Self-Referential Optimal Advising When Reactions are Delayed

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Abstract

Like social predictions also advices addressed to the relevant agents may influence their subject and consequently may be liable to self-referentiality effects. It is a well-known phenomenon that decisionmakers tend to delay the execution of a given advice the more the less urgent the underpinning arguments appear to be to them. Particularly, this can be observed in economic and in environmental policy. What should a professional adviser do? It is the purpose of this study to provide an analytical framework in which a professional adviser's objectives are analyzed. Naturally, his[1] first objective is to choose such an advice and such underpinning arguments that the advice really will be taken by the addressed agents (argument justification objective). This is closely related to the problem of the predictability of social events which for the first time has rigorously been analyzed by Grunberg and Modigliani in 1954. The adviser's second objective of being right with his underpinning arguments and his third objective, i.e. his potential self-interest in the ultimate outcome, will be taken into account in this study by means of a subjective utility function. This approach can be seen as complementary to the literature on strategic information transmission and credibility.

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[1]The reader is kindly requested to understand the male forms "he", or "his" as also encompassing the female forms "she" and "her".

1 Introduction

It is a commonly known phenomenon in social reality that decisionmakers tend to delay the execution of a given advice when it involves same uncomfortableness to them. If, in addition, the advice is underpinned by some arguments, the delay usually is the greater the less important and urgent the arguments appear to be. Facing such a delaying reaction regime the adviser encounters economic problems if he has *own objectives*. Actually, there are several reasonable objectives for a professional adviser: first of all he may strive *for both* that his advice will be taken by the target group and that his arguments will turn out to be correct. (The latter clearly means that he will give that argument which he considers to have the greatest probability of coming true.) Or, he may, willy-nilly, give priority to one of these two sub-objectives if they are not achievable at the same time. Or, he may even have in mind some other criterions determining a certain set of actions from which he desires the target group to make her choice. The adviser's *motivation* for the *first objective* is obvious: if he achieves it, he proves both to be successful in social affairs and to be well-versed in the specific field of the subject. Clearly, for a professional adviser his public reputation is of vital interest. Thus, the first objective will be called *the adviser's social reputation objective*. The *last objective*, on the other hand, can naturally be formalized by utility maximization through the adviser: the set of desired actions is formed by the maximizers of the adviser's subjective utility function. It appears to be reasonable to assume that the utility criterion is a mixture of the first objective and the adviser's potential self-interest in the final outcome.

Examples for this occur in the relationship between a consulting firm or an internal expert panel and the management of a firm, or in the relationships between an economic expert committee and the government, between advisory boards and executive councils of the European Community, or between disarmament negotiators and their governments when in the eyes of their governments they are going too far. Another field to which the preceding characterization particularly applies is that of environmental policy. Here the adviser for instance may be an official committee of experts, a private person, a political party or a group of members of parliament. The target group may be the local, national or international legislature, a branch of industry or commerce, or a group of consumers. Naturally, advisers will strive for the first objective addressed above in order to be taken seriously in future. In addition to that, however, advisers usually also have a self-interest in the ultimate outcome. A delaying reaction behaviour by the target group in dependence on the urgency of arguments is, for instance, well-known from advices which require protective measures, more rigorous limiting values of pollutants, incentives for activities which are beneficial to the environment, or voluntary self-restriction in certain production and consumption activities. Examples of topics in the public eye where this can be particularly observed are the destruction of the global ozone layer through propellants and production of coolants and foam materials, air pollution and pollution of rivers and seas, especially of the North Sea, the risk of radioactive contamination, animals tests, cutting down and dying of forests, toxic waste, polluted food and conservation, especially protection of species. Actually the delaying schedules passed by international conferences on the ozone layer problem and on the pollution of the North Sea speak for themselves.

Representing advices and arguments on one-dimensional scales in Section 2 the characterized reaction behaviour of the target group is analytically formalized in an intuitive way by a reaction function which is a *fiber-preserving* (or *fiberwise*) map from a subspace $\tilde{S}^1 \times \tilde{S}^1$ of the torus into the torus. The first factor $\tilde{S}^1$, the so-called *base space*, represents the arguments, whereas the advices and the ultimate outcome are represented by the points of the second factor $\tilde{S}^1$, the so-called *fiber space*. The advantages of the representation on the torus will become clear in Proposition 2 in Section 2. As it has been pointed out, the social reputation objective clearly plays a crucial role. However, there is the difficulty that its two sub-objectives in general are unlikely to be achievable at the same time. Obviously, the one sub-objective of being right with one's argument no matter whether the advice is taken, or not, is specifically related to the nature of the subject of the respective matter under consideration. Thus, the second Section will be mainly concerned with the other sub-objective of *argument justification*, i.e., choosing an advice which will be taken by the target group no matter which argument is needed for it. Actually, the argument justification sub-objective is reasonable for the adviser: if the execution of his advice is delayed he does not prove to be able to assert himself with the target group. If it is exceeded he risks to appear to be useless at all.

One may naturalIy be interested in sufficient conditions for the existence of a solution to the argument justification objective in the presented analytical framework. Section 2 provides several results of increasing generality. The solutions to the argument justification objective just turn out to be the fixed points of the addressed reaction function. Actually, only an *elementary fixed point result* is needed, namely the Intermediate Value Theorem which is one of the fundamental principles of mathematics.

The analysis is further extended to the question when an *arbitrarily picked advice* really will be taken by the target group. For this the representation using the torus $\tilde{S}^1 \times \tilde{S}^1$ turns out to be useful: it allows for generalization and unification. The main result of Section 2 is contained in

Proposition 2. Integrating the diverging objectives by maximizing a subjective utility function will be the theme of Section 3. All proofs are intuitive and elementary in that they rely on the Intermediate Value Theorem.

In a general context the argument justification objective has been called "the opportunistic principle" by Böge, 1974. The reason for this is that it, oversubtly, can be interpreted as just talking up to the target group. A famous example for this can be found in Saint-Exupery's novel "The Little Prince" in the King of the Asteroid: The King deliberately only orders his subjects to do what they anyway are just going to do. In effect, all orders by the king are always obeyed by his subjects.

Obviously, the problem of argument justification in advising is intimately related to the well-known problem of giving accurate public forecasts when the subject of the forecast is influenced by the forecast itself ("self-referential", or "reflexive forecasts", "forecasting with feedbacks"). The issue of self-referentiality in social predictions has been of major interest in economics since O. Morgenstern's pioneering study on the subject in 1928. In their path-breaking paper from 1954 Grunberg and Modigliani for the first time provided a rigorous existence analysis using Brouwer's Fixed Point Theorem. In the sequel there had been given further extensions and applications in the literature using various model frameworks (e.g. Devletoglou, 1961; Rothschild, 1964; Galatin, 1976; Jordan, 1980, and in particular the controversy by Kemp, Chiang, and Grunberg/Modigliani in the American Economic Review 1961/1962). Actually, the present study is in the tradition of Grunberg and Modigliani in that it uses a continuous reaction function of the decision-makers as a primitive concept. A survey on theoretical and applied literature on the topic can be found in Tamborini, 1997; Güth and Kliemt, 2004; Lehmann-Waffenschmidt, 1990, 1996; and Lehmann-Waffenschmidt and Sandri, 2006. Actually, there has also been major interest in this topic by other social sciences, for instance concerning election forecast or opinion research, and by philosophy for the methodological aspects (e.g. Stewart, 1975; Henshel, 1978).

There is another branch in game theoretical literature which is closely related to our approach (e.g. Kreps and Wilson, 1982; Milgrom and Roberts, 1982; Wilson, 1985 for a survey). These studies investigate the sequential strategic interactions between an adviser whose motives are uncertain (a "spy", or a "double agent") and a decisionmaker (a "representative of government"). In this context, it pays for an unfriendly adviser to build a reputation (credibility) by providing accurate and valuable information and performing useful services, and eventually to cash in on his reputation. However, the concern of this approach is different from ours as we analyse the situation of a professional adviser who never may afford of entirely cashing in on his reputation. In this sense the two approaches may well be viewed as being complementary to each other.

2 The Argument Justification Objective

This Section provides a formal model for analyzing the adviser's argument justification objective. Furthermore, it presents same results concerning the existence of solutions to this objective.

As already has been argued in the Introduction, the adviser's objective of argument justification is reasonable because otherwise he would damage his reputation: in the case of a delaying reaction behaviour the adviser does not assert himself with the target group, and in the case of an exceeding reaction behaviour he risks to appear to be useless. As it has been mentioned already, geometric intuition will be a good guide throughout the whole Section.

Let us start with the following *heuristics*. Think of a certain social problem for which the necessary measures are well-known through scientific work. However, what still really matters is the *date of putting these measures into action*. (Think for instance of disarmament, or economic policy, or of the list of environmental topics mentioned in the Introduction.) Thus, at the heart of an advice will be a date $t \geq t_0 = 0$ until which the measures should be put into action by the target group where t_0 denotes the present date. Furthermore, let the advice t be underpinned by arguments, for instance by specific results from research work, or by ethical arguments. In any way, the arguments essentially emphasize the *degree of importance and urgency* to become active. Actually the proverb "it is at the eleventh hour" indicates the way for mathematical representation: the degree of importance and urgency will be represented by a point on the clock face, that means by a point on the *unit circle* (unit sphere) $S^1 = \{(x_1, x_2) \in R^2 \mid \sqrt{x_1^2 + x_2^2} = 1\}$. Clockwise approaching the point $(0, 1)$, i.e. 12 o'clock, on S^1 represents *increasing urgency and importance* up to the *emergency case* at 12 o'clock.

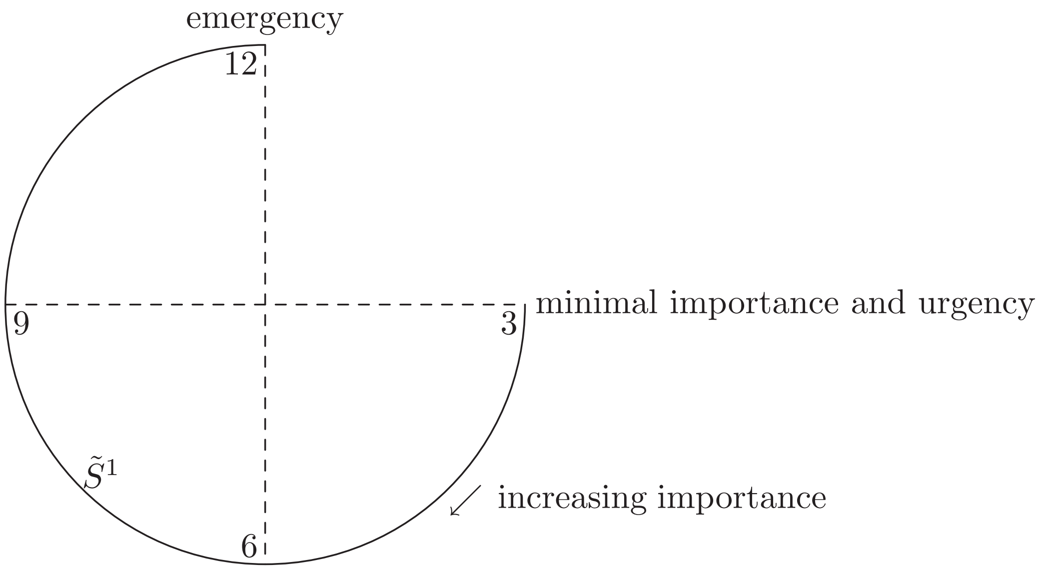
At first glance, the representation of the advice and the underpinning arguments by a time interval and a point on the clock face respectively might seem to be somewhat tautologous. However, it is not at all tautologous if an advice which refers to a date is further underpinned by an argument emphasizing the importance and the urgency of taking action.

Apparently, this stylization just provides an *ordinal ranking* of degrees of urgency and importance rather than a *cardinal* one. Thus, for instance, one may remove that part of S^1 which lies in the strictly positive quadrant of the plane. The remaining part

$$\tilde{S}^1 := S^1 \bigcap (R^2 \setminus R^2_{++}) = \{(x_1, x_2) \in R^2 \mid \sqrt{x_1^2 + x_2} = 1,\ x_1 \leq 0 \text{ } or \text{ } x_2 \leq 0\}$$

is the (ordinal) scale of importance and urgency expressed by the arguments, i.e., it represents the arguments (Figure 1).

Figure 1: The Space $\tilde{S}^1$ of Arguments



Clearly, *also the advices* t can be represented on the unit circle. At first glance this might appear to be somewhat roundabout. However, the significance of this kind of representing advices will turn out in the final result (Proposition 2) of this Section. Actually, it allows for an appealing generalization and unification of the assumptions of Proposition 1.

Let us proceed by taking a second copy of $\tilde{S}^1$. There is nothing unnatural with bounding advices from above by some positive real number T: Advisers a priori do not take advices $t > T$ into consideration, and furthermore any point of $[0, T]$ has to be a reasonable argument of the decisionmaker's *reaction function* which will be introduced later. Accordingly, let us map the interval of *admissible advices* $[0, T]$ onto $\tilde{S}^1$ by the canonical homeomorphism

$$\begin{aligned} \overline{\tau} : [0, T] &\overset{\approx}{\rightarrow} \tilde{S}^1 \subset R^2 \\ t &\mapsto (\overline{\tau}_1(t), \overline{\tau}_2(t)). \end{aligned}$$

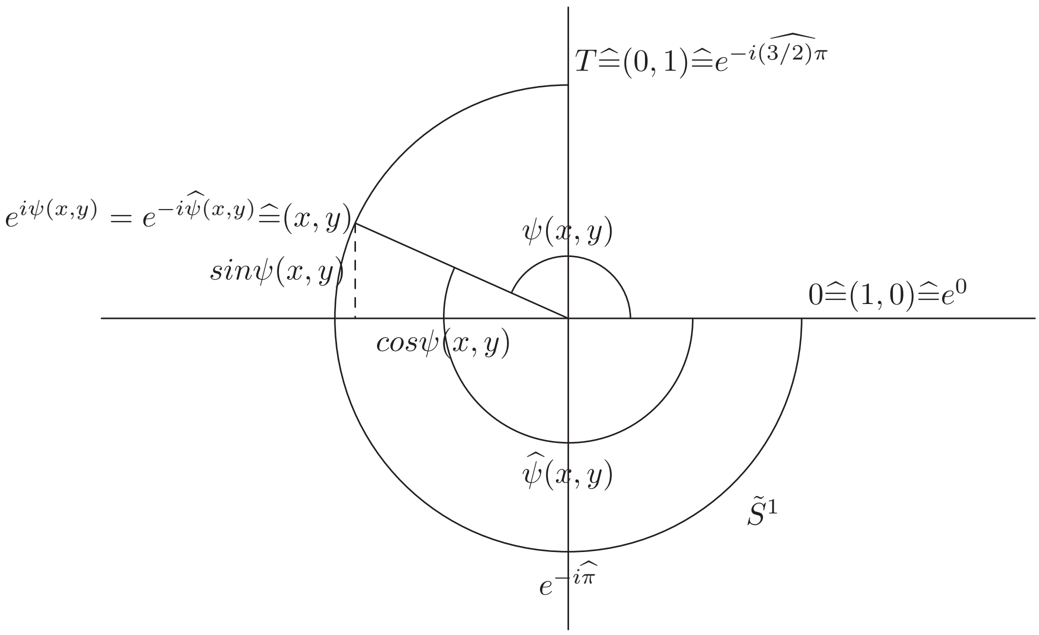
$\overline{\tau}$ maps 0 on $(1, 0)$ and T on $(0, 1)$ (cf. Figure 2 below).

For the sake of convience, from now on the points (x, y) of the unit circle will be represented by their *polar coordinates*

$$e^{i\psi(x,y)} := cos\psi(x, y) + i\ sin\psi(x, y)$$

where $cos\psi(x, y) = x$ and $sin\psi(x, y) = y$. Following the mathematical convention the angle $\psi(x, y) \in [0, 2\pi[$ denotes the *counter-clockwise angle* between the ray of positive reals and the ray from the origin through $(x, y) \in \tilde{S}^1$ (see Figure 2).

Figure 2: Polar Coordinate Representation of Advices and Arguments



Using also the *complementary angle*

$$\widehat{\psi}(x, y) := 2\pi - \psi(x, y)$$

one actually has *two representations* of every point $(x, y) \in S^1$ different from $(1, 0)$:

$$\begin{array}{ccc} e^{i\psi(x,y)} & = & e^{i[-(2\pi-\psi(x,y))]} = e^{-i\widehat{\psi}(x,y)} \\ \| & & \| \\ cos\psi(x, y) + i\ sin\psi(x, y) & = & cos[-(2\pi - \psi(x, y))] + i\ sin[-(2\pi - \psi(x, y))] \end{array}$$

This is well-known from the periodicity and symmetry properties of *sin* and *cos*.

Consequently, the canonical homeomorpism $\overline{\tau}$ can equivalently be represented by the following mapping:

$$\begin{array}{rcl} \tau : [0,T] & \stackrel{\approx}{\longrightarrow} & \tilde{S}^1 \\ t & \mapsto & e^{i\psi_t} = e^{-i\widehat{\psi}_t} \end{array}$$

where the angle functions ψ_t and $\widehat{\psi} = 2\pi - \psi_t$ are given in the intuitive way:

$$\begin{array}{rcl} \psi : [0,T] & \longrightarrow & [\pi/2, 2\pi] \\ t & \mapsto & \psi_t = \psi(\overline{\tau}(t)) = \psi(\overline{\tau}_1(t), \overline{\tau}_2(t)) \end{array}$$

and

$$\begin{array}{rcl} \widehat{\psi} : [0,T] & \longrightarrow & [0, 3/2\pi] \\ t & \mapsto & \widehat{\psi}_t = 2\pi - \psi(\overline{\tau}_1(t), \overline{\tau}_2(t)). \end{array}$$

This means, both angle functions are increasing linear functions in advices t:

$$\widehat{\psi}_{(t_1+t_2)} = \widehat{\psi}_{t_1} + \widehat{\psi}_{t_2}$$

and

$$\begin{array}{rcl} \psi_{(t_1+t_2)} & = & 2\pi - [(2\pi - \psi_{t_1}) + (2\pi - \psi_{t_2})] \\ & = & \psi_{t_1} + \psi_{t_2} - 2\pi \quad \forall t_1, t_2 \in [0,T] \text{ with } t_1 + t_2 \leq T. \end{array}$$

For its intuitive appeal and in order to simplify the notation, henceforth only the *complementary angle function* $\widehat{\psi}$ will be used. Accordingly,

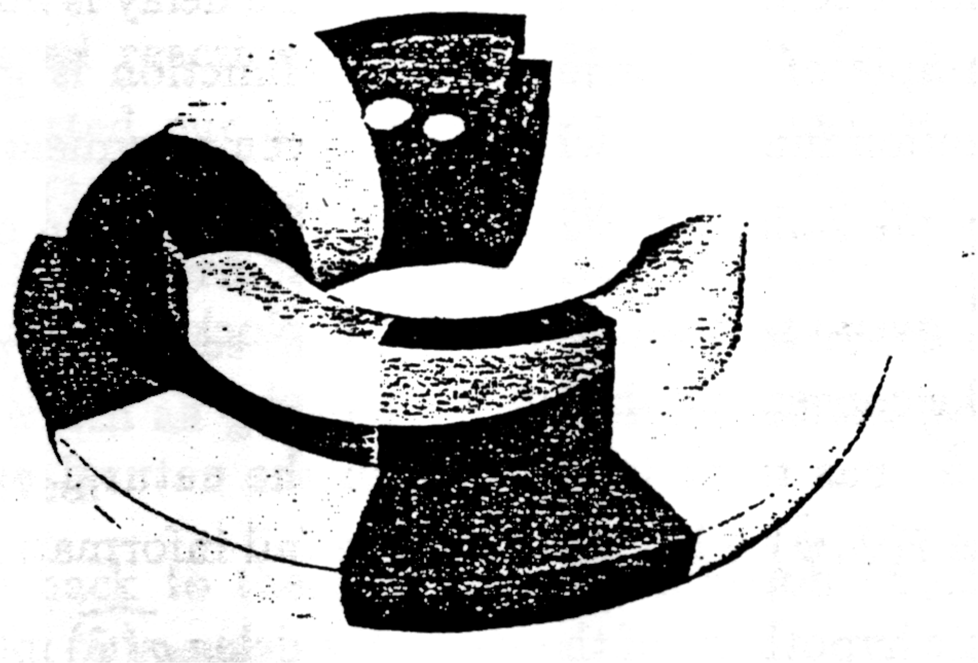
$$\tau(t) = e^{-i\widehat{\psi}_t}.$$

Furthermore, in the sequel *all angles* ϕ which are *clockwise measured* will be written as $\widehat{\phi} \in [0, 2\pi[$.

After these preparations the *basic hypothesis* of the study that the target group executes a given advice with decreasing delay when the adviser *increases* his *emphasis* on the importance and the urgency can be analytically represented in the following way: consider the Cartesian product $\tilde{S}^1 \times \tilde{S}^1$. It can be geometrically visualized as a subspace of the torus $S^1 \times S^1$ (see Figure 2):

Thus, an *argument* is represented by a point $e^{-i\widehat{\varphi}}$ of the first factor $\tilde{S}^1$, i.e. of the so-called *base space* of the product $\tilde{S}^1 \times \tilde{S}^1$. Correspondingly, an *advice* t is represented through the homeomorphism τ by a point $e^{-i\widehat{\psi}_t}$ of the second factor $\tilde{S}^1$, i.e. of the *fiber space over the base point* $e^{-i\widehat{\varphi}}$.

Figure 3: The Subspace $\tilde{S}^1 \times \tilde{S}^1$ of the Torus $S^1 \times S^1$



Given an argument $e^{-i\widehat{\varphi}}$ from the base space $\tilde{S}^1$ a *delayed reaction behaviour*, i.e. a *delayed reaction regime*, of the target group is representable by a continuous *reaction function*, which may be known or unknown to the adviser:

$$\begin{aligned}(id \times r_{\widehat{\varphi}}) : \left\{e^{-i\widehat{\varphi}}\right\} \times \tilde{S}^1 &\rightarrow \left\{e^{-i\widehat{\varphi}}\right\} \times S^1 \\ \left(e^{-i\widehat{\varphi}}, e^{-i\widehat{\psi}_t}\right) &\mapsto \left(e^{-i\widehat{\varphi}}, r_{\widehat{\varphi}}\left(e^{-i\widehat{\psi}_t}\right)\right) \\ &= \left(e^{-i\widehat{\varphi}}, e^{-i\overline{r}\left(\widehat{\varphi},\widehat{\psi}_t\right)}\right),\end{aligned}$$

where $\overline{r}\left(\widehat{\varphi}, \widehat{\psi}_t\right) > \widehat{\psi}_t$ for all advices $e^{-i\widehat{\psi}_t}$. (In order to simplify the notation we omit the $\widehat{}$ over $\overline{r}\left(\widehat{\varphi}, \widehat{\psi}_t\right)$. Thus, throughout the study $\overline{r}\left(\widehat{\varphi}, \widehat{\psi}_t\right)$ stands for $\widehat{\overline{r}\left(\widehat{\varphi}, \widehat{\psi}_t\right)}$.) Using a continuous reaction function we are in the tradition of Grunberg and Modigliani.

Definition 1. Given a pair $\left(e^{-i\widehat{\varphi}}, e^{-i\widehat{\psi}_t}\right)$ of an advice $e^{-i\widehat{\psi}_t}$ and an underpinning argument $e^{-i\widehat{\varphi}}$ the *ultimate outcome* under the *reaction function* $id \times r_{\widehat{\varphi}}$

is $r_{\widehat{\varphi}}\left(e^{-i\widehat{\psi}_t}\right) = e^{-i\overline{r}\left(\widehat{\varphi},\widehat{\psi}_t\right)}$.

Of course, in general one has $\overline{r}\left(\widehat{\varphi},\widehat{\psi}_t\right) \in \mathbb{R}_+$. Notice that an advice $t\left(i.e.\ e^{-i\widehat{\psi}_t}\right)$ is from the bounded interval $[0, T]$, whereas the ultimate outcome is not bounded from above, i.e. the delay is not restricted.

A first introductory example of a delaying reaction function is given by the "*uniformly σ-delayed reaction function*" which for a given argument $e^{-i\widehat{\varphi}}$ delays every advice $t \underset{\tau}{\widehat{=}} e^{-i\widehat{\psi}_t}$ by the same interval $[0, \sigma(\widehat{\varphi})] \widehat{=} e^{-i\widehat{\sigma(\widehat{\varphi})}}$. In other words, the whole fiber space $\tilde{S}^1$ over the base point $e^{-i\widehat{\varphi}}$ is clockwise rotated by the angle $\sigma(\widehat{\varphi})$. Let us take the convention that an image point $e^{-i\overline{r}\left(\widehat{\varphi},\widehat{\psi}\right)} \in S^1$ which lies outside the domain $\tilde{S}^1$ has to be interpreted in the natural *additive way.* Actually, the image angle $\overline{r}\left(\widehat{\varphi},\widehat{\psi}\right) \in \mathbb{R}_+$ contains the full information.

According to the basic hypothesis of the study the delay $\widehat{\sigma(\widehat{\varphi})}$ must decrease for increasingly important and urgent arguments $\widehat{\varphi}$. Analytically this can be represented by a continuous mapping from the base space

$$\begin{aligned} \widehat{\sigma} : \tilde{S}^1 &\rightarrow [0, 2\pi[\\ e^{-i\widehat{\varphi}} &\mapsto \widehat{\sigma(\widehat{\varphi})} \end{aligned}$$

which is *strictly monotonically decreasing* with clockwise increasing arguments. (Recall that the present date is represented by 0.) The additional property

$$\widehat{\sigma}\left(e^0\right) = \widehat{\sigma}(1, 0) > 0$$

ensures that $\widehat{\sigma}$ really generates a delay.

A simple example for $\widehat{\sigma}$ is given by the difference to the emergency argument:

$$\begin{aligned} \widehat{\sigma}_d : \tilde{S}^1 &\rightarrow [0, {}^3/{}_{2\pi}] \\ e^{-i\widehat{\varphi}} &\mapsto \widehat{{}^3/{}_{2\pi} - \widehat{\varphi}} \end{aligned}$$

Now we can state the following

Definition 2. A function $\widehat{\sigma} : \tilde{S}^1 \rightarrow [0, 2\pi[$ with the above properties is called a *delay function.* Given a delay function $\widehat{\sigma}$ the resulting fiberwise continuous reaction function

$$\begin{aligned} r_{\widehat{\sigma}} : \tilde{S}^1 \times \tilde{S}^1 &\rightarrow \tilde{S}^1 \times S^1 \\ \left(e^{-i\widehat{\varphi}}, e^{-i\widehat{\psi}}\right) &\mapsto \left(e^{-i\widehat{\varphi}}, e^{-i\left(\widehat{\psi}+\widehat{\sigma(\widehat{\varphi})}\right)}\right) \end{aligned}$$

is called the *uniformly σ-delayed reaction function.* The term "fiberwise" characterizes the particular property of $r_{\widehat{\sigma}}$ that it leaves all base points $e^{-i\widehat{\varphi}}$ unchanged.

Uniformly σ-delayed reaction functions are fairly peculiar. Nevertheless, we have started our analysis with these functions mainly for the following two reasons. (1) They provide a simple formalization of delayed reaction behaviour which furthermore is geometrically intuitive. (2) Besides this propaedeutical quality the class of uniformly σ-delayed reaction functions in fact is the germ from which all generalized classes of reaction functions used in this study will be derived.

Now let us come back to the argument justification objective. We will shorten notation by the following

Definition 3. An *accurately taken (underpinned) advice* is a pair $\left(e^{-i\widehat{\varphi}}, e^{-i\widehat{\psi}}\right)$ of an advice $e^{-i\widehat{\psi}}$ and an underpinning argument $e^{-i\widehat{\varphi}}$ by which the adviser's objective of argument justification is achieved. In other words, an accurately taken advice is a *fixed point of the target group's reaction function.*

It is a trivial observation that for a uniformly σ-delayed reaction function the set of accurately taken advices equals the fiber over $e^{-\widehat{3/2\pi}}$ if $\sigma\left(\widehat{3/2\pi}\right) = 0$, and is empty if $\sigma\left(\widehat{3/2\pi}\right) > 0$.

There is a remark in order: it might be argued that taking the set of fixed points of the reaction function as the set of solutions to the argument justification objective I would unnecessarily restrict the solution set. Indeed, points of $\tilde{S}^1 \times \tilde{S}^1$ which are *"approximate" fixed points* might well qualify as *solutions* in a broader sense. However, the following existence results concerning fixed points also throw light on the broadened solution set of approximate fixed points: particularly the points of the neighbourhoods of fixed points are approximate fixed points. This discussion, however, will turn out to be a special aspect of the more comprehensive analysis of the adviser's objectives in Section 3 below.

Evidently, a uniformly σ-delayed reaction function is not very satisfactory as representing real reaction behaviour. Actually, there is no reason why the target group for a given argument should delay *any* possible advice by the *same time interval.* For instance, one might think instead of a *proportional delay* by which, for a given argument, the advices t are delayed the more the greater is t. But, clearly, also this class of reaction functions is fairly restrictive. Looking more closely, it turns out to be a subclass of the class of *delayed reaction functions* which is obtained by *continuously fiberwise deforming* the functions of the reference-class of the uniformly σ-delayed reaction functions. The following definition makes this precise.

Definition 4. A uniformly σ-delayed reaction function

$$r_{\widehat{\sigma}} : \tilde{S}^1 \times \tilde{S}^1 \to \tilde{S}^1 \times S^1$$

is *continuously fiberwise deformed (perturbed) into a "delayed reaction function"*

$$\begin{array}{rcl} r : \tilde{S}^1 \times \tilde{S}^1 & \to & \tilde{S}^1 \times S^1 \\ \left(e^{-i\widehat{\varphi}}, e^{-i\widehat{\psi}}\right) & \mapsto & \left(e^{-i\widehat{\varphi}}, e^{-i\left(\overline{r}\left(\widehat{\varphi},\widehat{\psi}\right)\right)}\right) \end{array}$$

if r is a continuous deformation (perturbation) of $r_{\widehat{\sigma}}$ i.e. if there is a continuous map (homotopy) $R : \tilde{S}^1 \times \tilde{S}^1 \times [0,1] \to \tilde{S}^1 \times S^1$ with $R|_{\tilde{S}^1 \times \tilde{S}^1 \times \{0\}} = r_{\widehat{\sigma}}$ and $R|_{\tilde{S}^1 \times \tilde{S}^1 \times \{1\}} = r$, such that each *fiber mapping*

$$r_{e^{-i\widehat{\varphi}}} = r|_{\left\{e^{-i\widehat{\varphi}}\right\} \times \tilde{S}^1} : \left\{e^{-i\widehat{\varphi}}\right\} \times \tilde{S}^1 \to \left\{e^{-i\widehat{\varphi}}\right\} \times S^1$$

is a continuous deformation of the corresponding fiber mapping

$$r_{\widehat{\sigma}_{e^{-i\widehat{\varphi}}}} = r_{\widehat{\sigma}}|_{\left\{e^{-i\widehat{\varphi}}\right\} \times \tilde{S}^1}.$$

Furthermore, for any advice $e^{-i\widehat{\psi}}$ the function

$$\overline{r}\left(-, \widehat{\psi}\right) : \tilde{S}^1 \to S^1$$

is (weakly) monotonically decreasing with increasingly urgent arguments such that $\overline{r}\left(0, \widehat{\psi}\right) > \widehat{\psi}$ and $\overline{r}\left(\widehat{\varphi}, \widehat{\psi}\right) \geq 0$ for all $\widehat{\varphi} \in \tilde{S}^1$.

Remark. The last requirement guarantees that r (weakly) satisfies the basic hypothesis and in fact represents a *delayed* reaction behaviour. According to the definition a delayed reaction function r is just a continuous family of continuous fiber mappings $r_{e^{-i\widehat{\varphi}}}$ such that r is homotopic to $r_{\widehat{\sigma}}$ and each fiber mapping is generated by continuously perturbing the corresponding fiber mapping $r_{\widehat{\sigma}_{e^{-i\widehat{\varphi}}}}$ of the uniformly σ-delayed reaction function $r_{\widehat{\sigma}}$. Moreover, obviously every delay function can be derived from the difference delay function $\widehat{\sigma}_d$ by a continuous perturbation. Thus, the Definition 3 equivalently could have been stated *by using only the difference delay function* $\widehat{\sigma}_d$ as reference function, or "germ", of the class of delayed reaction functions.

A preliminary result on the solvability of the argument justification objective is the following

Proposition 1. Let

$$\begin{array}{rcl} r : \tilde{S}^1 \times \tilde{S}^1 & \rightarrow & \tilde{S}^1 \times S^1 \\ \left(e^{-i\widehat{\varphi}}, e^{-i\widehat{\psi}}\right) & \mapsto & \left(e^{-i\widehat{\varphi}}, e^{-i\left(\overline{r}\left(\widehat{\varphi},\widehat{\psi}\right)\right)}\right) \end{array}$$

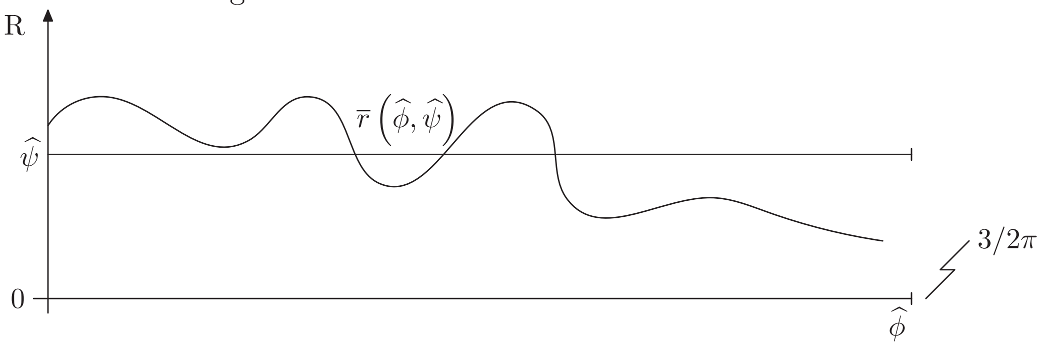
be a delayed reaction function with the additional boundary properties $\overline{r}\left(0, \widehat{\psi}\right) > \widehat{\psi}$ and $\overline{r}\left(\widehat{3/2\pi}, \widehat{\psi}\right) \leq \widehat{\psi}$ for all advices $e^{-i\widehat{\psi}} \in \tilde{S}^1$. Then *for every advice* $e^{-i\widehat{\psi}}$ there exists *at least one argument* $e^{-i\widehat{\varphi}\left(\widehat{\psi}\right)} \in \tilde{S}^1$ so that the advice $e^{-i\widehat{\psi}}$ is accurately taken by the target group. In other words, the point

$$\left(e^{-i\widehat{\varphi}\left(\widehat{\psi}\right)}, e^{-i\widehat{\psi}}\right)$$

is a *fiberwise fixed point* of the reaction function r.

Proof. The boundary properties and the continuity of r again admit the application of the Intermediate Value Theorem: choose an arbitrary advice $e^{-i\widehat{\psi}}$ and consider the orbit $\overline{r}\left(-, \widehat{\psi}\right)$ of image points under the family of fiber mappings $\left(r_{e^{-i\widehat{\varphi}}}\right)$ for $\widehat{\varphi}$ running from 0 to $\widehat{3/2\pi}$. Due to the assumptions the orbit is a continuous function from $\left[0, \widehat{3/2\pi}\right]$ into $\mathbb{R}_+$. Due to the boundary assumptions the Intermediate Value Theorem applies (cf. Figure 4). □

Figure 4: The Orbit of an Advice $e^{-i\widehat{\psi}}$ under $\overline{r}$



Actually, the boundary conditions of Proposition 1 can be significantly weakened so that a wide class of reaction functions with a *generalized delaying characteristic* obtains for which still the *existence of at least one accurately taken advice* can be ensured:

Corollary (to Proposition 1). Let

$$\begin{array}{rcl} \rho : \tilde{S}^1 \times \tilde{S}^1 & \to & \tilde{S}^1 \times S^1 \\ \left(e^{-i\widehat{\varphi}}, e^{-i\widehat{\psi}}\right) & \mapsto & \left(e^{-i\widehat{\varphi}}, e^{-i\left(\overline{\rho}\left(\widehat{\varphi},\widehat{\psi}\right)\right)}\right) \end{array}$$

be a continuous reaction function with the following property:
$\widehat{\psi} + \overline{\rho}\left(\widehat{\varphi}, \widehat{\psi}\right) \geq 0$ for all pairs $\left(e^{-i\widehat{\varphi}}, e^{-i\widehat{\psi}}\right) \in \tilde{S}^1 \times \tilde{S}^1$, and for *at least one advice* $e^{-i\widehat{\psi}_0} \in \tilde{S}^1$ there exists an argument $e^{-i\widehat{\varphi}_1\left(\widehat{\psi}_0\right)}$ with $\overline{\rho}\left(\widehat{\varphi}_1\left(\widehat{\psi}_0\right), \widehat{\psi}_0\right) > \widehat{\psi}$ and an argument $e^{-i\widehat{\varphi}_2\left(\widehat{\psi}_0\right)}$ with $\overline{\rho}\left(\widehat{\varphi}_2\left(\widehat{\psi}_0\right), \widehat{\psi}_0\right) \leq \widehat{\psi}$.
Then for the *generalized delaying reaction function* ρ there exists at least one accurately taken advice $\left(e^{-i\widehat{\varphi}}, e^{-i\widehat{\psi}_0}\right)$.

Proof. This follows immediately from Proposition 1. □

Nevertheless, the assumptions of Proposition 1 can even be generalized without weakening the result. Particularly, the rigid boundary conditions can be relaxed. This can be done in a unifying and intuitive way by making essential use of the torus representation of the reaction function. For this, we firstly need a preparatory definition which actually is intimately related to the uniformly σ-delayed reaction function type of Definition 1.

Definition 5. The *uniform simple twist of the torus* is the fiberwise continuous self-mapping

$$\begin{array}{rrcl} \alpha : & S^1 \times S^1 & \to & S^1 \times S^1 \\ & \left(e^{-i\widehat{\varphi}}, e^{-i\widehat{\psi}}\right) & \mapsto & \left(e^{-i\widehat{\varphi}}, e^{-i\left(\left(\widehat{\psi}+\widehat{\pi}\right)-\widehat{\varphi}\right)}\right) \end{array}$$

Remark. α just twists the torus once in the following geometrically intuitive way: at base point e^0 the fiber $\{e^0\} \times S^1$ is *clockwise rotated* ("delayed") by the angle π. Increasing the angle $\widehat{\varphi}$ of the base point the rotation angle *decreases* to zero at the base point $e^{-i\widehat{\pi}}$ and then *increases* into the opposite direction up to π if $\widehat{\varphi}$ is increased up to 2π. (Particularly, the *set of fixed points* of α is just the whole fiber over the base point $e^{-i\widehat{\pi}}$.) Indeed, α is almost an old friend: if the difference delay function $\widehat{\sigma}_d$ would have been defined using $\widehat{\pi}$ instead of $\widehat{3/2\pi}$, α would just be the extension of the reaction function $r_{\widehat{\sigma}_d}$ (cf. Definition 2) to the whole range of $S^1 \times S^1$. Thus it is reasonable to regard α as an *extended* (to $S^1 \times S^1$) *standard prototype formalization of delaying reaction behavior.*

We are now ready for the generalizing and unifying result which has been announced above. Particularly, it is desirable to admit more general reaction functions than in Proposition 1. Actually, this will be achieved by means of the mapping α. Roughly speaking, the reaction functions admissible for Proposition 2 are essentially the identity mapping, or they inherit the twist characteristic of α. But the twist characteristic is a prototype representation of delaying reaction behavior.

Proposition 2. Let a continuous fiberwise function

$$\begin{array}{rccc} \rho: & \tilde{S}^1 \times \tilde{S}^1 & \rightarrow & \tilde{S}^1 \times S^1 \\ & \left(e^{-i\widehat{\varphi}}, e^{-i\widehat{\psi}}\right) & \mapsto & \left(e^{-i\widehat{\varphi}}, e^{-i\overline{\rho}\left(\widehat{\varphi},\widehat{\psi}\right)}\right) \end{array}$$

with $\overline{\rho}\left(\widehat{\varphi},\widehat{\psi}\right) \geq 0$ be given which can continuously and fiberwise be extended to a continuous fiberwise self-mapping

$$\begin{array}{rccc} \rho_0: & S^1 \times S^1 & \rightarrow & S^1 \times S^1 \\ & \left(e^{-i\widehat{\varphi}}, e^{-i\widehat{\psi}}\right) & \mapsto & \left(e^{-i\widehat{\varphi}}, e^{-i\overline{\rho}_0\left(\widehat{\varphi},\widehat{\psi}\right)}\right) \end{array}$$

of the torus with the following properties:

1. ρ_0 is a fiberwise continuous deformation of the uniform simple twist α, and
2. ρ_0 has no fixed points over base points $e^{-i\widehat{\varphi}}$ with $\widehat{\varphi} \in \left]3/2\pi, 2\pi\right[= S^1 \backslash \tilde{S}^1$

Then the conclusion of Proposition 1 still holds, i.e. considering ρ as a reaction function *with a generalized delaying characteristic* for every advice $e^{-i\widehat{\psi}}$ there exists at least one argument $e^{-i\widehat{\varphi}\left(\widehat{\psi}\right)}$ such that $\left(e^{-i\widehat{\varphi}\left(\widehat{\psi}\right)}, e^{-i\widehat{\psi}}\right)$ is an accurately taken underpinned advice, i.e. is a fixed point of ρ.

Remark. Correspondingly to the remark after Definition 4 one could call the twist α of the torus $S^1 \times S^1$, or rather its restriction to $\tilde{S}^1 \times \tilde{S}^1$, the *germ of the class of the generalized reaction functions ρ which are considered in Proposition 2.* Actually one can easily imagine a large variety of intuitive mappings ρ which qualify for Proposition 2, since everything is playing in the $\mathbb{R}^3$. Since the final outcome $e^{-i\rho\left(\widehat{\varphi},\widehat{\psi}\right)}$ is not restricted to $\tilde{S}^1$ (to $[0, T]$) the use of the torus apparently is reasonable. However, there remains the question: what do the assumptions (1) and (2) mean economically? Roughly speaking the answer is that ρ either is essentially the identity mapping or it inherits the twist characteristic of α. More detailed: By (1) and (2) ρ is the restriction to $\tilde{S}^1 \times \tilde{S}^1$ of some fiberwise continuous perturbation of the uniform simple twist α which is fixed-point free over base points from $\left]3\pi/2, 2\pi\right[$. Consequently, either $\rho_0|_{\tilde{S}^1 \times S^1}$ is the *identity mapping*, and so is ρ, or they are both not. In the latter case, from the economic viewpoint one again can distinguish two sub-cases: either $\rho_0|_{\tilde{S}^1 \times S^1}$ and ρ are the *identity mapping* over the boundary of the base space, i.e. on $\{e^0\} \times S^1 \cup \left\{e^{-i\widehat{3/2\pi}}\right\} \times S^1$ and on $\{e^0\} \times \tilde{S}^1 \cup \left\{e^{-i\widehat{3/2\pi}}\right\} \times \tilde{S}^1$ respectively, or they are both not. In the *first case* ρ is just a fiberwise perturbation of the identity mapping on $\tilde{S}^1 \times \tilde{S}^1$, or it *inherits* – as in the general *second case* – the *twist characteristic of the uniform simple twist* α: consider the *orbit* $\left\{\left(e^{-i\widehat{\varphi}}, e^{-i\,\overline{\rho}_0\left(\widehat{\varphi},\widehat{\psi}\right)}\right) | \widehat{\varphi} \in S^1\right\} \subset S^1 \times S^1$ of any advice $e^{-i\widehat{\psi}} \in \tilde{S}^1$ and its projection on the fiber space $\tilde{S}^1$ of the product $\tilde{S}^1 \times \tilde{S}^1$. The assumptions (1) and (2) make sure that this projection is *onto*. Moreover, for all $\widehat{\varphi} \in S^1 \backslash \tilde{S}^1 =]3\pi/2, 2\pi[$ the image $e^{-i\,\overline{\rho}_0\left(\widehat{\varphi},\widehat{\psi}\right)}$ must be on *one* side of $e^{-i\widehat{\psi}}$, i.e. either $\overline{\rho}\left(\widehat{\varphi}, \widehat{\psi}\right)$ is greater than $\widehat{\psi}$ for all $\widehat{\varphi} \in S^1 \backslash \tilde{S}^1$, or it is smaller. On the other hand there are arguments $\widehat{\varphi}_1$ and $\widehat{\varphi}_2$ from $\tilde{S}^1$ such that $\overline{\rho}_0\left(\widehat{\varphi}_1, \widehat{\psi}\right) = \overline{\rho}\left(\widehat{\varphi}_1, \widehat{\psi}\right)$ is *equal to, or greater* than $\widehat{\psi}$, *and* $\overline{\rho}\left(\widehat{\varphi}_2, \widehat{\psi}\right)$ *is equal to, or smaller* than $\widehat{\psi}$. But this is just the essence of the twist characteristic of α, and α is the extended prototype formalization of delaying reaction behavior.

Now let us proceed to the

Proof (of Proposition 2). Let an arbitrary advice $e^{-i\widehat{\psi}} \in \tilde{S}^1$ be given. Due to the assumptions the orbit of image points of $e^{-i\widehat{\psi}}$ under ρ_0, i.e. the set $\left\{\left(e^{-i\widehat{\varphi}}, e^{-i\,\overline{\rho}_0\left(\widehat{\varphi},\widehat{\psi}\right)}\right) | \widehat{\varphi} \in S^1\right\} \subset S^1 \times S^1$, is closed and its projection $\left\{e^{-i\,\overline{\rho}_0\left(\widehat{\varphi},\widehat{\psi}\right)} | \widehat{\varphi} \in S^1\right\}$ on the fiber S^1 *covers the whole* S^1, i.e., is *onto*. (It is even homotopic to the identity mapping of the S^1.) From the Intermediate Value Theorem and the last assumption of Proposition 2 follows that there must be at least one fixed point

$$\left(e^{-i\widehat{\varphi}}, e^{-i\widehat{\psi}}\right)$$

of ρ_0 with $\widehat{\varphi} \in \tilde{S}^1$. But for $\left(e^{-i\widehat{\varphi}}, e^{-i\widehat{\psi}}\right) \in \tilde{S}^1 \times \tilde{S}^1$ the mapping ρ_0 is identical with ρ, and thus $\left(e^{-i\widehat{\varphi}}, e^{-i\widehat{\psi}}\right)$ is also a fixed point of ρ, i.e. it is an accurately taken advice. $\square$

Analogously to the Corollary to Proposition 1 we have the following obvious

Corollary (to Proposition 2). If assumption (2) in Proposition 2 is replaced by the assumption that at least for one advice $e^{-i\widehat{\psi}}$ there is no fixed point $\left(e^{-i\widehat{\varphi}}, e^{-i\widehat{\psi}}\right)$ of ρ with $e^{-i\widehat{\varphi}} \in]3\pi/2, 2\pi[$, then there exists at least one accurately taken advice $\left(e^{-i\widehat{\varphi}_0}, e^{-i\widehat{\psi}}\right)$.

3 Utility Maximization by the Adviser

So far, the adviser's objective of argument justification has been analyzed in isolation from his other objectives addressed in the Introduction. As it has been pointed out before, the argument justification objective together with the sub-objective of being right with one's argument forms the *social reputation objective*. In addition to that, however, the adviser may also have a personal self-interest in the ultimate outcome of the subject.

Clearly these diverse objectives are likely to be conflicting. A reasonab1e way to represent analytically the simultaneous striving for conflicting objectives is given by maximizing a subjective utility function. This means the adviser chooses an advice $e^{-i\widehat{\varphi}}$ and an argument $e^{-i\widehat{\psi}}$ which maximize his utility function.

Let us take one point at a time. First, the two sub-objectives of the social reputation objective shall be balanced with each other through a utility function. Then the analysis will be extended further to include also the adviser's self-interest in the ultimate outcome.

Formally, the two sub-objectives of the reputation objective mean the following: as it has been pointed out in the previous section, the argument justification sub-objective means the search for fixed points $\left(e^{-i\widehat{\varphi}_0}, e^{-i\widehat{\psi}_0}\right)$ of a given reaction function

$$r : \left(e^{-i\widehat{\varphi}}, e^{-i\widehat{\psi}}\right) \mapsto \left(e^{-i\widehat{\psi}_t}, e^{-i\,\overline{r}\left(\widehat{\varphi},\widehat{\psi}\right)}\right).$$

The sub-objective of being right with one's argument on the other hand means to put forward *that argument* $e^{-i\widehat{\varphi}_1}$ which one considers the most probable one. These two sub-objectives are obviously *conflicting*, or *inconsistent*, if and only if there is no fixed point in the fiber over the base point $e^{-i\widehat{\varphi}_1}$, i.e. no point $\left(e^{-i\widehat{\varphi}_1}, e^{-i\widehat{\psi}}\right)$, $\widehat{\psi} \in \tilde{S}^1$, with $\overline{r}\left(\widehat{\varphi}_1, \widehat{\psi}\right) = \widehat{\psi}$. (The problem that the reaction function possibly may be not (completely) known to the adviser will be dispelled by assuming that in this case r just denotes the reactions which are expected by the adviser.)

For our formalization below we need a concept of *distance* between two points $e^{-i\widehat{\chi}}$ and $e^{-i\widehat{\omega}}$ of S^1. Let us take the absolute difference $|\widehat{\chi} - \widehat{\omega}|$ for this. (Of course, there is no restriction to angles $\widehat{\chi}, \widehat{\omega}$ to be chosen from the interval $[0, 2\pi[$.)

1. A natural *first formalization* of the adviser's subjective utility function is provided by

$$\begin{array}{rccl} u^I : & \tilde{S}^1 \times \tilde{S}^1 & \rightarrow & \mathbb{R} \\ & \left(e^{-i\widehat{\varphi}}, e^{-i\widehat{\psi}}\right) & \mapsto & \overline{u}^I\left(\left|\widehat{\varphi} - \widehat{\varphi}_1\right|, \left|\widehat{\psi} - \overline{r}\left(\widehat{\varphi}, \widehat{\psi}\right)\right|\right) \end{array}$$

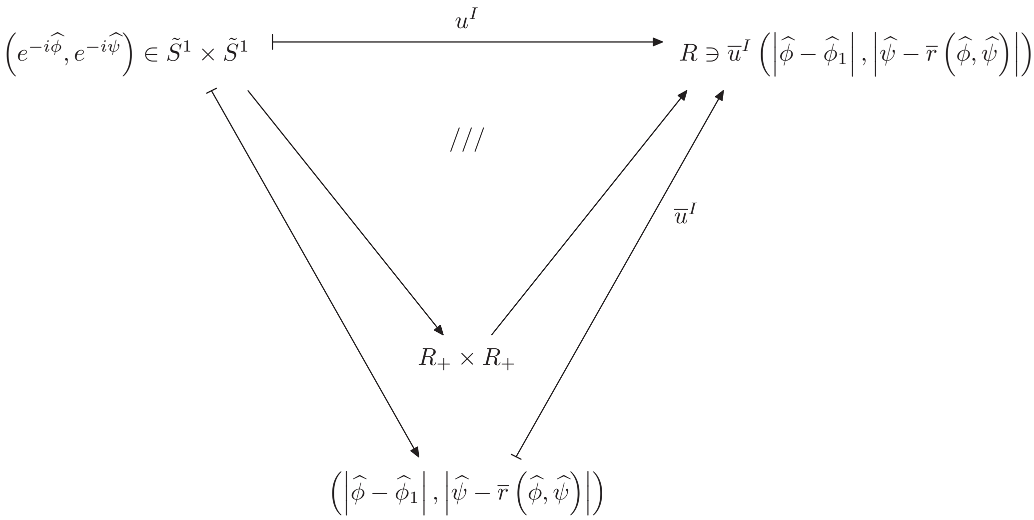
where the *partial functions* of $\overline{u}^I : \mathbb{R}_+ \times \mathbb{R}_+ \mapsto \mathbb{R}$,

$$\begin{array}{rcl} i.e. \quad \overline{u}^I_y(-) & := & \overline{u}^I(-, y) \\ and \quad \overline{u}^I_x(-) & := & \overline{u}^I(x, -) \end{array}$$

for arbitrary, but fixed, arguments $x, y \in \mathbb{R}_+$ are *decreasing*. (In differentiability terms one could write $\overline{u}^I_1(-, y) \underset{(-)}{\leq} 0$ and $\overline{u}^I_2(x, -) \underset{(-)}{\leq} 0$ where $\overline{u}^I_i$, $i = 1, 2$, as usual denotes the first partial derivative of $\overline{u}^I$ after the i-th argument.)

An equivalent representation of the utility function u^I can be given by the commutative diagram of Figure 5. Let us still have a closer look

Figure 5: Definition of u^I



at the properties of u^I. The symmetry properties of $\overline{u}^I$ just mean that the direction of deviation of the given argument $e^{-i\widehat{\varphi}}$ from the *favoured argument* $e^{-i\widehat{\varphi}_1}$ and of the given advice $e^{-i\widehat{\psi}}$ from the ultimate outcome $e^{-i\,\overline{r}\left(\widehat{\varphi}, \widehat{\psi}\right)}$ *does not matter*, but only the absolute values of the respective deviations.

However, it might be the case that the adviser prefers an argument $e^{-i\widehat{\varphi}}$ which *overstates* the importance and urgency, i.e. $\widehat{\varphi} > \widehat{\varphi}_1$, to the argument $e^{-i[\widehat{\varphi}_1-(\widehat{\varphi}-\widehat{\varphi}_1)]}$ which *understates* them by the same absolute value. Or, he may prefer an advice $e^{-i\widehat{\psi}}$ which is delayed under the argument $e^{-i\widehat{\varphi}}$, i.e. $\widehat{\psi} < \overline{r}\left(\widehat{\varphi}, \widehat{\psi}\right)$, to an advice $e^{-i\widehat{\chi}}$ which would for the argument $e^{-i\widehat{\varphi}}$ be exceeded by the target group by the same absolute value, i.e. $\overline{r}\left(\widehat{\varphi}, \widehat{\chi}\right) = \widehat{\chi} - \left[\overline{r}\left(\widehat{\varphi}, \widehat{\psi}\right) - \widehat{\psi}\right]$. Both motivations appear to be reasonable for an adviser who strives for avoiding to be held responsible for having advised carelessly.

On the contrary, however, the adviser may give priority to the *opposite view* in order to avoid to make a name for himself as a "Kassandra" and to wear out his arguments and advices for the future.

2. Formally, the latter considerations can be taken into account simply by removing the absolute bars of the arguments of $\overline{u}^I$. Thus, the following *second formalization* of the adviser's utility function obtains:

$$\begin{array}{rccc} u^{II}: & \tilde{S}^1 \times \tilde{S}^1 & \rightarrow & \mathbb{R} \\ & \left(e^{-i\widehat{\varphi}}, e^{-i\widehat{\psi}}\right) & \mapsto & \overline{u}^{II}\left(\widehat{\varphi} - \widehat{\varphi}_1, \widehat{\psi} - \overline{r}\left(\widehat{\varphi}, \widehat{\psi}\right)\right), \end{array}$$

where $\overline{u}^{II} : \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$.

(a) Let us now formalize the first addressed attitude of '*responsibility aversion*' by properties of the partial utility functions. In doing so we apparently have to take care of the fact that if we vary ψ then only the second argument of $\overline{u}^{II}$ varies, whereas, if we vary $\widehat{\varphi}$, then the first *and* the second argument of $\overline{u}^{II}$ vary. Therefore we formalize responsibility aversion by the following conditions:

i. fixing an arbitrary $\widehat{\varphi} \in]\widehat{\varphi}_1, 2/2\pi]$ then

$$\overline{u}^{II}\left(\widehat{\varphi} - \widehat{\varphi}_1, \widehat{\psi} - \overline{r}\left(\widehat{\varphi}, \widehat{\psi}\right)\right) < \overline{u}^{II}\left(\widehat{\varphi} - \widehat{\varphi}_1, \widehat{\chi} - \overline{r}\left(\widehat{\varphi}, \widehat{\chi}\right)\right)$$

for any $e^{-i\widehat{\psi}}, e^{-i\widehat{\chi}} \in \tilde{S}^1$ with $\widehat{\psi} > \overline{r}\left(\widehat{\varphi}, \widehat{\psi}\right)$ and $\widehat{\chi} - \overline{r}\left(\widehat{\varphi}, \widehat{\chi}\right) = -\left(\widehat{\psi} - \overline{r}\left(\widehat{\varphi}, \widehat{\psi}\right)\right) < 0$ (cf. Figure 6a).

ii. fixing an arbitrary $y \in \mathbb{R}$ then $\overline{u}^{II}(x, y) > \overline{u}^{II}(-x, y)$ for any $x \in \mathbb{R}$ (cf. Figure 6b).

Remark. One might think that it also should be possible to give a characterization directly by using the angles $\widehat{\varphi}$ and $\widehat{\psi}$. However, if one switches from $\widehat{\varphi} > \widehat{\varphi}_1$ to $2\widehat{\varphi}_1 - \widehat{\varphi}\,(< \widehat{\varphi}_1)$ which is equally far distant from $\widehat{\varphi}_1$, then the total net effect on $\overline{u}^{II}$ in general is ambiguous when $\overline{r}\left(-, \widehat{\psi}\right)$ is monotonically increasing with decreasing $\widehat{\varphi} \in [0, 3/2\pi]$: due to our assumption the switch in the first argument of $\overline{u}^{II}$ from $\widehat{\varphi}$ to $2\widehat{\varphi}_1 - \widehat{\varphi}$ has a *decreasing effect*. But the switch from $\widehat{\varphi}$ to $2\widehat{\varphi}_1 - \widehat{\varphi}$ also causes a change of the second argument of $\overline{u}^{II}$ from $\widehat{\psi} - \overline{r}\left(\widehat{\varphi}, \widehat{\psi}\right)$ to $\widehat{\psi} - \overline{r}\left(2\widehat{\varphi}_1 - \widehat{\varphi}, \widehat{\psi}\right)$. And due to i. above this change in the second argument of $\overline{u}^{II}$ clearly has an *increasing effect* when $\widehat{\psi} - \overline{r}\left(\widehat{\varphi}, \widehat{\psi}\right) > 0$ and $\widehat{\psi} - \overline{r}\left(2\widehat{\varphi}_1 - \widehat{\varphi}, \widehat{\psi}\right) > -\left(\widehat{\psi} - \overline{r}\left(\widehat{\varphi}, \widehat{\psi}\right)\right)$ (cf. Figure 5, replace $\widehat{\varphi}$ by $2\widehat{\varphi}_1 - \widehat{\varphi}$).

Using differentiability terms one alternatively could write

$$\left|\overline{u}_2^{II}\left(\widehat{\varphi} - \widehat{\varphi}_1, \widehat{\psi} - \overline{r}\left(\widehat{\varphi}, \widehat{\psi}\right)\right)\right| > \overline{u}_2^{II}\left(\widehat{\varphi} - \widehat{\varphi}_1, \widehat{\chi} - \overline{r}\left(\widehat{\varphi}, \widehat{\chi}\right)\right)$$

and

$$\left|\overline{u}_1^{II}(x, y)\right| < \overline{u}_1^{II}(-x, y)$$

(cf. Figure 6a,b).

(b) The second attitude of *Kassandra reputation aversion* is analogously formalized by the opposite relations in the above conditions. This is illustrated by Figure 7a,b.

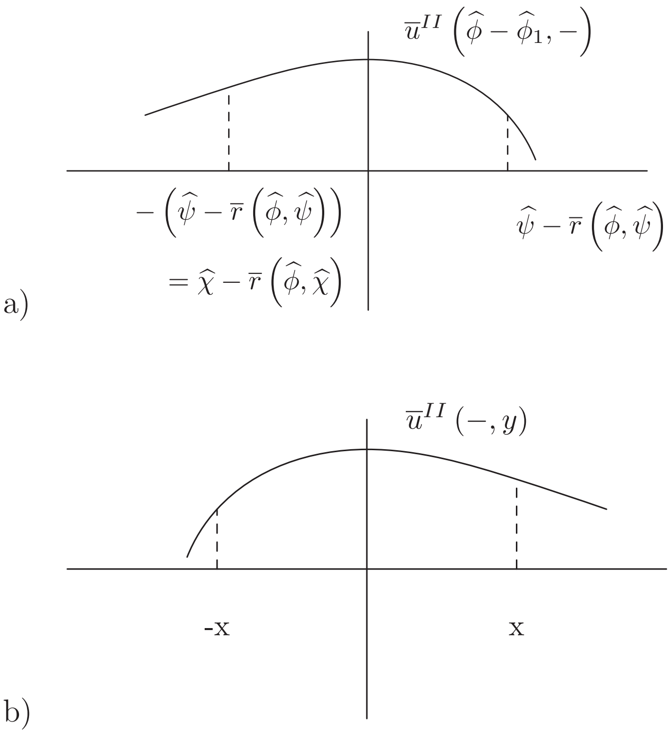
3. So far only the simultaneous striving for the two conflicting sub-objectives of the adviser's social reputation objective has been formalized. Now, the analytical formalization can be extended further by taking also into account a potential self-interest of the adviser in the final outcome $e^{-i\,\overline{r}\left(\widehat{\varphi}, \widehat{\psi}\right)}$.

Taken as an isolated motive the adviser's self interest will be represented in a natural way also by a utility function

$$\begin{array}{rrcl} u^s : & \mathbb{R} & \rightarrow & \mathbb{R} \\ & \overline{r}\left(\widehat{\varphi}, \widehat{\psi}\right) & \mapsto & u^s\left(\overline{r}\left(\widehat{\varphi}, \widehat{\psi}\right)\right) \end{array}$$

If, for instance, the adviser is a committed environmentalist, his utility function u^s will be *decreasing* in $\overline{r}\left(\widehat{\varphi}, \widehat{\psi}\right)$. If, on the contrary, he is biased by the interests of those who will be negatively affected by the realization of the measures, u^s will be an *increasing function.*

Figure 6: Partial Utility Functions of a Responsibility Averse Adviser



By means of u^s and the previously given u^{II} an *intuitive third formalization of the adviser's utility function* u^{III} can be provided:

$$\begin{array}{rccl} u^{III}: & \tilde{S}^1 \times \tilde{S}^1 & \rightarrow & \mathbb{R} \\ & \left(e^{-i\widehat{\varphi}}, e^{-i\widehat{\psi}}\right) & \rightarrow & \overline{u}^{III}\left[u^s\left(\overline{r}\left(\widehat{\varphi},\widehat{\psi}\right)\right), u^{II}\left(\widehat{\varphi}-\widehat{\varphi}_1, \widehat{\psi}-\overline{r}\left(\widehat{\varphi},\widehat{\psi}\right)\right)\right]. \end{array}$$

Clearly, the partial utility functions $\overline{u}^{III}(-,y)$ and $\overline{u}^{III}(x,-)$ are to be taken as *increasing* functions.

Two reasonable classes of type-III-utility functions shall be mentioned here. *First*, one may think of a simple *addition* of the two utility subfunctions, i.e.

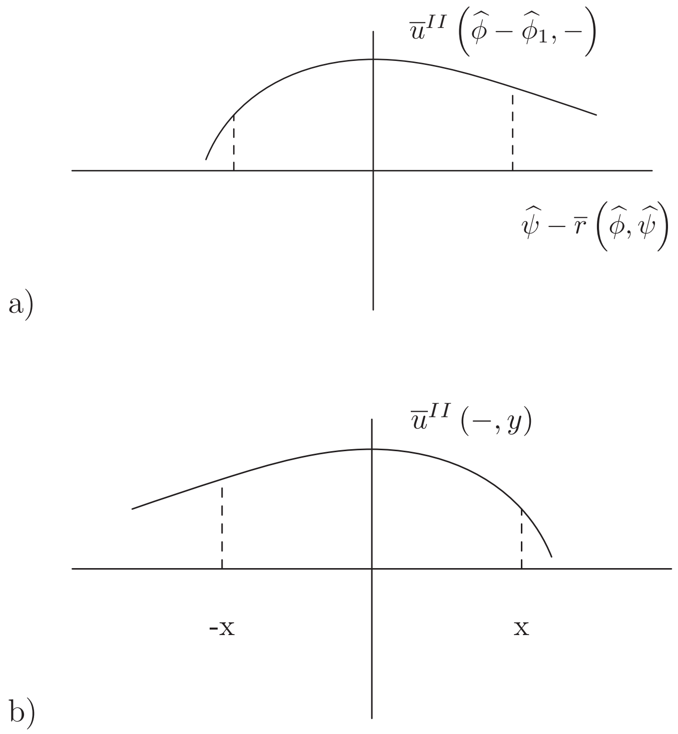
$$u^{III_1}\left(e^{-i\widehat{\varphi}}, e^{-i\widehat{\psi}}\right) = u^s\left(\overline{r}\left(\widehat{\varphi},\widehat{\psi}\right)\right) + u^{II}\left(\widehat{\varphi}-\widehat{\varphi}_1, \widehat{\psi}-\overline{r}\left(\widehat{\varphi},\widehat{\psi}\right)\right).$$

A *second* example is given by a *Cobb-Douglas type* utility function

$$u^{III_2}\left(e^{-i\widehat{\varphi}}, e^{-i\widehat{\psi}}\right) = \left(u^s\left(\overline{r}\left(\widehat{\varphi},\widehat{\psi}\right)\right)\right)^{\alpha_1} \cdot \left(u^{II}\left(\widehat{\varphi}-\widehat{\varphi}_1, \widehat{\psi}-\overline{r}\left(\widehat{\varphi},\widehat{\psi}\right)\right)\right)^{\alpha_2}$$

with the positive weights α_1 and α_2 summing up to $+1$.

Figure 7: Partial Utility Functions of a Kassandra Reputation Aversed Adviser



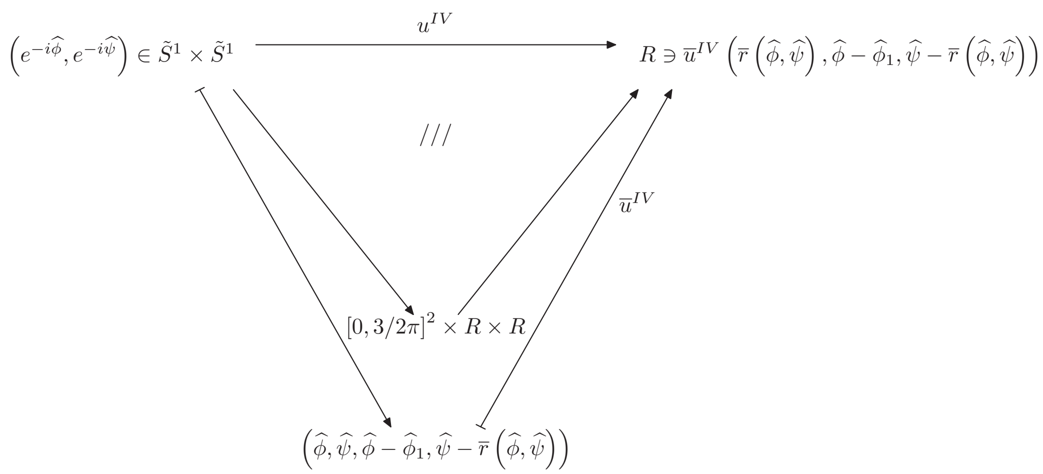
To interpret these two examples, the first one allows for a *total substitution* of striving for the reputation objective for striving for the self-interest objective and vice versa, whereas the second multiplicative Cobb-Douglas-type function does not.

4. The separable utility function u^{III}, however, turns out to be a special case of the following generalized *fourth formalization* u^{IV}:

$$\begin{array}{rrcl} u^{IV}: & \tilde{S}^1\times\tilde{S}^1 & \rightarrow & \mathbb{R} \\ & \left(e^{-i\widehat{\varphi}},e^{-i\widehat{\psi}}\right) & \mapsto & \overline{u}^{IV}\left(\overline{r}\left(\widehat{\varphi},\widehat{\psi}\right),\widehat{\varphi}-\widehat{\varphi}_1,\widehat{\psi}-\overline{r}\left(\widehat{\varphi},\widehat{\psi}\right)\right). \end{array}$$

The partial utility functions of $\overline{u}^{IV}$ have already been characterized by the previous considerations (see Fig. 8).

Figure 8: Definition of u^{IV}



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