

An Affine Macro-Finance Term Structure Model for the Euro Area

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First draft: November 13, 2006

Preliminary and incomplete, comments welcome

Abstract

A joint model of macroeconomic and term structure dynamics is specified and estimated for the euro area. The model comprises a backward-looking Phillips curve, a dynamic aggregate demand equation, a monetary policy rule and a specification of the dynamics of trend growth and the natural real rate of interest. Given the linear dynamics of the macroeconomic state variables, bonds are priced under the assumption of no arbitrage. Yields of all maturities are affine functions of the state vector. The estimated model is used for policy experiments: the responses to a cost-push shock, a shock to the output gap and a monetary policy shock are in general stronger for short-term rates than for long-term yields. The response to a shock to potential output growth is different in nature, though: for the first three years after the shock, the response is stronger the longer the time to maturity. For all shocks, impulse responses of bond yields are fairly persistent, which reflects the persistence of their macroeconomic driving forces.

Keywords: affine term structure models, monetary policy, euro area

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1 Introduction

Starting from the seminal contributions of Vasiček (1977) and Cox, Ingersoll, and Ross (1985), there is a large and growing literature that tries to explore the nature of the dynamics of interest rates and the relationship of yields with different maturities in an arbitrage-free framework. Within this literature, models from the affine class have become particularly prominent.¹ These models are characterized by a solution that expresses bond yields as an affine function of the state vector. This property follows from assuming that state dynamics are linear, that innovations are Gaussian and that the pricing kernel is a linear function of the state vector.

In the empirical finance literature, the state vector driving the maturity spectrum of bond yields has usually been assumed to be some low-dimensional vector of latent factors. Empirically, these factors could then be interpreted as level, slope or curvature according to how they impact on different maturity ranges of the term structure. Affine multifactor models with latent factors may be used for determining arbitrage-free bond prices, for pricing interest-rate derivatives, and for forecasting.

In these models, bond yields are essentially explained by bond yields themselves.² From an economic perspective, however, the macroeconomic factors that stand behind the dynamics of short and long-term interest rates are of interest. In order to establish this link between interest rates and key macroeconomic variables, a recent strand of the literature combines the principle of arbitrage-free valuation with elements from dynamic macro models. Most of these combined approaches are nested within the class of affine multifactor models. In contrast to the finance literature, however, some or all of the factors are no longer unspecified, but rather identified as macroeconomic variables such as inflation or real activity. These macro-finance models then allow to shed light on the ultimate macroeconomic sources of risk, and they make it possible to assess the impact of macroeconomic shocks on bond yields of any maturity.

The fast-growing number of affine term structure models with macroeconomic variables in the literature divides - amongst other things - along the line of how the 'macroeconomy' is specified. One strand of the literature starts from a reduced form VAR representation of the macroeconomic data, excluding interest rate themselves. The VAR is linked to the term structure by a Taylor-type monetary policy rule: movements in the short term interest rate are traced back to movements in inflation, a real activity component, and some unobservable components, see, e.g., Ang and Piazzesi (2003), Fendel (2004) or Ang, Dong, and Piazzesi (2005). Some models are augmented with a law of motion for the inflation

¹See Duffie and Kan (1996) and Dai and Singleton (2000).

²See, e.g., Babbs and Nowman (1998), Cassola and Luis (2003), Duan and Simonato (1999) or de Jong (2000) for empirical applications that estimate the latent factor process from a panel of observed bond yields.

target (Lildholdt, Panigirtzoglou, and Peacock (2005)) or long-run macroeconomic expectations (Dewachter and Lyrio (2006)). Other papers utilize a more structural macroeconomic framework (Bekaert, Cho, and Moreno (2005), Hördahl, Tristani, and Vestin (2006), Hördahl, Tristani, and Vestin (2005a), Hördahl, Tristani, and Vestin (2005b) or Rudebusch and Wu (2004)), some of them incorporating elements of equilibrium models with rational expectations.

The macroeconomic model underlying the term structure dynamics in this paper follows the lines of Laubach and Williams (2003) and Mesonnier and Renne (2004).³ Its core elements are a 'backward-looking' Phillips curve and aggregate demand (IS) equation. Monetary policy is represented by a Taylor-type rule that allows for interest-rate smoothing and persistent policy shocks. The model also incorporates a specification of potential output growth and the natural real rate of interest. This allows to analyse the impact of shocks to these key macroeconomic variables which is not accounted for in most other papers of the macro-finance term structure literature.

The model is estimated on the basis of artificial macroeconomic euro area data from 1981 to 2006, bond yields enter the econometric model as of 1998. To my knowledge, the only other paper that explores the joint dynamics of the macroeconomy and the term structure in the euro area is Hördahl et al. (2005a). However, their model - which is smaller and comprises rational expectations - differs from the one considered here, and they use a different estimation period (1991 - 2004).

The fit of the model to observed yields and macro variables turns out to be fairly good, so it can be used for policy analysis. The high persistence of the macroeconomic variables is mirrored in the impulse responses of bond yields to macroeconomic shocks. This is particularly noticeable for a shock to the trend growth rate of potential output which has a strong and long-lasting effect on all yields. For this shock, it is long-term rates that react most strongly on impact. The other shocks (inflation, output gap, monetary policy), in contrast, affect short-term rates more strongly than long-term yields. However, since the initial response at the short end of the yield curve may be quite dynamic, longer-term yields can react more strongly than the one-year rate during the first few quarters after the shock.

The remainder of the paper is structured as follows. Section 2 outlines the set up of the macro model and - based on that- derives arbitrage-free term structure dynamics. Section 3 shows how to bring the theoretical model into state space form and describes the estimation approach as well as the data. Parameter estimates, the fit of the model and impulse responses are discussed in section 4, the last section concludes and gives an outlook on possible extensions and refinements.

³Note that these papers do not consider term structure implications.

2 The Model

2.1 The Macroeconomic Module

This subsection introduces a small structural macroeconomic model, containing inflation, the output gap, the one-period nominal and real interest rate, the natural real rate of interest, and potential output growth. In the next subsection, arbitrage-free bond yields will then be computed as functions of these macroeconomic state variables.

The model that I use is based on Mesonnier and Renne (2004) (MR), who employ it for estimating the natural real rate of interest in the euro area. Their specification can in turn be interpreted as a modification of the models by Rudebusch and Svensson (1999) and Laubach and Williams (2003). The MR model consists of a dynamic supply schedule (backward-looking Phillips curve), a dynamic demand specification (backward-looking IS equation) and the joint dynamics of potential output growth and the natural real rate of interest. These are represented by the following equations, the time frequency is quarterly.

$$\pi_{t+1} = c_\pi + \alpha_1 \pi_t + \alpha_2 \pi_{t-1} + \alpha_3 \pi_{t-2} + \beta z_t + \epsilon_{t+1}^\pi \quad (2.1)$$

$$z_{t+1} = \psi_z z_t + (1 + L)\gamma(i_t - \pi_{t+1|1} - r_t^*) + \epsilon_{t+1}^z \quad (2.2)$$

$$r_t^* = c_r + \theta_r a_t \quad (2.3)$$

$$\Delta y_t^* = c_y + \theta_y a_t + \epsilon_t^y \quad (2.4)$$

$$a_{t+1} = \psi_a a_t + \epsilon_{t+1}^a \quad (2.5)$$

$$y_t = y_t^* + z_t \quad (2.6)$$

The Phillips curve equation (2.1) relates current inflation π to its own lags and the previous period's output gap z . The latter is defined in (2.6) as the difference between log actual output y and log potential output y^* . Inflation can also be affected by idiosyncratic, serially uncorrelated cost-push shocks ϵ^π . Unlike MR, it will not be assumed that the α_i in (2.1) sum to unity, but rather that their sum is smaller than one. Thus, since the output gap z should be zero on average, I have to include the constant c_π to allow the unconditional expectation of inflation to differ from zero.

The IS equation (2.2) describes the dynamics of the output gap. Besides depending on the last quarter's output gap and idiosyncratic demand shocks ϵ^z , it is linked to $(i_t - \pi_{t+1|1} - r_t^*)$ and its lag.⁴ The expression $i_t - \pi_{t+1|1}$ represents the model-consistent (ex-ante) real interest rate, i.e. the difference between the nominal one-quarter interest rate i_t and the one-step-ahead expectation of inflation $\pi_{t+1|t} \equiv E_t(\pi_{t+1})$. The variable r_t^* is the natural, neutral or equilibrium real interest rate (NRI). The notion of a natural real interest rate goes back to Wicksell (1898) and has gained revived prominence in the literature

⁴ L is the lag-operator.

of New-Keynesian models.⁵ In these models, that are characterized by price rigidities, the NRI represents the real rate in the hypothetical equilibrium with perfectly flexible prices. The NRI is a function of real shocks and represents an important benchmark for monetary policy. Real rates exceeding the NRI represent a contractionary monetary policy stance, whereas a real interest rate below the NRI stands for an expansionary stance. This property carries over to the - not explicitly microfounded - model considered here. When the real rate is below (above) the NRI, the negative (positive) real-rate gap ($i_t - \pi_{t+1|1} - r_t^*$) stimulates (decreases) demand⁶ and - ceteris paribus - increases (decreases) inflation via the Phillips curve.

In a hypothetical world without additional demand and cost-push shocks, monetary policy could steer nominal rates in a way that equalizes the actual real rate to its natural counterpart and would thus permanently stabilize output gap and inflation fluctuations. However, the presence of idiosyncratic shocks implies that the task of monetary policy is not that trivial. Shocks to the NRI and idiosyncratic supply or demand shocks occur simultaneously, all exerting pressures on inflation and the output gap, that may differ in size, direction and persistence, thereby creating a trade-off for monetary policy.

In line with its definition, the NRI is assumed to share a common trend with potential output. Moreover, consistent with a standard Ramsey-type growth model, the steady state of the NRI should be a function of the steady state of potential-output growth (as well as the intertemporal elasticity of substitution in consumption and the time preference of households). This is reflected in equations (2.3) and (2.4). The NRI r_t^* and potential output growth Δy_t^* share a common persistent component a_t , the dynamics of which is given by (2.5). In the following, a_t will be referred to as the trend growth rate. The additional transitory shock ϵ^y is specific to potential output growth; NRI-specific shocks are also conceivable, but I will follow MR and abstract from those: as a_t , r_t^* , Δy_t^* are all unobservable, with specification (2.3) - (2.4) it is already hard to distinguish statistically between the persistent component a_t and the transitory ϵ_t^y . The problem would be aggravated by including an additional NRI-shock.⁷ Finally, the steady state values⁸ of the NRI and potential output growth are given by c_r and c_y , respectively.

Unlike MR who treat the short-term nominal interest rate as exogenous, I close the

⁵See Woodford (2003). See, e.g., Amato (2005) for a discussion of the concept of the NRI.

⁶Note that the parameter γ is typically negative.

⁷The main thing to note is that the current specification is sufficient to make sure that while sharing the common trend a_t , the NRI and potential output growth are not perfectly correlated with each other. The variance of ϵ^y determines the covariance of the two variables. Moreover, one can show that there is an observationally equivalent specification that allows the NRI to have an idiosyncratic component, while potential output growth features none.

⁸Here and in the following, the notion of a steady state refers to the situation in which all shocks are zero. Since the considered model is linear, the steady state of a variable coincides with its unconditional expectation.

model with a monetary policy rule of the following form:

$$i_t = \phi_i i_{t-1} + (1 - \phi_i)(c_i + \phi_\pi \pi_t + \phi_g \Delta y_t) + \nu_t. \quad (2.7)$$

The form of this reaction function is fairly common in the literature. The current policy rate is a convex combination of a target interest rate

$$i_t^* = c_i + \phi_\pi \pi_t + \phi_g \Delta y_t$$

and the previous period's rate i_{t-1} . The monetary policy shock ν_t captures influences on the short rate that are independent of the systematic components i_{t-1} and i_t^* .

The target interest rate i_t^* is a linear function of contemporaneous inflation π_t and output growth Δy_t . This particular measure of real activity is also used in the monetary policy rules in Ang et al. (2005). However, in most specifications in the literature some sort of output gap is used instead. To do the same here in a way that is model-consistent, I would either have to assume that the policy maker in fact observes z_t or that he uses an estimate of it. For instance, if one supposes that the central bank knows the true model (2.1) - (2.6), it could compute the conditional expectation of z_t based on observed current and past inflation, interest rates and output growth. In order to keep the model simple, however, I abstract from those considerations and will stick to the specification (2.7) which has the advantage that the central bank reacts to observable variables only.

The monetary policy shock in (2.7) is allowed to be persistent as well,

$$\nu_t = \psi_\nu \nu_{t-1} + \epsilon_t^\nu. \quad (2.8)$$

This is motivated by the observation that the level of the short-term interest rate i_t is highly persistent⁹, and the persistence inherited from inflation and real activity is not sufficient to fully capture that: regressing i_t on π_t and Δy_t would generate residuals with strong remaining serial correlation. However, it is a priori not clear how to appropriately account for the high persistence. Setting ϕ_i in (2.7) equal to zero, all persistence would have to be captured by ψ_ν in (2.8), implying that it is monetary policy shocks themselves that are persistent. Constraining instead ν_t to be white noise, persistence would have to be attributed fully to interest-rate smoothing by the central bank. The question of how to 'distribute' persistence of i_t to interest-rate smoothing and policy shocks lies at the heart of the discussion about 'monetary policy gradualism'.¹⁰ I try to be as agnostic as possible about it and let the data decide. It will turn out that both ψ_ν and ϕ_i can be estimated with satisfying precision.

As it stands, (2.7) implicitly assumes a constant inflation and growth objective as one may rewrite (2.7) as

$$i_t = \phi_i i_{t-1} + (1 - \phi_i)[\tilde{c}_i + \phi_\pi (\pi_t - \pi^*) + \phi_g (\Delta y_t - (\Delta y)^*)] + \nu_t$$

⁹The first-order autocorrelation is about 0.97.

¹⁰See, e.g., Rudebusch (2002), Gerlach-Kristen (2004) or Rudebusch (2005).

where π^* and $(\Delta y)^*$ represent the inflation and output growth target. In principle, it is preferable to have both objectives to be time-varying. However, with the term structure application in view, this would require to formulate a complete law of motion of these time-varying objectives. Under the no-arbitrage condition, any long-term bond yield is a risk-adjusted expectation of the average of future short rates. Thus, in order to compute this expectation consistent with the model, the dynamics of the short rate have to be fully specified. Since these depend via the monetary policy rule on the inflation and the growth target, one would have to specify the dynamics of those as well. As in Hördahl et al. (2006) I have tried to model the inflation target as a (near-)random walk, which, however did not lead to satisfactory results.¹¹ Hence, I will stick to the rule (2.7) - (2.8) that abstracts from time-varying targets. That this might be a reasonable choice is confirmed by the residuals of the estimated policy rule that show no signs of misspecification. However, I cannot rule out that time variation in the inflation or growth objective - that I do not explicitly account for - is picked up by monetary policy shocks, which in turn drives up their estimated persistence.

The model is completed by stipulating that the five shocks are contemporaneously uncorrelated. Moreover, for pricing bonds and for estimating the model, it will be assumed that they are all normally distributed. Hence for the vector $\epsilon_t = (\epsilon_t^\pi, \epsilon_t^a, \epsilon_t^z, \epsilon_t^y, \epsilon_t^\nu)$,

$$\epsilon_t \sim N(0, Q), \quad \text{with } Q = \text{diag}(\sigma_\pi^2, \sigma_a^2, \sigma_z^2, \sigma_y^2, \sigma_\nu^2), \quad (2.9)$$

where the σ_i denote the standard deviations of the respective shocks, and $\text{diag}(x)$ denotes a square matrix with the vector x building the main diagonal and zeros elsewhere.

The structure of the system (2.1) - (2.8) allows for a convenient Markovian representation of the model, that will be useful when employing it below for pricing bonds. Define the 12×1 -vector X_t as

$$X_t = (\pi_t, \pi_{t-1}, \pi_{t-2}, \pi_{t-3}, g_t, i_t, i_{t-1}, a_t, a_{t-1}, z_t, z_{t-1}, \nu_t)'$$

where here and in the following $g_t \equiv \Delta y_t$ for notational convenience. Then one can write (2.1) - (2.8) as

$$\mathcal{K}_0 X_t = c_0 + \mathcal{K}_1 X_{t-1} + R_0 \epsilon_t,$$

where \mathcal{K}_0 and \mathcal{K}_1 are 12×12 , c is 12×1 , and R is 12×5 .¹² The matrix \mathcal{K}_0 is not diagonal, since the monetary policy rule and the presence of expected inflation in (2.2) imply contemporaneous relationships between the elements of X_t . However, the equation can be multiplied through by the inverse of \mathcal{K}_0 to obtain

$$X_t = c + \mathcal{K} X_{t-1} + R \epsilon_t, \quad (2.10)$$

with $\mathcal{K} = \mathcal{K}_0^{-1} \mathcal{K}_1$, $c = \mathcal{K}_0^{-1} c_0$ and $R = \mathcal{K}_0^{-1} R_0$.

¹¹Maybe this could be attributed to the particular dynamics of inflation within the relatively short period since 1981, with a distinct downward trend at the beginning and a rather 'flat' evolution since about 1999, see figure 1.

¹²[To be completed: putting the detailed structure of the system matrices into the appendix.]

2.2 Pricing Long-Term Bonds

Taking the structural macroeconomic model, compactly represented by the SVAR(1) (2.10), as a basis, I will now derive arbitrage-free prices of long-term bonds. Let P_t^n denote the time t price of a pure discount bond paying one unit of account at time $t + n$ with certainty. Then the family of bond price processes is arbitrage-free if and only if there exists a sequence of strictly positive random variables $\{M_t\}$ such that

$$P_t^n = E_t(M_{t+1}P_{t+1}^{n-1}), \quad (2.11)$$

for all t and n .¹³ The random variable M_t is called the stochastic discount factor (SDF) or pricing kernel. Bond prices are related to yields y_t^n via

$$y_t^n = -\frac{1}{n} \ln P_t^n. \quad (2.12)$$

The joint macro-finance model will belong to the affine class of term structure models.¹⁴ Discrete-time models from this family are characterized by four components: first, the short-term interest rate is an affine function of factors; second, the evolution of the factor vector is a linear autoregressive process; third, market prices of risk are affine functions of the factors; and fourth, there is a pricing kernel which is an exponentially-affine function of the short rate and 'priced' factor innovations.

Here, the factor vector is given by X_t and the short rate is a particularly simple transformation, namely

$$i_t = \delta' X_t, \quad (2.13)$$

where δ is a 12×1 -vector with a one on the fifth position, that picks i_t from X_t , and zeros elsewhere. The factor process is given by (2.10) which is rewritten here slightly using a normalization of shock variances

$$X_t = c + \mathcal{K}X_{t-1} + \Sigma v_t, \quad v_t \sim N(0, I_5) \quad (2.14)$$

i.e. $\Sigma = RQ^{0.5}$, and I_5 denotes the 5×5 -identity matrix.

The market price of risk vector λ_t is also an affine function of X_t ,

$$\lambda_t = \lambda_0 + \lambda_1 X_t, \quad (2.15)$$

where λ_0 and λ_1 are a vector and a matrix of appropriate dimensions.

Finally, the pricing kernel is an exponential-affine function of the vector of factors and its innovations,

$$M_{t+1} = \exp(-0.5\lambda_t' \lambda_t - i_t - \lambda_t' v_{t+1}). \quad (2.16)$$

¹³See Irlle (1998) for a more rigorous statement and a proof of the equivalence.

¹⁴Cf. Duffie and Kan (1996) and Dai and Singleton (2000). See Backus, Foresi, and Telmer (1998) for an introduction to the discrete-time version.

Solving (2.11) given the specified dynamics of the pricing kernel, leads to a solution function mapping the factor vector into bond prices,

$$P_t^n = \exp\left(\tilde{A}_n + \tilde{B}'_n X_t\right), \quad (2.17)$$

where \tilde{A}_n and \tilde{B}_n satisfy the difference equations

$$\tilde{A}_{n+1} = \tilde{A}_n - \tilde{B}'_n \Sigma \lambda_0 + \frac{1}{2} \tilde{B}'_n \Sigma \Sigma' \tilde{B}_n \quad (2.18)$$

$$\tilde{B}'_{n+1} = \tilde{B}'_n (\mathcal{K} - \Sigma \lambda_1) - \delta', \quad (2.19)$$

with initial condition $\tilde{A}_0 = 0$ and $\tilde{B}_0 = 0$.

The exponential-affine form for bond prices in (2.17) implies that continuously compounded yields are affine functions of the state vector X_t ,

$$y_t^n = A_n + B'_n X_t \quad (2.20)$$

with $A_n = -\tilde{A}_n/n$ and $B_n = -\tilde{B}_n/n$. Note that for the one-period interest rate y_t^1

$$y_t^1 = \delta' X_t = i_t \quad (2.21)$$

as expected.

For models of the affine class, risk premia of various types are also affine functions of the factor vector. One-quarter forward premia frp_t^n , being defined as the difference between the implied forward rate and the expected one-quarter spot rate, are defined as

$$\begin{aligned} frp_t^n &= f_{n,t} - E_t(y_{t+n}^1) \\ &= \ln P_t^n - \ln P_t^{n+1} - E_t(y_{t+n}^1) \\ &= -nA_n + (n+1)A_{n+1} - A_1 \\ &\quad (-nB'_n + (n+1)B'_{n+1} - B'_1 \mathcal{K}^n) X_t. \end{aligned} \quad (2.22)$$

Yield risk premia (or term premia) yp_t^n are given by the average of forward premia

$$\begin{aligned} yp_t^n &= \frac{1}{n} \sum_{i=1}^{n-1} frp_t^i \\ &= A_n + B'_n X_t - A_1 - B'_1 (I_d - \mathcal{K}^n) (I_d - \mathcal{K})^{-1} X_t. \end{aligned} \quad (2.23)$$

It can be shown that the latter expression is just equal to the difference between the n -period yield at time t and the average of expected future one-period rates, i.e. the hypothetical yield that would prevail under the condition of the pure expectations hypothesis.¹⁵

¹⁵Cf. Cochrane (2001).

3 Data and Estimation Approach

3.1 Macroeconomic and Bond Yield Data

Since the beginning of stage three of European Monetary Union (EMU) in 1999, 30 quarters have elapsed by now. Hence, estimating models for the euro area with quarterly data still requires compromises of some sort. One may either stick to a relatively short sample period by not taking too many data points before 1999 into account, or one has to rely on artificial euro area data. The approach chosen here will be a mixture of these two possibilities.

As macroeconomic data, I will employ inflation, output growth and the short-term interest rate. An empirical proxy for the output gap will not be used, instead z_t is kept as a latent variable in the model. The data are quarterly, cover the period 1981Q2 - 2006Q2 and come from the database of the Area Wide Model (AWM).¹⁶ These are artificial euro area data that have by now been utilized in several empirical studies. The data set is updated until 2006Q2 by Bundesbank staff. Inflation, π_t , is hundred times the annualized quarter-to-quarter change of the seasonally adjusted log HICP, output growth Δy_t is hundred times the quarter-to-quarter change (not annualized) of seasonally adjusted log real GDP. The interest rate i_t is a monthly average of the three-month money market rate.

For bond yields, the Statistical Data Warehouse of the ECB¹⁷ provides artificial two-, three-, five-, seven- and ten-year government bond yields for the euro area. In principal, these could be used for estimation. However, this would not really be consistent with the model set up. The artificial yields are weighted averages of the euro area member country yields, thus, the here supposed no-arbitrage relation is unlikely to hold between those yields. Consequently, yield data will only be employed as of 1998. From 1999 on, these are zero-coupon swap rates from Bloomberg with maturities of one, two, three, five, seven, and ten years. For the year 1998 for which these data had not been available, I use the corresponding yields for Germany. All data are shown in figure 1. The different sample periods for macro- and yield-data can be adequately accounted for within the state space framework as explained in the following.

3.2 State Space Form

For estimation, the model is brought into state space form.¹⁸ The state vector is given by X_t , the transition equation is (2.10). The measurement vector until $T^* = 1997Q4$ contains

¹⁶See Fagan, Henry, and Mestre (2001).

¹⁷Publicly available via <http://sdw.ecb.int>.

¹⁸See Hamilton (1994) for state space models and the Kalman filter in general, and Lemke (2006) for estimating term structure models in a state space framework. Estimation and numerical computations have been conducted using GAUSS employing also its TSM and MAXLIK package.

inflation, output growth and the short rate,

$$Y_t^1 = (\pi_t, g_t, i_t),$$

hence the measurement equation for $t = 1, \dots, T^*$ is given by

$$Y_t^1 = Z_1 X_t,$$

where Z_1 is a 3×12 matrix that selects π_t , g_t and i_t from X_t . Note that the measurement equation contains no error term.

For $t = T^* + 1, \dots, T$, ($T = 2006Q2$) bond yields $y_t^{n_j}$ with maturities $(n_1, n_2, \dots, n_6)' = (4, 8, 12, 20, 28, 40)'$, measured in quarters, are also included in the measurement equation. Bond yields are related to the state vector via (2.20). Stacking these relations, one obtains

$$\begin{pmatrix} y_t^{n_1} \\ \vdots \\ y_t^{n_6} \end{pmatrix} = \begin{pmatrix} A_{n_1} \\ \vdots \\ A_{n_6} \end{pmatrix} + \begin{pmatrix} B'_{n_1} \\ \vdots \\ B'_{n_6} \end{pmatrix} X_t. \quad (3.1)$$

The right-hand side contains the model solution, i.e. arbitrage-free yields. However, since the macroeconomic factors will not be able to price bonds of all maturities perfectly, a vector of measurement errors is added to the latter relation. Written in compact notation,

$$Y_t^2 = d_2 + Z_2 X_t + \xi_t. \quad (3.2)$$

For the distribution of the vector ξ_t of measurement errors I choose the simple specification

$$\xi_t \sim N(0, h^2 I_6). \quad (3.3)$$

This is not an innocuous assumption since it implies that the difference between theoretical and observed yields has the same variance for all maturities. Alternatively, one may specify a different error variance for each maturity, which, however, would come at the cost of additional free parameters that would have to be estimated.

Summing up, the state space model consists of the transition equation (2.10) and the measurement equation with time-varying dimension

$$Y_t = d_t + Z_t X_t + w_t \quad (3.4)$$

where

$$Y_t = Y_t^1, \quad d_t = 0_{3 \times 1}, \quad Z_t = Z_1, \quad w_t = 0_{3 \times 1}, \quad \text{for } t = 1, \dots, T^*$$

and

$$Y_t' = (Y_t^{1'}, Y_t^{2'}) \quad d_t' = (0'_{3 \times 1}, d_2') \quad Z_t' = (Z_1' | Z_2'), \quad w_t' = (0'_{3 \times 1}, \xi_t'), \quad \text{for } t = T^* + 1, \dots, T.$$

Denote by $\mathcal{Y}_s = \{1, Y_1, \dots, Y_s\}$ a sequence of observations of the measurement vector. Since the model is linear and the innovation of the transition equation and the measurement error are both Gaussian, the filtering densities $p(X_t | \mathcal{Y}_t)$ as well as the prediction

densities $p(X_t|\mathcal{Y}_{t-1})$ and $p(Y_t|\mathcal{Y}_{t-1})$ are Gaussian and can be computed by the Kalman filter.

It should be noted that the state vector X_t contains elements that are included in \mathcal{Y}_{t-1} , namely inflation, output growth, interest rate and their lags. For these, the respective part of the filtering density $p(X_t|\mathcal{Y}_t)$ is degenerate as expected. For instance, $E(\pi_t|\mathcal{Y}_t) = \pi_t$ and $Var(\pi_t|\mathcal{Y}_t) = 0$. For the truly unobservable variables, z_t and a_t , the Kalman filter delivers conditional expectations and the filtering error variance is strictly positive.

Unknown model parameters, collected in a vector ψ , can be estimated by maximum likelihood. The log-likelihood is given by

$$l(\psi; \mathcal{Y}_T) = \sum_{t=1}^T \ln p(Y_t|\mathcal{Y}_{t-1}). \quad (3.5)$$

Concerning the computational procedure, the sequences of $p(Y_t|\mathcal{Y}_{t-1})$ and $p(X_t|\mathcal{Y}_t)$ until $t = T^*$ can be obtained by running the Kalman filter on the 'small' state space model in which the measurement equation contains only the three macroeconomic variables. The final filtering density, $p(X_{T^*}|\mathcal{Y}_{T^*})$ is thus the conditional density of X_{T^*} given observations of the macroeconomic variables up to this point. From $T^* + 1$ on, the 'bigger' state space model where the measurement also contains the bond yields is utilized. The density $p(X_{T^*}|\mathcal{Y}_{T^*})$ serves then to initialize the filtering process with the larger measurement vector. The first prediction density after T^* will be $p(Y_{T^*+1}|\mathcal{Y}_{T^*})$ where Y_{T^*+1} consists of the vector of macroeconomic variables and the first observation of the yield vector, whereas \mathcal{Y}_{T^*} contains only observations of inflation, output growth and the short rate.

Finally, since the transition equation is stable and time-invariant, the Kalman filter at $t = 0$ can be initialized by the unconditional mean and variance of the state vector X_t , i.e.

$$X_0 \sim N(\mu_x, V_x) \quad \text{with } \mu_x = (I_{12} - \mathcal{K})^{-1}c \quad \text{and } vec(V_x) = (I_{144} - \mathcal{K} \otimes \mathcal{K})^{-1}vec(Q).$$

3.3 Calibration and Estimation

The total number of unknown parameters is very large. The macroeconomic module alone has 22 parameters (including variances of shocks), the term structure component comes with additional parameters in λ_0 (5×1) and λ_1 (5×12) governing the market price of risk, as well as the parameter h , the standard deviation of the measurement error. Hence, I will restrict and calibrate some parameters a priori before estimating the remaining ones.

First, I set $c_y = 0.49$ and $c_r = 2.71$, which corresponds to an (annualized) potential output growth of 1.96%, and a long-run natural real interest rate of 2.71%, respectively. These values have been obtained by estimating the macro-module with the interest rate

specification switched off, they are also similar in magnitude to those obtained by Mesonnier and Renne (2004) for the sample until 2002Q4.¹⁹ The remaining constants c_π and c_i cannot be chosen independently. Having calibrated c_r and c_y , I include the Phillips-curve constant c_π in the set of parameters to be estimated. Assuming that the output gap is zero on average, $E(z_t) = 0$, equations (2.1) - (2.6) fully determine the unconditional expectations of π_t , Δy_t , and i_t . Hence, the constant c_i results as a function of these steady-state values. Second, the variance of σ_a^2 is normalized to unity in order to achieve identification. Finally, the calibration of Mesonnier and Renne (2004) is used who fix the variance ratio $\sigma_y/\sigma_z = 0.5$ and the ratio $\theta_r/\theta_y = 16$.²⁰

The remaining parameters are estimated in two steps. First, the parameters of the macroeconomic module are estimated, then these are fixed at their point estimates and the term structure parameters - market prices of risk and measurement error - are estimated. Concerning the market prices of risk, it is usually assumed that λ_1 in (2.15) is different from zero, i.e. some of the market prices - the components of λ_t - are in fact time-varying. However, since the time series of yields included in the estimation process is relatively short, it turned out that time-varying market prices of risk cannot be estimated with satisfactory precision. Thus, as Fendel (2004) and Cassola and Luis (2003) that use a much longer sample in their studies for Germany, I treat market prices of risk as constant. As for the five elements in λ_0 , estimating them all simultaneously yielded insignificant estimates, a result that is common in the literature.²¹ Since there are only five such parameters I estimate the model for each permutation of restrictions, that constrain either one or two of the elements in λ_0 to be zero. The specification ultimately used corresponds to the one with the lowest value of the Akaike criterion.

4 Results

4.1 Estimation Results

The parameter estimates of the two-step estimation procedure are given in table 1. First of all, all of the parameters appear reasonable with respect to sign and size. For those parameters that have also been estimated by Mesonnier and Renne (2004), the results can be compared. In doing so, however, one has to note three differences between their estimation and the one conducted here: first, they assume that the α_i coefficients of lagged inflation in the Phillips curve (2.1) sum to one, while I estimate them without that restriction and add a constant to that equation. Second, they treat the short-term

¹⁹They obtain $c_y = 0.52$ and $c_r = 3.1$.

²⁰See their paper for justifications of these values and robustness analyses.

²¹Ang and Piazzesi (2003) and Hördahl et al. (2006), for instance, use a heuristic iterative procedure to restrict some market-price-of-risk parameters to zero based on t-statistics.

interest rate as exogenous, while here it is endogenized. Third, their sample is from 1979Q1 - 2002Q4, while the one considered here dates from 1981Q2 - 2006Q2.

The lag parameters of inflation sum to 0.7, thus the decision to relax the unit root assumption appears reasonable.²² The autoregressive parameters of trend growth a_t and the output gap z_t are higher than in the other study. The estimates of the key transmission parameters β (impact of the output gap in the Phillips curve) and γ (impact of the real interest rate gap in the IS equation)²³ are very similar to those of Mesonnier and Renne in terms of size and estimation precision. This differs from the results by Hördahl et al. (2006) who find the respective parameters in their model to be insignificantly different from zero. However, they use monthly instead of quarterly data and the model mixes backward- and forward-looking elements, which prevents a direct comparison of the results.

The reaction parameter on inflation in the monetary policy rule is slightly exceeding unity and significant. The parameter governing the reaction to output growth is slightly greater than 2 (i.e. corresponding to about 0.5 for annualized productivity growth) but is estimated fairly imprecisely. There is a distinct degree of interest rate smoothing indicated by an estimated ϕ_i of 0.93. It is also possible to estimate the persistence of monetary policy shocks quite precisely, finding the autoregressive parameter ψ_ν to be about 0.33. As a plausibility check we estimated the policy rule also as a single equation by nonlinear least squares, specifying the error to be an AR(1). This yielded very similar results, in terms of size and precision of the estimated parameters. Point estimates of ϕ_π and ϕ_g are 1.39 and 2.20, respectively, i.e. slightly higher than the system estimates. The autoregressive parameters ψ_ν and ϕ_i are estimated as 0.93 and 0.33, respectively.

For a further heuristic check of the plausibility of the estimates, figure 3 shows the Kalman-smoothed estimate of z_t , the model-implied output gap, together with an output gap measure resulting from HP-filtering and that provided by the OECD. As already mentioned, no proxy for the output gap has been used within the estimation process. Against this background, the estimated z_t process tracks the dynamics of the two empirical measures quite well. However, there are distinct differences in levels during certain episodes; but the OECD gap and the HP-implied gap - both widely used in empirical studies - also differ from each other significantly from time to time. While the solid bold line ('Macro model') is based on Kalman smoothing that only uses the state space model with the macroeconomic variables in the measurement equation, the dashed bold line ('Macro TS model') additionally uses term structure information from 1998 on. Compared to the pure-macro case, it implies a slightly higher gap most of the time. However, the dynamics of the estimated gap do hardly change. While one may have expected a priori that the latent factor z_t may change in a peculiar fashion in order to fit long-term bond yields, the results show that its estimated evolution is not very much affected by the inclusion of

²²Also, all tests reject a unit root in inflation for the estimation period.

²³The γ here corresponds to λ in Mesonnier and Renne.

long-term interest rates in the measurement vector.

As to the term structure parameters, two of the three market-price-of-risk parameters that are estimated are significant. These parameters govern the size and maturity structure of risk premia. For the small sample since 1998, yield risk premia turn out to be very small and even slightly negative at the short end of the maturity spectrum. For instance, for maturities of one, five and ten years, I obtain yield risk premia - as defined in (2.23) - of -12, -7, and 7 basis points respectively.²⁴ Risk premia of such a small magnitude raise the question whether bonds should be rather priced under the assumption of market prices of risk being equal to zero, i.e. $\lambda_1 = 0$ $\lambda_0 = 0$ in (2.15). Using this specification, however, would markedly deteriorate the fit of bond yields. Thus, for the following analyses, λ_0 is set as provided by the ML estimates in table 1. The standard deviation of the measurement error for bond yields is precisely estimated and amounts to about 29 basis points. This is comparable to the results of HTV, who allow for maturity-dependent measurement errors that exhibit standard deviations of between 23 and 28 basis points.

As an additional measure of fit, figure 2 plots the actual yields versus model-implied yields $\hat{y}_{t|T}^n$ for selected maturities, where

$$\hat{y}_{t|T}^n = \hat{A}_n + \hat{B}_n' \hat{X}_{t|T}. \quad (4.1)$$

That is, A_n and B_n in (2.20) are replaced by their estimates (which are in turn based on the ML estimates of structural parameters) and $\hat{X}_{t|T}$ is the Kalman-smoothed estimated of the state vector.²⁵ It is worthwhile emphasizing that unlike e.g. Fendel (2004) or Ang and Piazzesi (2003), the specification in this paper does not use additional latent 'term structure factors'. Rather, bond prices are functions only of those variables that play a well-defined role within the macroeconomic model. Figure 2 shows that the dynamics of the yields are well traced by the macroeconomic factors. However, the result for the maturity of one year, in particular, suggests that an additional term structure factor or a change in the specification of the macro-module may be required to improve the model's fit.²⁶ The results of Fendel (2004) employing such a latent factor, however, show that there are also episodes of persistent deviations of model-implied yield from observed ones. Unfortunately, Hördahl et al. (2006), Ang and Piazzesi (2003) and most other macro-finance papers on the term structure do not show comparable graphs.

Figure 4 shows the mean yield curve implied by the model (line) and the average of the corresponding yields from the data (circles). The model-implied mean yield curve is the average of the yields as computed in (4.1). The figure reveals that average yields are fitted well along the whole maturity spectrum.²⁷

²⁴Experimenting with time-varying market prices of risk showed that the ten-year premium fluctuates between -8 and 20 basis points.

²⁵As mentioned above, the smoothing sets those elements of the state vector which are observable - i.e. inflation, output growth, the interest rate and their lags - automatically equal to their observed values.

²⁶This is also reflected in one-step-ahead forecast errors which show some remaining autocorrelation.

²⁷Note that one advantage of the arbitrage-free approach to term structure modeling is the possibility

4.2 Impulse Response Analysis

The estimated macro-term-structure model can be used for various policy experiments. In the following I will show impulse responses of key macroeconomic variables and selected bond yields to the shocks of the model. As in Hördahl et al. (2006), the shocks have a direct structural interpretation. Before considering the results, it is useful to know that the estimated \mathcal{K} matrix in (2.10) contains only stable roots, but some of them come in complex conjugate pairs. This implies the familiar result in the dynamic macroeconomics literature that some of the impulse responses will not take a direct way back to zero but will rather cross the zero line once before dying out.²⁸

I will consider responses to an inflation shock ϵ^π , a shock to the trend growth rate ϵ^a , an output gap shock ϵ^z , and a monetary policy shock ϵ^ν .²⁹ Shocks via ϵ^y will not be considered, since this idiosyncratic component of potential output growth does not have a very useful interpretation: as discussed above, it mainly serves to govern the strength of the comovement of the natural rate of interest (NRI) and potential output growth.

The size of all shocks will be one percentage point, which helps to facilitate visual inspection of the different responses to a specific shock. However, for each figure I supply the estimated standard deviation of the respective shock which is meant to give a hint on the 'typical' magnitude of that shock. The exception is ϵ^a which is set to 3.472 rather than to unity, which corresponds to a shock to annualized potential output growth of one half percentage point.³⁰

Starting with a shock to inflation, figure 5, this has the initial effect of raising current inflation π_0 but also expected inflation $\pi_{1|0}$ for the next period. Abstracting for a moment from changes in the policy rate i , this decreases the real interest rate in (2.2). Since the NRI r^* is not affected by the shock, this leads to a negative real rate gap and - as γ is negative - to an increase of the output gap in the next period. As potential output growth is unaffected, actual output growth changes one-to-one with changes in the output gap.³¹ Thus, monetary policy will increase i as a response to both higher inflation and output growth. However, the interest rate response is subdued due to the strong interest rate smoothing. For the following periods, inflation will remain elevated due to its own persistence and due to positive impulses from the output gap which are themselves persistent. The latter feedback mechanism is also the reason for the lively responses of inflation in the first five quarters.

to compute yields for any maturity, and not only for those maturities that have been included in the estimation process.

²⁸[To be completed: Construct confidence intervals of impulse responses via Bootstrap.]

²⁹I do not distinguish in terminology between the monetary policy shock ν_t in (2.7) and the shock ϵ^ν to that shock in (2.8).

³⁰See equations (2.5) and (2.4) above and note that θ_y is estimated as 0.036. Then $3.472 \cdot 0.036 \cdot 4 = 0.5$.

³¹From (2.6), $\Delta z_t = \Delta y_t - \Delta y_t^*$.

For interpreting the responses of long-term interest rates, it is simplest to think in terms of the expectations hypothesis. This is a particularly good approximation in the case considered here as risk premia are time-invariant and small. In general, the response to the inflation shock is smaller, the longer the time to maturity. However, since the one-year rate mirrors the hump-shaped response of the short rate while the longer-term rates do not, the one-year yield does not react the strongest on impact. Corresponding to the muted response of the short rate, the responses of long-term yields are also relatively small, the maximum of about 15 basis points is exhibited by the one-year rate after six quarters.

The shock to trend growth, figure 6, has a very persistent effect on the economy as ψ_a in (2.5) is estimated as 0.97. First of all, the shock increases actual output growth on impact by as much as potential output growth. Due to the lag structure of the model, the output gap does not react immediately. Moreover, the shock to trend growth increases the NRI r_t^* and thus generates a negative real-interest-rate gap in the IS equation. This in turn raises the output gap in the next period, which then feeds through to inflation, providing in turn an additional stimulus to the output gap via inflation expectations. Due to both channels that have an impact on the real-rate gap - an elevated NRI that goes back to steady state very slowly and an increase in inflation expectations - there is a strong pressure driving the output gap upwards, which in turn fuels inflation further. In order to counterbalance this process, monetary policy has to raise interest rates strongly. However, since it is constrained by the strong smoothing term in the policy rule, interest rates rise quite slowly but for a fairly prolonged time.

This slow but very persistent increase in the short rate is reflected in the reaction of longer-term yields. The lifetime of the one-year bond in period 0 only covers periods within which the short rate will not have been increased by much yet. For longer maturities, however, the expected high short rates in the future are incorporated in the bond yield. This implies that the initial effect of the shock increases with time to maturity. As time goes by, the yield spread becomes smaller, and the transition back to steady state will eventually be characterized by a yield spread which is negative.³²

As a response to an output gap shock, figure 7, actual output growth also increases, inducing the central bank to raise the short rate by $(1 - \phi_i) \cdot \phi_g$. Due to the relatively high ψ_z in the IS equation (2.2), the output gap is quite persistent and goes back to zero quite slowly. Simultaneously, an elevated output gap has its usual impact on inflation which gives again rise to an additional stimulus to the output gap via the real interest rate. In order to reduce inflation, the monetary authority increases the policy rate. However, following the prescribed rule (2.7), there is a counterbalancing effect resulting from actual output growth being slightly negative as the output gap goes back down to steady state.

³²Strictly speaking, the yield spread is below its steady state value, which may or may not imply an inverted yield curve

The response of interest rates is similar to the inflation-shock case. Again, the relative magnitude of the response is quite small. The 'S-shaped' movement in the one-year rate reflects the slight 'S-shaped' response of the short rate (which is just less clearly visible due to the different scaling).

Finally, consider the effects of a contractionary monetary policy shock in figure 8. The output gap decreases via the real-rate channel, inflation only reacts in the second period after the shock due to its reaction to the negative output gap. The fact that the interest rate increases further for two periods after the shock can be explained as follows. First, inflation has not yet reacted and does not call for an interest rate reduction. The output gap has decreased implying a decrease in actual output growth, in turn requiring a decrease in the interest rate. However, this effect on the interest rate is very small. The important impact on the short rate comes via the smoothing channel combined with the persistence of the policy shock itself: the value of slightly more than 1.2 percentage points observed for the first period after the shock is the sum of ϕ_i and ψ_ν showing up in (2.7) and (2.8), respectively.

Long-term rates are monotonic decreasing, the impact of the shock is bigger for short-term than for long-term yields. For the first three quarters after the shock, the one-year yield exhibits an increase of more than one percentage point. Again, this is a direct consequence of the described temporary upward move of the short rate.

5 Summary and Outlook

In this paper I have presented a structural model that intends to capture the joint dynamics of the term structure and the macroeconomy for the euro area. The macroeconomic module has been estimated using quarterly data from 1981 - 2006. Parameter estimates of the term structure module (market prices of risk and variance of the measurement error) have been based on bond yield observations from 1998 - 2006. The estimated dynamics of inflation, the output gap and trend growth exhibit considerable persistence. The Taylor-type monetary policy rule is characterized by strong interest rate smoothing and monetary policy shocks which are also serially correlated.

For explaining long-term interest rates, I have not used any additional term-structure specific factor, as is often done in the literature. However, the macroeconomic state variables alone turn out to give an adequate fit of bond yields for the period 1998 - 2006. Yield risk premia are estimated to be quite small (below 10 basis points). This result may be partly owed to the fact that I have assumed constant market prices of risk for estimation. However, experimenting with time-varying term premia showed similar magnitudes on average.

The estimated model is well suited for policy analyses as it can trace out the effects of

nominal and real macroeconomic shocks on both macroeconomic variables and the yield curve. The impulse responses of macroeconomic variables are reasonable and qualitatively comparable to results from similar models.³³ The persistence of macroeconomic dynamics is mirrored in the reaction of bond yields to the macroeconomic impulses. Shocks to inflation, the output gap and the short rate affect short-term rates more than long-term yields. However, this ordering can be different in the first few periods after the shock. The response to a shock of potential output growth is different in nature. For the first three years after the shock, the response is the stronger the longer the time to maturity. Thereafter, the 'term structure of impulse responses' eventually becomes inverted before the impact of the shocks dies out.

There is a number of possible modifications and extensions at the current state of this paper. First, experiments with estimating jointly the monetary policy rule and term structure parameters³⁴ show that this implies a better fit of the yield dynamics, a much larger reaction coefficient on output growth in the monetary policy rule and a somewhat higher degree of persistence of the policy shock. The consequences of estimating macro and term structure parameters jointly should be further explored.

Second, it may be desirable to extend the period for which yields are included to some time before 1998. This will come at the cost of not using 'clean' euro area yields anymore, but might sharpen the inference on risk parameters and allow term premia to be estimated as time-varying. Alternatively, the short sample period for yields may be accounted for by using a Bayesian approach.

Third, it would be interesting to examine different specifications of the monetary policy rule. That might include rules that are forward-looking and rules that react to estimates of the output gap or the natural real rate of interest. Moreover, a time-varying inflation objective may be incorporated. Finally, given a standard objective function of monetary policy, the optimal interest rate rule within a certain class of reaction functions may be derived. All these variations may potentially lead to a better fit of the term structure and would also yield important insights about how the reactions of long-term bond rates depend on different characteristics of monetary policy behavior.

³³See, e.g., Rudebusch and Svensson (1999).

³⁴That is, in the second stage of the estimation procedure, all other macro-parameters are fixed but the parameters in (2.7) are estimated jointly with the market prices of risk and the measurement error variance.

Appendix

A Tables

Table 1: Parameter estimates

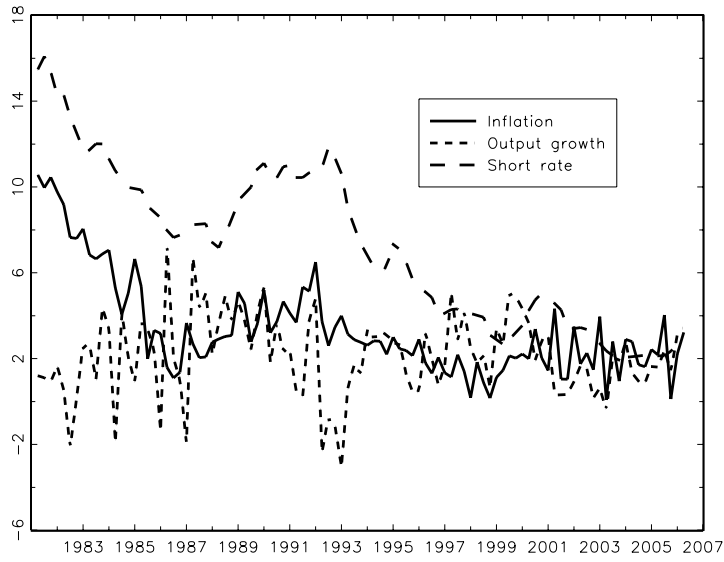
c_π	α_1	α_2	α_3	β	σ_π	ψ_z
0.614	0.310	0.120	0.274	0.174	1.036	0.876
(0.26)	(0.10)	(0.09)	(0.09)	(0.09)	(0.07)	(0.08)
γ	σ_z	ψ_a	σ_y	c_y	θ_y	c_r
-0.070	0.350	0.968	0.175	0.490	0.036	2.710
(0.03)	(0.03)	(0.09)	.	.	(0.02)	.
θ_r	c_i	ϕ_i	ϕ_π	ϕ_g	σ_ν	ψ_ν
0.578	1.671	0.931	1.019	2.039	0.455	0.333
.	.	(0.02)	(0.36)	(1.75)	(0.03)	(0.10)
$\lambda_{0,\pi}$	$\lambda_{0,a}$	$\lambda_{0,z}$	$\lambda_{0,y}$	$\lambda_{0,\nu}$	h	
0.000	0.000	-0.247	18.005	-0.639	0.287	
.	.	(0.23)	(7.22)	(0.30)	(0.02)	

ML-estimates of parameters of the macroeconomic module (first three rows of parameters) based on sample 1981Q2 - 2006Q2, term structure parameters based on sample 1998Q1 - 2006Q2 (fourth row). Asymptotic standard errors in parentheses, based on inverse Hessian. Parameters without standard errors are calibrated or functions of other estimated parameters, see main text for details.

B Figures

Figure 1: The data

(a) Inflation, output growth, short rate



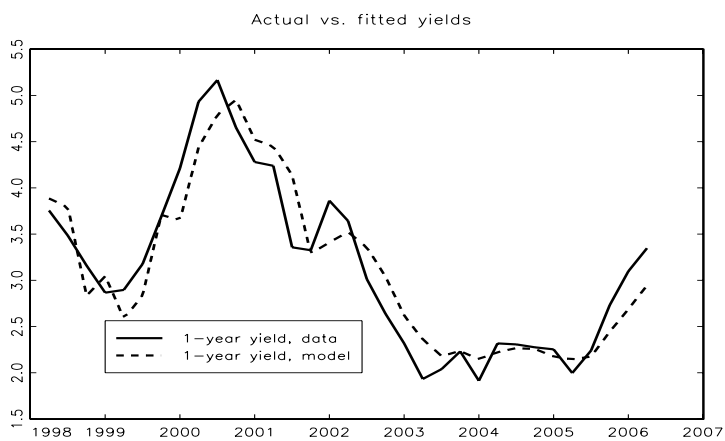
(b) One-, five-, and ten-year yields



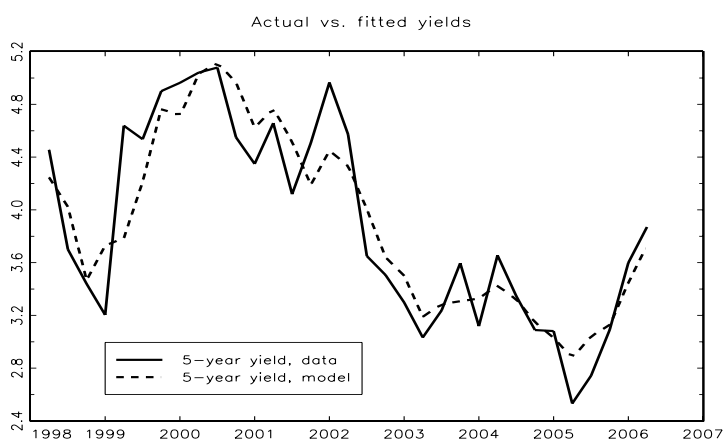
See the main text, section 3.1, for details.

Figure 2: Actual vs. model-implied yields

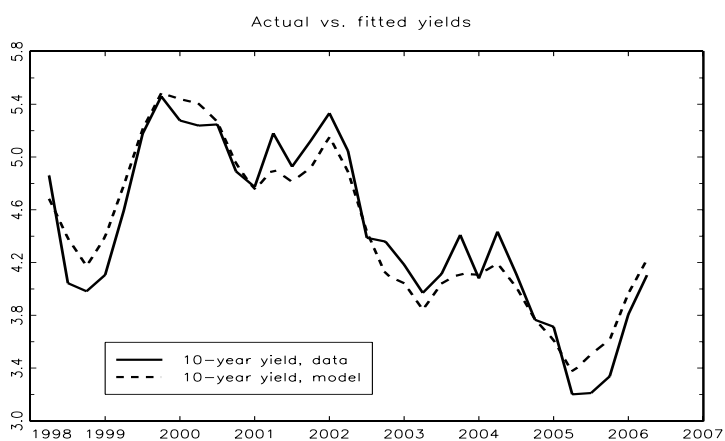
(a) 1-year yield



(b) 5-year yield

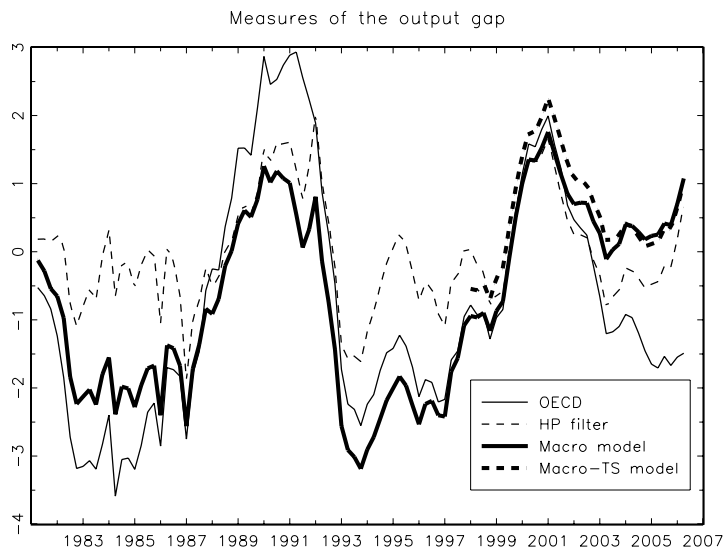


(c) 10-year yield



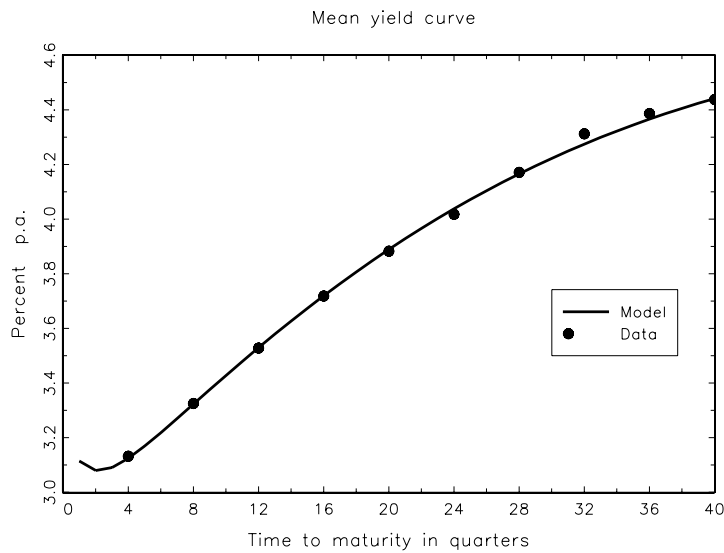
Model-implied yields are based on smoothed states.

Figure 3: Different measures of the output gap



'Macro model' refers to the smoothed output gap, based on observations of inflation, output growth and the short rate only. 'Macro-TS model' refers to the smoothed output gap, when bond yields are included in the measurement vector as well (as of 1998).

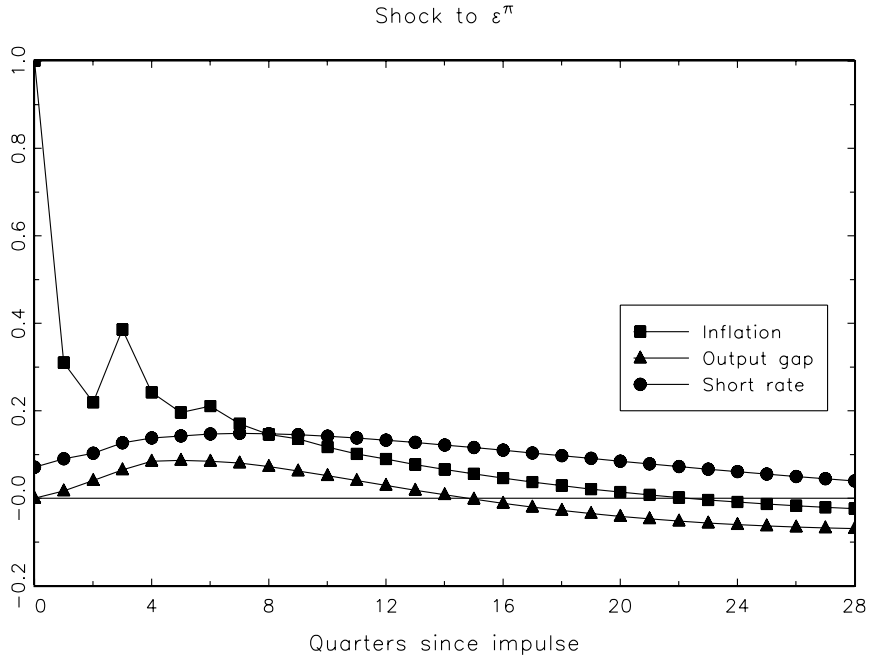
Figure 4: Actual and model-implied mean yield curve



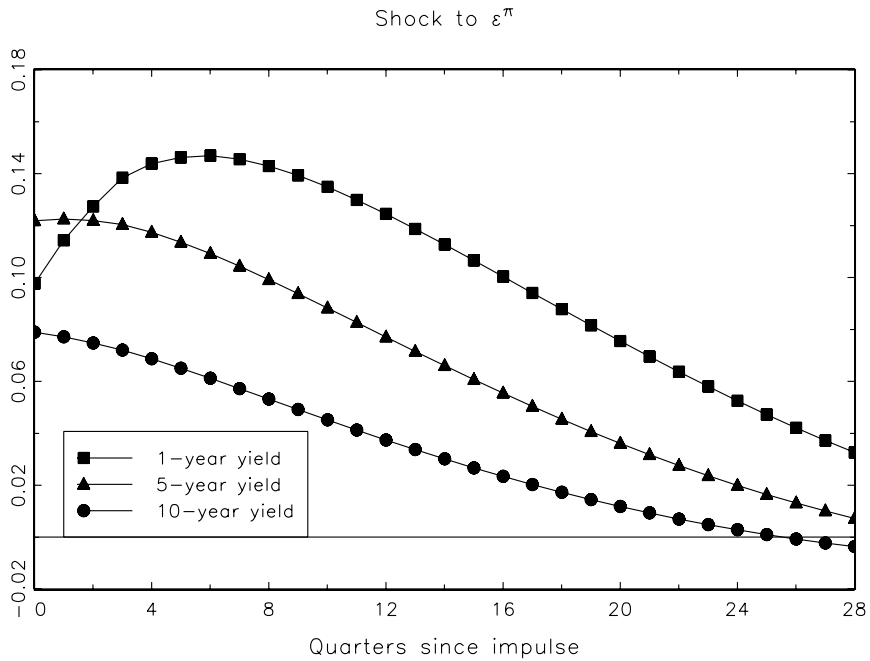
For each time to maturity, the solid circle represents the average of the corresponding yield over the period 1998Q1 - 2006Q2. The model counterpart is the average of the fitted yields (based on smoothed states).

Figure 5: Impulse response to an inflation shock

(a) Inflation, output gap, short rate



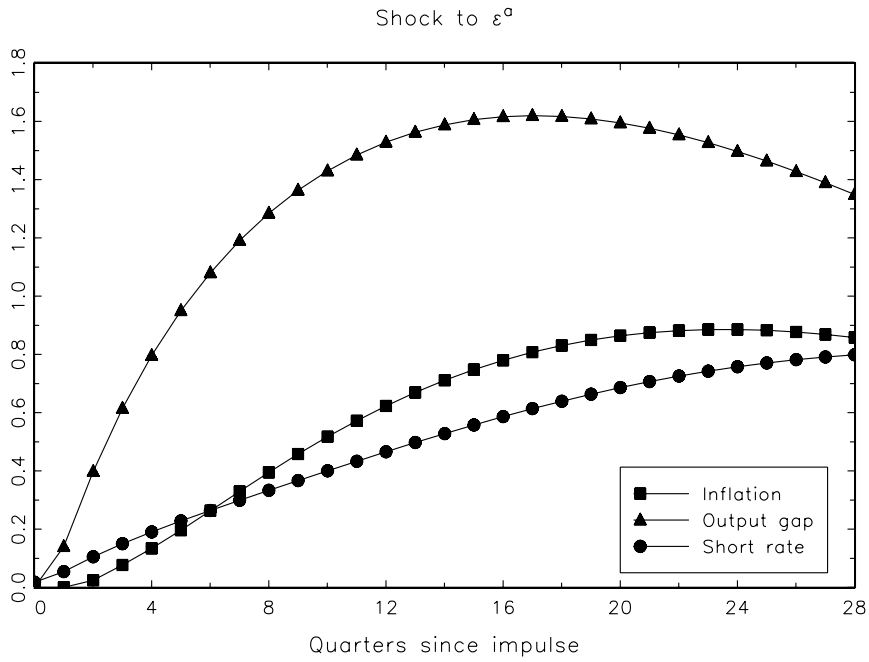
(b) One-, five-, ten-year yield



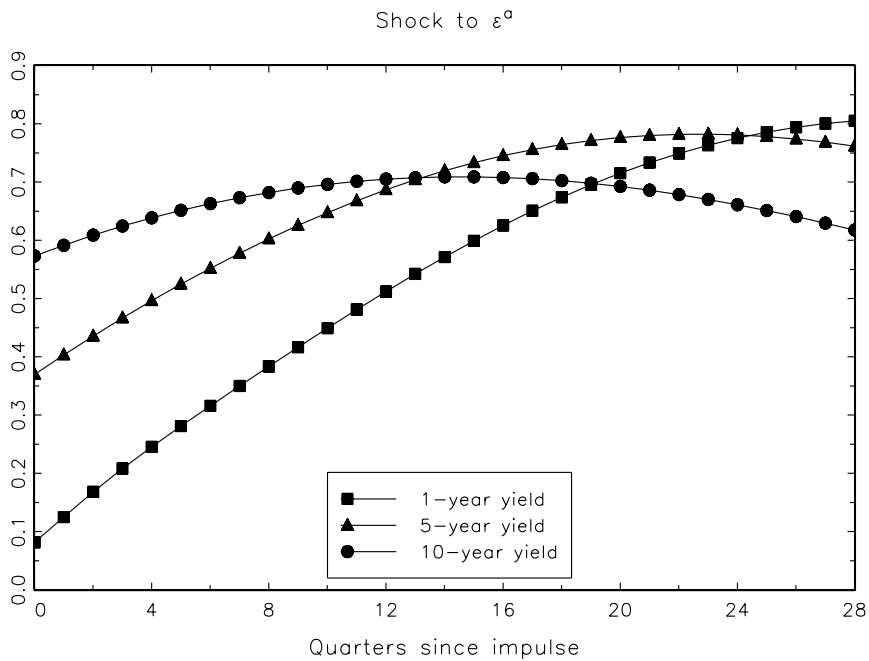
Response to a one-time shock to ϵ^π of 1%. (Stdd. dev of that shock is 1.04.) All responses in percentage points.

Figure 6: Impulse response to a trend-growth-rate shock

(a) Inflation, output gap, short rate



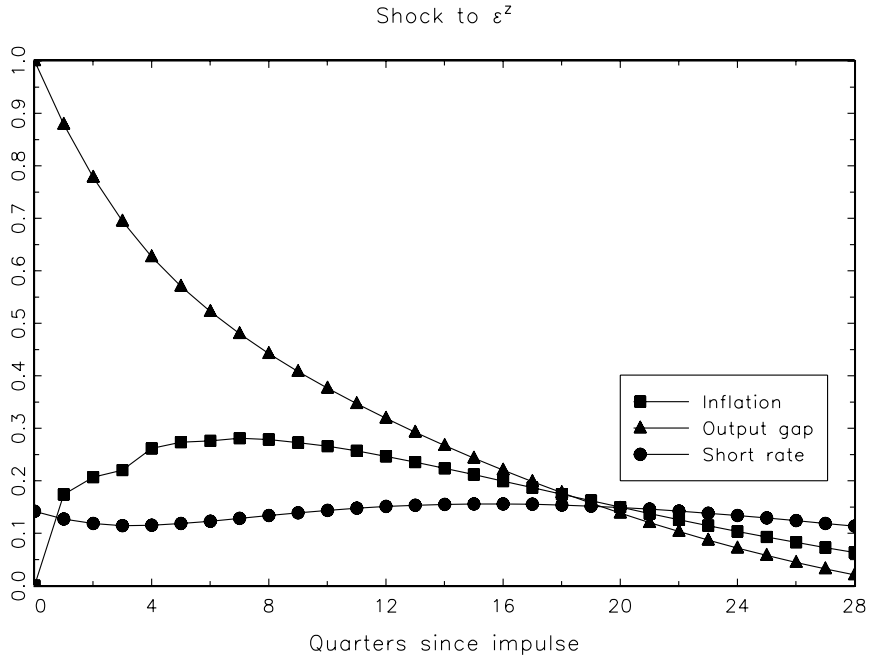
(b) One-, five-, ten-year yield



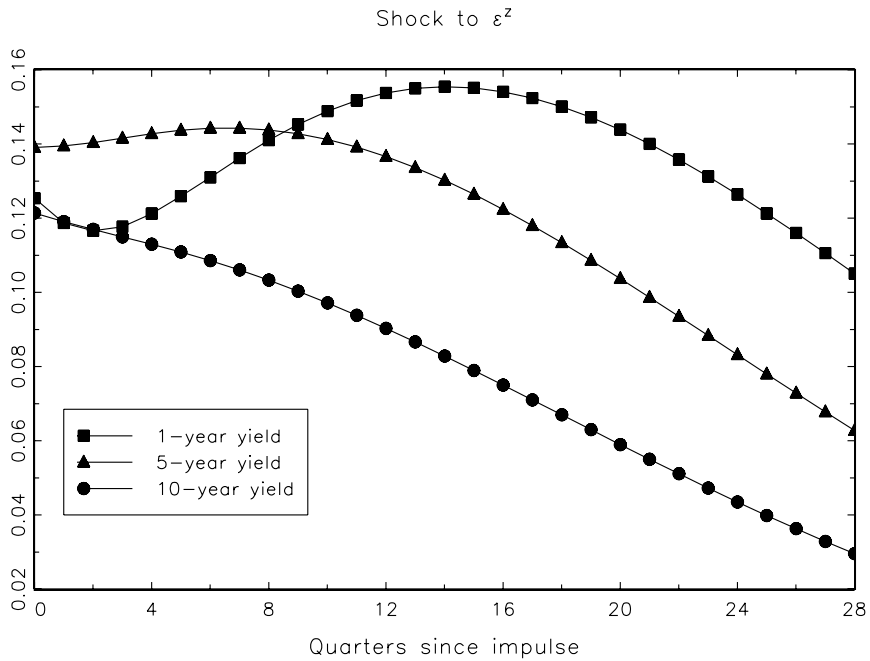
Response to a one-time shock to ϵ^a of 3.472. (Stdd. dev of that shock is 1.00.) Note, the loading of a_t on potential output growth Δy_t^* is $\theta_y = 0.036$. Thus, the shock increases *annualized* potential output growth on impact by 0.5 percentage points.) All responses in percentage points.

Figure 7: Impulse response to an output gap shock

(a) Inflation, output gap, short rate



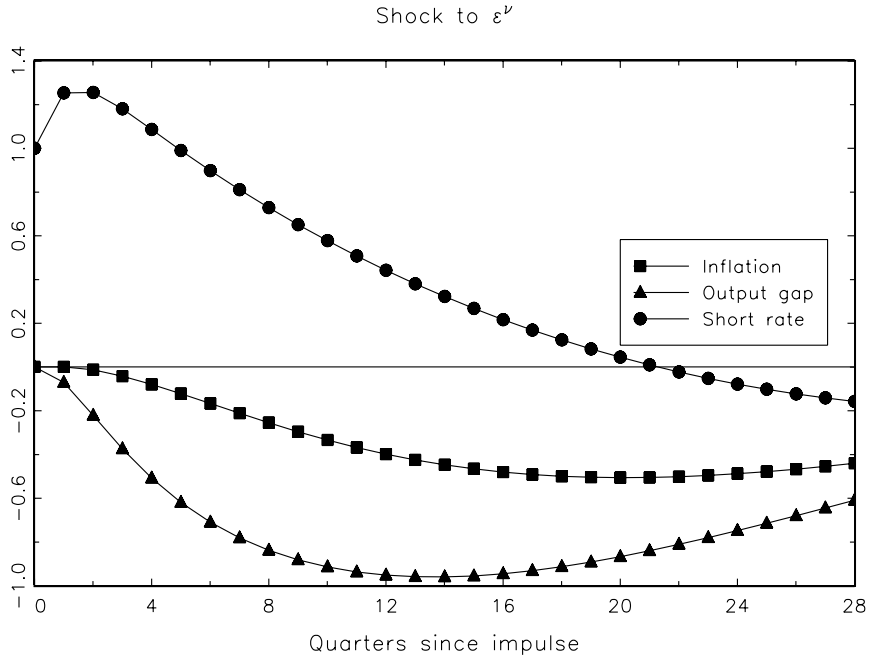
(b) One-, five-, ten-year yield



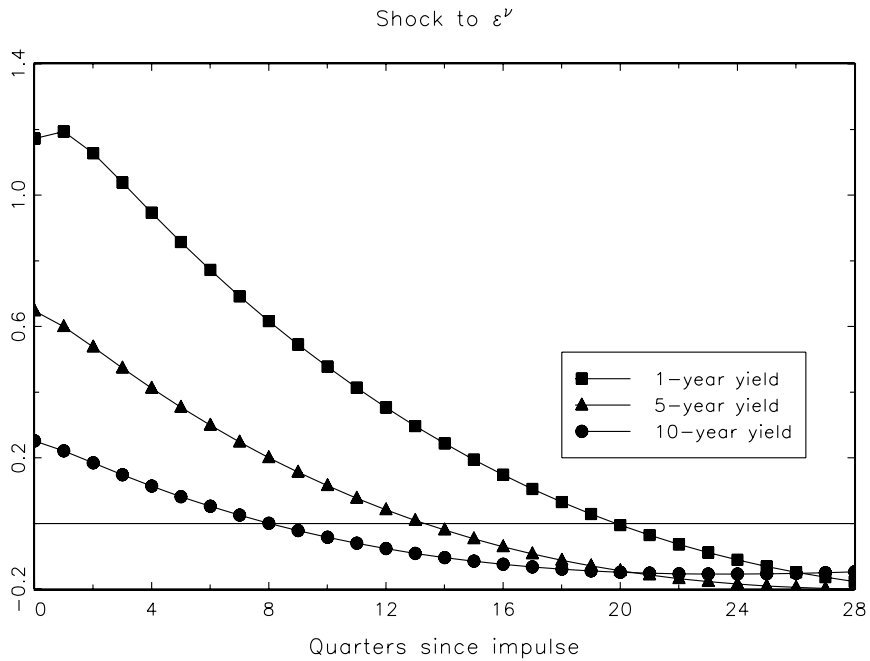
Response to a one-time shock to ϵ^z of 1%. (Note: stdd. dev of that shock is 0.35.) All responses in percentage points.

Figure 8: Impulse response to a monetary policy shock

(a) Inflation, output gap, short rate



(b) One-, five-, ten-year yield



Response to a one-time shock to ϵ^v of 1%. (Stdd. dev of that shock is 0.46.) All responses in percentage points.

**C The Elements of \mathcal{K} and Q in the SVAR representation
(2.10)**

[To be completed]

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