

# Optimal Public Provision of Nursing Homes and the Role of Information\*

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## Abstract

Increasing demand for long-term care poses three challenges to the policy-maker: (i) How should care be supplied, formally within a nursing home or informally within the family, to individuals differing in the severity of their dependence; and (ii) at what level? (iii) How can financial strain be mitigated for families with severely dependent members? The problems are aggravated when individual severity is the family's private information. We consider a theoretical model of formal and informal care under adverse selection. Informal carers are assumed altruistic towards dependent family members. Nursing homes provide more effective care for severe cases but impose a disutility from being institutionalised on all cases. The regulator sets a transfer to redistribute consumption and, where relevant, to finance a public nursing home. We derive the allocations under full and asymmetric information

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with and without nursing home, respectively, and examine under which conditions a nursing home improves social welfare. Our main result is that by imposing a utility loss without offering greater effectiveness in the care for mildly dependent cases, the nursing home facilitates self-selection and possibly eliminates distortions in caring levels and transfers. Informational asymmetries may thus lead to care being provided too often within institutions rather than within a family context.

*Keywords:* adverse selection, long-term care, nursing home, redistribution

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# 1 Introduction

In recent years population ageing has moved into a top place of the policy agenda of most industrialised countries. As more and more people live to a high age, the ailments and frailty coming with oldest age are going to increase the demand for long-term care at population level even though it is recognised that most of the life years gained are healthy years.<sup>1</sup> It is therefore generally accepted that social expenditure for long-term care is going to remain significant. Current (year 2000) spending levels within OECD countries range between 0.2 per cent and 3 percent of GDP with the bulk lying between 0.5 per cent and 1.6 per cent of GDP (OECD 2005). Public expenditure on long-term care amounts to a share of 10 to 20 per cent of public health care spending. The increasing demand for long-term care poses three challenges to the policy-maker: (i) How should care be supplied, formally within a nursing home or informally within the family, to individuals differing in the severity of their dependence; and (ii) at what level? (iii) How can financial strain be mitigated for families with severely dependent members?

Issues (i) and (ii) receive already a considerable amount of attention in the policy arena. It is generally recognised that a trend towards a more flexible provision of care within a home or community context is welcome and, as levels of disability are declining, also feasible. Lakdawalla and Philipson (2002) argue that the market for long-term care is prone not only to exhibit an increase in demand but also an increasing supply as the younger old take over, at the point of retirement, caring responsibilities for the oldest old. Indeed, the supply of informal care explains very well the decrease in per capita output within the US market for long-term care over the period 1971-95. This notwithstanding, the market for nursing home care will continue to exist at significant levels as the scope for within family provision of care is limited both by the degree of disability and by the time constraints faced by informal carers. The mix and the interplay between formal and informal care is going to remain on the agenda. While issue (iii) has received perhaps less attention in the policy debate, policy-makers in a number of countries have come to

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<sup>1</sup>It is expected that as an OECD average one in four persons will belong to the group 65+ by the year 2040. At the same time the share of the oldest old (80+) within this group is going to increase from around one in five to one in three (OECD 2005).

recognise the financial strain on families who care for dependent members and seek to mitigate it by reimbursing informal carers for private expenditure and time allocated to the provision of care.<sup>2</sup> Where families with severely dependent members receive considerable financial transfers problems are prone to arise whenever the degree of dependency is the family's private information. It is easy to envisage that families try to exaggerate the degree of dependency and their efforts in the provision of care in order to become eligible for higher benefits. While there are a number of mechanisms for the policy-maker to reveal the degree of dependency, the inefficiencies related to them place a bound on the scope of redistribution and generally lead to a distorted allocation of care. Combining issues (iii) and (i) raises the question under which form of provision - formal or informal - the informational problems are greater or, in other words, whether one form of provision turns out to be superior on informational grounds.

In order to address these questions we consider a theoretical model of formal and informal care under adverse selection.<sup>3</sup> Informal carers are assumed altruistic towards dependent family members. Nursing homes provide more effective care for severe cases but impose a disutility from being institutionalised on all cases. The regulator sets a transfer to redistribute consumption and, where relevant, to finance a public nursing home. We derive the allocations under full and asymmetric information with and without nursing home, respectively, and examine under which conditions a nursing home improves social welfare. Under complete information the regulator fully compensates for the consumption loss suffered by families providing for severe cases. Institutional care for severe cases increases social welfare if and only if its effectiveness is sufficiently large relative to the disutility from institutionalisation. Asymmetric information generally leads to distorted caring levels and transfers both with and without nursing home and limits redistribution

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<sup>2</sup>These benefits may come both in cash or in-kind. See OECD (2005: Table 1.1) for an overview of different public long-term care programmes.

<sup>3</sup>We should stress that we do not consider adverse selection within an ex-ante insurance context, where the propensity to become severely dependent is private information. We rather consider a context of ex-post moral hazard, where individuals have an incentive to misreport the degree of (realised) dependency in order to receive greater benefits. In the concluding section 5 we comment on how our model can be read in the context of long-term-care insurance.

towards families with severely dependent members. By imposing a utility loss without offering more effective care for mildly dependent cases, the nursing home facilitates self-selection and possibly eliminates distortions in caring levels and transfers. We provide a rationale based on informational asymmetries for why care may be provided too often within institutions rather than within a family context.

Our model relates to two strands of literature. First, it contributes to the (theoretical) economics of long-term care.<sup>4</sup> Our work is closest in spirit to Pestieau and Sato (2004) and Jousten et al. (2005). Pestieau and Sato (2004) consider the mix in the provision of formal and informal care to dependent parents when their children differ in their productivity in the labour market. The study develops an optimal policy comprising the public provision of a nursing home, a subsidy paid to children providing informal care and a flat tax on earnings. While the model thus addresses issues of the right mix in the allocation of care under redistributive concerns it is set out under complete information only. Jousten et al. (2005) deals with the allocation of care within the family or within a nursing home when children differ in the degree of altruism towards their parents. While they study the optimal policy (transfer/provision of public nursing home) under asymmetric information as we do, their model differs in a number of respects: First, the adverse selection problem arises with respect to the degree of the children's altruism (perfect or not there at all); second their assumptions about the technology of the nursing home technology make it always inferior to the provision of care within the family; third, in their model the nursing home is always provided (only the level of care provided within the nursing home is endogenous), whereas in our model the decision of whether to provide a given level of care within a nursing home or within the family becomes an additional choice variable. Corresponding to these differences in the set-up our results are rather different. For instance, Jousten et al (2005) find that the level (quality) of nursing home care is distorted downwards in order to make the nursing home an unattractive option for altruistic children who would provide care for their parents themselves but may be willing to send them to a nursing home if this guarantees them a financial transfer. In our model, the provision of care for severely dependent parents is distorted upwards (within

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<sup>4</sup>See Norton (2000) for an overview on the economics of long-term care.

the nursing home or the family) as this makes it a more costly option for the children of only mildly dependent parents.

Second, our work contributes to the literature on the use of public expenditure for redistributive purposes (Nichols and Zeckhauser 1982, Blackorby and Donaldson 1988, Besley and Coate 1991, Boadway and Marchand 1995). This work studies how public expenditure has to be structured in order to allow for redistribution towards the poor subject to self-selection constraints. In Besley and Coate (1991) self-selection is achieved through quality reductions in publicly provided private goods (where the rich prefer a higher quality). In Boadway and Marchand (1995), individuals seek to attain an optimal provision by amending a public provision by private purchases. Here, the over-provision of the publicly provided good fully crowds out the private provision of a rich individual who then selects pure private provision. Our model goes beyond this literature in that it distinguishes the *level* of provision from the *technology* of provision. Technology is relevant in the following sense: For any level of care, provision within the nursing home is more effective for highly dependent types; while a nursing home leads to a direct utility loss. As it turns out, the planner facilitates self-selection (and redistribution) not only through distortions in the levels of provision (as was hitherto known) but also through distortions in the technology choice. In particular, a desire to reduce informational rents may lead to the choice of an ineffective or even 'hurtful' technology. The latter corresponds to a 'self-selection through ordeal' argument as was proposed but not formalised by Nichols and Zeckhauser (1982).

The remainder of the paper is structured as follows. The next section introduces the model, section 3 characterises the first-best provision for the family and nursing home, respectively, and provides a condition under which public nursing homes should be introduced. Section 4 provides the solution under asymmetric information leading up to our main result regarding the differences in the provision of nursing homes under first and second-best circumstances. Section 5 offers concluding remarks. Some of the more technical proofs are relegated to an appendix.

## 2 The model

We consider a model economy where an individual cares for a disabled relative. To fix ideas we suppose a setting where one parent has one child.<sup>5</sup> Parents are considered to have problems with activities of daily living (ADLs), i.e. they are in need of care. In general there are three different forms of care: (i) care provided within a family context (informal or *family care*), (ii) care purchased by the parent on the market (formal or *market care*), and (iii) inpatient care provided by a nursing home (*nursing home care*). For the purpose of the current paper we will not distinguish between family and market care but concentrate on the peculiarities of nursing home care and its implications for public policy.

There are two severity types,  $H$  (high) and  $L$  (low), and the share of high severity parents is denoted  $h \in (0, 1)$ . Let  $a \geq 0$  denote the level of care or *attention* a parent receives measured in money. Then utility of the parent derived from family or market care is given by

$$v_i^F(a) = v_i(a), \quad (1)$$

where the subscript  $i \in \{L, H\}$  refers to the severity type and the superscript  $F$  to family care. As usual we let  $v'_i > 0$  and  $v''_i < 0$ . The following assumptions are crucial:

$$\forall a \geq 0 : v_L(a) > v_H(a), \quad (2)$$

$$\forall a \geq 0 : v'_L(a) < v'_H(a). \quad (3)$$

Both assumptions are intuitive. Equation (2) simply is the definition of high severity. Attention or care is assumed to be more productive when the parent in need is a high severity type (equation (3)). An additional unit help is, at all care levels, more valuable for parents having major problems in performing ADLs.

If care is provided in a nursing home, the parent utility is

$$v_H^N(n) = \mu v_H(n) - \bar{v}, \quad (4)$$

$$v_L^N(n) = v_L(n) - \bar{v}, \quad (5)$$

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<sup>5</sup>Other obvious settings include an individual caring for her or his spouse or a parent caring for a disabled child.

where  $n \geq 0$  is the amount per patient spent by the nursing home. The parameter  $\bar{v} \geq 0$  captures the disutility parents suffer when being moved from their accustomed environment to an institution.<sup>6</sup> We consider this loss to be independent from severity. In contrast, the effectiveness/productivity of the nursing home hinges on severity: for any given level of care provided to  $H$ -types, the nursing home is at least as productive as family care, i.e.  $\mu \geq 1$ . Whereas the nursing home has no productivity advantage for  $L$ -types. Thus, from a parent perspective, low severity types should never be taken care of in a nursing home. The overall costs of the nursing home are  $\theta n$ , where  $\theta \in [0, 1]$  is the share of parents that are taken care of in a nursing home, and financed by taxation.

The utility of the child is given by

$$U_i = u(c) + \beta v_i(a). \quad (6)$$

All children have an identical endowment  $y > 0$ . They can provide informal care,  $a$ , to their parent and have net income (consumption)  $c = y - a - T$ , where  $T$  is a tax (subsidy) paid to (by) the government. The consumption utility is given by the first term of equation (6). As is standard we assume

$$u' = \frac{du}{dc} > 0 \text{ and } u'' = \frac{d^2u}{dc^2} < 0.$$

The second term of equation (6) captures that children typically care about the wellbeing of their parents. The parameter  $\beta$  will in general lie between 0 and 1. To avoid a conflict of interest between children and the social planner we consider children to be perfect altruists, i.e.  $\beta = 1$ .<sup>7</sup> Note that our model then has an alternative interpretation with  $u$  as the consumption utility of the “parent” and  $a$  the monetary resources (s)he allocates to care. These resources may buy care on the private market ( $a$  is formal care) or may be a transfer to the child (bequest) that stimulates attention ( $a$  is informal care). Thus,

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<sup>6</sup>When parents were living with their children,  $\bar{v}$ , can be understood as the dread from separation. However, in many issues, family or ambulatory care is provided to a parent who is still living in their own property. In this case  $\bar{v}$  is the disutility from giving up their ‘own place’.

<sup>7</sup>Jousten et al. (2005) consider the degree of altruism  $\beta$  to be private information of the child. Using an optimal taxation model they derive the second-best optimal long-term care insurance.

our model is more general than it may at first appear. In particular, when adopting the alternative interpretation, altruism is not necessary for our results.

For  $\beta = 1$  the social welfare function is given by

$$W = hU_H + (1 - h)U_L. \quad (7)$$

The social planner is now maximizes (6) subject to the budget constraint

$$hT_H + (1 - h)T_L = \theta n. \quad (8)$$

With type-dependent income taxes and type-contigent nursing home care provision, however, the planner's ability to directly observe severity levels is crucial. In the following Section 3 severity is considered observable and the optimum of the maximization problem will be referred to as first-best. In Section 4 severity is private information of the family. Type contigent contracts thus have to be incentive compatible and a second-best optimal long-term care arrangement results.

### 3 First-best long-term care

We start out by deriving the optimal long-term care arrangement without nursing homes ( $\theta = 0$ ). We then derive the optimal policy with nursing homes when all high severity parents are allocated to a nursing home ( $\theta = h$ ). Finally, we ask whether and when the provision of nursing homes is efficient.

#### 3.1 The optimal policy without nursing homes (case 1)

The social planner maximizes the objective function  $W_F$ , given in equation (7), with respect to  $a_H$ ,  $a_L$ ,  $T_H$ , and  $T_L$  subject to the budget constraint  $hT_H + (1 - h)T_L = 0$ . The index 'F' indicates that we are in the case with family care only. Using the budget constraint to substitute for  $T_H$  we get

$$W_F = h \left( u\left(y - a_H + \frac{1-h}{h}T_L\right) + v_H(a_H) \right) + (1-h) \left( u(y - a_L - T_L) + v_L(a_L) \right). \quad (9)$$

The first-order conditions are

$$\begin{aligned}\frac{\partial W_F}{\partial a_H} &= h(-u'_H + v'_H) = 0, \\ \frac{\partial W_F}{\partial a_L} &= (1-h)(-u'_L + v'_L) = 0, \\ \frac{\partial W_F}{\partial T_L} &= (1-h)u'_H - (1-h)u'_L = 0,\end{aligned}$$

with  $u'_i := u'(c_i)$ . This is the standard optimal taxation result that can be summarized as

$$v'_H(a_H^F) = u'(c_H^F) = u'(c_L^F) = v'_L(a_L^F), \quad (10)$$

where the super-script ‘ $F$ ’ is used to denote the first-best values with family care only.

**Lemma 1** *The first-best policy without nursing homes has the following properties:*

- (i)  $a_H^F > a_L^F$ ,
- (ii)  $c_H^F = c_L^F = y - ha_H^F - (1-h)a_L^F$  and
- (iii)  $T_L^F = h(a_H^F - a_L^F) > 0 > (1-h)(a_L^F - a_H^F) = T_H^F$ .

The results of the proposition are quite intuitive: The first result (i) simply states that more family care is provided, when care is more productive, i.e. when  $H$ -types need assistance. Like in optimal direct taxation models with perfect information, the utilitarian social planner eliminates income inequality and, as shown in (ii), identical consumption levels obtain. Obviously, income equalization with type-dependent care levels can only be achieved through redistributive taxation, where children of  $H$ -types receive a net transfer and  $L$ -types have to pay taxes.

To see the structural equivalence to the standard optimal direct taxation framework (e.g. Stiglitz 1982), consider that  $y$  is the time endowment of an individual. If the individual refrains from supplying labour it receives utility  $v_i(y)$ . With labour supply  $c_i$ , however, the individual gets  $u(c_i) + v_i(y - c_i)$ . Let  $a_i \equiv y - c_i$  then we are in the framework considered here, though, the interpretation is different. Note that in the optimal direct taxation interpretation of the model the  $L$ -types are the more productive individuals since they can provide an additional unit labour at lower cost than  $H$ -types,  $v'_L < v'_H$ .

### 3.2 The optimal policy with nursing homes (case 2)

As argued above, given the utility of  $L$ -types from nursing home care (5) it cannot be efficient to allocate low severity patients to a nursing home. With perfect information we thus have  $\theta = h$ . Using (1), (4), (6) and substituting  $T_H = n - \frac{1-h}{h}T_L$  from the budget constraint the social objective is

$$W_N = h \left( u\left(y + \frac{1-h}{h}T_L - n\right) + \mu v_H(n) - \bar{v} \right) + (1-h) (u(y - a_L - T_L) + v_L(a_L)), \quad (11)$$

where the index ‘ $N$ ’ is used to denote the case with nursing homes for the  $H$ -types and family care for the  $L$ -types.

The first-order conditions are

$$\frac{\partial W_N}{\partial n} = h (-u'_H + \mu v'_H) = 0, \quad (12)$$

$$\frac{\partial W_N}{\partial a_L} = (1-h) (-u'_L + v'_L) = 0, \quad (13)$$

$$\frac{\partial W_N}{\partial T_L} = (1-h)u'_H - (1-h)u'_L = 0, \quad (14)$$

implying

$$\mu v'_H(n^N) = u'(c_H^N) = u'(c_L^N) = v'_L(a_L^N), \quad (15)$$

where we use the super-script ‘ $N$ ’ to denote the first-best variables when  $H$ -types are allocated to a nursing home.

**Lemma 2** *If  $H$ -types are allocated to a nursing home and if  $\mu > 1$ , then the optimal policy has the following properties:*

(i)  $n^N > a_H^F > a_L^F > a_L^N$ ,

(ii)  $c_H^N = c_L^N = y - hn^N - (1-h)a_L^N$  and

(iii)  $T_H^N = hn^N + (1-h)a_L^N > h(n^N - a_L^N) = T_L^N > 0$ .

(iv) For  $\mu = 1$ , we have  $n^N = a_H^F > a_L^F = a_L^N$  and  $c_H^N = c_L^N = c_H^F = c_L^F$ .

Again, the results are intuitive. (i) Due to the higher productivity of nursing homes as compared to the family,  $H$ -types receive more care with nursing homes than without. In turn, this implies a higher care level for  $H$ -types than for  $L$ -types. (ii) The social

planner continues to equalize consumption levels. (iii) Since nursing homes improve the care technology the overall resources spend for care are higher and, consequently, the consumption levels are lower than without nursing home care.

### 3.3 First-best public provision of nursing home

Let  $W_N^*$  and  $W_F^*$  denote the maximised welfare functions with and without nursing home respectively. The social welfare gain achieved by the introduction of publicly provided nursing homes is then given by  $\Delta^* := W_N^* - W_F^*$ . First-best nursing home provision is then characterized by the following decision rule: provide nursing home care to  $H$ -types if and only if  $\Delta^* \geq 0$ . Using (9) and (11) and observing that consumption is equalised across types independent of whether nursing home care is available or not, we can write

$$\Delta^*(\bar{v}, \mu) = \begin{aligned} & u(c_H^N) - u(c_H^F) + h [\mu v_H(n^N) - \bar{v} - v_H(a_H^F)] \\ & + (1-h) [v_L(a_L^N) - v_L(a_L^F)] \end{aligned} \quad (16)$$

**Lemma 3** *The function  $\Delta^*(\bar{v}, \mu)$  has the following properties:*

- (i)  $\frac{d\Delta^*}{d\bar{v}} = -h < 0$ ,
- (ii)  $\frac{d\Delta^*}{d\mu} = h v_H(n^N) > 0$ ,
- (iii)  $\lim_{\bar{v} \rightarrow \infty} \Delta^*(\bar{v}, \mu) = -\infty$ ,
- (iv)  $\lim_{\mu \rightarrow \infty} \Delta^*(\bar{v}, \mu) = \infty$
- (v)  $\Delta^*(0, \mu) > (=) 0$  if  $\mu > (=) 1$ ,
- (vi)  $\frac{d\bar{v}}{d\mu} |_{\Delta^*=0} = v_H(n^N) > 0$ ,
- (vii)  $\frac{d^2\bar{v}}{d\mu^2} |_{\Delta^*=0} = v'_H(n^N) \frac{dn^N}{d\mu} > 0$

**Proof.** See Appendix. ■

The first four properties are rather intuitive. When the disutility of nursing home care increases, the provision of nursing homes becomes less desirable (i). This holds also in the limit where costs are prohibitive (iii). If there is no disutility, however, social welfare can be increased whenever nursing homes improve the productivity of care, i.e. when  $\mu > 1$  (v). The social planner is indifferent between nursing homes and family care when  $\bar{v} = 0$  and  $\mu = 1$ . An increase in the productivity of nursing homes makes them socially more desirable (ii), and again, this holds in the limit (iv). The social planner is indifferent

between family care only and nursing home provision for  $H$ -types whenever  $\Delta^* = 0$ . For every  $\mu \geq 1$  define

$$\bar{v}^*(\mu) := \arg_{\bar{v}} \{\Delta^*(\bar{v}, \mu) = 0\}. \quad (17)$$

Strict monotonicity of  $\Delta^*$  in its arguments guarantees that  $\bar{v}^*(\mu)$  is a singleton. Thus,  $\bar{v}^* : [1, \infty) \rightarrow [0, \infty)$ ,  $\mu \mapsto \bar{v}^*(\mu)$  is a function that is, due to the properties (vi) and (vii), increasing and convex in  $\mu$ .

**Proposition 1** *The function  $\bar{v}^*(\mu)$  defines a locus in the nursing home technology space  $(\bar{v}, \mu)$  such that  $\Delta^* = 0$  on the locus,  $\Delta^* < 0$  for all pairs  $(\bar{v}, \mu)$  with  $\bar{v} > \bar{v}^*(\mu)$  and  $\Delta^* > 0$  for all pairs  $(\bar{v}, \mu)$  with  $\bar{v} < \bar{v}^*(\mu)$ .*

The proof follows directly from the properties of  $\Delta^*(\bar{v}, \mu)$  and  $\bar{v}^*(\mu)$  given in Lemma 1. This proposition tells us that the public provision of nursing homes is efficient whenever the productivity gain,  $\mu - 1$ , is large enough as compared to the disutility of being institutionalised,  $\bar{v}$ . This result is illustrated in Figure 1 below where nursing home provision is efficient to the south-east of  $\bar{v}^*(\mu)$  (the shaded area in Figure 1) and inefficient to the north-west of  $\bar{v}^*(\mu)$ .

## 4 Care provision under asymmetric information

### 4.1 The second-best without nursing home (case 3)

In the following, we assume that severity is the family's private information and unknown to the planner. When designing the transfer system, the social planner then needs to obey the incentive compatibility (IC) constraints

$$u(y - a_H - T_H) + v_H(a_H) \geq u(y - a_L - T_L) + v_H(a_L) \quad (\text{ICH})$$

$$u(y - a_L - T_L) + v_L(a_L) \geq u(y - a_H - T_H) + v_L(a_H). \quad (\text{ICL})$$

for the  $H$ -type and  $L$ -type, respectively. Recall from the first-best that  $a_H^F > a_L^F$  and  $c_H^F = c_L^F$ . From this, it follows that (ICL) is violated in first-best, whereas (ICH) holds as

a strict inequality. From (ICH) and (ICL) we obtain the monotonicity (M) condition

$$\begin{aligned}
[v_H(a_H) - v_H(a_L)] - [v_L(a_H) - v_L(a_L)] &\geq 0 \\
&\Leftrightarrow (v'_H - v'_L)(a_H - a_L) \geq 0 \\
&\Leftrightarrow a_H - a_L \geq 0, \tag{M}
\end{aligned}$$

where the last inequality follows from our assumption  $v'_H \geq v'_L$ . It will turn out that (M) will generally be satisfied for our problem. The social planner maximises (9) subject to the budget constraint  $hT_H + (1-h)T_L = 0$ , (ICH) and (ICL). Denoting by  $\psi_H$  and  $\psi_L$ , with  $\psi_H \geq 0; \psi_L \geq 0$ , the Langrangean multiplier of (ICH) and (ICL), respectively, and substituting for  $T_H = -\frac{1-h}{h}T_L$  we obtain the following set of first-order conditions for  $a_H$ ,  $a_L$  and  $T_L$ .

$$(h + \psi_H)(-u'_H + v'_H(a_H)) - \psi_L(-u'_H + v'_L(a_H)) = 0, \tag{18}$$

$$(1 - h + \psi_L)(-u'_L + v'_L(a_L)) - \psi_H(-u'_L + v'_H(a_L)) = 0, \tag{19}$$

$$(1 - h)(u'_H - u'_L) + (\psi_H - \psi_L) \left( \frac{1-h}{h}u'_H + u'_L \right) = 0. \tag{20}$$

Denoting second-best variables with a ' $\widehat{\cdot}$ ', and continuing to use ' $F$ ' for the case in which care is provided within the family only, we can characterise the solution as follows.

**Lemma 4** *When care is provided within the family, the second-best allocation under asymmetric information has the following properties:*

(i) *General structure: The second-best allocation involves  $\widehat{\psi}_L > \widehat{\psi}_H = 0; \widehat{c}_H^F < \widehat{c}_L^F$  and  $\widehat{a}_H^F > \widehat{a}_L^F$ .*

(ii) *Distortions:  $\widehat{a}_L^F$  is conditionally efficient, i.e. satisfies  $u'_L = v'_L(\widehat{a}_L^F)$  and  $\widehat{a}_H^F$  is distorted upwards, i.e. satisfies  $u'_H > v'_H(\widehat{a}_H^F)$ .*

**Proof.** See Appendix. ■

Under adverse selection, the planner has to concede an information rent to  $L$ -types in order to induce them to reveal truthfully their identity. The rent drives a wedge between the consumption levels allocated to the two types, where  $L$ -types are allowed a greater consumption. Thus as is common in models of income taxation, asymmetric information

reduces the scope for redistribution. The planner extracts some of the  $L$ -type's rent and, thereby, increase the scope for redistribution, by raising the level of informal care for the  $H$ -type. This works as excessive home care makes the allocation unattractive for the  $L$  type.<sup>8</sup>

It is difficult to make any statements as to how the second-best levels of care and consumption compare to the first-best. Intuitively, one would expect perhaps that the transfer paid by the  $L$ -type is lowered, such that  $\widehat{T}_L^F < T_L^F$  and, thus,  $\widehat{c}_L^F > c_L^F = c_H^F > \widehat{c}_H^F$ .<sup>9</sup> However, it is undetermined whether the level of care allocated to the  $H$ -types,  $\widehat{a}_H^F$ , is above or below its first-best level,  $a_H^F$ . While  $\widehat{c}_H^F < c_H^F$  would suggest  $\widehat{a}_H^F < a_H^F$ , the upward distortion for efficiency reasons may imply the opposite. But then it cannot be ruled out that extreme distortions in  $\widehat{a}_H^F$  overturn the 'intuitive' solution. For instance, for  $\widehat{a}_H^F \gg a_H^F$ , it is possible that  $\widehat{T}_L^F > T_L^F$  so that  $\widehat{c}_H^F < \widehat{c}_L^F < c_L^F$ . Here,  $H$ -types are forced to provide so much (excessive) care that the transfer paid by the  $L$ -types is increased. Consequently consumption is lower for both types. Alternatively, for  $\widehat{a}_H^F < a_H^F$  we may find  $c_H^F < \widehat{c}_H^F < \widehat{c}_L^F$ . In this case, care is under-provided to  $H$ -patients but the scope for consumption increases for both types.<sup>10</sup>

## 4.2 The second-best with nursing home (case 4)

We now turn to the remaining case, where care is provided within a nursing home under asymmetric information about severity. Recall that it is suboptimal to accommodate  $L$  types in the nursing home. Thus, assuming that the nursing home cannot turn down

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<sup>8</sup>Note that either effort  $a_i$  or gross income  $y - a_i$  is observable/contractible. The second option concurs with the income taxation literature. However, strictly speaking we would have the choice of leisure as a further confounding variable. The first option may appear to be at odds with the usual assumption of unobservable effort. Yet, in our model, observable effort does not relax the information problem (about severity). For instance, in Germany transfers to the family for the provision/purchase of home care in are linked to a measure of the hours of care provided/purchased.

<sup>9</sup>This would also imply  $\widehat{a}_L^F > a_L^F$ .

<sup>10</sup>As the comparison between the second-best and first-best levels is not substantive for our subsequent analysis, we do not provide a formal derivation of these results. A graphical representation of the solution, which shows the scope for the different outcomes is available from the authors upon request.

patients, the following IC constraints must hold

$$u(y - T_H) + \mu v_H(n) - \bar{v} \geq u(y - a_L - T_L) + v_H(a_L), \quad (\text{ICHN})$$

$$u(y - a_L - T_L) + v_L(a_L) \geq u(y - T_H) + v_L(n) - \bar{v}. \quad (\text{ICLN})$$

The constraint (ICHN) must hold as the planner cannot force even severe types into a nursing home. (ICHN) and (ICLN) imply the monotonicity condition

$$\mu v_H(n) - v_H(a_L) \geq v_L(n) - v_L(a_L), \quad (\text{MN})$$

which is satisfied if  $n \geq a_L$ . The social planner maximises (11) subject to the budget constraint  $h(T_H - n) + (1-h)T_L = 0$ , (ICHN) and (ICLN). Continuing to use  $\psi_i, i = H, L$  as Lagrangean multiplier on constraint (ICiN), the first-order conditions for  $n, a_L$  and  $T_L$  are given by

$$(h + \psi_H)(-u'_H + \mu v'_H(n)) - \psi_L(-u'_H + v'_L(n)) = 0, \quad (21)$$

(19) and (20). The situation turns out to be more complex than in the case without nursing home. This is because the direct utility loss from nursing care,  $\bar{v}$ , may induce countervailing incentives in the following sense.<sup>11</sup> If  $\bar{v}$  is low the situation is similar to case 3, where  $L$ -types have an incentive to mimic  $H$ -types in order to obtain a greater financial transfer. However, in contrast to the family setting, posing as an  $H$ -type now comes with a direct utility loss for the parent when being institutionalised. This relaxes (ICLN), and for intermediate values of  $\bar{v}$  both (ICLN) and (ICHN) may be slack. However, if  $\bar{v}$  grows large enough then (ICHN) binds. Here, children of  $H$ -types have to be given an incentive to send their parents to a nursing home despite the direct disutility  $\bar{v}$ . The following lemma provides a more formal characterisation of the three regimes.

**Lemma 5** *Consider the first-best allocation  $\{n^N, a_L^N, c_H^N, c_L^N\}$ . The second-best allocation then involves (i) regime 1 with  $\hat{\psi}_L > \hat{\psi}_H = 0$  if and only if  $\bar{v} < v_L(n^N) - v_L(a_L^N)$ ; (ii) regime 2 with  $\hat{\psi}_L = \hat{\psi}_H = 0$  if and only if  $\bar{v} \in [v_L(n^N) - v_L(a_L^N), \mu v_H(n^N) - v_H(a_L^N)]$ ; and (iii) regime 3 with  $0 = \hat{\psi}_L < \hat{\psi}_H$  if and only if  $\bar{v} > \mu v_H(n^N) - v_H(a_L^N)$ .*

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<sup>11</sup>See Laffont and Martimort (2002: section 3.1) on countervailing incentives.

**Proof.** Note that  $n^N > a_L^N$  so that (MN) is satisfied and the interval referred to in part (ii) of the Lemma is non-empty. The Lemma follows immediately from inserting the inequalities in (i) and (iii) into (ICHN) and (ICLN), respectively. ■

The second-best allocation within each of the regimes is then characterised as follows.

**Lemma 6** *When H-types are cared for in a nursing home, the second-best allocation under asymmetric information has the following properties:*

*Regime 1: (i)  $\widehat{c}_H^N < \widehat{c}_L^N$ ; (ii)  $\widehat{a}_L^N$  conditionally efficient; and (iii)  $\widehat{n}^N$  distorted upwards.*

*Regime 2:  $\widehat{c}_i^N = c_i^N$ ,  $\widehat{a}_L^N = a_L^N$ ,  $\widehat{n}^N = n^N$ , i.e. the first-best.*

*Regime 3: (i)  $\widehat{c}_H^N > \widehat{c}_L^N$ ; (ii)  $\widehat{n}^N$  conditionally efficient; and (iii)  $\widehat{a}_L^N$  could be distorted upwards or downwards.*

**Proof.** Regime 1: As  $\widehat{\psi}_L > \widehat{\psi}_H = 0$  the proof follows in analogy to the proof of part (ii) of Lemma 4.

Regime 2: For  $\widehat{\psi}_L = \widehat{\psi}_H = 0$  the first-best is realised.

Regime 3: Here,  $0 = \widehat{\psi}_L < \widehat{\psi}_H$ . It then follows from (20) that  $\widehat{c}_H^N > \widehat{c}_L^N$ . Conditional efficiency of  $\widehat{n}^N$  follows from (21), where  $\widehat{\psi}_L = 0$ . Finally, using  $\widehat{\psi}_L = 0$  in (19) we obtain  $(1 - h)(u'_L - v'_L(a_L)) = \psi_H(u'_L - v'_H(a_L))$ . Noting that  $u'_L - \mu v'_H(n) = u'_L - u'_H > 0$  and that  $v'_H(a_L) \leq \mu v'_H(n)$  depends on  $\mu$ , we see that neither  $u'_L > \max\{v'_H(a_L), \mu v'_H(n)\}$  nor  $v'_H(a_L) > u'_L > \mu v'_H(n)$  can be ruled out. ■

The allocation thus depends on the direct disutility of parents sent to a nursing home,  $\bar{v}$ . For low levels of  $\bar{v}$  (regime 1) the allocation is similar to the one without nursing home. Note, however, the upward distortion in the provision of nursing care. This stands in contrast to the finding by Jousten et al. (2005), where care is underprovided in the nursing home. As for the case of family care, we cannot make statements about the deviation from their first-best values of the second-best choices. In particular, we cannot assess whether  $\widehat{n}^N$  lies above or below the first-best level  $n^N$ . Whereas rent extraction calls for an upward distortion,  $\widehat{n}^N < n^N$  cannot be ruled out. This situation may arise if the marginal utility of nursing care for the  $L$ -type,  $v'_L(\widehat{n}^N)$  is relatively high as compared to the marginal utility of consumption  $u'_H$ . In this case, there is not much scope to extract rents by way of over-provision of care. The excess provision of nursing care required to

allow for more redistribution may then turn out to be so high that both types lose out on consumption. In this case, it may be more effective to under-provide care  $\widehat{n}^N \ll n^N$  and, thereby, grant more consumption to both types.

For high levels of  $\bar{v}$  (regime 3) the incentive problem is reversed, where the children of  $H$  types would have to be given an incentive to send their parents to a nursing home. This is done by granting them a higher level of consumption (i.e. reducing the tax/fee for nursing care) and allocating an efficient quality of nursing care. Furthermore, family care is rendered unattractive for  $H$  types by imposing a distortion in the level of care that is subsidised. The direction of the distortion depends on the preferences of the  $H$ -type. If for the level of consumption  $\widehat{c}_L^N$  granted to  $H$ -children, when mimicking an  $L$  type,  $H$ -parents still have a high propensity to benefit even from family care, i.e. if  $v'_H(\widehat{a}_L^N) > u'_L$ , then the level of family care,  $\widehat{a}_L^N$ , is distorted downwards from its efficient level. The converse is true if  $H$ -parents do not stand to gain too much from family care, i.e. if  $v'_H(\widehat{a}_L^N) < u'_L$ .

Notably, for intermediate levels of  $\bar{v}$  (regime 2) the first-best is attainable. As they do not benefit from the greater effectiveness of the nursing home,  $L$  types stand to benefit less from a nursing home than  $H$  types. But then if the disutility from nursing care  $\bar{v}$  is sufficiently large but not too large, the nursing home becomes an unattractive option to  $L$  types, while retaining attraction for  $H$  types. Here, natural separation is feasible.

### 4.3 Second-best public provision of nursing home

Proceeding along similar lines as for the first best, let  $\widehat{W}_F$  and  $\widehat{W}_N$  denote the maximised second-best welfare functions with and without nursing home respectively. The net welfare gain from the introduction of a nursing home in a second-best environment is then given by  $\Delta^{**} := \widehat{W}_N - \widehat{W}_F$ , where a nursing home should be introduced to provide care for  $H$  types if and only if  $\Delta^{**} \geq 0$ . Inserting the respective second-best variables we can write

$$\begin{aligned} \Delta^{**}(\bar{v}, \mu) &= h \left[ u(\widehat{c}_H^N) + \mu v_H(\widehat{n}^N) - \bar{v} - u(\widehat{c}_H^F) - v_H(\widehat{a}_H^F) \right] \\ &\quad + (1 - h) \left[ u(\widehat{c}_L^N) + v_L(\widehat{a}_L^N) - u(\widehat{c}_L^F) - v_L(\widehat{a}_L^F) \right]. \end{aligned} \quad (22)$$

Note that due to informational rents, there is no longer an equalisation of consumption across types. The second best-allocation  $\{\widehat{n}^N, \widehat{a}_L^N, \widehat{c}_H^N, \widehat{c}_L^N\}$  depends on both  $\bar{v}$  and  $\mu$  in a

non-trivial way. Furthermore, the precise relationships change across regimes 1-3. Thus, it is difficult to characterise the locus for which  $\Delta^{**}(\bar{v}, \mu) = 0$  and compare it to the locus  $\bar{v}^*(\mu)$ , where the planner is indifferent in the first-best between whether care is provided within the family or within an institution. In order to gain some leeway, we draw on a graphical representation in  $(\mu, \bar{v})$  space. This is illustrated in Figure 1, in which we have already included the locus  $\bar{v}^*(\mu)$ . Recall that nursing care is preferable from a first-best point of view to the SE of  $\bar{v}^*(\mu)$ , i.e. in the shaded area in Figure 1.

**Insert Figure 1 here**

Noting that the allocation in the first-best does not depend on  $\bar{v}$  so that  $\frac{dn^N}{d\bar{v}} = \frac{da_L^N}{d\bar{v}} = 0$  we can make use of the following definitions. Let

$$\bar{v}^-(\mu) := v_L(n^N) - v_L(a_L^N) \tag{23}$$

$$\bar{v}^+(\mu) := \mu v_H(n^N) - v_H(a_L^N) \tag{24}$$

define the boundaries between regimes 1 and 2 [equation (23)] and regimes 2 and 3 [equation (24)], respectively. The following lemma provides a further characterisation of the boundaries  $\bar{v}^-(\mu)$  and  $\bar{v}^+(\mu)$ .

**Lemma 7** (i)  $\frac{\partial \bar{v}^-}{\partial \mu} = v'_L(n^N) \frac{dn^N}{d\mu} - v'_L(a_L^N) \frac{da_L^N}{d\mu} > 0$  and  $\frac{\partial \bar{v}^+}{\partial \mu} = v_H(n^N) + \mu v'_H(n^N) \frac{dn^N}{d\mu} - v'_H(a_L^N) \frac{da_L^N}{d\mu} > 0$ .

(ii)  $\bar{v}^+(\mu) > \max\{\bar{v}^*(\mu), \bar{v}^-(\mu)\}$  for all  $\mu$

(iii) If  $v_i''' \leq 0$ , then there exists a unique  $\mu^*$  with  $\mu^* \in (1, \infty)$  such that  $\bar{v}^-(\mu) \geq \bar{v}^*(\mu) \Leftrightarrow \mu \leq \mu^*$ .

**Proof.** See Appendix. ■

The lemma confirms the clear-cut ordering of regimes 1-3 in  $(\mu, \bar{v})$  space: Regime 1 for  $\bar{v} < \bar{v}^+(\mu)$ , regime 2 for  $\bar{v} \in [\bar{v}^-(\mu), \bar{v}^+(\mu)]$  and regime 3 for  $\bar{v} > \bar{v}^-(\mu)$ . More importantly, it establishes that the  $\bar{v}^*(\mu)$  locus crosses  $\bar{v}^-(\mu)$  only once and from below and never crosses  $\bar{v}^+(\mu)$ . Hence, whenever regime 3 is realised in second-best, a nursing home cannot be optimal from a first-best perspective. Indeed, we will show below that regime 3 never arises even in a second-best context. It is thus convenient to characterise the  $\Delta^{**}(\bar{v}, \mu) = 0$  locus for regimes 2 and 1 and to ignore regime 3.

**Lemma 8** Consider regimes 1 and 2. The function  $\Delta^{**}(\bar{v}, \mu)$  has the following properties

- (i)  $\frac{d\Delta^{**}}{d\bar{v}} = \frac{dV(\bar{v}, \mu)}{d\bar{v}} = -\left(h - \widehat{\psi}_L\right) < 0$ ,
- (ii)  $\frac{d\Delta^{**}}{d\mu} = \frac{dV(\bar{v}, \mu)}{d\mu} = hv_H(\widehat{n}^N) > 0$ ,
- (iii)  $\lim_{\bar{v} \rightarrow \infty} \Delta^{**}(\bar{v}, \mu) = -\infty$ ,
- (iv)  $\lim_{\mu \rightarrow \infty} \Delta^{**}(\bar{v}, \mu) = \infty$
- (v)  $\Delta^{**}(0, \mu) > (=)0$  if  $\mu > (=)1$ ,
- (vi)  $\frac{d\bar{v}}{d\mu} \Big|_{\Delta^{**}=0} = \frac{hv_H(\widehat{n}^N)}{h - \widehat{\psi}_L} > 0$ .

**Proof.** In order to show (i) and (ii) we recall that  $\widehat{W}_F$  is constant in  $(\bar{v}, \mu)$  and make use of the value function

$$V(\bar{v}, \mu) = \widehat{W}_N(\bar{v}, \mu) + \widehat{\psi}_L \left[ u(y - \widehat{a}_L^N - \widehat{T}_L^N) + v_L(\widehat{a}_L^N) - u(y - \widehat{T}_H^N) - v_L(\widehat{n}^N) + \bar{v} \right],$$

with  $\widehat{\psi}_L > 0$  in regime 1 and  $\widehat{\psi}_L = 0$  in regime 2. We can then write  $\Delta^{**}(\bar{v}, \mu) = V(\bar{v}, \mu) - \widehat{W}_F$  and, using the envelope theorem, we obtain the derivatives stated in (i) and (ii). To show that  $-(h - \widehat{\psi}_L) < 0$  in part (i) rearrange (20), with  $\widehat{\psi}_H = 0$ , to obtain  $\widehat{\psi}_L = \frac{(1-h)(u'_H - u'_L)}{\frac{1-h}{h}u'_H + u'_L}$ . But then  $h - \widehat{\psi}_L = \frac{u'_L}{\frac{1-h}{h}u'_H + u'_L} > 0$ . Parts (ii)-(vi) follow in analogy to the proof of the corresponding parts in Lemma 3. ■

The properties are similar to those established with regard to  $\Delta^*(\bar{v}, \mu)$  in Lemma 3. From part (v) we note that similar to the first-best, the planner is indifferent between a nursing home and a family setting for  $\bar{v} = 0$  and  $\mu = 1$ . This is because the allocations in the nursing home and family setting are equivalent, both involving precisely the same distortions. For every  $\mu \geq 1$  define

$$\bar{v}^{**}(\mu) := \arg_{\bar{v}} \{ \Delta^{**}(\bar{v}, \mu) = 0 \}. \quad (25)$$

Strict monotonicity of  $\Delta^{**}$  in its arguments guarantees that  $\bar{v}^{**}(\mu)$  is a singleton. Thus,  $\bar{v}^{**} : [1, \infty) \rightarrow [0, \infty)$ ,  $\mu \mapsto \bar{v}^{**}(\mu)$  is a function that is, due to property (vi), increasing in  $\mu$ . For regime 1 we also note that in the limit  $(\bar{v}, \mu) \rightarrow (\bar{v}^-(\mu), \mu)$  we have  $\widehat{\psi}_L \rightarrow 0$  and  $\widehat{x}^N \rightarrow x^N$  with  $x \in \{c_L, c_H, a_L, n\}$ . Thus,  $\bar{v}^{**}(\mu)$  is continuous and continuously differentiable at the point  $\bar{v}^{**}(\mu) = \bar{v}^-(\mu)$  should this exist. Similar to the first-best case we then obtain the following proposition

**Proposition 2** *The function  $\bar{v}^{**}(\mu)$  defines a locus in the nursing home technology space  $(\bar{v}, \mu)$  such that  $\Delta^{**} = 0$  on the locus,  $\Delta^{**} < 0$  for all pairs  $(\bar{v}, \mu)$  with  $\bar{v} > \bar{v}^{**}(\mu)$  and  $\Delta^{**} > 0$  for all pairs  $(\bar{v}, \mu)$  with  $\bar{v} < \bar{v}^{**}(\mu)$ .*

As in the case of the first-best, the public provision of nursing homes is efficient in a second-best context if and only if the productivity gain,  $\mu - 1$ , is large enough as compared to the disutility of being insitutionalised,  $\bar{v}$ . This does not answer yet the more interesting questions as to how the second-best locus  $\bar{v}^{**}(\mu)$  relates to the first-best locus  $\bar{v}^*(\mu)$  and what this tells us about discrepancies in the provision of nursing homes that arise from the informational asymmetry. In order to do so, we examine in turn regime 2 and then regime 1.

Consider thus a pair  $(\mu, \bar{v})$ , satisfying  $\bar{v} \in [\bar{v}^-(\mu), \bar{v}^+(\mu)]$  implying that  $(\mu, \bar{v})$  give rise to regime 2. Furthermore, assume  $\mu \geq \mu^*$ , implying from Lemma 7 that  $\bar{v}^*(\mu) \geq \bar{v}^-(\mu)$ . Finally, define  $\Delta_F := (W_F^* - \widehat{W}_F)$ . Note that  $\Delta_F > 0$  by definition of first- and second-best welfare and that  $\Delta_F$  is a constant in  $(\mu, \bar{v})$ -space because both of its components relate to the provision within the family. We can then establish the following

**Lemma 9** *Let  $\mu \geq \mu^*$ . Then  $\bar{v}^{**}(\mu) - \bar{v}^*(\mu) = \Delta_F h^{-1} > 0$  is a constant for all  $\mu$ .*

**Proof.** Consider  $\Delta^*(\bar{v}, \mu) = (W_N^* - W_F^*) = 0$ . This is equivalent to  $h\bar{v} = (W_N^* + h\bar{v} - W_F^*)$  or  $\bar{v} = (W_N^* + h\bar{v} - W_F^*) h^{-1}$ . By construction  $\bar{v} = \bar{v}^*(\mu)$  so that we can write  $\bar{v}^*(\mu) = (W_N^* + h\bar{v} - W_F^*) h^{-1}$ .<sup>12</sup> Similarly we can write  $\bar{v}^* * (\mu) := (\widehat{W}_N + h\bar{v} - \widehat{W}_F) h^{-1}$ . Then we find  $\bar{v}^* * (\mu) - \bar{v}^*(\mu) = [(\widehat{W}_N - W_N^*) - (\widehat{W}_F - W_F^*)] h^{-1} = (W_F^* - \widehat{W}_F) h^{-1} = \Delta_F h^{-1} > 0$  for all  $\mu$ . Here, the second equality follows from the fact that with a nursing home the first-best allocation is realised within regime 2 so that  $\widehat{W}_N = W_N^*$ . ■

Hence, it follows that within regime 2, the locus for  $\bar{v}^{**}(\mu)$  lies strictly above the locus  $\bar{v}^*(\mu)$ . The distance  $\Delta_F h^{-1}$  between the two schedules is determined by the extent of the informational inefficiency in the family context.<sup>13</sup> In the region between  $\bar{v}^{**}(\mu)$  and

<sup>12</sup>This expression for  $\bar{v}^*(\mu)$  is obviously just implicit as the RHS depends on  $\bar{v}$ . We use the expression for mathematical expedience but caution not to interpret it as a closed form for  $\bar{v}^*(\mu)$ .

<sup>13</sup>Note that the lemma also implies that  $\frac{\partial \bar{v}^*}{\partial \mu} = \frac{\partial \bar{v}^{**}}{\partial \mu}$  holds in regime 2. This is readily verified from parts (vi) of Lemmas 3 and 8, respectively, when observing that  $\widehat{n}^N = n^N$  and  $\widehat{\psi}_L = 0$  in regime 2.

$\bar{v}^*(\mu)$  it is thus efficient to provide a nursing home in a second-best but not in a first-best context. The reason is that admission to a nursing home allows a separation of types at no cost and, thus, first-best redistribution. By continuity this finding should extend to some  $\mu \leq \mu^*$ .

We turn now to regime 1, i.e. we consider  $(\mu, \bar{v})$  that satisfy  $\bar{v} \leq \bar{v}^-(\mu)$ . From parts (v) of Lemmas 3 and 8 we know that  $\bar{v}^{**}(1) = \bar{v}^*(1) = 0$ , hence the two schedules coincide at the origin. Whereas we also know the slopes  $\frac{\partial \bar{v}^{**}}{\partial \mu} = \frac{h v_H(\hat{n}^N)}{h - \hat{\psi}_L}$  and  $\frac{\partial \bar{v}^*}{\partial \mu} = v_H(n^N)$  (from parts (vi) of Lemmas 3 and 8) it is difficult to compare them because generally  $\hat{n}^N \neq n^N$ . Nonetheless, we can pin down a few more characteristics of the  $\bar{v}^{**}(\mu)$  schedule.

**Lemma 10** (i) *Let  $v_i''' \leq 0$ . Then there exists a unique  $\mu^{**}$  with  $\mu^{**} \in (1, \mu^*)$  such that  $\bar{v}^-(\mu) \geq \bar{v}^{**}(\mu) \Leftrightarrow \mu \leq \mu^{**}$ .*

(ii) *There exists a  $\mu^{***}$  with  $\mu^{***} \in [1, \mu^{**})$  such that  $\bar{v}^{**}(\mu) > \bar{v}^*(\mu)$  if  $\mu > \mu^{***}$*

**Proof.** See Appendix. ■

We have now established the position of the  $\bar{v}^{**}(\mu)$  locus in  $(\mu, \bar{v})$  space as far as we possibly can. Figures 2a and 2b provide an illustration.

### Insert Figures 2a and 2b here

Specifically, we have established that the  $\bar{v}^{**}(\mu)$  schedule crosses regime 1 for  $\mu \in (1, \mu^{**})$  and that it continues to lie above the  $\bar{v}^*(\mu)$  locus at least for a range of intermediate  $\mu \in [\mu^{***}, \mu^{**}]$ . These may include all  $\mu > \mu^{***} = 1$ , as depicted in Figure 2a. A sufficient condition for this to happen is that  $\frac{\partial \bar{v}^{**}}{\partial \mu} = \frac{h v_H(\hat{n}^N)}{h - \hat{\psi}_L} \geq v_H(n^N) = \frac{\partial \bar{v}^*}{\partial \mu}$  for all  $\mu \in [1, \mu^{**})$ . It is readily verified that this is always true if  $\hat{n}^N \geq n^N$ . However, for  $\hat{n}^N < n^N$  we cannot rule out that  $\bar{v}^*(\mu)$  and  $\bar{v}^{**}(\mu)$  intersect for some  $\mu \in (1, \mu^{**})$ . This case is depicted in Figure 2b.<sup>14</sup> The following proposition summarises our main result.

**Proposition 3** (i) *There exists a  $\mu^{***} > 1$  such that for all  $\mu > \mu^{***}$  and for all  $\bar{v} \in [\bar{v}^*(\mu), \bar{v}^{**}(\mu)]$  public provision of a nursing home is optimal under asymmetric information but not under symmetric information.* (ii) *The reverse case where public provision*

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<sup>14</sup>In fact, there may be more than one points of intersection.

*of a nursing home is optimal under symmetric information but not under asymmetric information can arise only if  $\hat{n}^N < n^N$ ,  $\mu < \mu^{***}$  and  $\bar{v} \in [\bar{v}^{**}(\mu), \bar{v}^*(\mu)]$*

As long as the nursing home is sufficiently effective ( $\mu > \mu^{***}$ ) we have over-provision of nursing care in the following sense: As long as the disutility  $\bar{v}$  is not too high, nursing care is offered in order to facilitate redistribution under asymmetric information, where it would not be an efficient mode of care under complete information. This corresponds to the medium-shaded area in Figures 2a and 2b (the light shaded area still corresponding to the area where nursing homes are offered only in a first-best setting). The case for over-provision is obvious for regime 2, where provision of a nursing home allows the implementation of the first-best solution even under asymmetric information. However, the argument extends to regime 1, where the provision of a nursing home gives rise to an informational problem and distortions which are similar to the family context. Nevertheless, the direct disutility from being institutionalised,  $\bar{v}$ , relaxes the binding incentive constraint (ICLN) and thereby allows a greater extent of redistribution. If  $\hat{n}^N \geq n^N$  there will always be a tendency towards overprovision of nursing homes within regime 1 (see Figure 2a). We cannot rule out, however, a scenario as depicted in Figure 2b. For the dark-shaded area, the productivity of the nursing home is low and the level of nursing care is reduced under asymmetric information below the first-best, i.e.  $\hat{n}^N < n^N$ . In such a case, a nursing home is not publicly provided under asymmetric information where it would be under symmetric information. For a sharp reduction in nursing care, the productivity advantage of the nursing home diminishes by so much that it becomes unattractive in a second-best setting despite its greater scope for redistribution.

## 5 Conclusions

We have derived the allocation of long-term care and redistributive transfers both in the absence and in the presence of a nursing home and both under complete and asymmetric information about the degree of dependency. We have characterised the allocation in terms of the nursing home technology: its effectiveness/productivity of care and the direct utility loss associated with institutional care. Unsurprisingly, nursing homes should be provided

publicly if and only if they are sufficiently effective in relation to the loss of utility due to institutionalisation. Under asymmetric information this rule becomes biased, however, usually in favour of the nursing home. This is because the direct utility loss of nursing care provides a disincentive for the children of less dependent parents to dress them up as severe cases. Thus, informational rents are lower and the scope for redistribution towards families with severely dependent parents is greater. Only if the level of care offered in the nursing home is severely biased downwards under asymmetric information can a situation arise, where nursing homes are not provided in a second-best where they should in a first-best. While we have derived our arguments within a rather general two-type model of adverse selection, a number of comments are due on possible limitations and the scope for extensions.

First, we note that nursing home technology is biased, in a sense, in favour of the severely dependent  $H$ -types. This is because for the utility specification  $\mu v_H(n)$  effectiveness,  $\mu$ , and level of care,  $n$ , are technological complements. Thus, the gains from greater effectiveness fully accrue to  $H$ -types both through higher  $\mu$  and through the associated increase in  $n$ .  $L$ -types tend to lose out as they have to co-finance the greater provision  $n$  by way of a higher net transfer to the planner. The question of who stands to benefit from a greater effectiveness may have a bearing, however, on the equilibrium structure. To see this recall that in the presence of nursing care only two regimes could arise under asymmetric information: Regime 1, where (the children of)  $L$ -types seek to mimic  $H$ -types; and regime 2, where there is natural separation. In principle, a third regime, regime 3, is possible, where the children of  $H$ -types have to be given an incentive to send their parents to a nursing home rather than mimic  $L$ -types. As it turns out this regime cannot arise as an equilibrium outcome. This is because it corresponds to combinations of (low effectiveness and high disutility) for which the provision of family care is always optimal even under asymmetric information. However, a conflict of interest between the policy-maker and (the children of)  $H$ -types may arise when the benefits of greater effectiveness  $\mu$  do not fully accrue to  $H$ -types. Thus, consider a set-up where for a utility function  $v_H(\mu n)$ , effectiveness,  $\mu$ , and level of care,  $n$ , are substitutes. For a greater  $\mu$  a given level of utility  $v_H(\mu, n)$  could then be attained at lower cost  $n$ , allowing a reduction

of transfers to both  $H$ - and  $L$ -types. If the share of  $L$ -types is sufficiently large, it cannot be ruled out then that in a first-best setting the planner may wish to introduce a nursing home for  $H$ -types even if this is not in their private interest. Regime 3 may then turn up as part of the equilibrium structure.

Second, we cast our argument in a model of tax financed long-term care. While this corresponds to the institutional set-up of a number of countries (e.g. Norway, Spain, Sweden or the UK), other countries rely at least partially on long-term care insurance (e.g. Germany, Japan, Switzerland or the USA). Our model can be interpreted in the context of long-term care insurance when we adopt the following interpretation. Assume that for an elderly person there are only two states:  $L$  and  $H$ . At the point of signing the insurance contract the probability of becoming severely dependent,  $h$ , is identical for everyone. Thus, the premium is the same for everyone and amounts to  $\bar{T} = ha_H + (1 - h)a_L$  without nursing home and  $\bar{T} = hn + (1 - h)a_L$  with a nursing home, respectively. Under complete information about severity, insurance benefits would then be given by  $B_i = a_i i = H, L$ , in the absence of a nursing home, and  $B_L = a_L$  and  $B_H = 0$  if nursing home care is offered free of charge.<sup>15</sup> The adverse selection problem then arises ex-post, when it comes to assigning insurance benefits according to unobservable severity. Here, the benefits have to be designed in a way that rules out misreporting.<sup>16</sup>

Three, in our model the planner can only learn about the degree of dependency of individuals from the reports made by family members (or, equivalently, on the basis of their choices from the menu of long-term care contracts). In reality, many health care systems rely on the direct verification of dependency levels by experts. In Germany, for instance, a dependency level and the corresponding level of care and transfers are ascribed only after the dependent person has been examined by an expert physician. In the extreme, the planner could always implement the first best if a perfect and costless

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<sup>15</sup>If nursing home care is offered at a fee that covers its marginal cost, the premium is  $\bar{T} = (1 - h)a_L$  and benefits are  $B_L = a_L$  and  $B_H = n$ .

<sup>16</sup>Of course, adverse selection may also arise ex-ante with respect to the remaining healthy and/or unhealthy life-expectancy. Sloan and Norton (1997) provide empirical evidence suggesting that this form of adverse selection is one cause for the low level of private long-term-care insurance in the US.

audit was available. More generally, the availability of an imperfect and/or costly audit will relax the incentive compatibility constraints and thereby mitigate the associated inefficiency.<sup>17</sup> While this argument immediately extends to our set-up, we conjecture that the availability of an imperfect audit would not fundamentally change our main result. Thus, the planner would presumably apply the audit both when care is solely provided within the family *and* when a public nursing home is provided. While the audit would clearly increase the efficiency in both cases, it would turn over the tendency towards over-provision of nursing homes under asymmetric information only if the increase in efficiency was *significantly* greater for an audit performed in a family care context. We do not see reasons for why this should be the case.

## 6 Appendix

**Proof of Lemma 3:** To begin with note that  $W_F^*$  does not depend on  $(\bar{v}, \mu)$  as the parameters relating to the nursing home are irrelevant in the context of family care. Hence, the following holds.

- (i)  $\frac{\partial \Delta^*}{\partial \bar{v}} = -h < 0$  as changes in  $\bar{v}$  do not affect any of the optimal choices  $\{c_L^N, c_H^N, a_L^N, n^N\}$
- (ii) With nursing homes the envelope theorem applies, i.e.  $\frac{\partial \Delta^*}{\partial \mu} = \frac{\partial W_N}{\partial \mu} \Big|_{x=x^N} = hv_H(n^N) > 0$ , where  $x \in \{c_L, c_H, a_L, n\}$ .
- (iii) Follows for  $h > 0$  as all choices  $\{c_L^N, c_H^N, a_L^N, n^N\}$  and  $W_F^*$  are finite.
- (iv) From (15) we obtain  $\lim_{\mu \rightarrow \infty} n^N = \frac{y}{h}$  and  $\lim_{\mu \rightarrow \infty} a_L^N = \lim_{\mu \rightarrow \infty} c_H^N = 0$ . This is because in the limit all income is allocated to nursing care. But then,  $\lim_{\mu \rightarrow \infty} W_N^*(\bar{v}, \mu) = h(\mu v_H(\frac{y}{h}) - \bar{v}) = \infty$ . Since  $W_F^*$  is finite we obtain  $\lim_{\mu \rightarrow \infty} \Delta^*(\bar{v}, \mu) = \infty$
- (v)  $\Delta^*(0, \mu) = W_N^*(0, \mu) - W_F^*(0, \mu) = u(c_H^N) - u(c_H^F) + h[\mu v_H(n^N) - v_H(a_H^F)] + (1-h)[v_L(a_L^N) - v_L(a_L^F)]$   
 $\geq W_N(0, \mu) \Big|_{(c_L, c_H, a_L, n)=(c_L^F, c_H^F, a_L^F, a_H^F)} - W_F^*(0, \mu) = h(\mu - 1)v_H(a_H^F) \geq 0$ . The first equality is just the definition of  $\Delta^*$ , evaluated at  $\bar{v} = 0$  and  $\mu \geq 0$ . The second equation uses

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<sup>17</sup>See Laffont and Martimort (2002: section 3.6) for a brief exposition of the issue and seminal literature. Cremer et al. (2004) consider the effects of an audit in the determination of disability benefits as a route to (early) retirement.

the objective function of the respective cases given in equations (9) and (11). The first inequality results since we depart from the optimal values of the endogenous variables in the nursing home case. The rest follows by substitution.

(vi) Total differentiation of equation (16) yields  $d\Delta^* = \frac{\partial \Delta^*}{\partial \mu} d\mu + \frac{\partial \Delta^*}{\partial \bar{v}} d\bar{v} = hv_H (n^N) d\mu - hd\bar{v}$ . For  $d\Delta^* = 0$  we obtain the implicit relationship between  $\mu$  and  $\bar{v}$  as given in the lemma.

(vii) follows from repeated differentiation of  $d\Delta^*$ .

**Proof of Lemma 4:** Part (i): We first show by contradiction that an allocation with  $\psi_H > 0$  cannot be an equilibrium. To see this, consider first  $\psi_H > 0$  and  $\psi_L > 0$ . As both (ICH) and (ICL) bind, it follows that  $v_H(a_H) - v_H(a_L) = v_L(a_H) - v_L(a_L)$ . Since  $v'_H > v'_L$ , the equation can be true if and only if  $a_H = a_L = a$ . But then from (ICH) and (ICL),  $c_H = c_L = c$ . The latter implies  $u'_H = u'_L = u'$  and thus  $\psi_H = \psi_L = \psi$  as from (20). Substituting into (18) and (19) and solving each equation for  $u'$  we obtain  $u' = v'_H(a) + \frac{\psi}{h}(v'_H(a) - v'_L(a)) = v'_L(a) - \frac{\psi}{1-h}(v'_H(a) - v'_L(a))$ . As is readily verified, the second equation implies  $v'_H(a) = v'_L(a)$ , a contradiction. Next, consider  $\psi_H > 0$  and  $\psi_L = 0$ . Then, from (20),  $u'_H < u'_L$  and, thus,  $c_H > c_L$ . From (ICL), it then follows that  $a_H < a_L$ . But this contradicts (M). Hence,  $\psi_H = 0$

Noting that for  $\psi_L = \psi_H = 0$  implies  $c_H = c_L$  and  $a_H > a_L$ , which violates (ICL). Therefore,  $\hat{\psi}_L > \hat{\psi}_H = 0$ . Using this in (20), it follows that  $u'_H > u'_L$  and, thus,  $\hat{c}_H^F < \hat{c}_L^F$ . In turn, it must be true from (ICH) that  $\hat{a}_H^F > \hat{a}_L^F$ , which completes the proof.

Part (ii): From (19) we have  $u'_L = v'_L(a_L)$  implying conditional efficiency. Furthermore,  $h(u'_H - v'_H(a_H)) = \psi_L(u'_H - v'_L(a_H)) \geq \psi_L(u'_H - v'_L(a_L)) = \psi_L(u'_H - u'_L) > 0$ , where the first equality follows from (18), the first inequality follows under observation of  $\hat{a}_H^F > \hat{a}_L^F$  and  $v''_L \leq 0$ . The second equality follows from (19) and the last inequality follows from  $\hat{c}_H^F < \hat{c}_L^F$ . But then  $u'_H > v'_H(a_H)$ , implying the upward distortion.

**Proof of Lemma 7:** (i) Follows from straightforward differentiation of (23) and (24), respectively, while observing  $\frac{dn^N}{d\bar{v}} = \frac{da_L^N}{d\bar{v}} = 0$ ,  $\frac{dn^N}{d\mu} > 0$  and  $\frac{da_L^N}{d\mu} < 0$  are obtained from comparative static analysis of the system (12)-(14).

(ii) Recalling  $\frac{\partial \bar{v}^*}{\partial \mu} = v_H(n^N)$  from property (vi) in Lemma 3 and using the result in

part (i) of the present Lemma it is readily checked that  $\max \left\{ \frac{\partial \bar{v}^-}{\partial \mu}, \frac{\partial \bar{v}^*}{\partial \mu} \right\} < \frac{\partial \bar{v}^+}{\partial \mu}$  for all  $\mu$ . Furthermore,  $\bar{v}^+(1) = v_H(n^N) - v_H(a_L^N) > v_L(n^N) - v_L(a_L^N) = \bar{v}^-(1) > \bar{v}^*(1) = 0$ , where the first inequality follows under observation of  $v'_H > v'_L$  and  $n^N > a_L^N$ . But then,  $\bar{v}^+(\mu) > \max \{ \bar{v}^*(\mu), \bar{v}^-(\mu) \}$  for all  $\mu$ .

(iii) Define  $\Delta_{\bar{v}^*}(\mu) := \bar{v}^-(\mu) - \bar{v}^*(\mu)$ . We seek to show that  $\Delta_{\bar{v}^*}(\mu) \geq 0 \Leftrightarrow \mu \leq \mu^*$ . As the continuity of the functions  $\bar{v}^-(\mu)$  and  $\bar{v}^*(\mu)$  implies the continuity of  $\Delta_{\bar{v}^*}(\mu)$  it is then sufficient to establish (a)  $\Delta_{\bar{v}^*}(1) > 0$ ; (b)  $\lim_{\mu \rightarrow \infty} \Delta_{\bar{v}^*}(\mu) < 0$ ; and (c)  $\Delta_{\bar{v}^*}(\mu) = 0 \implies \frac{d\Delta_{\bar{v}^*}(\mu)}{d\mu} < 0$ . (a), (b) and (c) together imply a unique root  $\mu^* := \arg_{\mu \in (1, \infty)} \{ \Delta_{\bar{v}^*}(\mu) = 0 \}$ . (a) has already been established as part of the proof of part (ii). (b) follows as  $\lim_{\mu \rightarrow \infty} \Delta_{\bar{v}^*}(\mu) = v_L(\frac{y}{h}) - \lim_{\mu \rightarrow \infty} \bar{v}^*(\mu) < 0$ , where  $\lim_{\mu \rightarrow \infty} \bar{v}^*(\mu) = \infty$ . Here,  $\lim_{\mu \rightarrow \infty} \bar{v}^-(\mu) = v_L(\frac{y}{h})$  follows from the fact that for  $\mu \rightarrow \infty$  the whole income is spent on nursing care for the  $H$ -types so that  $\lim_{\mu \rightarrow \infty} n = \frac{y}{h}$ . Further,  $\lim_{\mu \rightarrow \infty} \bar{v}^*(\mu) = \infty$  follows from the fact that  $\lim_{\mu \rightarrow \infty} \Delta^*(\bar{v}, \mu) = \infty = -\lim_{\bar{v} \rightarrow \infty} \Delta^*(\bar{v}, \mu)$  according to properties (iii) and (iv) in Lemma 3.

To prove (c), consider

$$\begin{aligned} \frac{d\Delta_{\bar{v}^*}(\mu)}{d\mu} \Big|_{\Delta_{\bar{v}^*}(\mu)=0} &= \frac{\partial \bar{v}^-}{\partial \mu} \Big|_{\bar{v}^-(\mu)=\bar{v}^*(\mu)} - \frac{\partial \bar{v}^*}{\partial \mu} \Big|_{\bar{v}^-(\mu)=\bar{v}^*(\mu)} \\ &= v'_L(n^N) \frac{dn^N}{d\mu} - v'_L(a_L^N) \frac{da_L^N}{d\mu} - v_H(n^N) < 0, \end{aligned} \quad (26)$$

where  $n^N$  and  $a_L^N$  are the values realised at  $(\mu, \bar{v}^-(\mu))$ . Since  $v''_H \leq 0$  it follows that  $v_H(n^N) \geq n^N v'_H(n^N)$ . Furthermore, observe  $v'_L(a_L^N) = \mu v'_H(n^N)$ . Hence, it is sufficient for the inequality in (26) that  $v'_L(n^N) \frac{dn^N}{d\mu} - \left[ \mu \frac{da_L^N}{d\mu} + n^N \right] v'_H(n^N) < 0$ . Observing  $v'_L(n^N) < v'_H(n^N)$  it is then sufficient for (26) that  $\frac{dn^N}{d\mu} \leq \left[ n^N + \mu \frac{da_L^N}{d\mu} \right]$ . From comparative statics of the system (12)-(14) we obtain  $\frac{dn^N}{d\mu} = \frac{-v'_H(n^N)A}{\mu v''_H A + u''_H u''_L v''_L} > 0$ , where  $A := u''_L v''_L + \frac{1-h}{h} u''_H (u''_L + v''_L) > 0$ , and  $\frac{da_L^N}{d\mu} = \frac{-v'_H(n^N) u''_H u''_L}{\mu v''_H A + u''_H u''_L v''_L} < 0$ . Using these expressions one can show after some manipulations that

$$\begin{aligned} \frac{dn^N}{d\mu} &\leq \left[ n^N + \mu \frac{da_L^N}{d\mu} \right] \\ &\Leftrightarrow (n^N \mu v''_H + v'_H(n^N)) A + (n^N v''_L + \mu v'_H(n^N)) u''_H u''_L \leq 0. \end{aligned}$$

Noting  $A > 0$  and  $u''_H u''_L > 0$  it follows that the second equality holds if  $n^N \mu v''_H + v'_H(n^N) \leq 0$  and  $n^N v''_L + \mu v'_H(n^N) \leq 0$ . Recalling  $\mu \geq 1$  we have  $n^N \mu v''_H + v'_H(n^N) \leq$

$n^N v_H'' + v_H' (n^N) \leq 0$ , where the last inequality holds if  $v_H''' \leq 0$ . Substituting  $\mu v_H' (n^N) = v_L' (a_L^N)$  and observing  $n^N > a_L^N$  we have  $n^N v_L'' + \mu v_H' (n^N) < a_L^N v_L'' + v_L' (a_L^N) \leq 0$ , where the last inequality holds if  $v_L''' \leq 0$ . Hence,  $v_i''' \leq 0$  is sufficient for the inequality in (26). But then, (a)-(c) hold, which implies a unique root  $\mu^* := \arg_{\mu \in (1, \infty)} \{\Delta_{\bar{v}^*}(\mu) = 0\}$ .

**Proof of Lemma 10:** (i) The proof is analogous to the proof of part (iii) of Lemma 8. Defining  $\Delta_{\bar{v}^{**}}(\mu) := \bar{v}^-(\mu) - \bar{v}^{**}(\mu)$ , we seek to show that  $\Delta_{\bar{v}^{**}}(\mu) \geq 0 \Leftrightarrow \mu \leq \mu^{**}$ . As the continuity of the functions  $\bar{v}^-(\mu)$  and  $\bar{v}^{**}(\mu)$  implies the continuity of  $\Delta_{\bar{v}^{**}}(\mu)$  it is then sufficient to establish (a)  $\Delta_{\bar{v}^{**}}(1) > 0$ ; (b)  $\Delta_{\bar{v}^{**}}(\mu^*) < 0$ ; and (c)  $\Delta_{\bar{v}^{**}}(\mu) = 0 \implies \frac{d\Delta_{\bar{v}^{**}}(\mu)}{d\mu} < 0$ . (a), (b) and (c) together imply a unique root  $\mu^{**} := \arg_{\mu \in (1, \mu^*)} \{\Delta_{\bar{v}^{**}}(\mu) = 0\}$ .

(a) follows as  $\bar{v}^{**}(1) = 0 < \bar{v}^-(1)$ .

(b) We have  $\Delta_{\bar{v}^{**}}(\mu^*) = \bar{v}^-(\mu^*) - \bar{v}^{**}(\mu^*) = -[\bar{v}^{**}(\mu^*) - \bar{v}^*(\mu^*)] = -\Delta_F h^{-1} < 0$ ; where the second equality follows as  $\bar{v}^-(\mu^*) = \bar{v}^*(\mu^*)$  by definition of  $\mu^*$ ; and where the third equality and the inequality follow from Lemma 9.

(c) Consider

$$\begin{aligned} \frac{d\Delta_{\bar{v}^{**}}(\mu)}{d\mu} \Big|_{\Delta_{\bar{v}^{**}}(\mu)=0} &= \frac{\partial \bar{v}^-}{\partial \mu} \Big|_{\bar{v}^-(\mu)=\bar{v}^{**}(\mu)} - \frac{\partial \bar{v}^{**}}{\partial \mu} \Big|_{\bar{v}^-(\mu)=\bar{v}^{**}(\mu)} \\ &= v_L' (n^N) \frac{dn^N}{d\mu} - v_L' (a_L^N) \frac{da_L^N}{d\mu} - \frac{h v_H (\hat{n}^N)}{h - \hat{\psi}_L} \Big|_{\bar{v}^-(\mu)=\bar{v}^{**}(\mu)} \\ &= v_L' (n^N) \frac{dn^N}{d\mu} - v_L' (a_L^N) \frac{da_L^N}{d\mu} - v_H (n^N) \end{aligned}$$

The third equality holds as  $\hat{\psi}_L = 0$  and  $\hat{n}^N = n^N$  for  $\bar{v}^{**}(\mu) = \bar{v}^-(\mu)$ . Now we can apply the proof of (c) in part (iii) of Lemma 7 to show that the expression in the third line is negative if  $v_i''' \leq 0$ . But then, (a)-(c) hold, which implies a unique root  $\mu^{**} := \arg_{\mu \in (1, \mu^*)} \{\Delta_{\bar{v}^{**}}(\mu) = 0\}$ .

(ii) We note that  $\bar{v}^{**}(\mu^{**}) - \bar{v}^*(\mu^{**}) = \left[ \left( \widehat{W}_N - W_N^* \right) \Big|_{\bar{v}=\bar{v}^-(\mu^{**}), \mu=\mu^{**}} + \Delta_F \right] h^{-1} = \Delta_F h^{-1} > 0$ , where  $\left( \widehat{W}_N - W_N^* \right) \Big|_{\bar{v}=\bar{v}^-(\mu^{**}), \mu=\mu^{**}} = 0$  follows from the fact that for  $(\bar{v}^-(\mu^{**}), \mu^{**})$  we have  $\hat{\psi}_L = 0$  and  $\hat{x}^N = x^N$  with  $x \in \{c_L, c_H, a_L, n\}$ . At the same time  $\bar{v}^{**}(1) - \bar{v}^*(1) = 0$ . We can distinguish two cases. Either  $\bar{v}^{**}(\mu) - \bar{v}^*(\mu) > 0 \forall \mu \in (1, \mu^{**})$ . In this case,  $\mu^{***} = 1$ . Alternatively, we can have  $\bar{v}^{**}(\mu) - \bar{v}^*(\mu) < 0$  for some  $\mu \in (1, \mu^{**})$ . But then by continuity there must exist an  $\mu^{***} > 1$  such that  $\bar{v}^{**}(\mu) - \bar{v}^*(\mu) > 0 \forall \mu \in (\mu^{***}, \mu^{**})$ . This completes the proof.

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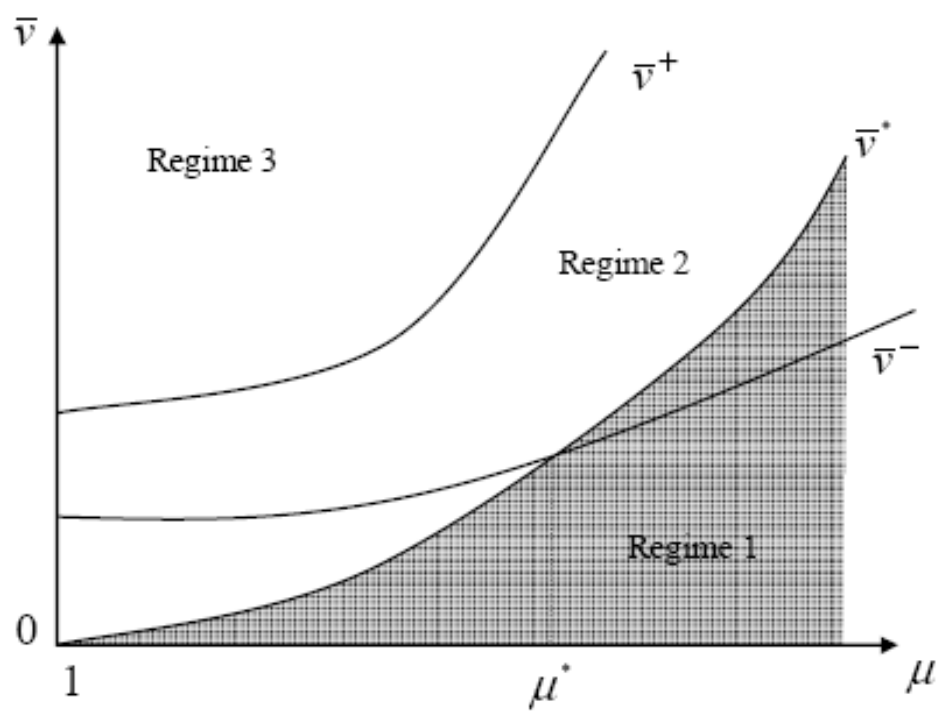


Figure 1

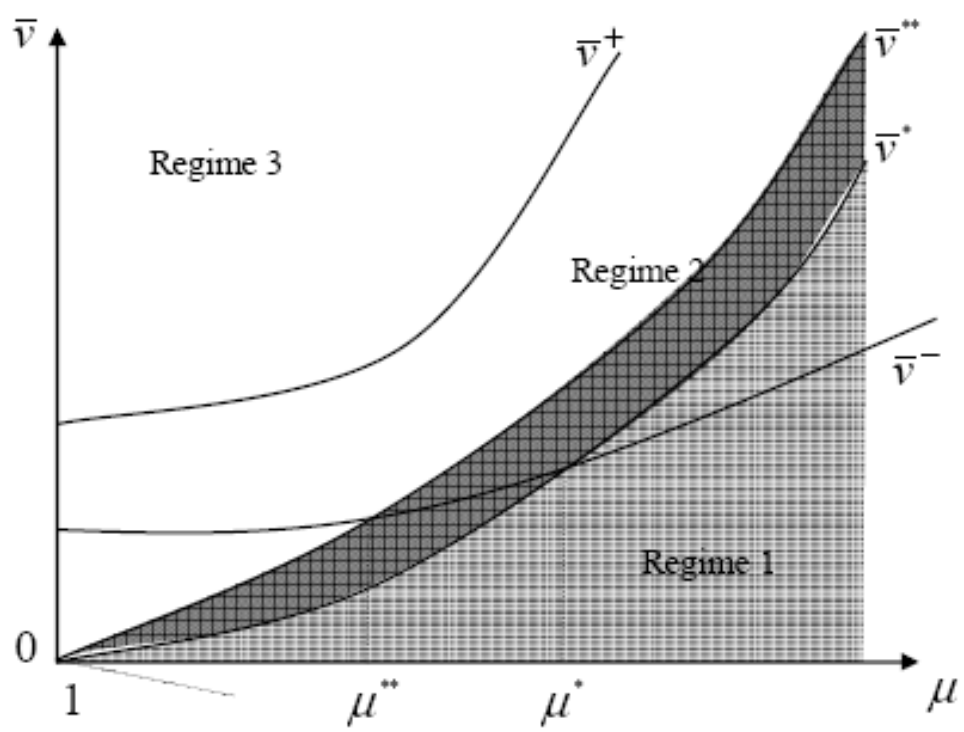


Figure 2a

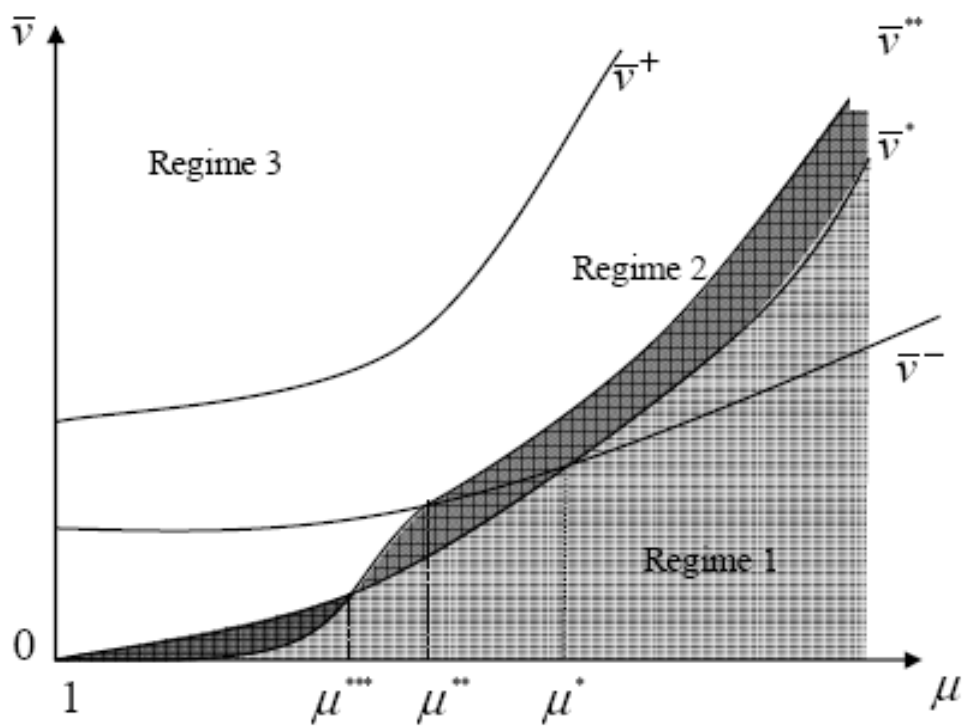


Figure 2b