

# The Taylor Rule and Interest Rate Uncertainty: An empirical investigation

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## Abstract

We use a Taylor rule with time-varying policy coefficients in combination with an unobserved components model for the output gap to estimate the uncertainty about future values of the Federal Funds Rate. We separate ex-ante interest rate uncertainty into three components: 1) uncertainty about the Fed's future policy coefficients, 2) uncertainty about future economic fundamentals, and 3) residual uncertainty. The results show important changes in interest-rate uncertainty over time. While for one-quarter forecasts uncertainty about the Fed's policy reaction is more important than uncertainty about economic fundamentals this result is reversed for the two-quarter forecast horizon.

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# 1 Introduction

Participants in financial markets devote considerable effort to predict how central banks will set interest rates in the future. At the same time, an essential task of central banks' communication policy is to "guide" expectations about its future policy moves. First, influencing expectations of future interest rates provides the central bank with some leverage over longer-term interest rates through the term structure. Second, in their communication central banks often try to keep uncertainty about future interest rates low. Rising uncertainty about future moves by the central bank can result in an increase in the volatility of money market rates. Increasing volatility of short term interest rates can destabilize the financial sector and, if the volatility increase is transmitted through the yield curve (Ayuso et al. (1997)), can cause the volatility of longer-term interest rates to rise as well, which can negatively affect economic performance.<sup>1</sup> From the financial sector's point of view estimates of uncertainty about future interest rates are important for the pricing of financial derivatives and for risk management.

The Taylor rule (Taylor (1993)) is often used as an approximate description of how central banks set short-term interest rates in response to (expected) economic conditions. Even though the central banks certainly do not mechanically follow the Taylor rule when deciding about monetary policy, financial market participants often use Taylor-type rules as a tool to form expectations about how the central bank will set interest rates in the future. Using interest rate rules to forecast policy makes it necessary to form expectations of economic conditions the central bank will have to react to in the future. The uncertainty about forecasts of the information the central bank will have to act upon is one source of uncertainty about future interest rates (uncertainty about future fundamentals).

However, the parameters in simple interest rate rules like Taylor-type rules have been shown to change over time. One reason for this is that a central bank does not always react in the same way to the same economic conditions because of shifts in the weights

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<sup>1</sup>The empirical effects of interest-rate volatility on real growth in the U.S. are studied in Muellbauer and Nunziata (2004). Byrne and Davis (2005) investigate the effects of long-run interest rate uncertainty on investment in the G7 economies.

attached to the different goals in the central bank's objective function. Another possible explanation is that simple interest rate rules are only very crude approximations to optimal monetary policy reaction functions. The information set central banks base their policy decisions on is much richer than, for example, the (forecasts of) output gap and inflation considered in the Taylor rule. Consequently, situations with identical (forecast) values of the output gap and inflation can be significantly different economically if judged by the much larger optimal information set. Thus, the central bank not necessarily has to react to (apparently) identical economic situations in the same way and this would show up in changing policy rule parameters. A third reason for changing responses of monetary policy can be changes in the monetary policy transmission mechanism, that is in the structure of the economy. Uncertainty about how the central bank will react to given economic conditions in the future (parameter uncertainty) is the second component of interest rate uncertainty. Finally, the remaining component of interest rate uncertainty (residual uncertainty) is related to the error term in empirically estimated Taylor rules, i.e. represents the approximation error of the Taylor rule relative to actual monetary policy.<sup>2</sup>

We build an empirical model of U.S. monetary policy that allows us to separate the different components of uncertainty about future interest rates. Our model consists of a Taylor rule with time-varying coefficients which describes how the Federal Funds rate responds to the contemporaneously unobservable economic conditions. Changes in the systematic reaction of monetary policy is captured by modeling the Taylor rule parameters as driftless random walks (e.g. Cooley and Prescott (1978)). We also consider a modified version of the model that allows for heteroskedasticity in the Taylor rule residual (Boivin (2006)). The current state of the economy is modeled using an unobserved component model of output, inflation and the output gap as in Kuttner (1994).

Our model is thus an extension to the growing empirical literature on time-varying monetary policy rules. Evidence of pronounced changes in Taylor-type interest rate rules for the U.S. was provided by Clarida, Gali and Gertler (2000) using split-sample regression analysis. They show a strong shift in the conduct of monetary policy re-

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<sup>2</sup>The latter two components, i.e. parameter uncertainty and residual uncertainty could be reduced by the central bank following the approximating policy rule more strictly.

lated to the appointment of Fed Chairman Volcker in 1979. More recently Boivin (2006) estimates forward-looking Taylor rules with time-varying parameters and reports important but gradual changes in the coefficients. Trecroci and Vassali (2006) show that time-varying monetary policy reaction functions for the U.S., the U.K., Germany, France and Italy perform superior to constant parameter rules in accounting for observed changes in policy rates.<sup>3</sup> Trehan and Wu (2004) estimate a time-varying parameter Taylor rule for the U.S. focussing on changes in the equilibrium real interest rate.

An important issue in empirical estimations of monetary policy rules is that estimation on ex-post data, that would not have been available to policymakers, can lead to distorted policy coefficients (e.g. Orphanides (2001)).<sup>4</sup> In this paper, we approach the real-time data issue by assuming that the Fed is unable to contemporaneously observe the relevant economic variables and thus has to rely on estimates of the state of the economy. These estimates are derived from an unobserved components model and are subject to revision if new information becomes available.

The empirical fact of important time-variation in uncertainty about short-term U.S. interest rates has been documented, for example, by Fornari (2005) using implied volatilities from swaptions. Sun (2005) finds evidence for regime shifts in the volatility of short-term interest rates in the U.S. as well as in other countries. Empirical models of interest rate uncertainty mostly have derived their measures of uncertainty as conditional variances from ARCH/ GARCH models (e.g. Chudrewicz (2002), Lanne and Saikkonen (2003)) or from stochastic volatility models or variants thereof (e.g. Caporale and Cipollini (2002)). An advantage of our approach is that our measure of interest rate uncertainty is not only derived from the time series of historical interest rate changes but from the way financial markets perceive monetary policy to be made. Thus, we can directly relate various components of interest rate uncertainty to their economic sources, i.e. changes in the behavior of the Fed and uncertainty about future economic conditions. Our approach is partially related to the study of Favero and Mosca (2001) on the expectations hypothesis of the term structure. They estimate

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<sup>3</sup>Time-varying Taylor rules have also been estimated for the Deutsche Bundesbank by Kuzin (2005) and using a regime-switching model by Assenmacher-Wesche (2006).

<sup>4</sup>See also Orphanides (2002,2003) for a discussion of the results in Clarida, Galì and Gertler (2000).

monetary policy reaction functions for the three-month rate and combine interest-rate forecasts derived from these with a term-structure relationship. The effect of current and future short-term interest rates on the six-month rate is allowed to depend on uncertainty about monetary policy. They show that the expectation hypothesis cannot be rejected in periods of low monetary policy uncertainty.

Our paper is structured as follows. Section 2 specifies the Taylor rule and the output-inflation model and describes the estimation procedure. Section 3 presents the empirical estimates for our model while section 4 contains the results for interest rate uncertainty. Finally, section 5 concludes.

## 2 A model of policy and economic fundamentals

### 2.1 The Taylor rule

Under the assumption that the central bank sets the short-term interest rate in response to the current or expected state of the economy uncertainty about future short-term interest rates stems from (i) uncertainty about the future state of the economy and (ii) uncertainty about the policy response to the state of the economy. The first type of uncertainty concerns forecasts made about future values of the variables contained in the central bank's reaction function while the second type concerns future values of the coefficients in the reaction function.

We assume that the central bank follows a Taylor-type rule in setting its policy rate. It is not necessary that the central bank exactly follows such a rule but that financial participants perceive the central bank to do it or use a Taylor rule to describe the setting of the short-run interest rate.

$$i_t = \bar{r}_t + \alpha_\pi(\pi_t - \bar{\pi}) + \alpha_z z_t, \quad (1)$$

where  $\bar{r}_t$  is the (time-varying) equilibrium real interest rate,  $\pi_t$  is the inflation rate,  $\bar{\pi}$  the target value for the inflation rate, and  $z_t$  is the output gap. The interest rate rule can be rewritten as

$$i_t = \alpha_{0,t} + \alpha_\pi \pi_t + \alpha_z z_t, \quad (2)$$

where  $\alpha_{0,t} = \bar{r}_t - \alpha_\pi \bar{p} \bar{i}$ .

In empirical work it is standard practice to allow for the gradual adjustment of the interest rate to its target level by including a lagged interest rate term, i.e.

$$i_t = (1 - \rho)(\alpha_{0,t} + \alpha_\pi \pi_t + \alpha_z z_t) + \rho i_{t-1}. \quad (3)$$

If we allow for a time-varying response to economic conditions we can rewrite (3) as

$$i_t = \beta_{0,t} + \beta_{\pi,t} \pi_t + \beta_{z,t} z_t + \rho_t i_{t-1} + \epsilon_t, \quad (4)$$

where  $\epsilon_t$  captures the component of the policy rate unrelated to inflation, the output gap or the lagged interest rate.

We assume that the central bank cannot observe the contemporaneous values of inflation and the output gap and therefore has to rely on estimates based on last period's data

$$i_t = \beta_{0,t} + \beta_{\pi,t} \pi_{t||t-1} + \beta_{z,t} z_{t||t-1} + \rho_t i_{t-1} + \epsilon_t, \quad (5)$$

where  $x_{t||t-1}$  denotes the conditional expectation of a variable in period  $t$  based on information available in period  $t - 1$ .

## 2.2 Output gap and inflation forecasts

The dynamics of the output gap and inflation rate are jointly modeled using the unobserved components model described in Kuttner (1994). The output equation is based on Watson (1986) and decomposes the log of real GDP ( $y$ ) into a random walk and a stationary AR(2) component

$$y_t = n_t + z_t \quad (6)$$

$$z_t = \phi_1 z_{t-1} + \phi_2 z_{t-2} + e_t^z \quad (7)$$

$$n_t = \mu_y + n_{t-1} + e_t^n. \quad (8)$$

$n$  is the trend component and follows a random walk with drift  $\mu_y$  while  $z$  is the (log) deviation of real GDP from potential output, i.e. the output gap .

Inflation dynamics are modelled as an ARIMA process in which the change in the rate of inflation depends on the lagged output gap

$$\Delta\pi_t = \mu_\pi + \gamma z_{t-1} + \nu_t + \delta_1 \nu_{t-1} + \delta_2 \nu_{t-2} + \delta_3 \nu_{t-3} + \delta_4 \nu_{t-4} \quad (9)$$

Writing the model (6)-(9) in state-space form we get the observation equation

$$Y_t = \mu + H\tilde{x}_t + e_t, \quad (10)$$

$$Y_t = \begin{bmatrix} \Delta y_t \\ \Delta \pi_t \end{bmatrix}, \mu = \begin{bmatrix} \mu_y \\ \mu_\pi \end{bmatrix}, e_t = \begin{bmatrix} e_t^n \\ 0 \end{bmatrix}$$

$$H = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \gamma & 1 & \delta_1 & \delta_2 & \delta_3 & \delta_4 \end{bmatrix}$$

$$\mathbf{E}e_t e_t' = \Sigma_Y = \begin{bmatrix} \sigma_{e,n}^2 & 0 \\ 0 & 0 \end{bmatrix},$$

and the transition equation for the state variables

$$\tilde{x}_{t+1} = F\tilde{x}_t + \zeta_{t+1} \quad (11)$$

$$\tilde{x}_t = \begin{bmatrix} z_t \\ z_{t-1} \\ \nu_t \\ \nu_{t-1} \\ \nu_{t-2} \\ \nu_{t-3} \\ \nu_{t-4} \end{bmatrix}, \zeta_t = \begin{bmatrix} e_t^z \\ 0 \\ e_t^\nu \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$F = \begin{bmatrix} \phi_1 & \phi_2 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{E}\zeta_t\zeta_t' = \Sigma_\zeta = \begin{bmatrix} \sigma_{e,z}^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{e,\nu}^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

We assume that the shocks  $e^\nu$ ,  $e^n$  and  $e^z$  are serially and mutually uncorrelated.

This model can be estimated by maximum likelihood using the Kalman filter. From the Kalman filter we obtain estimates of the current value of the output gap based on information from the previous period as  $z_{t|t-1}$  which is the first element of  $\tilde{x}_{t|t-1}$ , the forecast of the state vector based on information from the last period. We use this estimate, together with the inflation forecast  $\pi_{t|t-1} = \pi_{t-1} + \Delta\pi_{t|t-1}$  as exogenous variables in the estimation of the Taylor rule (5).

The interest rate rule model can be written in state-space form as

$$i_t = x_t' \beta_t + \epsilon_t, \quad (12)$$

$$x_t' = \begin{bmatrix} 1 & \pi_{t||t-1} & z_{t||t-1} & i_{t-1} \end{bmatrix}$$

$$\mathbf{E}\epsilon_t^2 = \sigma_\epsilon^2. \quad (13)$$

The time-varying parameters follow a random walk

$$\beta_{t+1} = \beta_t + w_{t+1} \quad (14)$$

$$\beta_t = \begin{bmatrix} \beta_{0,t} \\ \beta_{\pi,t} \\ \beta_{z,t} \\ \rho_t \end{bmatrix}, w_t = \begin{bmatrix} w_t^c \\ w_t^\pi \\ w_t^z \\ w_t^i \end{bmatrix}$$

$$\mathbf{E}w_t w_t' = \Sigma_w.$$

The shocks  $w$  and  $\epsilon$  are serially and mutually uncorrelated, as well as uncorrelated with any shocks in the output gap - inflation model. However, we allow for correlation among the shocks in  $w$ .<sup>5</sup> The parameters of this model can again be estimated by maximum likelihood and applying the Kalman filter. We will interpret the estimates of the time-varying parameters  $\beta$  as representing market participants' view of the currently relevant central bank reaction function.

### 2.3 Estimation

Using the estimates from the output gap - inflation model to estimate the interest rate rule parameters requires two basic assumptions. First, we assume that the contemporaneous value of  $x_t$  that underlies the central bank's decision is known to the public. That is, we assume that the central bank publishes or announces its estimates of the contemporaneous inflation rate and the output gap that enter the interest rate decision. Second,  $x_t$  must be exogenous to  $\beta_t$ . For example, our model does not allow for asymmetries in the interest rate response to the output gap or inflation, i.e. for

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<sup>5</sup>This is a consequence of the standardization implicit in the estimation approach that will be used.

the  $\beta$  parameters to vary systematically with changes in the estimated output gap or inflation rate.

Since the parameters of the interest rate rule follow a random walk we have to deal with the "pile-up" problem as discussed in Stock (1994). Basically, if the variances in  $\Sigma_w$  are small their maximum likelihood estimates will be biased toward zero. Following Stock and Watson's (1998) median unbiased estimation procedure we rewrite the observation equation as

$$\Delta\beta_t = w_t = \tau\eta_t, \tau = \lambda/T \quad (15)$$

with  $\eta$  being a vector of mutually uncorrelated shocks that are also assumed to be uncorrelated with the policy shock  $\epsilon$ .  $T$  is the number of observations.<sup>6</sup>  $\tau$  can be inferred from performing a conventional Quandt (1960) likelihood ratio test for stability: From the resulting test statistic  $QLR_T$  we can retrieve an estimate of  $\lambda$  by using Table 3 in Stock and Watson (1998), p. 354. In order to use this table we have to impose the normalization<sup>7</sup>

$$\Sigma_\eta = \frac{\sigma_\epsilon^2}{\mathbf{E}x_t x_t'}, \quad (16)$$

where, for estimation, we replace  $\mathbf{E}x_t x_t'$  by  $\frac{1}{T} \sum_{i=1}^T x_i x_i'$ . Thus, the overall estimation procedure for the model (10)-(14) is as follows:

1. Perform the  $QLR_T$ -test and obtain the estimate for  $\lambda$ .
2. Impose the restriction

$$\Sigma_w = \left(\frac{\lambda}{T}\right)^2 \frac{\sigma_\epsilon^2}{\frac{1}{T} \sum_{i=1}^T x_i x_i'} \quad (17)$$

3. Run the Kalman filter to estimate the remaining free parameter  $\sigma_\epsilon$  by maximum likelihood.

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<sup>6</sup>This procedure for the estimation of the variances is also applied in Boivin (1996).

<sup>7</sup>See Stock and Watson (1998), p. 351.

output equation		inflation equation		output gap equation	
$\mu_y$	0.82 (0.04)	$\mu_\pi$	0.01 (0.08)	$\phi_1$	1.46 (0.03)
		$\gamma$	0.13 (0.04)	$\phi_2$	-0.53 (0.06)
		$\delta_1$	-0.04 (0.04)		
		$\delta_2$	0.69 (0.06)		
		$\delta_3$	0.01 (0.01)		
		$\delta_4$	-0.11 (0.02)		
$\sigma_{e,n}$	0.60 (0.07)	$\sigma_{e,\nu}$	0.59 (0.03)	$\sigma_{e,z}$	0.55 (0.08)
SE	0.87	SE	0.61		

Standard errors in parentheses. Estimation from 206 quarterly observations from 1955:1 to 2006:2. Log-likelihood value is:-257.29.

Table 1: Parameter estimates for the output gap - inflation model

### 3 Estimation results

The output gap-inflation model is estimated on U.S. quarterly data from 1955Q1 to 2006Q2. All data were obtained from FRED II, the database of the Federal Reserve Bank of St. Louis. The inflation rate is defined as 100 times the annual difference of the log Consumer Price Index. Table 1 shows the coefficient estimates.  $\gamma$  is positive and significant. Thus, if output exceeds its potential ( $z > 0$ ) inflation accelerates. The point estimate implies that an output gap of 1%, i.e. output exceeding its potential by 1%, leads to an increase in the annual rate of inflation by 0.13%.<sup>8</sup>

Figure 1 shows the estimated time series of potential output (actual output minus the estimated output gap) and the log of real GDP. As in Kuttner (1994) potential output exhibits substantial short-run fluctuations.

« insert Figure 1 »

The deviations of actual output from its potential, i.e. the output gap, are displayed in Figure 2 along with 1.69 standard error bounds which correspond to 90% confidence

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<sup>8</sup>We also estimated a specification which allows for a direct effect of output growth on the change in the inflation rate. In contrast to Kuttner (1994) the relevant coefficient always turned out to be insignificant.

$\sigma_\epsilon$	$\sigma_{w,0}$	$\sigma_{w,\pi}$	$\sigma_{w,z}$	$\sigma_{w,i}$	SE
0.87	0.05	0.01	0.02	0.00	118.99
(0.04)					

Standard errors in parentheses. Estimation from 205 quarterly observations from 1955:2 to 2006:2. Log-likelihood value is:-290.51.

Table 2: Parameter estimates for Taylor rule

intervals. These error bounds are constructed using the Monte Carlo approach from Hamilton (1994) and reflect both the Kalman filter uncertainty and the uncertainty about the parameter estimates.

« insert Figure 2 »

The next variable that enters the interest rate rule is the inflation forecast  $\pi_{t|t-1}$  which is shown in Figure 3 together with the actual inflation rates.

« insert Figure 3 »

Using the time series for  $z_{t|t-1}$  and the inflation forecasts  $\pi_{t|t-1}$  we estimate the parameters of the time-varying Taylor rule as described in section 2. Since the standard deviations for the time-varying response parameters were obtained using the approach by Stock and Watson (1998), we cannot provide standard deviations for the estimates.

The fit of the time-varying Taylor rule is displayed in Figure 4. The estimated model provides a reasonable approximation to the actually observed interest rates with a sum of squared residuals of 197.93. As is to be expected from the random walk specification for the response parameters the fitted interest rate lags the observed one by one period.

« insert Figure 4 »

Figure 5 shows the one-sided estimates of the time varying long-run coefficients of the Taylor rule ( $\beta_{j,t|t}^{lr} = \frac{\beta_{j,t|t}}{1-\rho_{t|t}}$ ,  $j = 0, \pi, z$ ) in the first three panels and the time varying estimate of the coefficient on the lagged policy rate in the last panel. We interpret these one-sided estimates as representing the view of market participants of the Fed's policy reaction function. As in Boivin (2006) the interest rate rule parameters exhibit

strong variation over time.<sup>9</sup>

« insert Figure 5 »

## 4 Interest rate uncertainty

### 4.1 The one-period case

Uncertainty about the Federal Funds Rate in the next quarter can be defined as

$$\mathbf{E}_t \left[ (i_{t+1} - \hat{i}_{t+1|t})^2 | \Omega_t \right], \quad (18)$$

where

$$\hat{i}_{t+1|t} = \mathbf{E}_t [i_{t+1} | \Omega_t] = \mathbf{E}_t [x'_{t+1} \beta_{t+1} | \Omega_t]. \quad (19)$$

$\Omega_t$  represents the information available to market participants immediately *after* the policy rate is set at time  $t$ . This information set consists of the estimated coefficients in Tables 1 and 2 – that is we assume that the public knows the model the central bank uses to estimate the output gap and the current inflation rate – and all past values of  $y$  and  $\pi$  but *not* the current values of output and inflation  $y_t$  and  $\pi_t$  which cannot be observed contemporaneously. It also contains past and current values of  $i$  and the current values of the central bank’s estimates of the output gap  $z_{t|t-1}$  and of the inflation rate  $\pi_{t|t-1}$ .<sup>10</sup>

We assume  $\beta$  and  $x$  to be uncorrelated.<sup>11</sup> Thus,

$$\hat{i}_{t+1|t} = \mathbf{E}_t [x'_{t+1} | \Omega_t] \mathbf{E}_t [\beta_{t+1} | \Omega_t] = \hat{x}'_{t+1|t} \beta_{t+1|t}. \quad (20)$$

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<sup>9</sup>Note that these are not the smoothed parameter estimates  $\beta_{j,t|T}^{br}$ , which do exhibit much more gradual changes.

<sup>10</sup>An alternative assumption would be that market participants accept the model as a relatively accurate representation of the way the central bank acquires and uses its information.

<sup>11</sup>This assumption is implied by using the Kalman filter.

Note that since  $x_t = (1 \quad \pi_{t|t-1} \quad z_{t|t-1} \quad i_{t-1})$  the forecast of  $x_{t+1}$  based on  $\Omega_t$  is  $\hat{x}_{t+1|t} = (1 \quad \pi_{t+1|t-1} \quad z_{t+1|t-1} \quad i_t)$ . However the forecast of  $\beta_{t+1}$  based on  $\Omega_t$  is  $\beta_{t+1|t}$  as  $i_t$  is part of the information set.

« insert Figure 6 »

Hence, we have

$$\begin{aligned} \mathbf{E}_t \left[ (i_{t+1} - \hat{i}_{t+1|t})^2 | \Omega_t \right] &= \mathbf{E}_t \left[ (x'_{t+1} \beta_{t+1} - \hat{x}'_{t+1|t} \beta_{t+1|t})^2 | \Omega_t \right] \\ &= \mathbf{E}_t \left[ \beta'_{t+1} x_{t+1} x'_{t+1} \beta_{t+1} | \Omega_t \right] - \beta'_{t+1|t} \hat{x}_{t+1|t} \hat{x}'_{t+1|t} \beta_{t+1|t} + \sigma_\epsilon^2. \end{aligned} \quad (21)$$

This can be written as

$$\begin{aligned} \mathbf{E}_t \left[ (i_{t+1} - \hat{i}_{t+1|t})^2 | \Omega_t \right] &= \hat{x}'_{t+1|t} \mathbf{E}_t \left[ (\beta_{t+1} - \beta_{t+1|t})(\beta_{t+1} - \beta_{t+1|t})' | \Omega_t \right] \hat{x}_{t+1|t} \\ &\quad + \beta'_{t+1|t} \mathbf{E}_t \left[ (x_{t+1} - \hat{x}_{t+1|t})(x_{t+1} - \hat{x}_{t+1|t})' | \Omega_t \right] \beta_{t+1|t} \\ &= \hat{x}'_{t+1|t} P_{\beta,t+1|t} \hat{x}_{t+1|t} + \beta'_{t+1|t} P_{x,t+1|t} \beta_{t+1|t} + \sigma_\epsilon^2. \end{aligned} \quad (22)$$

$P_{\beta,t+1|t} = \mathbf{E}_t \left[ (\beta_{t+1} - \beta_{t+1|t})(\beta_{t+1} - \beta_{t+1|t})' \right]$  is obtained from the Kalman filter. This component of the overall interest rate forecast uncertainty represents uncertainty due to changes in the way the Fed reacts to the fundamental variables in its reaction function. The uncertainty due to this component of future interest rates rises if there is a numerical increase in the variables that enter the policy rule. The reason is that even when uncertainty about the  $\beta$ -parameters remains unchanged, uncertainty about the size of the interest rate response of the central bank increases when the absolute values of the variables the  $\beta$ -coefficients are multiplied with go up.

$P_{x,t+1|t} = \mathbf{E}_t \left[ (x_{t+1} - x_{t+1|t})(x_{t+1} - x_{t+1|t})' | \Omega_t \right]$  represents the uncertainty about the forecast of the economic variables the interest rate responds to. A detailed derivation of this expression can be found in the Appendix.

The results are presented in Figure 7. The Figure shows the overall interest rate uncertainty (22) together with two of its components<sup>12</sup>

$$\beta_{unc} = \hat{x}'_{t+1|t} P_{\beta,t+1|t} \hat{x}_{t+1|t}$$

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<sup>12</sup>The high initial values result from the Kalman filter initialization where we have chosen a relatively

and

$$x_{unc} = \beta'_{t+1|t} P_{x,t+1|t} \beta'_{t+1|t}.$$

« insert Figure 7 »

The Figure shows that most of the interest rate uncertainty is caused by the variance of the policy shock  $\sigma_\epsilon^2$ . There are three episodes in which an increase in the uncertainty about the policy response ( $\beta_{unc}$ ) leads to a burst in overall interest rate uncertainty. These episodes occurred at the end of the 1960s, in the mid 1970s and in the early 1980s. The latter two episodes coincide with an increase in the forecast uncertainty of the output gap and inflation. After the mid-1980s and particularly during the 1990s uncertainty about the policy parameters remains relatively low and rises again somewhat after 2001. Uncertainty about future values of output gap almost always plays a minor role relative to the interest rate rule parameter uncertainty. Uncertainty about economic fundamentals rises over the 1970s and drops again in the early 1980s to a level comparably to that of the 1960s. In the early 1990s it rises somewhat but remains far below the levels of the 1970s.

## 4.2 The two-period case

We can define uncertainty about the interest rate set two periods in the future as

$$\mathbf{E}_t \left[ (i_{t+2} - \hat{i}_{t+2|t})^2 | \Omega_t \right], \quad (23)$$

where

$$\begin{aligned} \hat{i}_{t+2|t} &= \mathbf{E}_t [i_{t+2} | \Omega_t] = \mathbf{E}_t [x'_{t+2} \beta_{t+2} | \Omega_t] \\ &= \mathbf{E}_t [x'_{t+2} | \Omega_t] \mathbf{E}_t [\beta_{t+2} | \Omega_t] = \hat{x}'_{t+2|t} \beta_{t+2|t}. \end{aligned} \quad (24)$$

« insert Figure 8 »

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high starting variance for the  $\beta$  vector in order to discount possible effects from the choice of the starting values.

We get

$$\begin{aligned}
\mathbf{E}_t \left[ (i_{t+2} - \hat{i}_{t+2|t})^2 | \Omega_t \right] &= \mathbf{E}_t \left[ (x'_{t+2} \beta_{t+2} - \hat{x}'_{t+2|t} \hat{\beta}_{t+2|t})^2 | \Omega_t \right] \\
&= \mathbf{E}_t \left[ \beta'_{t+2} x_{t+2} x'_{t+2} \beta_{t+2} | \Omega_t \right] - \beta'_{t+2|t} \hat{x}_{t+2|t} \hat{x}'_{t+2|t} \beta_{t+2|t} + \sigma_\epsilon^2,
\end{aligned} \tag{25}$$

and, finally,

$$\begin{aligned}
\mathbf{E}_t \left[ (i_{t+2} - \hat{i}_{t+2|t})^2 | \Omega_t \right] &= \hat{x}'_{t+2|t} \mathbf{E}_t \left[ (\beta_{t+2} - \beta_{t+2|t})(\beta_{t+2} - \beta_{t+2|t})' | \Omega_t \right] \hat{x}_{t+2|t} \\
&\quad + \beta'_{t+2|t} \mathbf{E}_t \left[ (x_{t+2} - \hat{x}_{t+2|t})(x_{t+2} - \hat{x}_{t+2|t})' | \Omega_t \right] \beta_{t+2|t} \\
&= \hat{x}'_{t+2|t} P_{\beta, t+2|t} \hat{x}_{t+2|t} + \beta'_{t+2|t} P_{x, t+2|t} \beta_{t+2|t} + \sigma_\epsilon^2.
\end{aligned} \tag{26}$$

$P_{\beta, t+2|t} = \mathbf{E}_t \left[ (\beta_{t+2} - \beta_{t+2|t})(\beta_{t+2} - \beta_{t+2|t})' | \Omega_t \right]$  can be computed by noting (14) as  $P_{\beta, t+2|t} = P_{\beta, t+1|t} + \Sigma_w$ . For the derivation of  $P_{x, t+2|t}$  refer to the appendix.

Figure 9 shows the two-period forecast uncertainty about the federal funds rate along with its two components. It is obvious that interest rate uncertainty is markedly higher over the longer forecast horizon. Furthermore, while over one period parameter uncertainty is more important than uncertainty about the future state of the economy the latter one turns out to be always more important for the longer forecast horizon. Uncertainty about the interest rate rule parameters never exceeds uncertainty about future economic fundamentals. Some smaller spikes in uncertainty about the central bank's future responses to economic conditions can be observed in the early mid-to late 1960s, the mid 1970s, and in the late 1970s to the early 1980s. Parameter uncertainty also increases in late 2001 after having been relatively low throughout the 1990s. Uncertainty about future economic conditions rises strongly in the early 1970s, mid 1970s and in the early 1980s and does only partially return to its previous lower levels. It comes down somewhat again in the early 1980s. In the early 1990s and 2000s we observe again two persistent increases in uncertainty about future economic conditions.

« insert Figure 9 »

1955:1-1979:3	1979:4-1982:3	1982:4-2006:2
0.76	2.15	0.48

Table 3: Estimates for  $\sigma_\epsilon$  for subsamples (OLS)

### 4.3 Heteroskedasticity

Many studies have presented evidence for shifts in the variance of the policy rule’s error term associated with the Volcker disinflation period at the Fed (e.g. Boivin (2006), Clarida, Gali and Gertler (2000)). To accomodate this heteroskedasticity we split the sample into three subperiods with break points in 1979:4 and 1982:4. We follow Boivin (2006) and estimate  $\sigma_\epsilon$  across the different regimes from the OLS residuals and use these estimates in the procedure outlined in section 2.1 to estimate the variances of the time-varying parameters and to run the Kalman filter.

The estimated value for  $\sigma_\epsilon$  is extremely high for the 1984:4 to 1982:4 period. For the most recent subsample it is about two thirds of the estimate for the first subsample. Figures 10 and 11 show that overall interest rate uncertainty was significantly higher in the pre 1979:4-period than in the post 1982:4-period. For the 1979:4 to 1982:4 subperiod most of the drastic increase in uncertainty is due to the increased variance of the error term  $\epsilon$ , but uncertainty about the interest rate rule parameters increased substantially as well. Uncertainty about the Taylor rule parameters is on average higher from 1960-79 than from 1982 on. As in the homoskedastic case, there are two spikes in parameter uncertainty at the end of 1960s and in the mid 1970s. In the final subsample uncertainty about the Fed’s response parameters is very low. Uncertainty about next quarter’s fundamentals starts out very low and slowly edges upward throughout the 1970s. We observe a strong rise in the mid 1980s when uncertainty about future economic conditions makes up most of overall interest rate uncertainty. After the mid 1980s uncertainty about future fundamentals is relatively low again. However, on average this component of overall interest rate uncertainty is higher after the late 1980s than in the 1960s and early 1970s.

« insert Figures 10+11 »

Figures 12 and 13 show the results for the two-period interest rate forecasts. The overall impression is similar to the one-period case but on a higher level. As in the homoskedastic case, uncertainty about the future state of the economy is more important than uncertainty about the policy parameters.

« insert Figures 12+13 »

## 5 Conclusion

We have constructed an empirical model of monetary policy in the U.S. that enables us to separate the uncertainty perceived by market participants about future interest rates into its basic components: uncertainty about the state of the economy in the future and uncertainty about how the Fed will react to future economic conditions.

Our results show that there is considerable time variation in the parameters of the policy rule. For forecast horizons up to one quarter uncertainty about the future values of these parameters is most of the time more important than uncertainty about the future state of the economy. For a forecast horizon of two quarters uncertainty about future economic conditions dominates uncertainty about future policy parameters. According to our model uncertainty about future interest rates is highly variable with peaks in the late 1960s/early 1970s, mid 1970s and late 1970s/early 1980s. Recently, uncertainty about future policy reactions has increased again and over the longer forecast horizon uncertainty about future economic conditions has also gone up.

We also accounted for shifts in the error variance of the interest rate rule. We derived a strong increase in interest rate uncertainty in the 1979-1982 period, driven by a surge in the error variance of the policy rule. The estimates from the modified model show uncertainty about future interest rates to have been exceptionally low in the post 1990 period.

## Appendix A: The Kalman filter equations

The estimates of the unobserved component  $\tilde{x}_{t|t-1} = E_{t-1}[\tilde{x}_t]$  and its covariance matrix  $P_{\tilde{x},t|t-1} = E_{t-1}[(\tilde{x}_t - \tilde{x}_{t|t-1})(\tilde{x}_t - \tilde{x}_{t|t-1})']$  are formed recursively

$$\tilde{x}_{t|t-1} = F\tilde{x}_{t-1|t-1}, \quad (\text{A1})$$

$$P_{\tilde{x},t|t-1} = FP_{\tilde{x},t-1|t-1}F' + \Sigma_\zeta, \quad (\text{A2})$$

with  $\tilde{x}_{t|t} = E_t[\tilde{x}_t]$  and its covariance matrix  $P_{\tilde{x},t|t} = E_t[(\tilde{x}_t - \tilde{x}_{t|t})(\tilde{x}_t - \tilde{x}_{t|t})']$ .

After the information on  $Y_t$  has become available, the estimates are updated as

$$\begin{aligned} \tilde{x}_{t|t} &= \tilde{x}_{t|t-1} + K_{t|t-1}(Y_t - Y_{t|t-1}) \\ &= \tilde{x}_{t|t-1} + K_{t|t-1}(Y_t - \mu - H\tilde{x}_{t|t-1}) \\ &= \tilde{x}_{t|t-1} + K_{t|t-1}(H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t) \end{aligned} \quad (\text{A3})$$

$$P_{\tilde{x},t|t} = P_{\tilde{x},t|t-1} - K_{t|t-1}HP_{\tilde{x},t|t-1}, \quad (\text{A4})$$

with  $K_{t|t-1} = P_{\tilde{x},t|t-1}H'[HP_{\tilde{x},t|t-1}H' + \Sigma_Y]^{-1}$ .

The second second row of (A3) is used to form the estimate of  $\tilde{x}_{t|t}$  while the third row is used in the computation of the expressions for interest rate uncertainty.

The forecasting and updating equations for the Taylor rule coefficients are

$$\beta_{t|t-1} = \beta_{t-1|t-1}, \quad (\text{A5})$$

$$P_{\beta,t|t-1} = P_{\beta,t-1|t-1} + \Sigma_w, \quad (\text{A6})$$

After the information on  $i_t$  has become available, the estimates are updated as

$$\begin{aligned}
\beta_{t|t} &= \beta_{t|t-1} + P_{\beta,t|t-1}x_t[x_t'P_{\beta,t|t-1}x_t + \sigma_\epsilon^2]^{-1}(i_t - i_{t|t-1}) \\
&= \beta_{t|t-1} + P_{\beta,t|t-1}x_t[x_t'P_{\beta,t|t-1}x_t + \sigma_\epsilon^2]^{-1}(i_t - x_t'\beta_{t|t-1}), \tag{A7}
\end{aligned}$$

$$P_{\beta,t|t} = P_{\beta,t|t-1} - P_{\beta,t|t-1}x_t[x_t'P_{\beta,t|t-1}x_t + \sigma_\epsilon^2]^{-1}x_t'P_{\beta,t|t-1}. \tag{A8}$$

Since we require only the one-sided estimates for  $\tilde{x}$  and  $\beta$  we do not reproduce the equations for the smoothing algorithm.<sup>13</sup>

## Appendix B: Uncertainty measures

### Uncertainty about economic conditions in the one-period case

**Derivation of (22):** Define We can use a Taylor-Approximation to write

$$\begin{aligned}
\mathbf{E} [\beta'_{t+1}x_{t+1}x'_{t+1}\beta_{t+1}|\Omega_t] &\approx \beta'_{t+1|t}\hat{x}_{t+1|t}\hat{x}'_{t+1|t}\beta_{t+1|t} \\
&\quad + 2\mathbf{E}(\beta_{t+1} - \beta_{t+1|t}|\Omega_t)' \hat{x}_{t+1|t}\hat{x}'_{t+1|t}\beta_{t+1|t} \\
&\quad + 2\mathbf{E}(x_{t+1} - \hat{x}_{t+1|t}|\Omega_t)' \beta_{t+1|t}\beta'_{t+1|t}\hat{x}_{t+1|t} \\
&\quad + \hat{x}'_{t+1|t}\mathbf{E}[(\beta_{t+1} - \beta_{t+1|t})(\beta_{t+1} - \beta_{t+1|t})'|\Omega_t]\hat{x}_{t+1|t} \\
&\quad + \beta'_{t+1|t}\mathbf{E}[(x_{t+1} - \hat{x}_{t+1|t})(x_{t+1} - \hat{x}_{t+1|t})'|\Omega_t]\beta_{t+1|t} \\
&\quad + 4\hat{x}'_{t+1|t}\mathbf{E}[(\beta_{t+1} - \beta_{t+1|t})(x_{t+1} - \hat{x}_{t+1|t})'|\Omega_t]\beta_{t+1|t} \\
&\approx \beta'_{t+1|t}\hat{x}_{t+1|t}\hat{x}'_{t+1|t}\beta_{t+1|t} \tag{B1} \\
&\quad + \hat{x}'_{t+1|t}\mathbf{E}[(\beta_{t+1} - \beta_{t+1|t})(\beta_{t+1} - \beta_{t+1|t})'|\Omega_t]\hat{x}_{t+1|t} \\
&\quad + \beta'_{t+1|t}\mathbf{E}[(x_{t+1} - \hat{x}_{t+1|t})(x_{t+1} - \hat{x}_{t+1|t})'|\Omega_t]\beta_{t+1|t}.
\end{aligned}$$

Substituting this expression into (21) yields (22).

Since  $x_{t+1} = (1 \ \pi_{t+1|t} \ z_{t+1|t} \ i_t)$  and  $\hat{x}_{t+1} = (1 \ \pi_{t+1|t-1} \ z_{t+1|t-1} \ i_t)$  we can write

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<sup>13</sup>For more details on the Kalman filter see, for example, Hamilton (1996) or Kim and Nelson (1999).

$$\begin{aligned}
P_{x,t+2|t} &= \mathbf{E}_t [(x_{t+1} - x_{t+1|t})(x_{t+1} - x_{t+1|t})' | \Omega_t] \\
&= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & p_{\pi,\pi,t+1} & p_{\pi,z,t+1} & 0 \\ 0 & p_{\pi,z,t+1} & p_{z,z,t+1} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{B2}
\end{aligned}$$

The individual elements can be derived as follows: We start with  $p_{\pi,\pi,t+1} = \mathbf{E} [(\pi_{t+1|t} - \pi_{t+1|t-1})^2 | \Omega_t]$ . The inflation forecast the central bank will react to in the next period is  $\pi_{t+1|t} = \pi_t + \Delta\pi_{t+1|t}$ . Since  $\pi_t$  is not known at the time the forecast  $\hat{x}_{t+1|t}$  is made  $\pi_{t+1|t-1} = \pi_{t|t-1} + \Delta\pi_{t+1|t-1}$ . Thus

$$\begin{aligned}
\pi_{t+1|t} - \pi_{t+1|t-1} &= (\pi_t - \pi_{t|t-1}) + (\Delta\pi_{t+1|t} - \Delta\pi_{t+1|t-1}) \\
&= (\Delta\pi_t - \Delta\pi_{t|t-1}) + (\Delta\pi_{t+1|t} - \Delta\pi_{t+1|t-1}) \\
&= \mathbf{1}'_2 [(Y_t - Y_{t|t-1}) + (Y_{t+1|t} - Y_{t+1|t-1})], \tag{B3}
\end{aligned}$$

with  $\mathbf{1}_2 = (0 \ 1)'$ . Using (10), (11), (A3), and (A4) we get

$$\begin{aligned}
(Y_t - Y_{t|t}) + (Y_{t+1|t} - Y_{t+1|t-1}) &= H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t + H(\tilde{x}_{t+1|t} - \tilde{x}_{t+1|t-1}) \\
&= H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t + HF(\tilde{x}_{t|t} - \tilde{x}_{t|t-1}) \tag{B4} \\
&= H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t \\
&\quad + HF K_{t|t-1} (H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t).
\end{aligned}$$

Using this expression we get

$$\begin{aligned}
p_{\pi,\pi,t+1} &= \mathbf{E} [(\pi_{t+1|t} - \pi_{t+1|t-1})^2 | \Omega_t] \\
&= \mathbf{1}'_2 \mathbf{E} \left[ ((Y_t - Y_{t|t-1}) + (Y_{t+1|t} - Y_{t+1|t-1})) \right. \\
&\quad \left. ((Y_t - Y_{t|t-1}) + (Y_{t+1|t} - Y_{t+1|t-1}))' | \Omega_t \right] \mathbf{1}_2 \tag{B5} \\
&= \mathbf{1}'_2 (I + HF K_{t|t-1}) [HP_{\tilde{x},t|t-1} H' + \Sigma_Y] (I + HF K_{t|t-1})' \mathbf{1}_2.
\end{aligned}$$

At the time the policy rate in period  $t$  is announced, uncertainty about  $\pi_{t+1|t}$ , the estimate of inflation the Fed will react to in the next period and which is forecast as  $\pi_{t+1|t-1}$ , stems from two sources: first,  $(\Delta\pi_t - \Delta\pi_{t|t-1})$ , the second element of  $(Y_t - Y_{t|t-1})$ , is the error made in estimating the change in the inflation rate from the previous to the current period. Second,  $(\Delta\pi_{t+1|t} - \Delta\pi_{t+1|t-1})$ , the (2,1) element of  $(Y_{t+1|t} - Y_{t+1|t-1})$ , is the difference between the change in inflation over the next period estimated by the central bank at the time it has to set  $i_{t+1}$  – and thus formed with knowledge of  $\pi_t$  – and the estimate of next period’s change in inflation that is formed by the public now without knowing  $\pi_t$ .

Next we compute  $p_{z,z,t+1} = \mathbf{E}[(z_{t+1|t} - z_{t+1|t-1})^2|\Omega_t]$ . Since  $z_t$  is the (1,1) element of  $\tilde{x}_t$ ,

$$\begin{aligned} z_{t+1|t} - z_{t+1|t-1} &= \mathbf{1}'_1(\tilde{x}_{t+1|t} - \tilde{x}_{t+1|t-1}) \\ &= \mathbf{1}'_1 F(\tilde{x}_{t|t} - \tilde{x}_{t|t-1}) \\ &= \mathbf{1}'_1 F K_{t|t-1} (H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t). \end{aligned} \tag{B6}$$

Thus, we arrive at

$$\begin{aligned} p_{z,z,t+1} &= \mathbf{E}[(z_{t+1|t} - z_{t+1|t-1})^2|\Omega_t] \\ &= \mathbf{1}'_1 \mathbf{E}[(\tilde{x}_{t+1|t} - \tilde{x}_{t+1|t-1})(\tilde{x}_{t+1|t} - \tilde{x}_{t+1|t-1})'|\Omega_t] \mathbf{1}_1 \\ &= \mathbf{1}'_1 F K_{t|t-1} H P_{\tilde{x},t|t-1} F' \mathbf{1}_1, \end{aligned} \tag{B7}$$

with  $\mathbf{1}_1 = (1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)'$ . Uncertainty about the Fed’s estimate of the output gap is due to the fact that when policy is set next period additional information in form of observations of  $\pi_t$  and  $y_t$  will be available.

Finally, combining (B3) and (B4) with (B6) we get

$$\begin{aligned} p_{\pi,z,t+1} &= \mathbf{E}[(\pi_{t+1|t} - \pi_{t+1|t-1})(z_{t+1|t} - z_{t+1|t-1})|\Omega_t] \\ &= \mathbf{1}'_2 (H P_{\tilde{x},t|t-1} F' + H F K_{t|t-1} H P_{\tilde{x},t|t-1} F') \mathbf{1}_1. \end{aligned} \tag{B8}$$

All these expressions can be evaluated using the parameter estimates from section 3 and the results from the Kalman filter.

## 5.1 Uncertainty about economic conditions in the two-period case

$$\begin{aligned}
P_{x,t+2|t} &= \mathbf{E}_t [(x_{t+2} - \hat{x}_{t+2|t})(x_{t+2} - \hat{x}_{t+2|t})' | \Omega_t] \\
&= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & p_{\pi,\pi,t+2} & p_{\pi,z,t+2} & p_{\pi,i,t+2} \\ 0 & p_{\pi,z,t+2} & p_{z,z,t+2} & p_{i,z,t+2} \\ 0 & p_{\pi,i,t+2} & p_{i,z,t+2} & p_{i,i} \end{bmatrix}. \tag{B9}
\end{aligned}$$

The inflation forecast the central bank will react to two periods in the future is  $\pi_{t+2|t+1} = \pi_{t+1} + \Delta\pi_{t+2|t+1} = \pi_{t-1} + \Delta\pi_t + \Delta\pi_{t+1} + \Delta\pi_{t+2|t+1}$ . The forecast of  $\pi_{t+2|t+1}$  based on information known at time  $t$  is  $\pi_{t+2|t-1} = \pi_{t-1} + \Delta\pi_{t|t-1} + \Delta\pi_{t+1|t-1} + \Delta\pi_{t+2|t-1}$ . Thus

$$\begin{aligned}
\pi_{t+2|t+1} - \pi_{t+2|t-1} &= (\Delta\pi_t - \Delta\pi_{t|t-1}) + (\Delta\pi_{t+1} - \Delta\pi_{t+1|t-1}) \\
&\quad + (\Delta\pi_{t+2|t+1} - \Delta\pi_{t+2|t-1}) \\
&= \mathbf{1}'_2 [(Y_t - Y_{t|t-1}) + (Y_{t+1} - Y_{t+1|t-1}) \\
&\quad + (Y_{t+2|t+1} - Y_{t+2|t-1})] \tag{B10}
\end{aligned}$$

with  $\mathbf{1}_2 = (0 \ 1)'$ . Using (10) and (11) we get

$$\begin{aligned}
(Y_t - Y_{t|t-1}) &+ (Y_{t+1} - Y_{t+1|t-1}) + (Y_{t+2|t+1} - Y_{t+2|t-1}) \\
&= H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t + H(\tilde{x}_{t+1} - \tilde{x}_{t+1|t-1}) \\
&\quad + H(\tilde{x}_{t+2|t+1} - \tilde{x}_{t+2|t-1}) \\
&= H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t + H(F(\tilde{x}_t - \tilde{x}_{t|t-1}) + \zeta_{t+1}) \\
&\quad + H[F(K_{t+1|t}HF + (F - K_{t+1|t}HF)K_{t|t-1}H)(\tilde{x}_t - \tilde{x}_{t|t-1}) \\
&\quad + F(F - K_{t+1|t}HF)K_{t+1|t}e_t + FK_{t+1|t}(H\zeta_{t+1} + e_{t+1})], \tag{B11}
\end{aligned}$$

where the expression in the last two lines of (B11) is derived in (B13). As a result we arrive at

$$\begin{aligned}
p_{\pi,\pi,t+2} &= \mathbf{E} \left[ (\pi_{t+2|t+1} - \pi_{t+2|t-1})^2 | \Omega_t \right] \\
&= \mathbf{1}'_2 \mathbf{E} \left[ \left( (Y_t - Y_{t|t-1}) + (Y_{t+1} - Y_{t+1|t-1}) + (Y_{t+2|t+1} - Y_{t+2|t-1}) \right) \right. \\
&\quad \left. \left( (Y_t - Y_{t|t-1}) + (Y_{t+1} - Y_{t+1|t-1}) + (Y_{t+2|t+1} - Y_{t+2|t-1}) \right)' | \Omega_t \right] \mathbf{1}_2 \\
&= \mathbf{1}'_2 \left[ H \left[ I + F \left( I + K_{t+1|t} H F + (F - K_{t+1|t} H F) K_{t|t-1} H \right) \right] P_{\tilde{x},t|t-1} \right. \\
&\quad \left[ I + F \left( I + K_{t+1|t} H F + (F - K_{t+1|t} H F) K_{t|t-1} H \right) \right]' H' \\
&\quad + H F \left( I + K_{t+1|t} H \right) \Sigma_\zeta \left( I + K_{t+1|t} H \right)' F' H' \\
&\quad H F \left( F - K_{t+1|t} H F \right) K_{t|t-1} \Sigma_Y K'_{t|t-1} \left( F - K_{t+1|t} H F \right)' F' H' \\
&\quad \left. + H F K_{t+1|t} \Sigma_Y K'_{t+1|t} F' H' \right] \mathbf{1}_2. \tag{B12}
\end{aligned}$$

For the squared forecasting error of the output gap estimate we have

$$\begin{aligned}
z_{t+2|t+1} - z_{t+2|t-1} &= \mathbf{1}'_1 (\tilde{x}_{t+2|t+1} - \tilde{x}_{t+2|t-1}) \\
&= \mathbf{1}'_1 F (\tilde{x}_{t+1|t} + K_{t+1|t} [H(\tilde{x}_{t+1} - \tilde{x}_{t+1|t}) + e_{t+1}] - \tilde{x}_{t|t-1}) \\
&= \mathbf{1}'_1 F [F(\tilde{x}_{t|t} - \tilde{x}_{t|t-1}) + K_{t+1|t} (H(F(\tilde{x}_t - \tilde{x}_{t|t}) + \zeta_{t+1}) + e_{t+1})] \\
&= \mathbf{1}'_1 F (F K_{t|t-1} [H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t] + K_{t+1|t} H F (\tilde{x}_t - (\tilde{x}_{t|t-1} \\
&\quad + K_{t|t-1} (H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t)) + K_{t+1|t} (H \zeta_{t+1} + e_{t+1})) \\
&= \mathbf{1}'_1 (F (K_{t+1|t} H F + (F - K_{t+1|t} H F) K_{t|t-1} H) (\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t) \\
&\quad + F (F - K_{t+1|t} H F) K_{t|t-1} e_t + F K_{t+1|t} (H \zeta_{t+1} + e_{t+1})). \tag{B13}
\end{aligned}$$

Thus,

$$\begin{aligned}
p_{z,z,t+2} &= \mathbf{E} \left[ (z_{t+2|t+1} - z_{t+2|t-1})^2 | \Omega_t \right] \\
&= \mathbf{1}'_1 \mathbf{E} \left[ (\tilde{x}_{t+2|t} - \tilde{x}_{t+2|t-1})(\tilde{x}_{t+2|t} - \tilde{x}_{t+2|t-1})' | \Omega_t \right] \mathbf{1}_1 \\
&= \mathbf{1}'_1 \left[ F (K_{t+1|t} H F + (F - K_{t+1|t} H F) K_{t|t-1} H) P_{\tilde{x},t|t-1} \right. \\
&\quad \left. (K_{t+1|t} H F + (F - K_{t+1|t} H F) K_{t|t-1} H)' F' \right. \\
&\quad \left. + F [(F - K_{t+1|t} H F) K_{t|t-1} + K_{t+1|t}] \Sigma_Y [(F - K_{t+1|t} H F) K_{t|t-1} + K_{t+1|t}]' F' \right. \\
&\quad \left. + F K_{t+1|t} H \Sigma_\zeta H' K'_{t+1|t} F' \right] \mathbf{1}_1.
\end{aligned} \tag{B14}$$

From (B10), (B11) and (B13) we derive

$$\begin{aligned}
p_{\pi,z,t+2} &= \mathbf{E} \left[ (\pi_{t+2|t+1} - \pi_{t+2|t-1})(z_{t+2|t+1} - z_{t+2|t-1}) | \Omega_t \right] \\
&= \mathbf{1}'_2 \left[ H (I + F(I + K_{t+1|t} H F + (F - K_{t+1|t} H F) K_{t|t-1} H) P_{\tilde{x},t|t-1} \right. \\
&\quad \left. (K_{t+1|t} H F + (F - K_{t+1|t} H F) K_{t|t-1} H)' F' \right. \\
&\quad \left. + H F (F - K_{t+1|t} H F) K_{t|t-1} \Sigma_Y K'_{t|t-1} (F - K_{t+1|t} H F)' F' \right. \\
&\quad \left. + H F K_{t+1|t} \Sigma_Y K'_{t+1|t} F' + H F K_{t+1|t} H \Sigma_\zeta H' K'_{t+1|t} F' \right] \mathbf{1}_1.
\end{aligned} \tag{B15}$$

Next are the correlations of the forecast errors for the output gap and inflation with the forecast error for the interest rate. The latter one is

$$\begin{aligned}
i_{t+1} - \hat{i}_{t+1|t} &= x'_{t+1} \beta_{t+1} - \hat{x}'_{t+1|t} \beta_{t+1|t} + \epsilon_{t+1} \\
&= x'_{t+1} (\beta_t + w_{t+1}) - \hat{x}'_{t+1|t} \beta_{t|t} + \epsilon_{t+1} \\
&= (x_{t+1} - \hat{x}_{t+1|t})' \beta_{t|t} + x'_{t+1} (\beta_t + w_{t+1} - \beta_{t|t}) + \epsilon_{t+1}.
\end{aligned} \tag{B16}$$

Since  $x'_{t+1} = (1 \ \pi_{t+1|t} \ z_{t+1|t} \ i_t)$  and  $\hat{x}'_{t+1|t} = (1 \ \pi_{t+1|t-1} \ z_{t+1|t-1} \ i_t)$  we can expand the above expression to

$$\begin{aligned}
i_{t+1} - \hat{i}_{t+1|t} &= (\pi_{t+1|t} - \pi_{t+1|t-1})\beta_{\pi,t|t} + (z_{t+1|t} - z_{t+1|t-1})\beta_{z,t|t} \\
&\quad + (\beta_{c,t} - \beta_{c,t|t}) + \pi_{t+1|t}(\beta_{\pi,t} - \beta_{\pi,t|t}) \\
&\quad + z_{t+1|t}(\beta_{z,t} - \beta_{z,t|t}) + i_t(\rho_t - \rho_{t|t}) \\
&\quad + x'_{t+1}w_{t+1} + \epsilon_{t+1}.
\end{aligned} \tag{B17}$$

The inflation forecast made for the interest rate setting in the next period is

$$\begin{aligned}
\pi_{t+1|t} &= \pi_t + \Delta\pi_{t+1|t} = \pi_t + \mathbf{1}'_2 Y_{t+1|t} \\
&= \pi_t + \mathbf{1}'_2 [\mu + H\tilde{x}_{t+1|t}] \\
&= \pi_t + \mathbf{1}'_2 [\mu + HF\tilde{x}_{t|t}] \\
&= \pi_t + \mathbf{1}'_2 [\mu + HF(\tilde{x}_{t|t-1} + K_{t|t-1}(H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t))],
\end{aligned} \tag{B18}$$

and  $(\pi_{t+1|t} - \pi_{t+1|t-1})$  is shown in (B3).

$$\begin{aligned}
z_{t+1|t} &= \mathbf{1}'_1 \tilde{x}_{t+1|t} \\
&= \mathbf{1}'_1 F \tilde{x}_{t|t} \\
&= \mathbf{1}'_1 F (\tilde{x}_{t|t-1} + K_{t|t-1}(H(\tilde{x}_t - \tilde{x}_{t|t-1}) + e_t)),
\end{aligned} \tag{B19}$$

and  $(z_{t+1|t} - z_{t+1|t-1})$  is shown in (B6).

Using these expressions, we get

$$\begin{aligned}
p_{\pi,i,t+2} &= \mathbf{E} [(\pi_{t+2|t+1} - \pi_{t+2|t-1})(i_{t+1} - i_{t+1|t})|\Omega_t] \\
&= \mathbf{1}'_2 [H(I + F(I + K_{t+1|t}HF + (F - K_{t+1|t}HF)K_{t|t-1}H))]P_{\tilde{x},t|t-1} \\
&\quad (\beta_{\pi,t|t}H(I + FK_{t|t-1}H) + FK_{t|t-1}H\beta_{z,t|t})' \mathbf{1}_2 \\
&\quad + \mathbf{1}'_2 HF(I + K_{t+1|t}H)FK_{t|t-1}\Sigma_Y [HFK_{t|t-1}\beta_{\pi,t|t} + FK_{t|t-1}\beta_{z,T|t}]' \mathbf{1}_2,
\end{aligned} \tag{B20}$$

and

$$\begin{aligned}
p_{i,z,t+2} &= \mathbf{E} \left[ (z_{t+2|t+1} - z_{t+2|t-1})(i_{t+1} - i_{t+1|t}) | \Omega_t \right] \\
&= \mathbf{1}'_1 F \left[ K_{t+1|t} H F + (I + K_{t+1|t} H) F K_{t|t-1} H \right] P_{\tilde{x},t|t-1} \\
&\quad \left[ H(I + F K_{t|t-1} H) \beta_{\pi,t|t} + F K_{t|t-1} H \beta_{z,t|t} \right]' \mathbf{1}_1 \\
&\quad + \mathbf{1}'_1 F (F + K_{t+1|t} H F) K_{t|t-1} \Sigma_Y \left[ H F K_{t|t-1} \beta_{\pi,t|t} + F K_{t|t-1} \beta_{z,t|t} \right] \mathbf{1}_1.
\end{aligned} \tag{B21}$$

Finally,  $p_{i,i} = \mathbf{E} \left[ (i_{t+1|t} - \hat{i}_{t+1|t-1})^2 | \Omega_t \right]$  is known from the one-step-ahead forecast uncertainty.

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Figure 1: Real GDP and Potential GDP

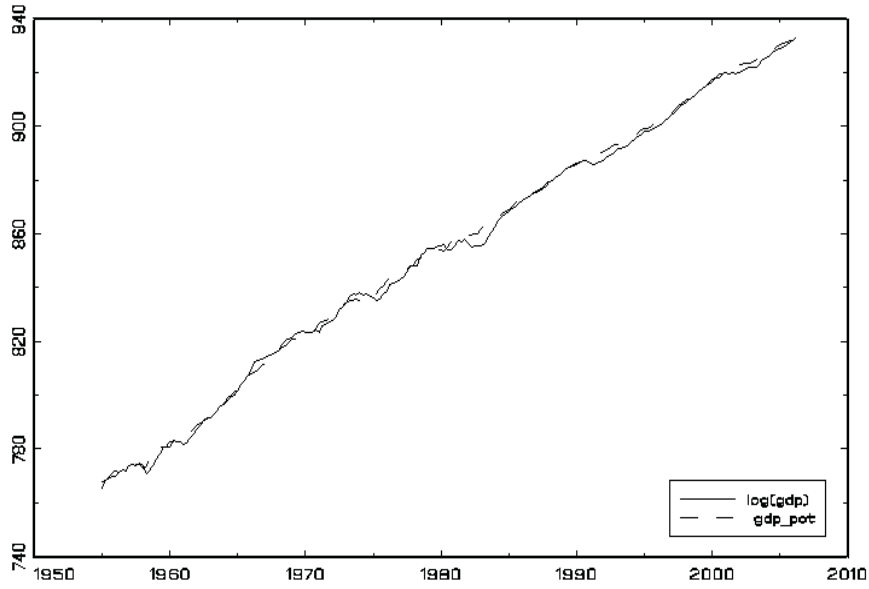


Figure 2: Output gap with 90% confidence bands

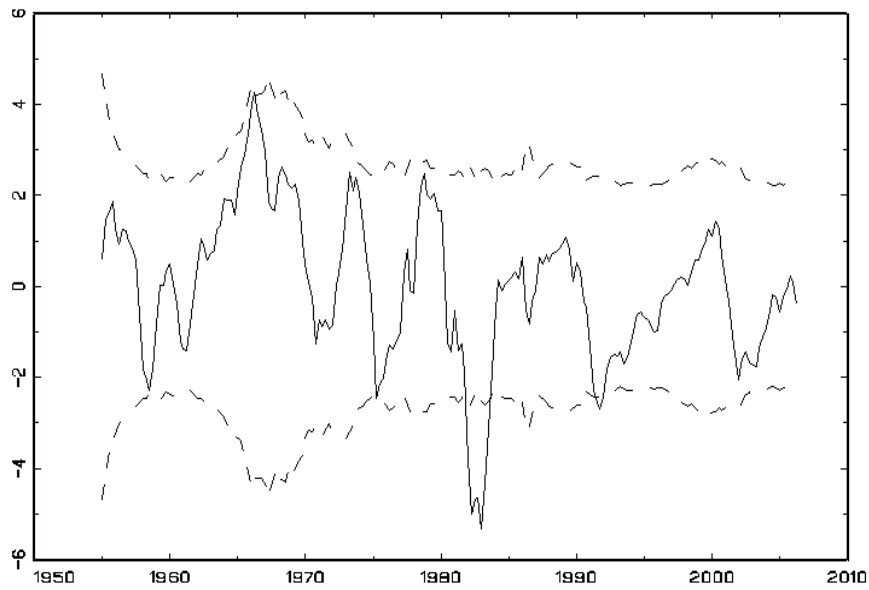


Figure 3: Actual and estimated inflation

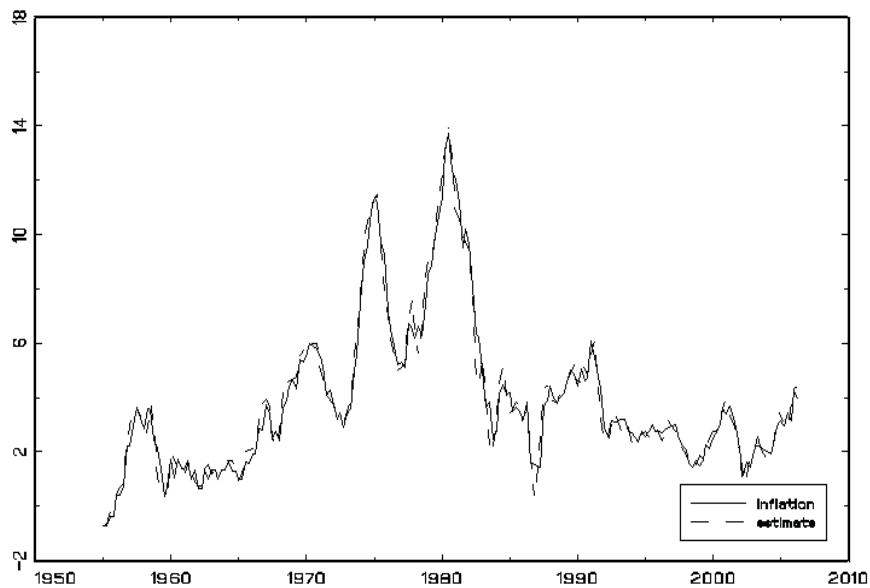


Figure 4: Actual and fitted Federal Funds Rate

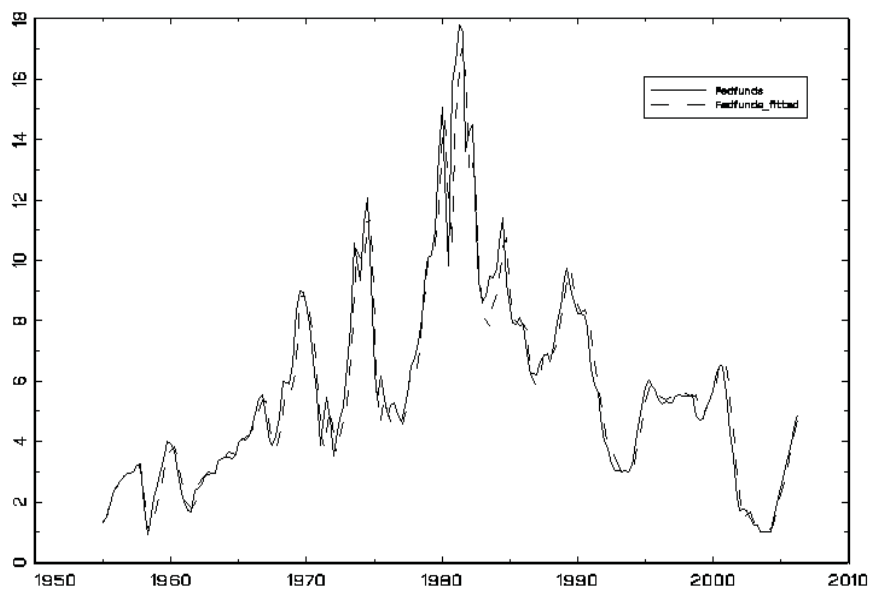


Figure 5: One-sided coefficient estimates

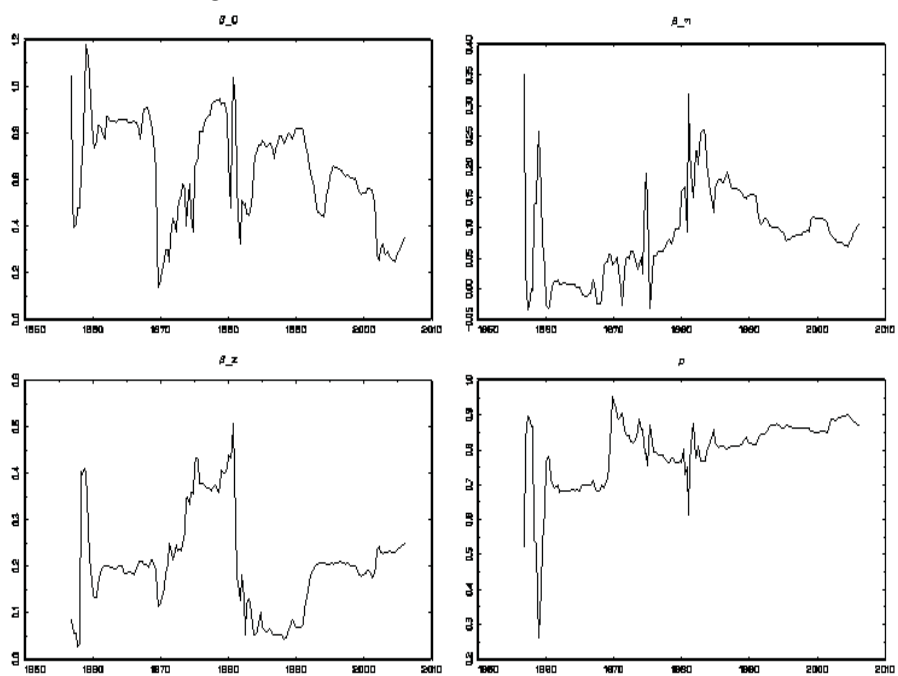


Figure 6: Federal Funds Rate forecasts and forecast errors

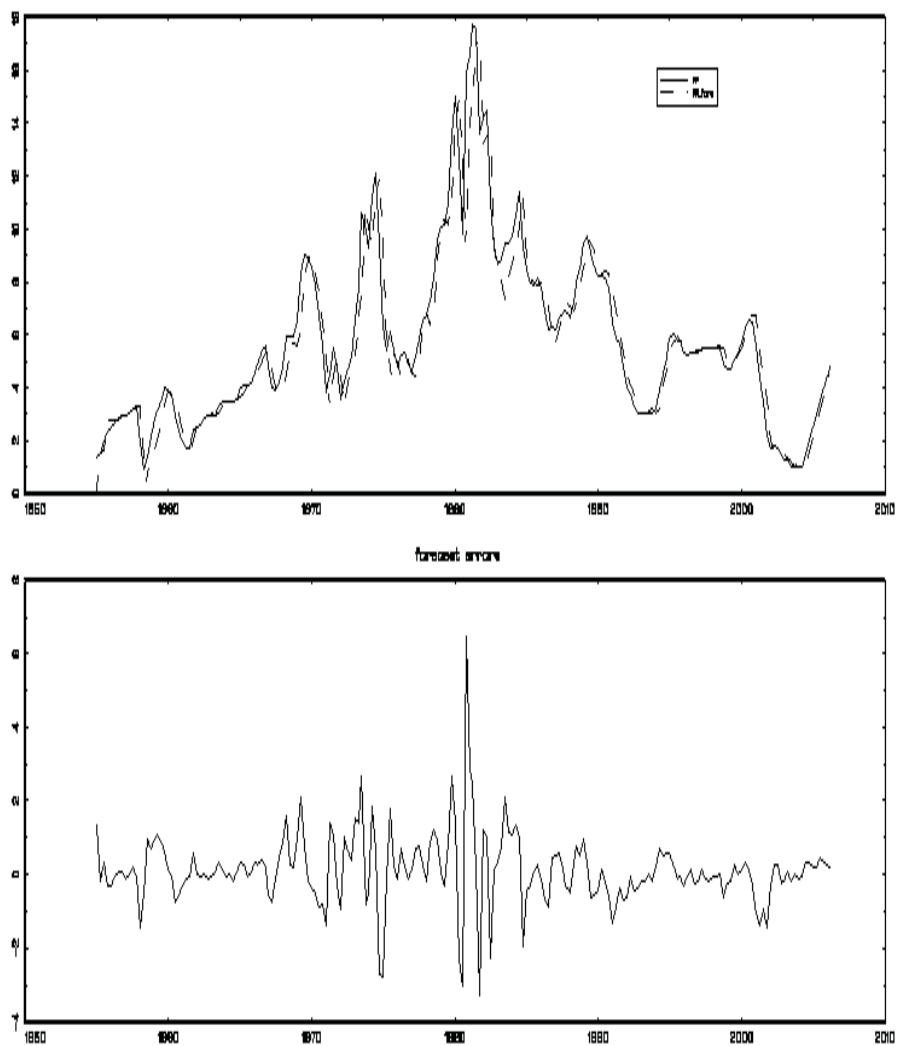


Figure 7: Uncertainty about one-quarter ahead Federal Funds Rate

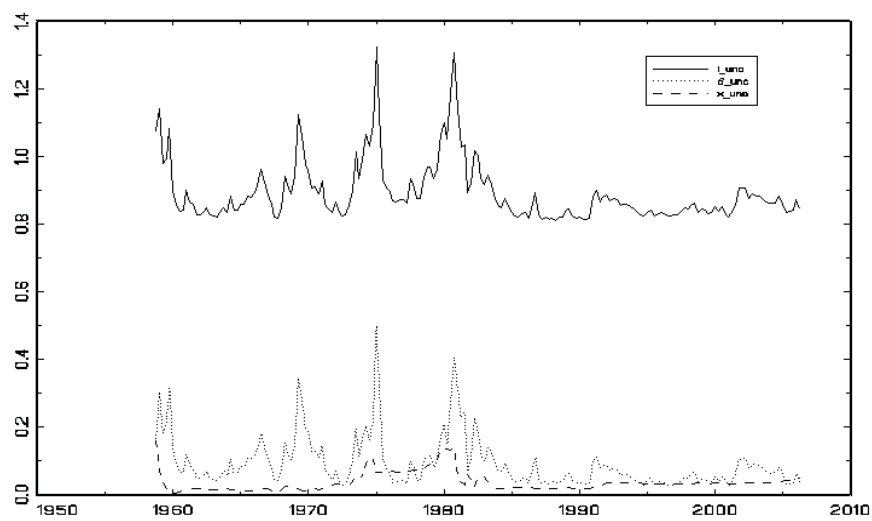


Figure 8: Federal Funds Rate forecasts and forecast errors (2 quarters)

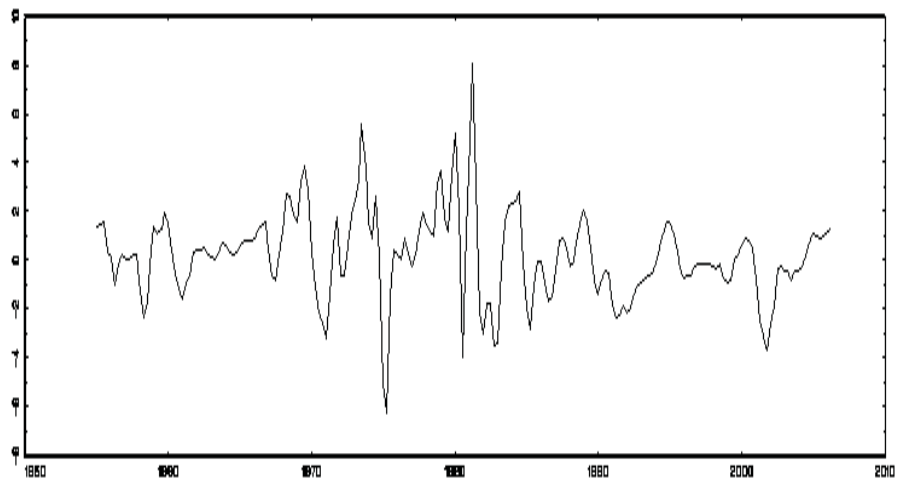
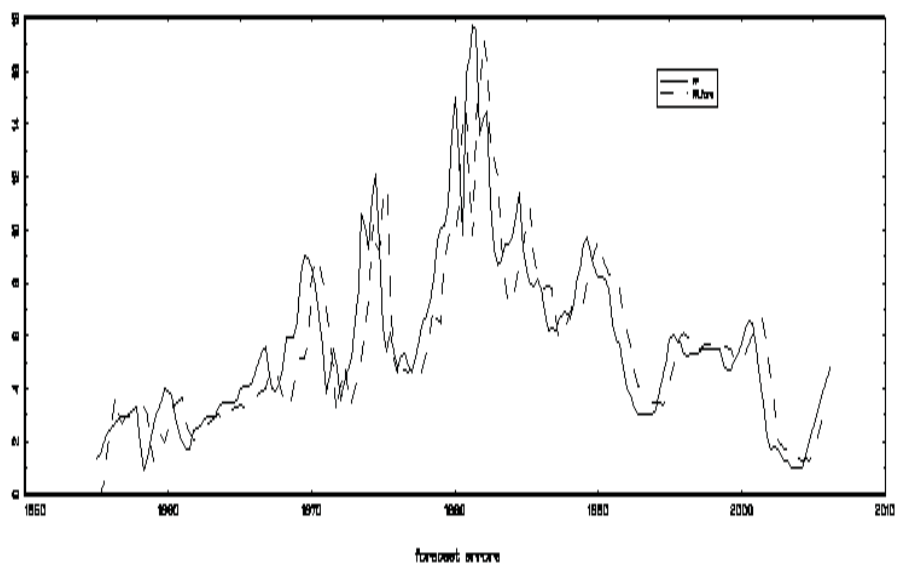


Figure 9: Uncertainty about two-quarter ahead Federal Funds Rate

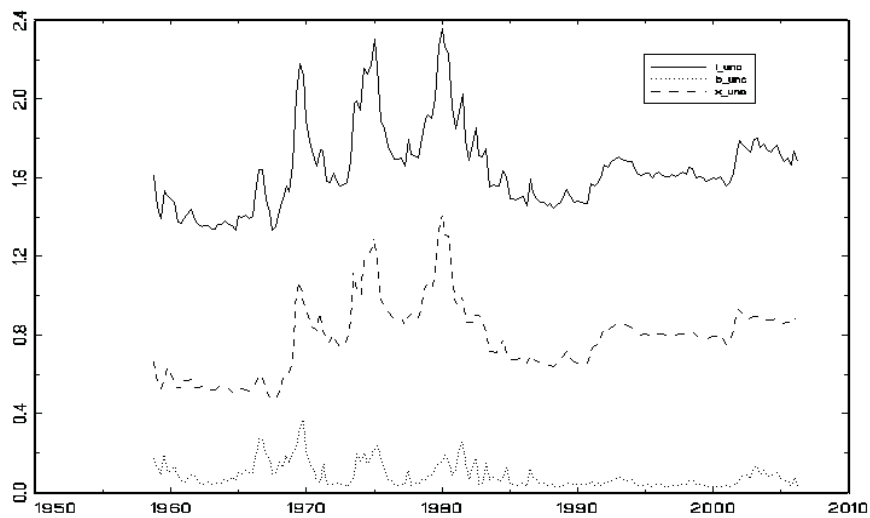


Figure 10: Uncertainty about one-quarter ahead Federal Funds Rate (heteroskedastic shocks)

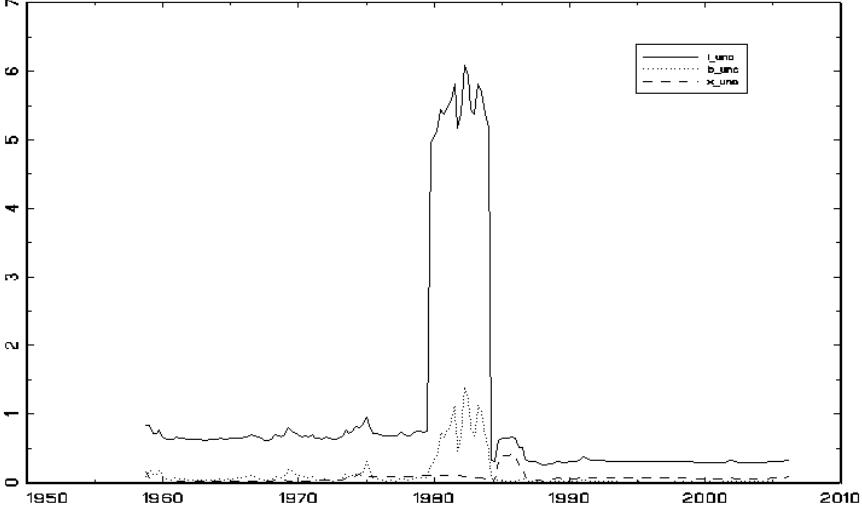


Figure 11: Uncertainty about one-quarter ahead Federal Funds Rate in subperiods(heteroskedastic shocks)

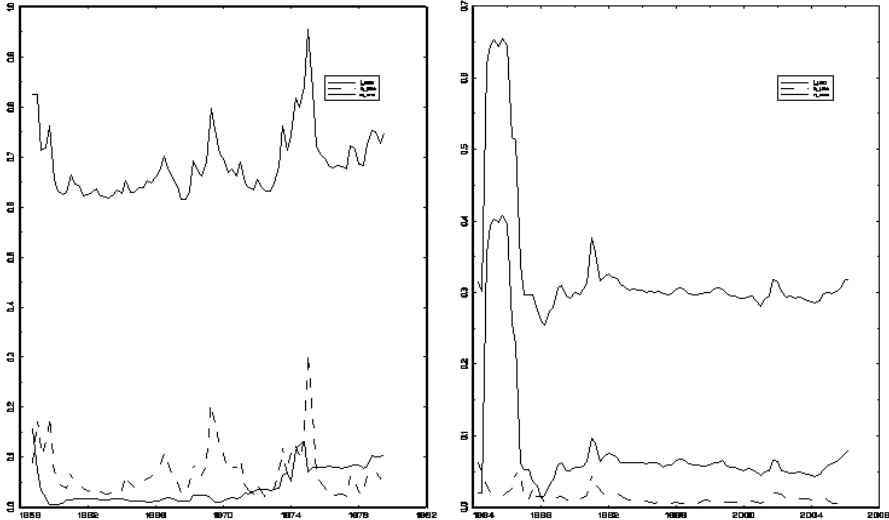


Figure 12: Uncertainty about one-quarter ahead Federal Funds Rate (heteroskedastic shocks)

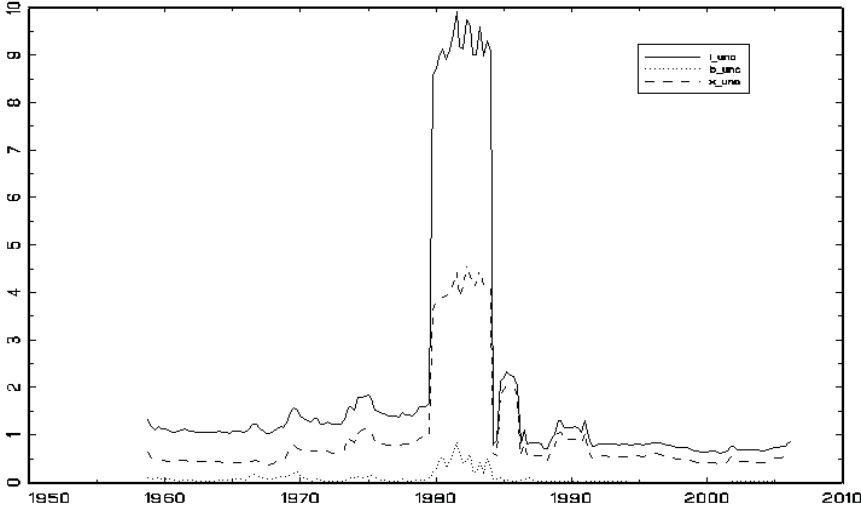


Figure 13: Uncertainty about one-quarter ahead Federal Funds Rate in subperiods(heteroskedastic shocks)

