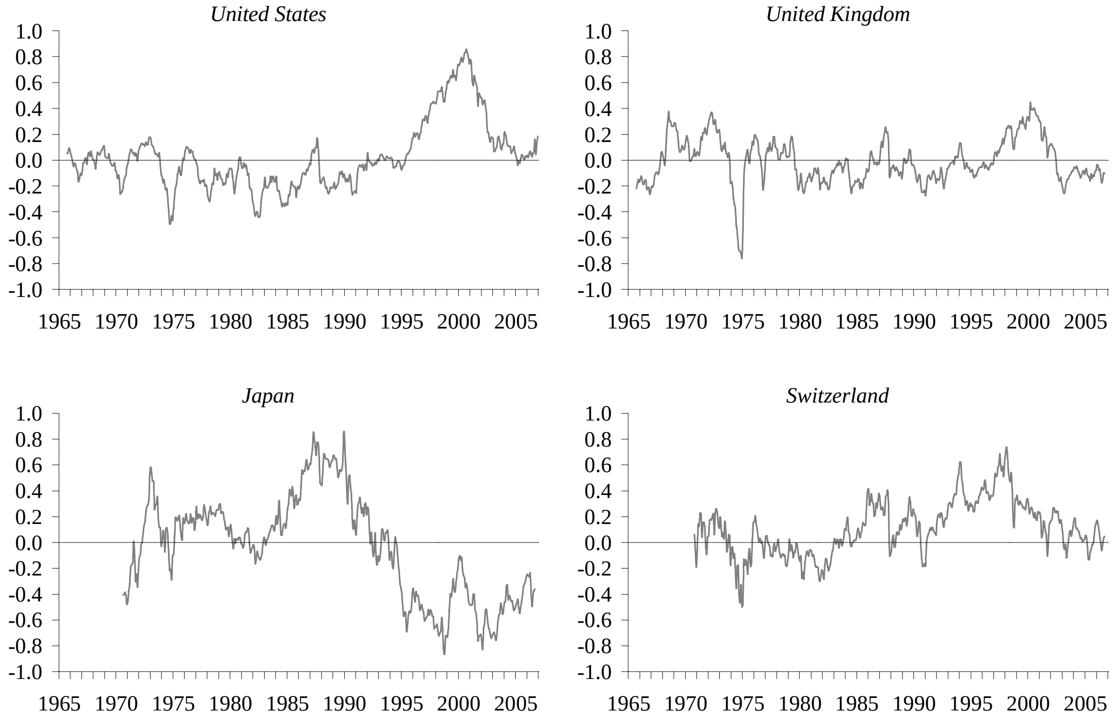
[21]Zhong *et al.* (2003) and Black, Fraser and Groenewold (2003) get a similar period of underval-

Figure 5: Fundamental Stock Prices and Fundamental PD Ratios



The black line is the observed stock price. The grey line is the estimated fundamental stock price. Both indexes are expressed in logarithm.

Figure 6: Gap Between the Observed and the Fundamental Prices



Each panel presents the difference between the (log) observed stock price and the estimated (log) fundamental stock price.

smaller overvaluation at the end of the nineties (and a bigger undervaluation after the crash in 1974). The Japanese market has known a long period of relatively important overvaluation at the end of the eighties and in the very beginning of the nineties. Since then, the C-CAPM model indicates that stocks are undervalued. Finally, the Swiss market displays a pattern similar to the United Kingdom with the exception that the main overvaluation period begins earlier in the nineties and decrease mostly in 1998. Note that the Swiss market has suffered more than other western countries of the 1998 crisis (Russian/ LTCM crisis). July 1998 has remained the historical maximum until the beginning of 2006 in Switzerland, whereas stock prices recovered much more rapidly

uation between the second half of the 70s and the first half of the 90s. The former uses *ex-post* data for sample going from 1871 through to 1997 (as in Shiller 1981); the latter use a general equilibrium framwork with samples going from 1947 trough to 2002 for quarterly data and from 1929 through to 2001 for annual data.

in other western countries.[22]

Finally, note that the volatility of the observed price is significantly greater than the volatility of the fundamental price in the United States and in Switzerland. This is in-line with the stock price volatility puzzle (Shiller 1981, LeRoy and Porter 1981). The volatility of the stock price in the United Kingdom is greater that the one of its fundamental value but the difference is not significant. Finally, in Japan, the volatility of the stock price is smaller than the one of the fundamental price.

6.2 Are stocks prices linked with their fundamental value?

As it is shown in the previous section, the prices can significantly diverge from their fundamental value for long periods. This conclusion naturally raises the question of the existence (or absence) of a link between the observed and the fundamental price. I study this question from two points of view. Firstly, I test for the presence of a unit root in the gap between the two prices. If the unit root is rejected, then the gap is mean reverting, which means that it tends to disappear after some time. In this case, the observed price eventually meets with its fundamental value.[23] Secondly, I check if the fundamental price can help to forecast future prices out-of-sample. This second point is investigated in details in the next section. The first approach is in-sample and the second is out-of-sample.

Table 2 presents the results of the Augmented Dickey-Fuller test and the Phillips-Perron test for unit root. The hypothesis of a unit root is rejected for all gaps and for both tests at a 10% confidence interval. The unit root is more strongly rejected for the United Kingdom and Switzerland. These results suggest that the gap is mean reverting and that there is a long term link between the price and its fundamental value. The

[22] More precisely, for the Swiss market, the record high of the 21 July 1998 (1931.49) has been beaten for one day (23 August 2000) before plunging until the beginning of 2003 with other western markets. On the other hand, the stock price index returned ot its level of July 1998 only five months later in the United States and ten months later in the United Kingdom.

[23] This approach is equivalent to testing for cointegration between the fundamental and the observed prices. In our case, the cointegrating vector is know *a priori* and equal to $\begin{bmatrix} 1 & -1 \end{bmatrix}$.

Table 2: Unit Root Tests for the Gaps

	ADF test	PP test
United States	-1.9209*	-2.0302**
United Kingdom	-4.2921***	-3.7850***
Japan	-1.7706*	-1.8606*
Switzerland	-2.5886***	-2.8800***

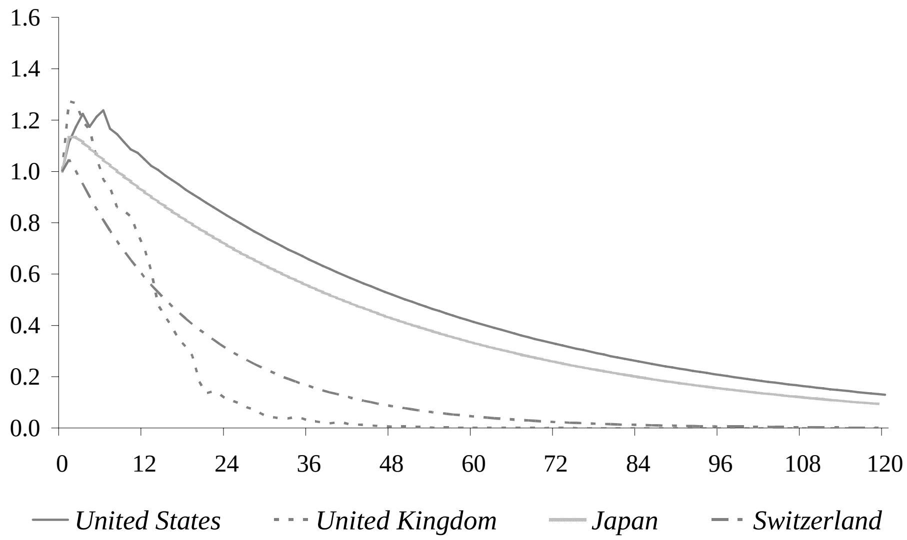
* (**, ***) denotes the rejection of the null hypothesis of unit root at a 10% (5%, 1%) confidence level. The test is performed with an equation without trend or constant. ADF means Augmented Dickey-Fuller and PP means Philipps-Perron.

link seems stronger in the United Kingdom and Switzerland than in the United States or in Japan.

In complement to the unit root tests, I estimate the impulse-response function of the different gaps. The results are presented in Figure 7. Each impulse-response function indicates how the gap evolves after a shock. As suggested by the unit root tests, all functions show that the gap generated by a random shock tends to disappear after some time. The gap vanishes more quickly in the United Kingdom and in Switzerland than in the United States or Japan. The half-life[24] of the gap is one year and two months in the United Kingdom and one year and 4 months in Switzerland against about 3.5 years in Japan and more that 4 years in the United States. This is a sign that there is a link between the observed and the fundamental price, but that this link is weak and that it takes time for the gap to die out after a shock. This suggest that the C-CAPM fundamental price does play a role in the *long term* evolution of the stock price, but that it performs rather poorly in explaining the short term dynamic of the stock price.

[24]The half-life corresponds to the number of months needed for the gap to wipe out half of the initial shock.

Figure 7: Impulse-Response Function for the Gaps



Impulse-response functions for the gaps between the fundamental and the observed price (expressed in months, horizontal axis).

7 Do fundamental prices help to forecast future prices?

This section analyses the ability of the information contained in the fundamental price to forecast future stock prices. As explained in the previous section, apart from its obvious practical applications, the forecast ability of the fundamental price is another way to study the link between the price and its fundamental value. If the fundamental price is able to give good out-of-sample forecasts, then it is a sign that there is a link between them.

Section 7.2 compares the accuracy of the forecasts made by a model based on the observed price only with those of different models that exploit the information contained in the fundamental price. These different models are presented in section 7.1. For all models, I compute out-of-sample forecasts. I also vary the forecast horizon h to see how the precision of the different models evolves with it. In section 7.3, I compare the forecasts of the C-CAPM fundamental model with other simpler fundamental models. Finally, in section 7.4, I analyze how the fundamental models performs in different periods to find out when the fundamental model is particularly accurate.

7.1 Forecasting models

The goal of the forecasting exercise is to determine if the information contained in the fundamental model does improve forecasts based solely on the observed prices. For that, I compare four forecasting models:

1. **Random walk with drift (benchmark model)**: for this model, I first estimate the growth rate of the observed price on the observable sample at time t. Then, I use the estimated growth rate to forecast the future price for different horizon starting at the price observed at time t.

2. **Fundamental random walk with drift**: similarly to the benchmark model, I first estimate the growth rate of the *fundamental* price on the observable sample at time t. Then, I use the estimated growth rate to forecast the future fundamental price for different horizon starting at the fundamental price at time t. I set the forecast for the future price equal to the forecast for the fundamental price.

3. **Error correction model**: in this model, I check if the gap observed at time t helps to predict the forecasting error at time $t+h$. For that, I estimate the following equation:

$$p_t - E_{t-h}(p_t) = \beta_0 + \beta_1 g_{t-h} \tag{34}$$

on the sample observable at time t, where $E_{t-h}(p_t)$ are the forecasting errors made by the random walk with a drift model.[25] I then correct the random walk with a drift forecasts with the estimated error.

[25] To estimate equation (34), I first make out-of-sample forecasts for a subsample $[t-k+1, t]$ of the entire sample $[0; t]$ available at time t by using the dynamic observed on $[0, t-k]$. I then use the forecast errors in equation (34).

4. **Fundamental price and gap dynamic**: this model combined the fundamental random walk dynamic with drift described above with the dynamic of the gap. The latter is estimated with the following equation:

$$\Delta g_t = \beta_0 + \beta_1 g_{t-1} + \beta_2 \Delta g_{t-1} + \varepsilon_t. \tag{35}$$

where g_t is the gap observed at time t. I first forecast the dynamic of the gap with the previous equation and then add the forecasted gap to the forecasted fundamental price obtained with the fundamental random walk with drift.

Note that, in addition to these models, I have also estimated models in which the observed price or the fundamental price is a random walk or a AR(1) process. However, their forecasts are constantly outperformed by those of the random walk with drift models.[26] Their results are therefore not presented here. Finally, remember that all forecasts are made out-of-sample. The forecast accuracy is measured by the mean absolute error.[27]

7.2 Forecast accuracy

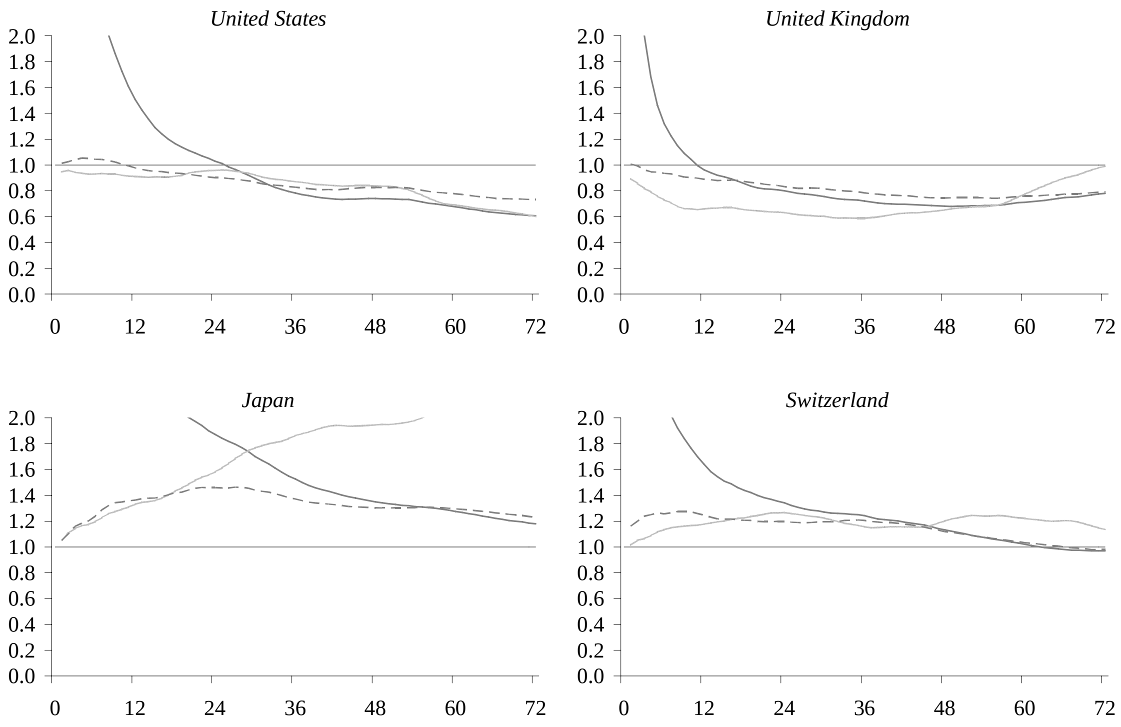
Figure 8 presents relative forecast accuracy of the different fundamental forecasting models with the benchmark for different forecast horizons (horizontal axis). A value below one indicates that the model is more accurate than the benchmark.

One can distinguish two groups: the United States and the United Kingdom on the one side and Japan and Switzerland on the other side. For the United States and the United Kingdom, using the fundamental price significantly improves forecast accuracy. For both countries, the fundamental random walk with a drift forecasts outperform the benchmark for forecast horizon longer than 2 years for the United

[26]The only exception is Japan for which the random walk and the AR(1) forecasts were slightly better than the random walk with a drift for both the observed and the fundamental price. However, using them as benchmark does not change the results presented in the next sections.

[27]Using the root mean square error to measure the forecast accuracy does not change the results.

Figure 8: Forecast Accuracy of Models Based on the Fundamental Price



Each panel compares the forecast accuracy of different models with the forecast accuracy of the random walk with drift model (benchmark). The black line corresponds to the fundamental random walk with drift model, the grey line to the error correction model and the dashed line to the fundamental price and gap dynamic model. Each panel gives the ratio of the model's average absolute errors over the benchmark's average absolute error. A value under 1 indicates that the model is more accurate than the benchmark. The comparison is made for different forecasting horizon (in months, horizontal axis).

States and than one year for the United Kingdom. For a horizon of 6 years and 4 years the improvement is of 40% and 30% for the United States and the United Kingdom respectively. In both countries, the error correction model constantly outperforms the benchmark. The fundamental price and gap dynamic model is also almost always better than the benchmark. The main finding is that by using the information contained in the fundamental price, it is possible to improve the forecasts of the benchmark *for all forecast horizons* for the United States and the United Kingdom. The performance of the fundamental price is particularly striking in the United Kingdom: the benchmark is outperformed by at least 20% for all horizons longer than 3 months!

For Japan and Switzerland, the forecasting accuracy of the benchmark model is generally worse than the one of the benchmark. The fundamental price performs particularly badly in Japan, where none of the models based on the fundamental price can beat the benchmark. For Switzerland, the fundamental random walk with a drift gives slightly more accurate forecasts than the benchmark for horizons longer than 5 years.

These results tend to attest the presence of a relatively strong link between the fundamental and the observed stock prices in the United States and in the United Kingdom. This link does not seem to exist in Japan and is at most weak in Switzerland.

7.3 Alternative fundamental models

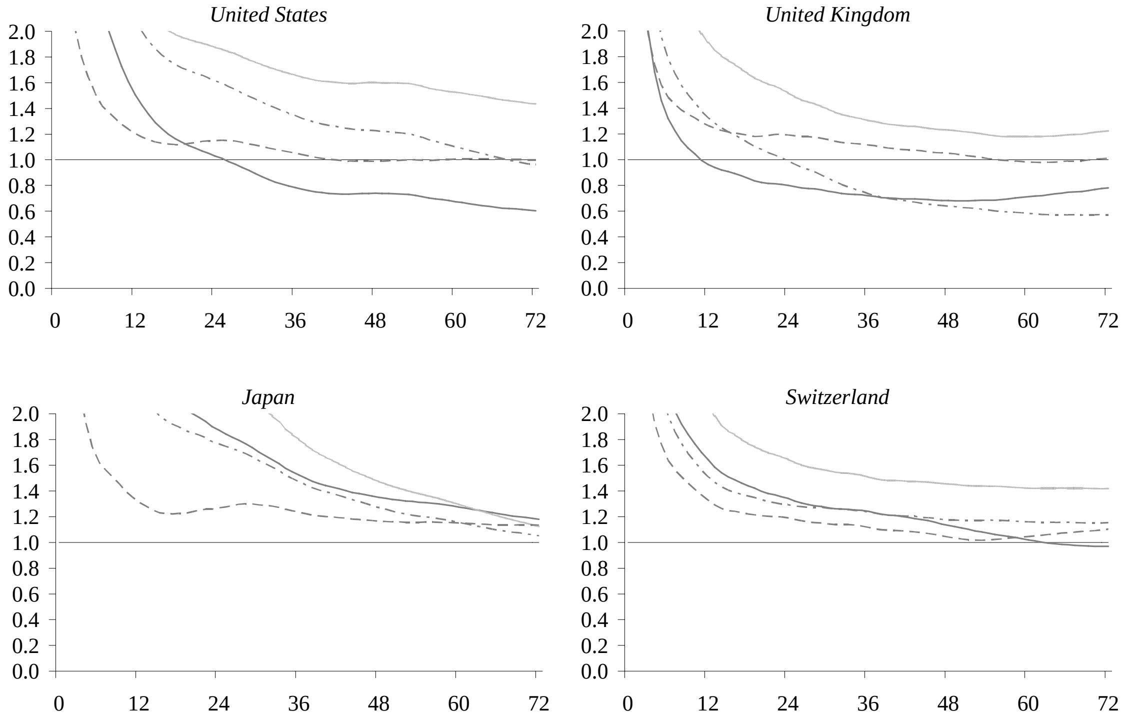
This section compares the forecast accuracy of the C-CAPM fundamental price developed in this paper with simpler had-hoc fundamental models. The goal of this exercise is to check if the effort made in constructing the C-CAPM fundamental price is worthy or if computationally less intensive models give similar results in terms of forecasting power. I compare the C-CAPM model with three simpler models:

1. **Trend model**: in this model, the fundamental price is determined by fitting a linear trend with the observed prices.
2. **Hodrick-Prescott filter model**: in this model the fundamental price is determined by estimating a Hodrick-Prescott filter with the observed price.[28]
3. **Moving average PD ratio**: in this model, the fundamental price is determined by the 10-year moving average of the PD ratio.

At every period, each fundamental price is estimated with the observed data only. I chose to compare the forecasts of the fundamental random walk with drift version of each model (cf. previous section). The alternative fundamental models correspond

[28]The smoothing parameter is set to 230'400, which corresponds to the hypothesis of an average gap of 20% between the fundamental and the observed price and an average annual change of 4% in the stock price long term trend.

Figure 9: Forecast Accuracy of Alternative Fundamental Models

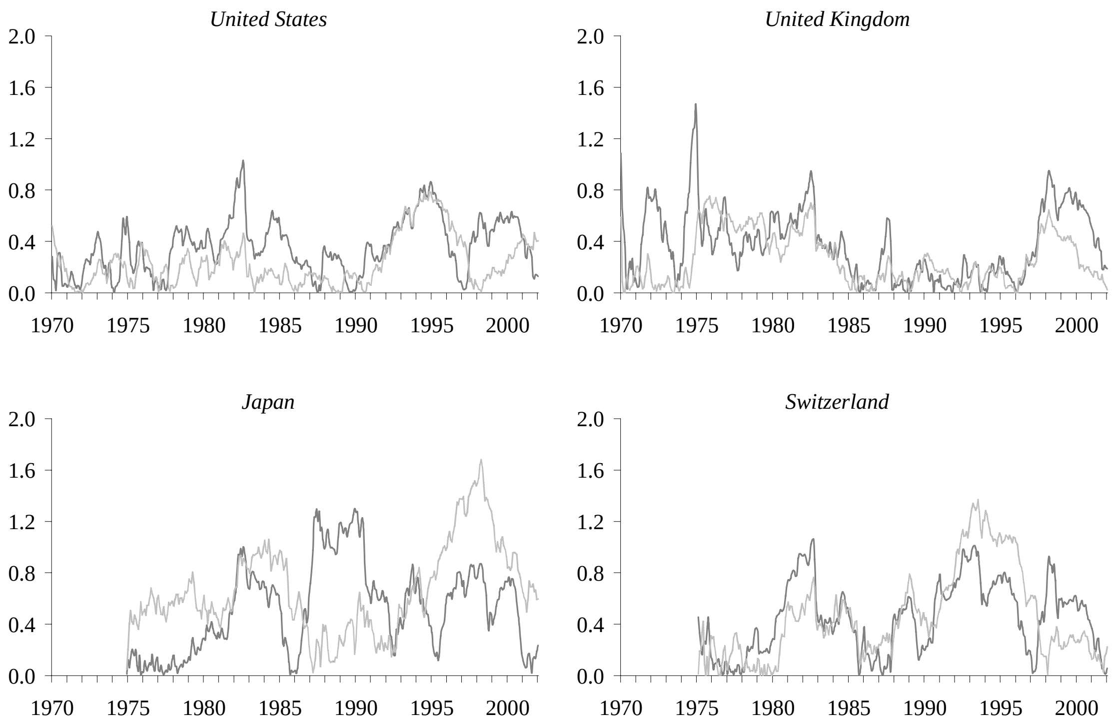


Each panel compares the forecast accuracy of different fundamental models with the forecast accuracy of the random walk with drift model (benchmark). The black line corresponds to the C-CAPM model, the grey line to the trend model, the dashed line to the Hodrick-Prescott filter model and the dot-dashed line to the moving average PD ratio model. Each panel presents the ratio of the model's average absolute errors over the benchmark's average absolute error. A value under 1 indicates that the model is more accurate than the benchmark. The comparison is made for different forecasting horizon (in months, horizontal axis).

to measures that are commonly used by practitioners to estimate imbalances on stock markets. For example, Borio and Lowe (2002) use the Hodrick-Prescott filter to identify bubble in different stock market. A constant PD ratio corresponds to the Gordon (1962) model. The moving average PD ratio can thus be considered as a Gordon model based on the only on the recent observed dynamic.

Figure 9 shows the forecast accuracy of the alternative fundamental models. With the exception of the United Kingdom, none of the alternative model can simultaneously outperform the benchmark or the C-CAPM fundamental model. This means that when the C-CAPM is more accurate than the benchmark, then it is also more accurate than simpler fundamental models. Thus the C-CAPM fundamental model developed in

Figure 10: Comparison of the Evolution of the forecast errors



Each panel compares the forecast accuracy of the random walk model with drift (black line) to the forecast accuracy of the fundamental random walk with drift model (grey line) for a forecast horizon of 60 months. The date at which the forecast is made is indicated on the horizontal axis. The accuracy is measured by the absolute error (vertical axis).

this paper adds some value to simpler ad-hoc fundamental models. For the United Kingdom, the forecasts of moving average PD ratio model are slightly better than the C-CAPM fundamental model for forecasts horizon longer than 4 years.

7.4 When are the fundamental forecasts the most accurate?

The two previous sections examine the forecasts accuracy of different models for the whole sample. However, it might happen that one model is better than another for a particular period. This section addresses this question and study more particularly if there is a link between the wideness of the gap between the fundamental and the observed prices and the precision of the forecasts.

Figure 10 gives an example of the evolution of the errors made by the random walk

TABLE 3: CORRELATION BETWEEN GAPS AND RELATIVE FORECAST ERRORS

Forecast horizon	1 year	2 years	3 years	4 years	5 years
United States	-0.3114*	-0.0080	0.1655*	0.2666*	0.4623*
United Kingdom	0.0373	0.3810*	0.5692*	0.5725*	0.5939*
Japan	-0.6479*	-0.5247*	-0.2999*	-0.2029*	-0.1706*
Switzerland	-0.5221*	0.3641*	-0.3565*	-0.3137*	-0.1870*

* denotes the rejection of the null hypothesis of a coefficient equal to 0 at a 1% confidence level. The correlation is measured with the Spearman coefficient. A positive value means that a (positive or negative) wider gap implies an increase of the accuracy of the fundamental forecasts in comparison to the forecasts based on the observed price

with drift and the fundamental random walk with drift models. It displays the error made at time t for a forecast horizon of 5 years. For example, in the United States, in the United Kingdom and in Switzerland, forecasts made in 1998 and 2001 and based on the fundamental model were more accurate than the one based on the observed price. At the same period, the gap between the observed and the fundamental prices was positive and relatively large in historical comparison. A similar conclusion applies to Japan for the period 1986-1990.

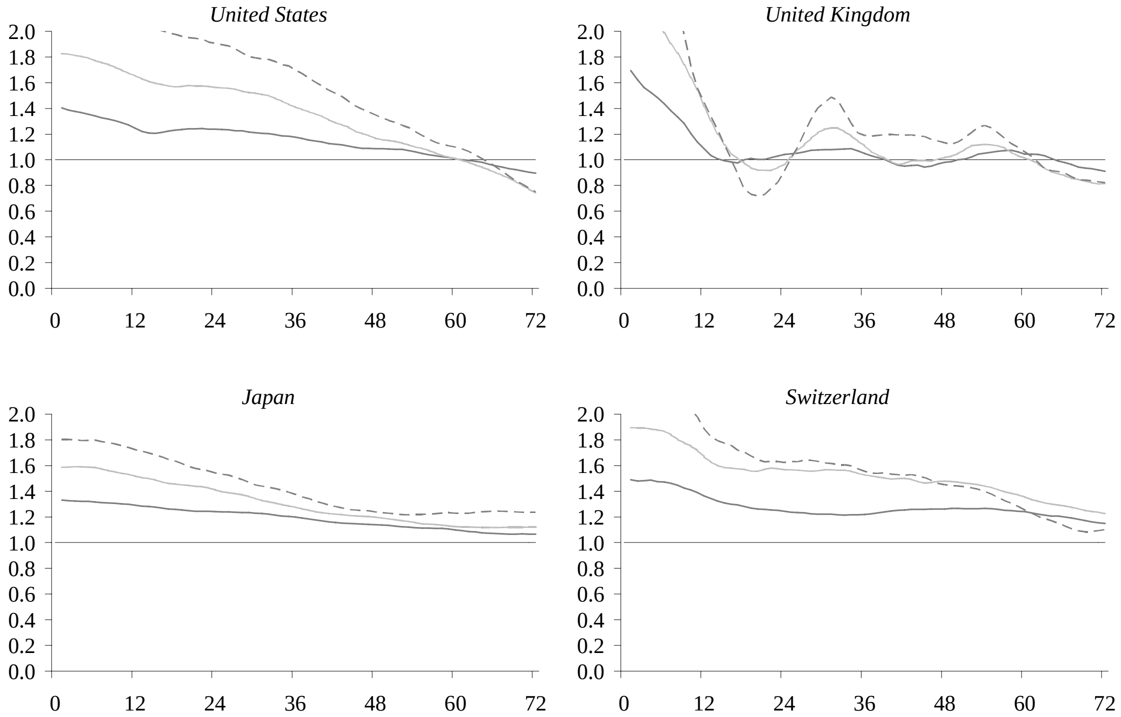
Table 3 gives more details about the relation between the wideness of the gap and the forecast error. It presents the Spearman correlation[29] between the absolute value of the gap and the forecast accuracy improvement made by using the fundamental model.[30] A positive correlation implies that, a (positive or negative) wider gap increases the accuracy of fundamental forecasts in comparison to forecast based on the observed price.

The correlation analysis shows that, in the United Stated and in the United Kingdom for horizon longer than 2 years, a wider gap implies more accurate forecast for the fundamental model. Since more accurate forecasts are a sign of a stronger link between the fundamental and the observed price, the positive correlation suggests that

[29]Since the form of the function linking the (absolute) gap with the forecast error is a priori not known, I prefer to use the Spearman rank-order correlation coefficient rather than the traditional (linear) correlation (Pearson coefficient). Spearman correlation coefficient is independent of the form taken by the function. Spearman rank-order correlation coefficient measures the linear correlation between the ranks of each observation.

[30]This variable takes a positive value when the fundamental price forecasts are better than the observed price forecasts and a negative value when they are worse than them.

Figure 11: Forecast Accuracy For Periods with Large Gaps



Each panel compares the forecast accuracy of the fundamental random walk with drift model for forecasts made in periods with large gaps to the forecast accuracy for the whole period. The black line is for forecasts made when the (absolute) value of the gap is greater than its median, the grey line when the (absolute) gap is greater than its 25% centile and the dashed line when the (absolute) gap is greater than its 10% centile. Each panel presents the ratio of the average absolute errors for the different sub-sample over the average absolute error for the whole sample. A value under 1 indicates that forecasts made in a particular sub-sample are more accurate than in the whole sample. The comparison is made for different forecasting horizon (in months, horizontal axis).

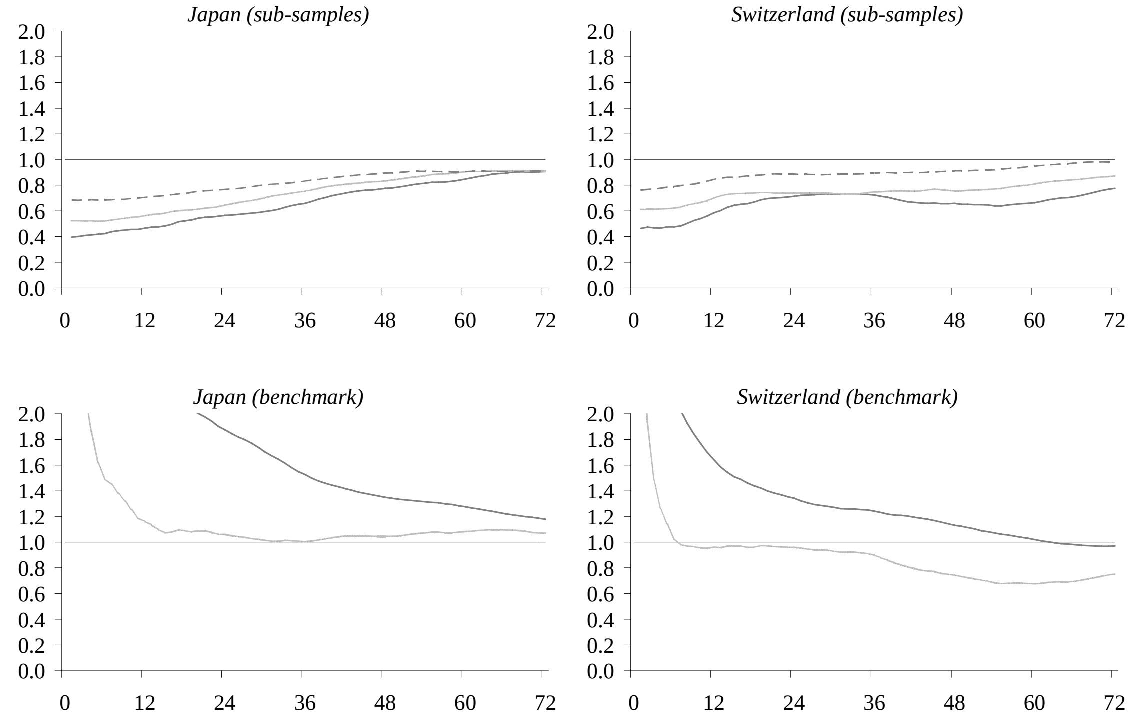
the further apart the two prices are, the more they tends to return towards each other in the long term. In other terms, the "attraction force" exerted by the fundamental price on the observed price get stronger with the distance between them. For Japan and Switzerland, the opposite is true: the fundamental price gives better forecasts when the gap is small. In this case, the fundamental price can be seen as a centre of gravity: the attraction force exerted by the fundamental price is stronger when the distance is short. If the observed price is far from the fundamental price, then the force that brings both prices together is weak or inexistent.

Building on the conclusions made for the United States and the United Kingdom, one could adopt the following strategy: if the gap is large, then one should use the fundamental price for the forecasts, otherwise one should use the observed price. Figure 11 compares the forecast accuracy of the fundamental random walk with drift for any value of the gap with those made when the gap is large. It shows that, for horizon longer than 5 years, the forecast accuracy improves of up to 25% and 20% in the United States and in the United Kingdom, respectively, if the forecast are made when the gap is large. Overall, in period where the gap is large, the fundamental forecasts are up to 55% more accurate than the benchmark for the United States and up to 35% for the United Kingdom. For comparison, for the same horizons and on the all sample, the improvement is of 40% for the United States and of 20% for United Kingdom. As suggested by the correlation analysis, there is no improvement for Japan and Switzerland.

For Japan and Switzerland, the negative correlation suggests the opposite strategy: one should use the fundamental price when the gap is small and the observed price when the gap is large. As the two upper panels of Figure 12 shows, this strategy improves the forecast accuracy of the fundamental random walk with drift model. However, as shown in the lower panels of Figure 12, the performance of this strategy against the benchmark are different in each country. In Japan, the difference between the fundamental model and the benchmark decreases during periods in which the fundamental and the observed prices are close, but the benchmark is still more accurate than the fundamental model. For Switzerland, restricting the use of the fundamental price model to periods in which both prices are close significantly improves its forecast accuracy. In these periods, it almost always outperforms the benchmark. The amelioration goes up to 30% for horizon of about 5 years.

The general conclusion is that, except for Japan, it is possible to significantly improve the forecast accuracy by using the C-CAPM fundamental price. This is true on the whole sample for the United States and for the United Kingdom and only in period in which the gap is small (smaller than is median) for Switzerland.

Figure 12: Forecast Accuracy for Periods with Small Gaps



The two upper panels compare the forecast accuracy of the fundamental random walk with drift model for forecasts made in periods with small gaps to the forecast accuracy for the whole period. The black line is for forecasts made when the (absolute) value of the gap is smaller than its median, the grey line when the (absolute) gap is smaller than its 25% centile and the dashed line when the (absolute) gap is smaller than its 10% centile. The two lower panels compares the forecast accuracy of the fundamental random walk with drift model for the whole period (black line) and for periods in which the (absolute) gap was smaller than its median (grey line) to the forecast accuracy of the random walk with drift model (benchmark). Each panel presents the ratio of the average absolute errors for the different periods over the one of the whole sample (upper panels) or the benchmark (lower panels). A value under 1 indicates that forecasts made in a particular sub-sample are more accurate than in the whole sample (upper panels) or than the benchmark (lower panels). The comparison is made for different forecasting horizon (in months, horizontal axis).

8 Conclusion

The results presented in this paper can be interpreted and exploited at different levels. The most obvious practical result is the ability of the C-CAPM fundamental model to forecast future prices for short horizon. As mentioned in the paper, the predictability of stock prices is not new. Several studies have shown that simple ratios such as dividend-price ratios or price-earnings ratios are useful to forecast stock prices in the long term. Typically, these ratios are able to forecasts price for horizons longer than five years. What is new here is that the C-CAPM fundamental model significantly shortens the horizons for which the forecast accuracy can be improved. Of course, the improvement depends on the market and on the period in which the forecast is made, but the results presented in this paper suggests that is generally possible to find a way to combine the observed and the fundamental prices to improve forecasts for horizons shorter than five years. While long term forecasts are of little interest for traders or portfolio managers, whose performance are evaluated on shorter intervals, forecast horizons in the range of those proposed in this paper might find their place in shorter term tactical portfolio allocation strategies. With that in mind, it would be useful to better study how significant and reliable the improvement made by the fundamental model is and if a portfolio based on fundamental forecasts would have generated excess returns in the past.

The C-CAPM fundamental model is also of interest for central banks and international organizations such as the IMF or the BIS. Indeed, such institutions show a growing interest in identifying imbalances on asset markets. Their fear is that such imbalances will eventually unwind and that their correction might have a significant impact on the economy. Thus, by detecting imbalances early enough, central banks hope to identify factors that could help them predicting the evolution of the economy and choosing an adequate policy. Two results indicate that the imbalances measured by the C-CAPM fundamental model would be relevant in this framework: firstly, there are strong empirical evidence of a link between the C-CAPM fundamental price and the observed stock price and secondly, the forecast horizon for which the fundamental model is helpful corresponds to the horizons, which is normally considered as pertinent for central banks policy (up to 3 or 5 years). In this context, an improvement of the model would be to relax assumption 5 and extend the set of variables used to forecast the fundamentals. As mentioned by Campbell (1999), consumption is not well forecasted by its own history. It is therefore unlikely that agents do only rely on its dynamic to form their expectations. Expanding the information set to other variables would refine the estimation of the gap between the price and its fundamental value and give a better measure of potential imbalances.

Finally, from a more academic point of view, the results presented in this paper seem to rehabilitate a bit the empirical relevance of the C-CAPM. Indeed, since the seminal paper of Mehra and Prescott (1986) documenting the equity premium puzzle raised by the C-CAPM, this model has been the target of multiple critiques. Numerous articles have contested or defended its use in the context of stock prices. In recent years, the trend seems to move toward modified versions of the C-CAPM such as habit formation or loss aversion utility models. Most of these studies are based on the observation of stock returns. This paper shed a new light on the C-CAPM by looking at stock prices instead of returns. From this point of view, the empirical evidence is kinder with the C-CAPM, at least for the long term evolution of stock prices. Note that, the methodology developed in this paper can be easily extended to any other linear SDF model. It would be interesting to check if models that perform better than the C-CAPM for stock returns give also better results for the long term dynamic of stock prices.

A Proofs

A.1 Second-order Taylor expansion of the log PD ratio

We start from the definition of the fundamental PD ratio in equation (5)

$$PD_t = E_t \sum_{i=1}^{\infty} \prod_{j=1}^{i} M_{t+j}\gamma_{t+j} = E_t\left(PD_t^*\right) = \sum^{s\in S} \lambda_s PD_{s,t}^* \qquad (36)$$

where $S = \{s_1, \ldots, s_N\}$ is the set of every possible states of nature s. Taking the log of this equation yields

$$\ln E_t \left(PD_t^*\right) = \ln \sum^{S} \lambda_s PD_{s,t}^* = \ln \sum^{S} \lambda_s e^{pd_{s,t}^*} = f\left(pd_{1,t}^*, \ldots, pd_{N,t}^*\right) \tag{37}$$

The second-order Taylor expansion of $f\left(pd_{1,t}^*, \ldots, pd_{N,t}^*\right)$ around $\overline{pd}_t^*$ is

$$\begin{aligned} & f\left(\overline{pd}_t^*, \ldots, \overline{pd}_t^*\right) + \sum^{s \in S} \left.\frac{\partial f}{\partial pd_{s,t}^*}\right|_{pd_{s,t}^* = \overline{pd}_t^*} \left(pd_{s,t}^* - \overline{pd}_t^*\right) + \\ & + \frac{1}{2} \sum^{s_1 \in S} \sum^{s_2 \in S} \left.\frac{\partial^2 f}{\partial pd_{s_1,t}^* \partial pd_{s_2,t}^*}\right|_{pd_{s,t}^* = \overline{pd}_t^*} \left(pd_{s_1,t}^* - \overline{pd}_t^*\right)\left(pd_{s_2,t}^* - \overline{pd}_t^*\right) + \\ & + R_t \end{aligned} \tag{38}$$

where $\overline{pd}_t^* = \sum^{S} \lambda_s e^{\overline{pd}_t^*}$ and R_t is the remainder of the Taylor expansion.

We have that

$$f\left(\overline{pd}_t^*, \ldots, \overline{pd}_t^*\right) = \ln \sum^{S} \lambda_s e^{\overline{pd}_t^*} = \overline{pd}_t^* \tag{39}$$

The first order term is

$$\begin{aligned} \sum^{s \in S} \left.\frac{\partial f}{\partial pd_{s,t}^*}\right|_{pd_{s,t}^* = \overline{pd}_t^*} \left(pd_{s,t}^* - \overline{pd}_t^*\right) &= \sum^{s \in S} \left.\frac{\lambda_s e^{\overline{pd}_t^*}}{\sum^{S} \lambda_s e^{\overline{pd}_t^*}}\right|_{pd_{s,t}^* = \overline{pd}_t^*} \left(pd_{s,t}^* - \overline{pd}_t^*\right) = \\ &= \sum^{s \in S} \lambda_s \left(pd_{s,t}^* - \overline{pd}_t^*\right) \end{aligned} \tag{40}$$

The second derivative in the second order term is

$$\frac{\partial^2 f}{\partial pd^*_{s_1,t}\partial pd^*_{s_2,t}} = -\frac{\lambda_{s_1} e^{pd^*_{s_1,t}} \lambda_{s_2} e^{pd^*_{s_2,t}}}{\left(\sum^S \lambda_s e^{pd^*_{s,t}}\right)^2} \text{ if } s_1 \neq s_2 \tag{41}$$

$$\frac{\partial^2 f}{\partial pd^*_{s_1,t}\partial pd^*_{s_2,t}} = \frac{\lambda_{s_1} e^{pd^*_{s_1,t}} \sum^S \lambda_s e^{pd^*_{s,t}} - \left(\lambda_{s_1} e^{pd^*_{s_1,t}}\right)^2}{\left(\sum^S \lambda_s e^{pd^*_{s,t}}\right)^2} \text{ if } s_1 = s_2 \tag{42}$$

Evaluated at $\overline{pd}^*_t$, the second derivative is

$$\frac{\partial^2 f}{\partial pd^*_{s_1,t}\partial pd^*_{s_2,t}} = -\lambda_{s_1}\lambda_{s_2} \text{ if } s_1 \neq s_2 \tag{43}$$

$$\frac{\partial^2 f}{\partial pd^*_{s_1,t}\partial pd^*_{s_2,t}} = \lambda_{s_1}\left(1 - \lambda_{s_1}\right) \text{ if } s_1 = s_2 \tag{44}$$

Given that, the second order term is

$$\sum^{s_1\in S}\sum^{s_2\in S} \left.\frac{\partial^2 f}{\partial pd^*_{s_1,t}\partial pd^*_{s_2,t}}\right|_{pd^*_{s,t}=\overline{pd}^*_t} \left(pd^*_{s_1,t} - \overline{pd}^*_t\right)\left(pd^*_{s_2,t} - \overline{pd}^*_t\right) = \sum^{s\in S} \lambda_{s_1}\left(pd^*_{s,t} - \overline{pd}^*_t\right)^2 \tag{45}$$

Recollecting all these results, we get that the Taylor expansion is

$$f\left(pd^*_{1,t}, \ldots, pd^*_{N,t}\right) = \overline{pd}^*_t + \sum^{s\in S} \lambda_s \left(pd^*_{s,t} - \overline{pd}^*_t\right) + \frac{1}{2}\sum^{s\in S} \lambda_{s_1}\left(pd^*_{s,t} - \overline{pd}^*_t\right)^2 + R_t \tag{46}$$

and thus

$$\ln E_t\left(PD^*_t\right) = E_t\left(pd^*_t\right) + \frac{1}{2}V_t\left(pd^*_t\right) + R_t \tag{47}$$

A.2 First order approximation of the logarithm of a sum

Campbell, Lo and McKinlay (1997) show that it possible to approximate the logarithm of a sum by the sum of logarithm. First consider

$$\ln(1 + PD^*) = \ln\left(1 + e^{pd^*}\right) \tag{48}$$

where $pd^* = \ln PD^*$. The second part of this equation can be approximated by a standard Taylor approximation around its mean. Define $f(x) = \ln\left(1 + e^{pd^*}\right)$ and take the Taylor approximation of it around its mean:

$$f(pd^*) \approx f\left(\overline{pd}^*\right) + f'\left(\overline{pd}^*\right)\left(pd^* - \overline{pd}^*\right) \tag{49}$$

with $f'\left(\overline{pd}^*\right) = e^{\overline{pd}^*}/\left(1 + e^{\overline{pd}^*}\right)$. Define $\rho = 1/\left(1 + e^{-\overline{pd}^*}\right)$ and plug it into the previous equation to get the final result

$$\ln(1 + PD^*) \simeq \kappa + \rho pd^* \tag{50}$$

with $\kappa = -\ln\rho - (1-\rho)\ln(1/\rho - 1)$.

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