Optimal discretionary policy and uncertainty about inflation persistence

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Abstract

This paper studies optimal discretionary policy with parameter uncertainty about inflation inertia within a hybrid New-Keynesian model estimated for the euro area by Smets (2003). We find that it is optimal for discretionary policy to respond more aggressively to cost-push shocks and real interest rate shocks in the presence of uncertainty about inflation inertia when the central bank faces a tradeoff between inflation and other target variables in its objective function. Using an empirically realistic measure of uncertainty about inflation inertia in the euro area, we quantify the effect of uncertainty on the optimal discretionary policy reaction to shocks. Moreover, in the cases where optimal policy is not certainty equivalent, we find that inflation returns slightly more gradually to equilibrium following a shock when the degree of inflation inertia is uncertain.

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1 Introduction

The question of how monetary policy should be set optimally when the structure of the economy exhibits inflation persistence is important for policy makers. A research network of economists from the national central banks of the euro area and the ECB have been investigating the empirical evidence for inflation persistence, its determinants and implications for monetary policy (see Altissimo, Ehrmann and Smets (2006) for a summary). The degree of inflation inertia is a key parameter for assessing the optimality of monetary policies. For example, Levin and Williams (2003) show that monetary policy rules which are optimal in a forward-looking model can perform badly in backward-looking models. Since there is little consensus about the degree of inflation inertia in the empirical literature, it is therefore of particular interest to study optimal policy when there is uncertainty about inflation inertia. Here and in the following, inflation inertia refers to the coefficient on lagged inflation in the New-Keynesian Phillips curve, ie in a structural, rather than reduced-form, model.[1] In recent studies, estimates of inflation inertia in structural models for the euro area vary widely, ranging from 0.04 to 0.72 (see Altissimo, Ehrmann and Smets (2006)).

This paper studies optimal discretionary monetary policy with parameter uncertainty about inflation inertia, within a forward-looking hybrid New-Keynesian model of the economy commonly used for monetary policy analysis.[2] We model parameter uncertainty about inflation inertia by assuming that the policy maker has a prior probability distribution of the parameter, and sets policy to minimize the expected loss based on this prior distribution.[3] A classic paper on optimal policy using such an approach to model parameter uncertainty is Brainard (1967), who found for static models that it is optimal for policy to react with greater caution in the presence of uncertainty about the impact of policy. By contrast, Craine (1979) found for a dynamic backward-looking model that optimal policy may become more

[1] We also refer to inflation inertia as endogenous inflation persistence, to distinguish it from extrinsic inflation persistence which is due to persistent extrinsic shocks.

[2] See Clarida, Gali and Gertler (1999) for monetary policy analysis using hybrid New-Keynesian models.

[3] This approach does not incorporate the implications of gradual learning about the structural parameters of the economy, which has been considered in the context of optimal policy for example in Wieland (2000), Beck and Wieland (2002) and Orphanides and Williams (2005).

aggressive in the presence of uncertainty about the transition dynamics. Another approach to investigating the implications of uncertainty is to study the effect of setting policy based on parameter values that are incorrect, but taken as certain in the central bank's optimisation problem.[4]

We derive optimal policy under discretion in the presence of uncertainty about inflation inertia by extending the solution method of Backus and Driffill (1986) for optimal discretionary policy in the case of certainty, to the case with parameter uncertainty. We study optimal discretionary policy under uncertainty about inflation inertia within a hybrid New-Keynesian model estimated for the euro area by Smets (2003). Using an empirically realistic measure of uncertainty about inflation inertia in the euro area, based on recent estimates of inflation inertia for the euro area, we quantify the effect of uncertainty on the optimal discretionary policy reaction to shocks.

In contrast to Söderström (2002), who considers the implications of uncertainty about inflation inertia within a purely backward-looking model, and Onatski and Williams (2003), who also consider a purely backward-looking model, this paper studies a forward-looking model. Considering a forward-looking model is useful, since in contrast to purely backward-looking models, such models contain forward-looking private sector expectations, which makes the treatment of private sector expectations consistent with the forward-looking behaviour of the policy maker in its optimisation problem (see Sargent 1999). In contrast to Kimura and Kurozumi (2003), who study uncertainty about inflation inertia for the case of optimal monetary policy under commitment, and Onatski and Williams (2003), who study optimised Taylor-type policy rules, we consider the effects of uncertainty about inflation inertia for optimal policy under discretion. While optimal policy under commitment is desirable from a normative viewpoint (see Woodford (1999)), from a positive viewpoint, central banks generally do not have a commitment technology available, and central bank behaviour is arguably best described as discretionary (see Issing et al. (2001)). This paper therefore considers optimal discretionary monetary policy with uncertainty about inflation inertia. Moreover, we consider *ad hoc* objective functions for the central bank, rather than micro-founded objective functions as in Kimura and Kurozumi (2003).[5]

We find that it is optimal for discretionary policy to respond more ag-

[4]See Walsh (2003), Angeloni, Coenen and Smets (2003), Coenen (2003), and Walsh (2004) for analysis using such an approach of misspecification.

[5]An overview of the implications for monetary policy design of inflation persistence,

gressively to cost-push shocks and real interest rate shocks in the presence of uncertainty about inflation inertia when the central bank faces a tradeoff between inflation and other target variables in its objective function. If the central bank's objective is to minimise inflation and output gap volatility, and if it also has concern for interest rate volatility or interest rate smoothing, then the optimal discretionary policy response to both cost-push shocks and real interest rate shocks is more aggressive in the presence of uncertainty about inflation inertia. If the central bank only cares about inflation and output gap volatility, however, then the optimal discretionary policy response to cost-push shocks is more aggressive in the presence of uncertainty about inflation inertia, while the optimal response to real interest rate shocks is the same as in the case of certainty. Finally, if the central bank's objective function only penalizes inflation volatility, then the optimal discretionary policy response to both cost-push shocks and real interest rate shocks does not depend on uncertainty about inflation inertia.

Using an empirically realistic measure of uncertainty about inflation inertia in the euro area, based on estimates of euro-area inflation inertia in recent studies[6], we find that the optimal discretionary policy response to cost-push shocks is up to about 20% larger in the case of uncertainty than in the case of certainty, for a relative weight in the central bank's objective function on the output gap ranging between 0 and 1, and for a relative weight on interest rate volatility or on the volatility of interest rate changes of 0 or 0.1.

Our results for the optimal discretionary policy response to cost-push shocks in a forward-looking model are consistent with the result of Söderström (2002) within a backward-looking model, who finds that it is optimal for policy to respond more aggressively to cost-push shocks when the central bank's objective function penalizes inflation and output volatility,[7] but that the response is certainty-equivalent if it only penalizes inflation volatility.[8] Our finding of a more aggressive optimal discretionary response to real inter-

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and of uncertainty about it, is presented in Levin and Moessner (2005), for both *ad hoc* and micro-founded objective functions.

[6]See Altissimo, Ehrmann and Smets (2006), Angeloni and Ehrmann (2004), Gali, Gertler and Lopez-Salido (2001), Jondeau and Le Bihan (2005), McAdam and Wilman (2004), Paloviita (2004) and Rumler (2005).

[7]See also Srour (1999) and Shuetrim and Thompson (1999).

[8]Also consistent with Söderström (2002)'s result for a backward-looking model, we find that the optimal discretionary policy response to real interest rate shocks is certainty-equivalent if the central bank cares only about inflation volatility.

est rate shocks when the central bank also cares about interest rate volatility is consistent with Kimura and Kurozumi's (2003) findings for optimal policy under commitment, using micro-founded loss functions.

In the cases where optimal discretionary policy is not certainty equivalent, we find moreover that inflation returns slightly more gradually to equilibrium following a shock, with the speed of convergence decreasing with uncertainty about inflation inertia.[9]

The paper is organised as follows. Section 2 describes the model of the economy, Section 3 describes the method used for determining optimal discretionary policy under uncertainty and presents the results. Finally, Section 4 concludes.

2 Model

We study optimal discretionary policy in the presence of uncertainty about inflation inertia within a hybrid New-Keynesian model commonly used for monetary policy analysis, and estimated for the euro area by Smets (2003),

$$\pi_t = \alpha y_t + \phi\pi_{t-1} + (1-\phi)E_t\pi_{t+1} + e_{ut}, \quad (1)$$

$$y_t = -\gamma(i_t - E_t\pi_{t+1}) + \theta y_{t-1} + (1-\theta)E_t y_{t+1} + e_{gt}. \quad (2)$$

The variables π_t, y_t and i_t denote deviations of the inflation rate, output and the short-term nominal interest rate from their steady-state values. Without endogenous persistence, equation (1) can be derived from optimising microeconomic behaviour of price-setting firms, with the assumption of monopolistic competition and sticky prices (see for example Goodfriend and King (1997), Rotemberg and Woodford (1997)), and equation (2) can be derived from an intertemporal consumption Euler equation. Equations (1) and (2) also include lagged output and inflation terms. The lagged inflation term in equation (1) may be motivated by the presence of partial price indexations (see Christiano, Eichenbaum and Evans (2001), Sbordone (2001)) or the presence of rule-of-thumb price-setters (see Gali and Gertler (1999)). The lagged output term in equation (2) may be motivated by habit persistence in consumption or the presence of rule-of-thumb consumers (see Campbell and Mankiw (1989), Fuhrer (2000)). There are two exogenous shocks in this

[9] This is also consistent with Kimura and Kurozumi (2003)'s finding for the case of commitment.

model, a shock e_{ut} to the inflation equation, and a shock e_{gt} to the output equation,

$$e_{ut+1} = \rho_u e_{ut} + \eta_{ut+1}, \quad e_{gt+1} = \rho_g e_{gt} + \eta_{gt+1}, \tag{3}$$

which are assumed to be serially uncorrelated. This is achieved by setting the autocorrelations of the shocks, ρ_u and ρ_g, to very small values (see Table A) . This allows us to study the implications of endogenous inflation persistence, rather than of persistence generated by exogenous shocks. Estimates of the model parameters for the euro area are given in Table A, where ϕ is the degree of endogenous inflation peristence, θ is the degree of endogenous output persistence, α is the slope of the New-Keynesian Phillips curve, γ is the interest elasticity of demand, and the discount factor is assumed to equal $\beta = 0.96$.

While optimal monetary policy under commitment is desirable from a normative viewpoint (see Woodford (1999)), central banks generally do not have a commitment technology available, and central bank behaviour is arguably best described as discretionary (see Issing et al. (2001)). We therefore consider optimal discretionary monetary policy with uncertainty about inflation inertia.

In order to solve for optimal discretionary policy in the presence of uncertainty about inflation inertia, we extend the approach for the case of certainty of Backus and Driffill (1986) (see also Söderlind (1999)), which is based on a formulation of the model in state-space form,

$$\begin{bmatrix} x_{1t+1} \\ E_t x_{2t+1} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_{1t} \\ x_{2t} \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u_t + \begin{bmatrix} \varepsilon_{t+1} \\ 0_{n_2 x1} \end{bmatrix}. \tag{4}$$

Here, the vector of endogenous variables, x_t, has been divided into predetermined variables, $x_{1t} = [y_{t-1,}\pi_{t-1}, e_{gt}, e_{ut}]'$, and jump variables, $x_{2t} = [y_{t,}\pi_t]'$; u_t is the vector of control variables, the nominal interest rate i_t. The errors ε_{t+1} are assumed to be i.i.d. shocks with zero mean, whose covariance matrix $\Sigma = E_t\left[\varepsilon'_{t+1}\varepsilon_{t+1}\right]$ is time-invariant, and which are uncorrelated with the predetermined variables x_{1t}; $0_{n_2 x1}$ is a zero matrix of size $n_2 x1$. The matrices of the hybrid New-Keynesian model for the euro area (see equations (1) to (3)) in state-space form are given by

$$A_{11} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_g & 0 \\ 0 & 0 & 0 & \rho_u \end{bmatrix}, A_{12} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \tag{5}$$

$$B_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, B_2 = \begin{bmatrix} \frac{\gamma}{(1-\theta)} \\ 0 \end{bmatrix}, \tag{6}$$

$$A_{21}(\phi) = \begin{bmatrix} 0 & \frac{-\theta}{1-\theta} & \frac{\gamma\phi}{(1-\theta)(1-\phi)} & -\frac{1}{1-\theta} & \frac{\gamma}{(1-\theta)(1-\phi)} \\ 0 & 0 & \frac{\phi}{(1-\phi)} & 0 & -\frac{1}{(1-\phi)} \end{bmatrix}, \tag{7}$$

$$A_{22}(\phi) = \begin{bmatrix} \frac{1}{(1-\theta)} + \frac{\gamma\alpha}{(1-\theta)(1-\phi)} & -\frac{\gamma}{(1-\theta)(1-\phi)} \\ -\frac{\alpha}{(1-\phi)} & \frac{1}{(1-\phi)} \end{bmatrix}. \tag{8}$$

We can see that only the two matrices of equations (7) and (8) depend on the degree of inflation inertia, while the remaining matrices do not depend on it.

3 Uncertainty about inflation inertia and optimal discretionary policy

The central bank minimises the expected discounted current and future values of the intertemporal loss function

$$E\left[\sum_{\tau=0}^{\infty} \beta^{\tau} L_{t+\tau} \mid I_t\right], \tag{9}$$

conditional on its information set, I_t, which does not include the actual value of the degree of inflation inertia, ϕ.The objective function of the central bank is assumed to be of the form

$$L_t = \pi_t^2 + \lambda^y y_t^2 + \lambda^i {i_t}^2, \tag{10}$$

penalizing the volatility of inflation around target and the volatility of the output gap, assigning a relative weight, λ^y, to output gap stabilisation, and allowing for the possibility of a concern for interest rate volatility on the part of the central bank, with a relative weight, λ^i, which may be zero or nonzero. The objective function may be written in the notation of equation (4) as

$$L(x_t, u_t) = x_t' Q x_t + x_t' U u_t + u_t' U' x_t + u_t' R u_t, \tag{11}$$

with appropriate constant matrices Q, U and R.

We model uncertainty about inflation inertia, ϕ, by assuming that the central bank has a uniform prior probability distribution for ϕ over the interval $[\bar{\phi}-\Delta, \bar{\phi}+\Delta]$, with mean $\bar{\phi} = 0.48$ equal to the estimated value (see Table A). The parameter Δ therefore quantifies the degree of uncertainty about inflation inertia. This allows us to explore whether optimal discretionary policy is certainty-equivalent or not.

Recent estimates of inflation inertia in structural models for the euro area vary widely, ranging from ϕ^l =0.04 to ϕ^u =0.72 in studies by Angeloni and Ehrmann (2004), Gali, Gertler and Lopez-Salido (2001), Jondeau and Le Bihan (2005), McAdam and Wilman (2004), Paloviita (2004) and Rumler (2005) (see Altissimo, Ehrmann and Smets (2006)). As an empirically realistic measure of uncertainty about inflation inertia in the euro area, we use the following value for Δ based on the range of these estimates,

$$\Delta^e = 0.5(\phi^u - \phi^l) = 0.34 \tag{12}$$

We approximate the central bank's uniform prior probability distribution by a discrete probability distribution with $n = 101$ discrete values, $\phi^j = \bar{\phi} + j\frac{\Delta}{k}$, $j = -k, ..., k$, and $k = \frac{n-1}{2}$, with each discrete value assigned the same probability of $\pi^j \equiv \pi(\phi^j) = \frac{1}{n}$, so that $\sum_{j=1}^{n} \pi^j = 1$. The private sector is assumed to know the actual value for the degree of inflation inertia, equal to the estimated value of $\bar{\phi} = 0.48$.

The solution of rational expectations models with partial information, where the current value of some variables are unobserved, has been considered in Pearlman, Currie and Levine (1986), Pearlman (1992) and Svensson and Woodford (2002). Following Pearlman (1992), the Bellman equation for the case of partial information in our case of uncertainty about inflation inertia may be written as

$$v(x_t) = \min_{u_t} \left\{ E[L(x_t, u_t) \mid I_t] + \beta E\left[v(x_{t+1}) \mid I_t\right] \right\}, \tag{13}$$

subject to equation (4) above, and taking private-sector expectations and x_{1t} as given. In the presence of parameter uncertainty, the central bank's information set I_t does not contain the degree of endogenous inflation persistence ϕ, and the central bank therefore needs to form expectations over it, based on its uniform prior probability distribution.

3.1 Solution method

We assume that the optimal feedback rule depends linearly on predetermined variables, and that the value function is a quadratic form in the predetermined variables, following Backus and Driffill (1986) (see also Soederlind (1999)),

$$u_t = -F_{1t}x_{1t}, \tag{14}$$

$$v(x_t) = x'_{1t}V_t x_{1t} + v_t. \tag{15}$$

Next, we rewrite the central bank's optimisation problem of equation (13) by substituting out the jump variables x_{2t}, following Backus and Driffill (1986). This is achieved by deriving a relationship between the jump variables on the one hand and the predetermined variables and the control variable on the other, based on the expectations formation of private agents, which is taken as given by the central bank. A linear relationship between jump variables and predetermined variables is assumed, according to which expectations by private agents are formed,

$$x_{2t+1} = C_{t+1}x_{1t+1}. \tag{16}$$

From the bottom row of equation (4), we have that

$$E_t x_{2t+1} = A_{21}(\phi)x_{1t} + A_{22}(\phi)x_{2t} + B_2 u_t. \tag{17}$$

Moreover, equation (16) together with the top row of equation (4) yields

$$E_t x_{2t+1} = C_{t+1}\left[A_{11}x_{1t} + A_{12}x_{2t} + B_1 u_t\right]. \tag{18}$$

Combining equations (17) and (18) yields

$$x_{2t} = D_t(\phi)x_{1t} + G_t(\phi)u_t, \tag{19}$$

where

$$D_t(\phi) = (A_{22}(\phi) - C_{t+1}A_{12})^{-1}(C_{t+1}A_{11} - A_{21}(\phi)), \tag{20}$$

$$G_t(\phi) = (A_{22}(\phi) - C_{t+1}A_{12})^{-1}(C_{t+1}B_1 - B_2). \tag{21}$$

Using the form for the value function of equation (15), using equation (19) to substitute out for the jump variables in terms of the predetermined and control variables, and using the discrete approximation for the uniform prior probability distribution, the central bank's optimisation problem of equation (13) may then be written as

$$x_{1t}'V_tx_{1t}+v_t=\min_{u_t}\left\{\begin{array}{c} x_{1t}'Q_t^*x_{1t}+x_{1t}'U_t^*u_t+u_t'U_t^{*\prime}x_{1t}+u_t'R_t^*u_t+ \\ \beta\sum_{j=1}^n\pi^jE_t^\varepsilon\left[\begin{array}{c}(A_t^{j*}x_{1t}+B_t^{j*}u_t+\varepsilon_{t+1})'V_{t+1} \\ (A_t^{j*}x_{1t}+B_t^{j*}u_t+\varepsilon_{t+1})+v_{t+1}\end{array}\right]\end{array}\right\} \tag{22}$$

where

$$A_t^{j*}\equiv A_{11}+A_{12}D_t(\phi^j), \tag{23}$$

$$B_t^{j*}\equiv B_1+A_{12}G_t(\phi^j), \tag{24}$$

Q_t^*, U_t^* and R_t^* are as given in the appendix, E_t^ε denotes expectations over the additive shocks, ε_{t+1}, and x_{1t} is taken as given. Since the central bank's information set does not contain the degree of inflation inertia, the central bank needs to form expectations over ϕ, based on its uniform prior probability distribution.

Since the shocks ε_{t+1} are uncorrelated with the predetermined variables x_{1t}, we can write equation (22) as

$$x_{1t}'V_tx_{1t}+v_t=\min_{u_t}\left\{\begin{array}{c} x_{1t}'Q_t^*x_{1t}+x_{1t}'U_t^*u_t+u_t'U_t^{*\prime}x_{1t}+u_t'R_t^*u_t+ \\ \beta\sum_{j=1}^n\pi^j(A_t^{j*}x_{1t}+B_t^{j*}u_t)'V_{t+1}(A_t^{j*}x_{1t}+B_t^{j*}u_t)+ \\ \beta\left(E_t^\varepsilon\left[\varepsilon_{t+1}'V_{t+1}\varepsilon_{t+1}\right]+v_{t+1}\right)\end{array}\right\} \tag{25}$$

The solution for the optimal feedback rule can then be derived from the first-order condition following from equation (25) as

$$F_{1t}=\left[R_t^*+\beta\sum_{j=1}^n\pi^j(B_t^{j*\prime}V_{t+1}B_t^{j*})\right]^{-1}\left[U_t^{*\prime}+\beta\sum_{j=1}^n\pi^j(B_t^{j*\prime}V_{t+1}A_t^{j*})\right]. \tag{26}$$

Substituting the optimal feedback rule (see equations (14) and (26)) back into the Bellman equation (25), and equating the terms quadratic in x_{1t} then yields an expression for the value function matrix of

$$V_t=Q_t^*-U_t^*F_{1t}-F_{1t}'U_t^{*\prime}+F_{1t}'R_t^*F_{1t}+\beta\sum_{j=1}^n\pi^j(A_t^{j*}-B_t^{j*}F_{1t})'V_{t+1}(A_t^{j*}-B_t^{j*}F_{1t}). \tag{27}$$

Private agents form expectations according to equations (17) and (18). Since private agents are assumed to know the actual value of the degree of inflation inertia, $\bar{\phi}$, we have that

$$x_{2t} = D_t(\bar{\phi})x_{1t} + G_t(\bar{\phi})u_t. \tag{28}$$

Together with equations (14) and (16), this implies that

$$C_t = D_t(\bar{\phi}) - G_t(\bar{\phi})F_{1t}. \tag{29}$$

Since the decision problem has an infinite horizon, the matrices may be independent of time t, and we can search for a stationary solution by iterating backwards in time on the set of coupled equations (26), (27) and (29) (see the appendix for more technical details).

3.2 Results

The algorithm described in Section 3.1 is used to determine optimal discretionary policy in the presence of uncertainty about inflation inertia within the hybrid New-Keynesian model estimated for the euro area by Smets (2003). The optimal discretionary monetary policy rule has the following form,

$$i_t = f_g e_{gt} + f_u e_{ut} + f_y y_{t-1} + f_\pi \pi_{t-1} \quad . \tag{30}$$

Coefficients of the optimal monetary policy rule are shown in Figure 1 as a function of uncertainty, Δ, about inflation inertia, for the uniform prior distribution for ϕ over the interval $[0.48 - \Delta, 0.48 + \Delta]$, as described above. Results are shown for $\lambda^y = 1$ and $\lambda^i = 0.1$. Figures 2 and 3 show the corresponding impulse responses to cost-push shocks and real interest rate shocks. We can see that in the presence of uncertainty, it is optimal for policy to respond more aggressively than in the case of certainty to both cost-push and real interest rate shocks. Cost-push shocks, e_{ut}, introduce an output-inflation tradeoff, and it is not optimal to perfectly offset them in the period of the shock. Since the transmission mechanism depends on the degree of inflation inertia, the optimal discretionary policy response depends on ϕ. In the case of uncertainty, it also depends on uncertainty about inflation inertia. The response to these shocks is more aggressive under uncertainty since a potentially high realization of inflation inertia would imply that the effect of cost-push shocks persists for longer, possibly requiring greater output contractions in future, and leading to an additional loss which is larger than the possible reduction in loss due to a realization of ϕ by the same amount below the mean. It is therefore optimal to prevent shocks from entering the system to a greater extent by reacting more aggressively to them initially.

In contrast to cost-push shocks, real interest rate shocks do not introduce an output-inflation tradeoff. When the degree of inflation inertia is certain, and if the central bank only cares about inflation and output volatility, it is therefore optimal to offset real interest rate shocks in the period of the shock. In that case, the optimal monetary policy response only depends on the interest elasticity of demand, γ, but not on the degree of inflation inertia. Consequently, uncertainty about inflation inertia does not affect the optimal discretionary policy response in that case. This is shown in Figure 4, where we can see that the reaction to real interest rate shocks is certainty-equivalent in the case of $\lambda^i = 0$. However, uncertainty about the degree of inflation inertia does affect the optimal response to real interest rate shocks when the central bank cares about interest rate volatility ($\lambda^i \neq 0$), since it is then no longer optimal to move interest rates in the period of the shock by the amount required to perfectly offset the real interest rate shock, since this would introduce too much interest rate volatility. Since the transmission of the real interest rate shock subsequently depends on the degree of inflation inertia, uncertainty about inflation inertia affects the optimal response to real interest rate shocks, as shown in Figures 1 and 3. We find that uncertainty about inflation inertia increases the optimal reponse to real interest rate shocks, but the response is still less aggressive than would be required to perfectly offset the shock.

Since cost-push shocks introduce an output-inflation tradeoff, the result of a more aggressive response to cost-push shocks carries over to the case when the central bank cares only about inflation and output volatility, as can be seen from Figure 4. If the central bank cares only about inflation volatility, however, then policy is certainty-equivalent in the presence of uncertainty about inflation inertia.

The results of a more aggressive response to both shocks for the case when the central bank cares about interest rate volatility also hold if the central bank instead has a concern for interest rate smoothing (ie with $\lambda^i {i_t}^2$ replaced by $\lambda^i (i_t - i_{t-1})^2$ in equation (10)), as shown in Figure 5.[10]

Next, we quantify the impact of uncertainty about inflation inertia on the

[10] Due to the concern for interest rate smoothing, the lagged interest rate is an additional predetermined variable, and the optimal policy rule therefore also contains a feedback of magnitude f_i on the lagged interest rate.

optimal discretionary policy response to shocks within the model of Smets (2003), using an empirically realistic measure of uncertainty about inflation inertia in the euro area based on recent estimates, $\Delta = 0.34$ (see equation (12) above). Table B presents the results for the relative changes in the coefficients in the optimal discretionary policy rule in equation (30) on cost-push shocks, f_u, and on real interest rate shocks, f_g, under uncertainty (for $\Delta = 0.34$) compared with the case of certainty ($\Delta = 0$), ie

$$r_u = \frac{f_u^{\Delta=0.34} - f_u^{\Delta=0}}{f_u^{\Delta=0}}, \quad r_g = \frac{f_g^{\Delta=0.34} - f_g^{\Delta=0}}{f_g^{\Delta=0}}, \tag{31}$$

respectively. Table B shows the range of values for r_u and r_g of equation (31) for a weight on the output gap in the central bank's objective function, λ^y, ranging between 0 and 1. These results are shown separately for the different forms of the central bank's objective function considered above, namely when the objective function penalizes only inflation and output gap volatility ($\lambda^i = 0$), and when it also penalizes the volatility of the level or of changes in interest rates ($\lambda^i = 0.1$). As shown in Table B, the optimal discretionary policy response to cost-push shocks, ie the magnitude of the coefficient f_u in equation (30), is up to about 20% larger in the case of uncertainty than in the case of certainty, for the empirically realistic measure of uncertainty about inflation inertia in the euro area and the different forms of the central bank's objective function considered here. The magnitude of the effect of uncertainty on the optimal discretionary policy response to real interest rate shocks is somewhat smaller than for cost-push shocks, as also shown in Table B.

In the cases where optimal discretionary policy is not certainty-equivalent, we find moreover that inflation returns slightly more gradually to equilibrium following a shock. When the central bank cares only about inflation and output gap volatility, the law of motion for inflation is given by

$$\pi_t = \rho_\pi \pi_{t-1} + h_u e_{ut}, \tag{32}$$

where ρ_π is the serial correlation of inflation.[11] The more gradual return of inflation to equilibrium under uncertainty following a cost-push shock can be seen from Figure 6, which shows the serial correlation of inflation as a

[11] Note that the reduced-form serial correlation in inflation depends on the parameters in the central bank's objective function. Benati (2005) finds empirical support for a dependence of reduced-form inflation persistence in the UK on the monetary policy regime.

function of the degree of uncertainty about inflation inertia. In the case when the central bank also has a concern for interest rate volatility, the impulse responses of inflation to cost-push shocks and real interest rate shocks shown in Figures 2 and 3 illustrate that inflation returns slightly more gradually to equilibrium under uncertainty than under certainty.

Our results for the optimal discretionary policy response to cost-push shocks in a forward-looking model are consistent with the result of Söderström (2002) within a backward-looking model, who finds that it is optimal for policy to respond more aggressively to cost-push shocks when the central bank's objective function penalizes inflation and output volatility, but that the response is certainty-equivalent if it only penalizes inflation volatility. Also consistent with Söderström (2002), we find that the optimal discretionary policy response to real interest rate shocks is certainty-equivalent if the central bank cares only about inflation volatility. However, our result of a certainty-equivalent response to real interest rate shocks when the central bank cares about both inflation and output volatility differs from Söderström (2002), who finds that a more aggressive response to demand shocks is optimal in that case. This difference is probably due to the fact that Söderström (2002) considers a purely backward-looking model, with somewhat different timing assumptions than the forward-looking model considered in this paper; in particular, he assumes a lag of one period in the effect of monetary policy on the ouput gap, while no such lag in the transmission mechanism is present in the model considered in this paper. Our finding for optimal discretionary policy of a more aggressive response to real interest rate shocks when the central bank also cares about interest rate volatility is consistent with Kimura and Kurozumi (2003)'s findings for optimal policy under commitment using micro-founded loss functions.

4 Conclusions

We derived optimal policy under discretion in the presence of uncertainty about inflation inertia by extending the solution method of Backus and Driffill (1986) for optimal discretionary policy in the case of certainty to the case with parameter uncertainty. We studied optimal discretionary policy under uncertainty about inflation inertia within a hybrid New-Keynesian model estimated for the euro area by Smets (2003). Using an empirically realistic measure of uncertainty about inflation inertia in the euro area, we quantified the effect of uncertainty on the optimal discretionary policy reaction to shocks.

When the central bank faces a tradeoff between inflation and other target variables in its objective function, we found that it is optimal for discretionary policy to respond more aggressively to cost-push shocks and real interest rate shocks in the presence of uncertainty about inflation inertia. If the central bank's objective is to minimise inflation and output gap volatility, and if it also has concern for interest rate volatility or interest rate smoothing, then the optimal policy response to both cost-push shocks and real interest rate shocks is more aggressive in the presence of uncertainty about inflation inertia. If the central bank only cares about inflation and output gap volatility, however, then the optimal policy response to cost-push shocks is more aggressive in the presence of uncertainty about inflation inertia, while the optimal response to real interest rate shocks remains independent of such uncertainty. Finally, if the central bank's objective function only penalizes inflation volatility, then the optimal policy response to both cost-push shocks and real interest rate shocks does not depend on uncertainty about inflation inertia.

Using an empirically realistic measure of uncertainty about inflation inertia in the euro area, we found that the optimal discretionary policy response to cost-push shocks is up to about 20% larger in the case of uncertainty than in the case of certainty, for a weight in the central bank's objective function on the output gap ranging between 0 and 1, and for a relative weight on interest rate volatility or on the volatility of interest rate changes of 0 or 0.1.

In future research, it would be interesting to apply our method to study optimal discretionary policy with uncertainty about inflation inertia in larger estimated models used for monetary policy analysis, such as the model of Smets and Wouters (2003) for the euro area, and in models with micro-foundations for inflation inertia and micro-founded objective functions (see Kimura and Kurozumi (2003), Gali and Gertler (1999), Woodford (2003), Steinsson (2003), and Amato and Laubach (2003)). Moreover, this paper assumed that uncertainty about inflation inertia is constant over time. In future research it would also be interesting to investigate the policy implications of gradual learning about the degree of inflation inertia.

5 Appendix

The matrices Q_t^*, U_t^* and R_t^* in equation (22) are given by

$$Q_t^* = \sum_{j=1}^{n} \pi^j \hat{Q}_t(\phi^j), U_t^* = \sum_{j=1}^{n} \pi^j \hat{U}_t(\phi^j), R_t^* = \sum_{j=1}^{n} \pi^j \hat{R}_t(\phi^j),$$

where

$$\hat{Q}_t(\phi^j) = Q_{11} + Q_{12} D_t(\phi^j) + D_t'(\phi^j) Q_{21} + D_t'(\phi^j) Q_{22} D_t(\phi^j),$$

$$\hat{U}_t(\phi^j) = Q_{12} G_t(\phi^j) + D_t'(\phi^j) Q_{22} G_t(\phi^j) + U_1 + D_t'(\phi^j) U_2,$$

$$\hat{R}(\phi^j) = R + G_t'(\phi^j) Q_{22} G_t(\phi^j) + G_t'(\phi^j) U_2 + U_2' G_t(\phi^j).$$

Here, the matrices Q and U have been partitioned conformably with x_{1t} and x_{2t}.

As a criterion for convergence of the matrices in the value function iteration, we choose the infinity-norm,

$$\|V\|_\infty = \max_{\{k,l=1,..,n_1\}} \left|(V_t)_{kl} - (V_{t+1})_{kl}\right|,$$

with a tolerance of 10^{-6}. Initial conditions are chosen as a zero-matrix of size $n_2 x n_1$for C_t, and as 0.01 times the unit matrix of size $n_1 x n_1$ for V_t. Equating the remaining terms in the Bellman equation (25), which are not quadratic in x_{1t} , yields an expression for the additional term in the value function,

$$v_t = \beta \left(tr \left[V_{t+1} \Sigma \right] + v_{t+1} \right), \tag{33}$$

where $\Sigma = E_t \left[\varepsilon_{t+1}' \varepsilon_{t+1} \right]$ is the covariance matrix of the shocks, as defined above. Given that a stationary solution for V was found, the stationary solution for v is given by

$$v = \frac{\beta}{1-\beta} tr \left[V \Sigma \right]. \tag{34}$$

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Tables

Table A: Parameters for the euro-area model (see Smets (2003)).

Estimated parameters (1977-1997)	
ϕ	0.48
θ	0.44
γ	0.06
α	0.18
Calibrated parameters	
ρ_u	10^{-10}
ρ_g	10^{-10}
β	0.96

Table B: Relative changes in the coefficients in the optimal discretionary policy rule on cost-push shocks and on real interest rate shocks under uncertainty (for $\Delta = 0.34$) compared with the case of certainty, for λ^y ranging between 0 and 1.

$L_t = \pi_t^2 + \lambda^y y_t^2 + \lambda^i {i_t}^2$, $\lambda^i = 0.1$	
Cost-push shock, r_u	20-23%
Real interest rate shock, r_g	9-16%
$L_t = \pi_t^2 + \lambda^y y_t^2$	
Cost-push shock, r_u	0-17%
Real interest rate shock, r_g	0
$L_t = \pi_t^2 + \lambda^y y_t^2 + \lambda^i (i_t - i_{t-1})^2$, $\lambda^i = 0.1$	
Cost-push shock, r_u	11-17%
Real interest rate shock, r_g	5-11%

Figures

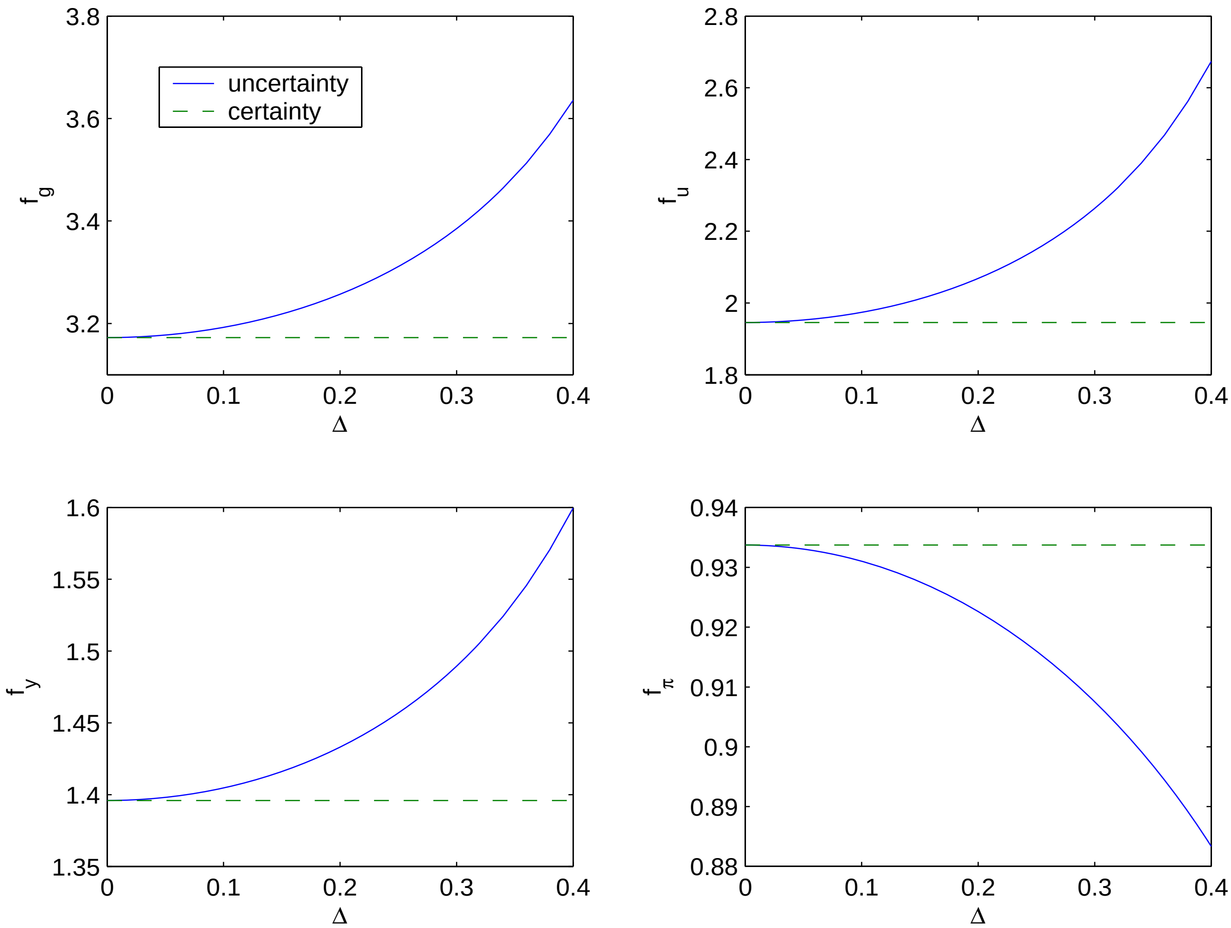

Figure 1: Coefficients of optimal monetary policy rule, as a function of uncertainty, Δ, about inflation inertia, with uniform prior distribution for ϕ over the interval $[0.48 - \Delta, 0.48 + \Delta]$; for $\lambda^y = 1$, $\lambda^i = 0.1$.

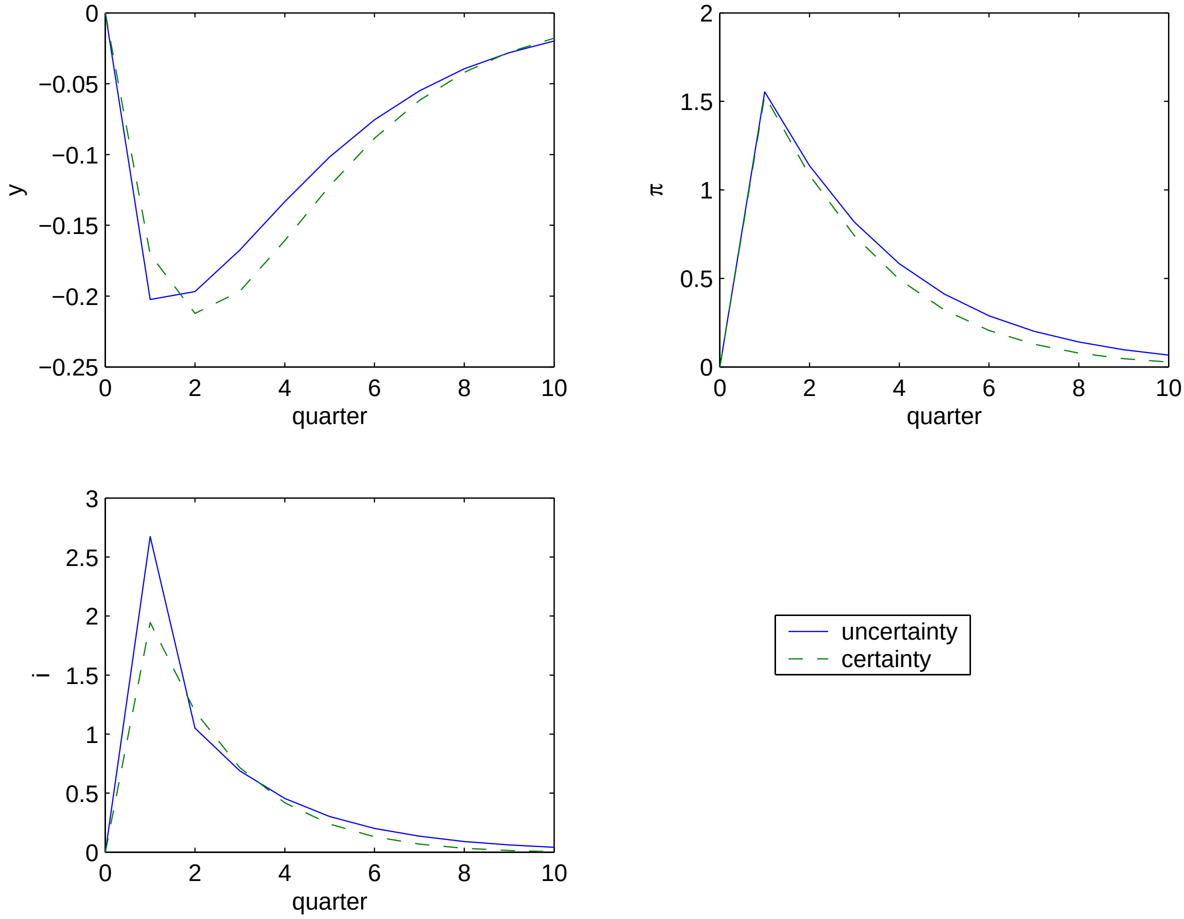

Figure 2: Impulse response to a unit cost-push shock, with uncertainty about inflation inertia in the form of a uniform prior distribution for ϕ over the interval $[0.48 - 0.4, 0.48 + 0.4]$; for $\lambda^y = 1$, $\lambda^i = 0.1$.

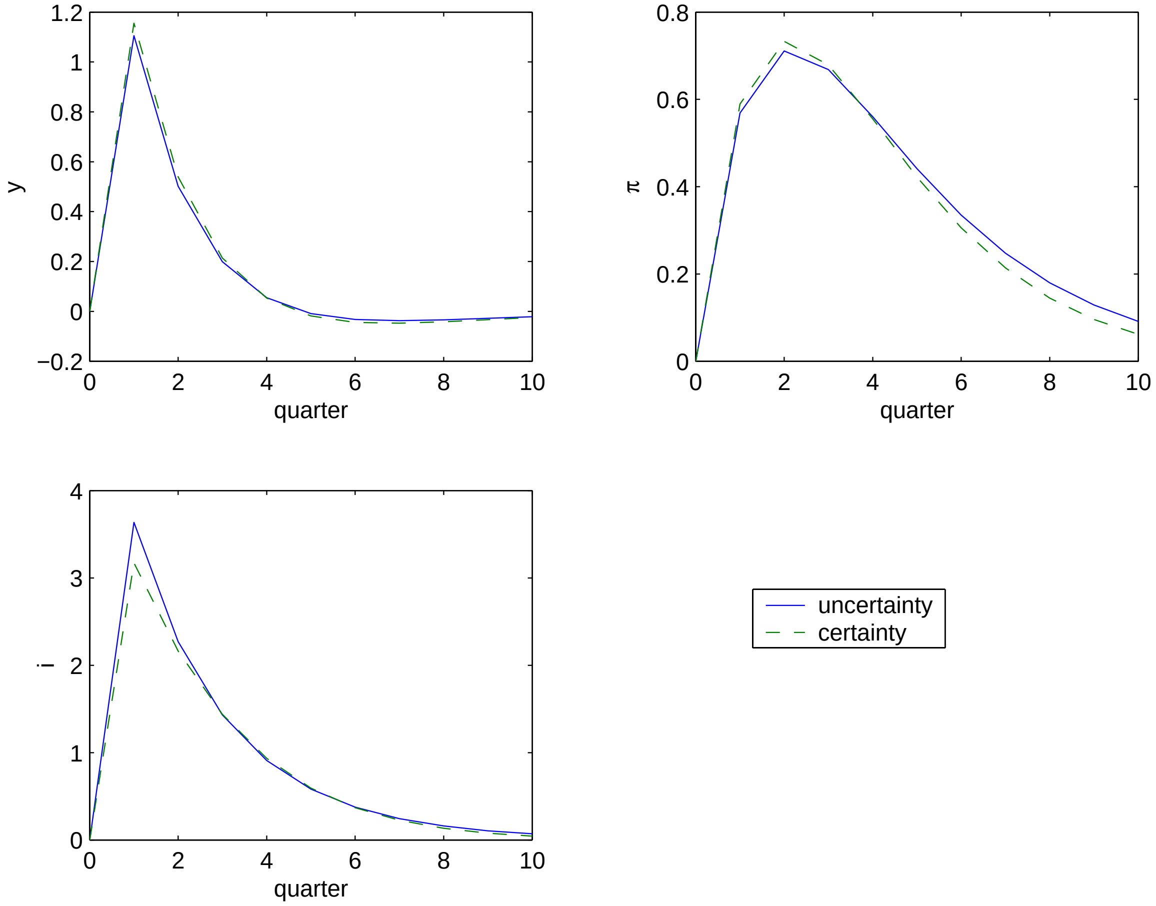

Figure 3: Impulse response to a unit real interest rate shock, with uncertainty about inflation inertia in the form of uniform prior distribution for ϕ over the interval $[0.48 - 0.4, 0.48 + 0.4]$; for $\lambda^y = 1$, $\lambda^i = 0.1$.

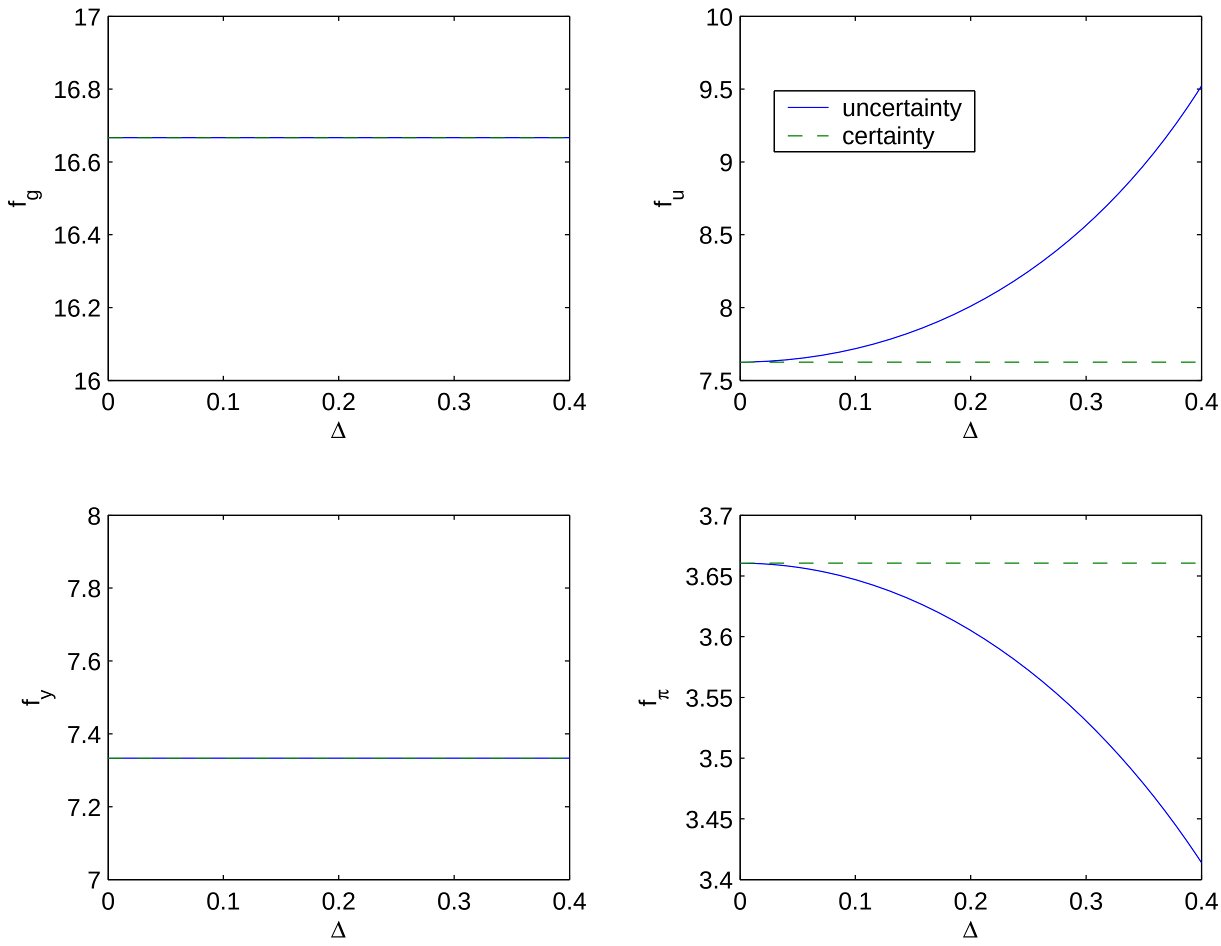

Figure 4: Coefficients of optimal monetary policy rule without a concern for interest rate volatility, as a function of uncertainty, Δ, about inflation inertia, with a uniform prior distribution for ϕ over the interval $[0.48 - \Delta, 0.48 + \Delta]$; for $\lambda^y = 1$, $\lambda^i = 0$.

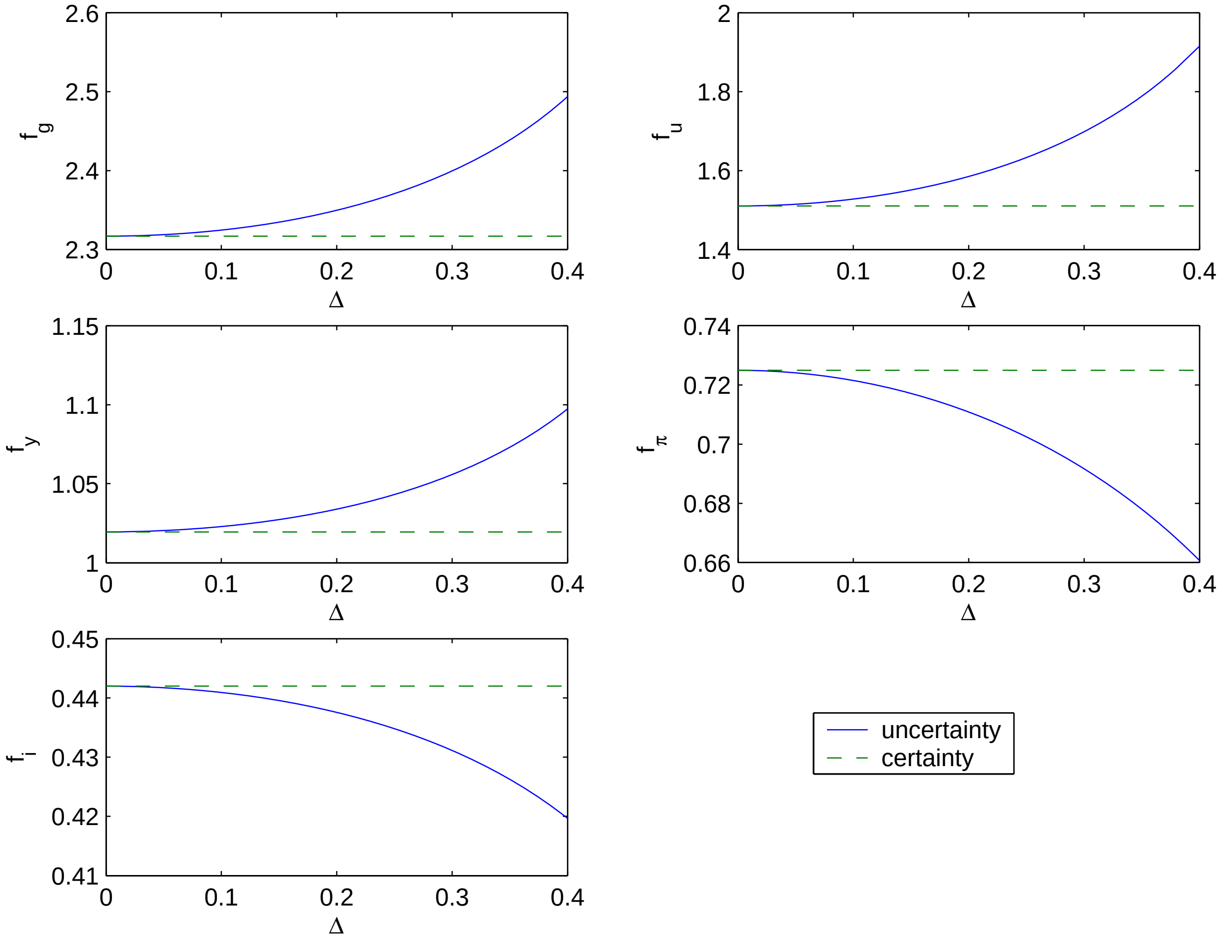

Figure 5: Coefficients of optimal monetary policy rule with a concern for interest rate smoothing, rather than interest rate volatility, as a function of uncertainty, Δ, about inflation inertia, with a uniform prior distribution for ϕ over the interval $[0.48-\Delta, 0.48+\Delta]$; for $\lambda^y = 1$, $\lambda^i = 0.1$.

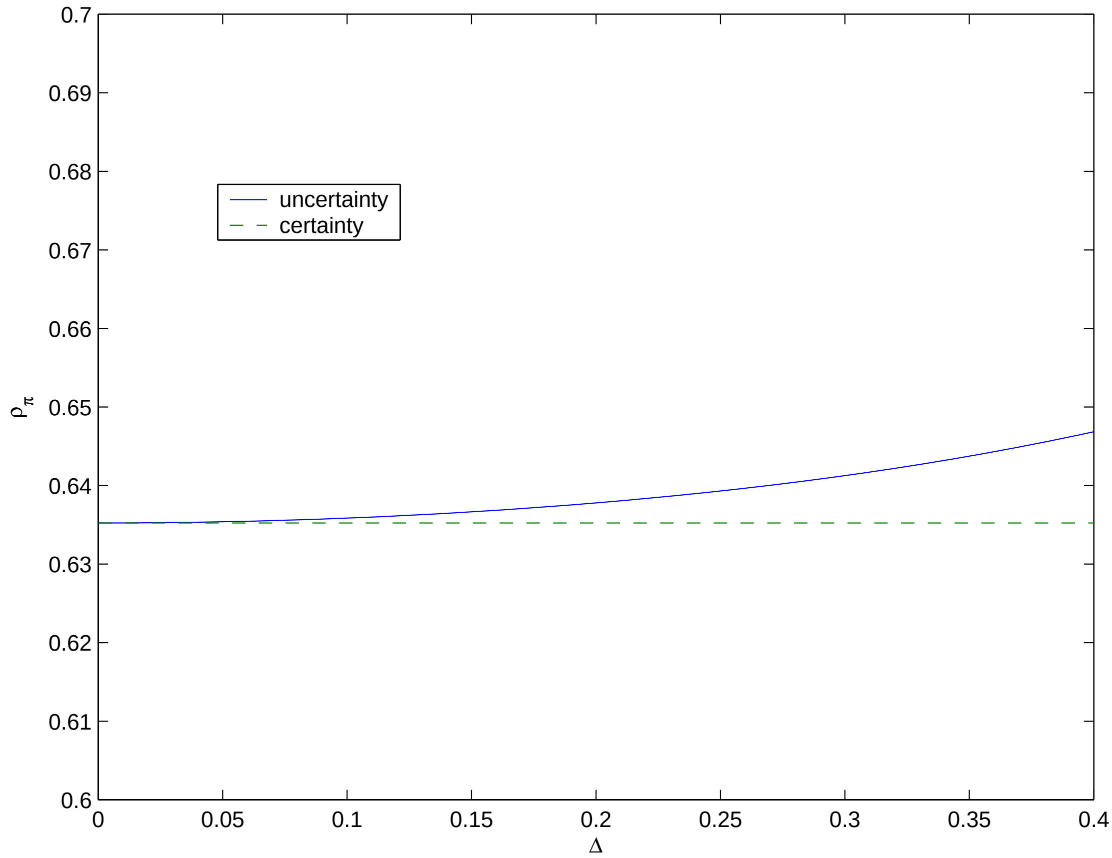

Figure 6: Serial correlation of inflation, ρ_π, as a function of uncertainty, Δ, about inflation inertia, for a uniform prior distribution for ϕ over the interval $[0.48-\Delta, 0.48+\Delta]$; for $\lambda^y = 1$, $\lambda^i = 0$ (i.e. without a concern for interest rate volatility).