The Cost of Unemployment Fluctuations Revisited*

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Abstract

The strength of the consumption risk when becoming unemployed depends on the re-employment probability and two further factors: the outside option (the value of home-production plus unemployment benefits) and the amount of savings the agent can use to smooth out consumption. We use the size of unemployment fluctuations over the business cycle to identify the size of a consumer's value of home-production. Especially for low-skilled consumers, who we assume are liquidity-constrained, the level of home-production matters for smoothing business cycle risk. We employ Bayesian techniques to estimate a New Keynesian model for post Volcker disinflation US data using six macroeconomic time-series. The estimated model replicates the fluctuations of unemployment over the two skill groups endogenously and implies reasonable business cycle statistics. We are currently in the process of extracting an approximation for the welfare cost of business cycles for each of the groups.

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1 Introduction

Unemployment is an important concern to most policy makers. Nevertheless, estimated New Keynesian DSGE models to date mostly do not spell out the functioning of the labor market, see for example the widely cited papers by Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2005). Models which explicitly introduce search and matching frictions, on the other hand, tend to be unable to generate fluctuations in key labor market variables consistent with the data. Finally, almost all macroeconomic models in the tradition of Mortensen and Pissarides (1994) rely either on full risk neutrality of workers or, similarly, on a full insurance assumption.[1] If individuals have the means to sufficiently self-insure against the state of unemployment or the technology to insure among each other, the welfare-enhancing role of stabilization policy might be very small as in Lucas (2003).

The purpose of this paper is twofold. We first develop a New Keynesian DSGE model similar to Smets and Wouters (2003) but with search and matching frictions in the labor market. The mechanism we employ is based on the work in real-business-cycle environments by Hagedorn and Manovskii (2006) and Jung (2005). The latter author shows that a high outside option of the worker combined with a suffiently low bargaining power enables a real-business-cycle model to endogenously track unemployment fluctuations within tight bounds. We illustrate that the functioning of this mechanism carries over to a New Keynesian model with nominal rigidities and a rich shock structure. To test the mechanism as tough as possible, in the current paper, and contrary to much of the literature, we do not allow for an explicit employment shock but rather ask the model to generate unemployment fluctuations endogenously. This robustifies the reliability of the model once we carry it towards welfare analysis. The model is estimated on six variables, excluding unemployment.

Econometrically, we contribute to the literature by conducting time-aggregation in the estimation process. For internal consistency, we build our economic model at a monthly frequency while our data is observed at a quarterly frequency. We therefore treat monthly observations of the endogenous variables as latent while we assume that only the quarterly averages are observable to the econometrician. We also add to the literature on validation of DSGE models. We explicitly

[1] Under this assumption, workers are perfectly insured against short-falls of consumption by a large "family". The full insurance assumption follows den Haan, Ramey, and Watson (2000) and is used widely, e.g. in Trigari (2006), Christoffel, Kuester, and Linzert (2006) and Krause and Lubik (2005), among others.

reserve some of the available time series to compare the model's prediction for these series with the actual data.

The second purpose of the paper is relaxing the assumption of complete markets and to introduce a potentially important welfare cost of the business cycle. As pointed out i.a. by Gruber (2001) most workers have the financial means to self-insure against occasional spells of unemployment. Still a sizeable fraction of the labor force (we use his lower bound estimate of 16%) does not have enough wealth even to cover 10% of the cost of an unemployment spell.[2] Yet the welfare costs implied by this observation will not least depend on the degree to which workers compensate their shortfall in purchases of market-traded consumption goods with those produced at home or with work in the shadow economy. The current model identifies this value of "home-production". The value of home-production is a crucial ingredient for the outside option of the worker and therefore for the amount of unemployment fluctuations observed in the data. Unemployment fluctuations therefore implicittly serve to identify the value of home-production.

In the model we associate this poorer than average group of workers with the low-skilled part of the population. Following the lead of Mankiw (2000) we assume that these workers are liquidity-constrained, i.e. they cannot participate in financial markets; and therefore cannot smooth consumption over time.[3] The assumption of liquidity-constrained workers, even though introduced in an admittedly ad hoc fashion,[4] enables us to keep the model tractable while still capturing the apparent empirical differences in financial wealth.[5] An advantage of this strict

[2] This is in line with other studies on the wealth distribution. Wolff (1998), for example, analyzes the Survey of Consumer Finances in 1995. He finds that 18.5% of households have zero or negative net worth, and that 28.7% of households have zero or negative financial (liquid) wealth. In addition, financial wealth is highly correlated with labor income. The households at the 40% (20%) lower end of the income distribution hold an amount of financial wealth that would enable them to sustain their consumption for an average of just 1 month (0 months).

[3] In fact, we assume the absence of any storage technology for this part of the population. This may not be a strong assumption since holding cash reserves can be pretty much subject to theft and durable consumption goods tend to be rather illiquid.

[4] Iacoviello (2005) makes a similar distinction between savers and spenders (in Mankiw's terminology). In his framework, a fraction of the households is more impatient and thus ends up facing a binding (albeit non-zero) borrowing constraint in equilibrium. For technical reasons, we are unable to impose borrowing constraints directly as the linearization used in the estimation procedure is ill-equipped to handle occasionally binding constraints.

[5] Johnson, Parker, and Souleles (2005) use the pre-announced 2001 U.S. tax rebates as a natural experiment. Examining answers to a set of questions in the Consumer Expenditure Survey directly targeted at these rebates, they find significant evidence of different spending behavior across different income and wealth groups. Responses are much larger for households with low liquid wealth or low income, consistent with liquidity constraints. Households with few liquid assets spend between 50% and more than 120% (point-estimates) of

formulation is that we can handle a richer state space and thus allow for more complex aggregate dynamics than, say, in Krusell and Smith (2002). This is important given that we ultimately want to use the model for deriving implications for optimal monetary policy. The same mechanism that generates the correct amount of fluctuations in the aggregate unemployment rate can be used to generate fluctuations in this constrained group of workers which are consistent with the data. In turn, this also indirectly identifies their value of home-production.

We use the estimate model to compute the welfare costs associated with business cycle fluctuations. The remainder of the paper is organized as follows. Section 2 lays out the theoretical model that we will estimate. Section 3 turns to the calibration and the estimation of the model. Section 4 discusses the welfare criteria used and presents estimates of the welfare costs of business cycle fluctuations. A final section concludes. We relegate a summary of the data and other estimation statistics to the Appendix.

the tax break on non-durable consumption, thus showing more hand-to-mouth, non-smoothing behavior than the average consumer. Standard errors, however, are large.

2 The Model

We incorporate search and matching frictions à la Mortensen and Pissarides (1994) into an otherwise standard New Keynesian business cycle model of the Smets and Wouters (2003) type. Consumers fall into two categories: those who can save into bonds, stocks, and physical capital and those who are prevented from access to asset markets.[6]

The model's production side features competitive factor markets in the only price-setting sector. The degree of nominal rigidity should thus be interpreted in this light.[7] One time period in the model refers to a calendar time of one month.

2.1 Preferences and Consumers' Constraints

There is a large number of identical families in the economy with measure $\nu \in [0,1]$. Each family consists of a measure of $1-u_t^{(1)}$ employed members and $u_t^{(1)}$ unemployed members. The families hold all assets in the economy. Consequently, profit income in the economy accrues entirely to this part of the population. The representative family pools the income of its working members, unemployment benefits of the unemployed members and financial income from financial assets which they hold via a mutual fund. There is also a measure $1-\nu$ (potentially zero) of individuals not associated with any family. These are assumed not to have access to financial markets. In period t, a measure $u_t^{(2)}$ of these constrained workers are unemployed. We assume that the mass of liquidity-constrained consumers does not vary over time and normalize the total mass of consumers in the economy to one. We index the different types by superscript $(o) \in \{1,2\}$.

Consumers have time-additive expected utility preferences. Preferences of consumer i can be represented by

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \mathrm{u}^{(o)} \left(c_{i,t}^{(o)}, c_{t-1}^{(o)}, \{h_{i,t}^{(o)}\} \right) \right\}, \tag{1}$$

where E_0 marks expectations conditional on period 0 information. $\mathrm{u}^{(o)}(c_{i,t}^{(o)}, c_{t-1}^{(o)}, h_{i,t}^{(o)})$, $o \in \{1,2\}$,

[6] For technical reasons it is important to ensure a sufficient degree of homogeneity in asset-holdings. For asset-holding workers we entertain a familiy structure which pools assets. The other group of workers must not hold any assets (not even currency) if the analysis is still to be viable.

[7] See, e.g., Eichenbaum and Fisher (2004) and Altig, Christiano, Eichenbaum, and Linde (2005) about firm-specific factor markets and the interpretation of a certain slope of the New Keynesian Phillips curve in this light.

is a standard period utility function of the form[8]

$$\mathtt{u}^{(o)}(c_{i,t}^{(o)}, c_{t-1}^{o}, h_{i,t}^{(o)}) = \frac{(c_{i,t}^{(o)} - \varrho^{(o)} c_{t-1}^{(o)})^{1-\sigma}}{1-\sigma} - \kappa^L \frac{\left(h_{i,t}^{(o)}\right)^{1+\varphi}}{1+\varphi}, \sigma > 0, \varphi > 0. \tag{2}$$

Here, $c_{i,t}^{(o)}$ denotes consumption of member i and $h_{i,t}^{(o)}$ are hours worked by member i in group o. Utility also depends on external habit persistence as in Abel (1990), indexed by parameter $\varrho^{(o)} \in [0,1)$. Note that external habit is assumed to take past aggregate consumption of the respective group of agents, $c_{t-1}^{(o)}$, as the reference point. κ^L is a positive scaling parameter of disutility of work.

2.1.1 Families/Financially Unconstrained Workers

The representative family maximizes the sum of expected utilities of its individual members,

$$\mathtt{W}_0^{(1)} = \int_0^1 E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \mathtt{u}^{(1)} \left(c_{i,t}^{(1)}, c_{t-1}^{1}, h_{i,t}^{(1)} \right) \right\} di. \tag{3}$$

Here "$\mathtt{W}_t^{(1)}$" stands for "welfare" of the family as of period t. Let $\mathtt{U}\left(c_t^{(1)}, c_{t-1}^{1}, u_t^{(1)}, \{h_{i,t}^{(1)}\}\right)$ denote the aggregate per-period utility function of the family:

$$\mathtt{U}\left(c_t^{(1)}, c_{t-1}^{(1)}, u_t^{(1)}, \{h_{i,t}^{(1)}\}\right) := \int_0^1 \mathtt{u}^{(1)}(c_{i,t}^{(1)}, c_{t-1}^{(1)}, h_{i,t}^{(1)}) di, \tag{4}$$

where $c_t^{(1)}$ is the average consumption level of family members and $\{h_{i,t}^{(1)}\}$ is shorthand for a potential distribution of hours contracts. We will later focus on a symmetric equilibrium in which each employed family member indeed works exactly the same hours and receives the same wage. Given its arguments, utility function $\mathtt{U}(\cdot,\cdot,\cdot,\cdot)$ gives the value of period family-utility when $c_t^{(1)}$ is optimally distributed among family members; see Appendix 6.1 for details.

Family members, i.e. workers of type 1, pool their income. Their budget constraint is thus given by

$$\begin{aligned} c_t^{(1)} + i_t + t_t = & \int_0^{1-u_t^{(1)}} w_{i,t}^{(1)} h_{i,t}^{(1)} di + u_t^{(1)} b^{(1)} + \frac{1}{\nu} \left[\frac{D_{t-1}}{P_t} R_{t-1} \epsilon_{t-1}^{b} - \frac{D_t}{P_t} \right] \\ & + \Psi_t + r_t^k z_t k_{t-1} - \psi(z_t) k_{t-1}, \end{aligned} \tag{5}$$

where i_t marks real investment per family member. So aggregate investment is given by $\nu\, i_t$. Both i_t and $c_t^{(1)}$ are choice variables of the family. $w_{i,t}^{(1)} h_{i,t}^{(1)}$ is wage per hour times hours worked

[8] See Jung (2005) for the more general but also less tractable case of utility allowing for balanced growth.

by individual household member i. t_t are lump-sum taxes per capita payable by the family. $b^{(1)}$ are real unemployment benefits paid to unemployed family members. The family holds $\frac{1}{\nu}D_t$ units of a risk-free one-period nominal bond (government debt) with gross nominal return $R_t\epsilon_t^b$. ϵ_t^b denotes a serially correlated shock to the risk premium. It drives a wedge between the return on assets held by households and the interest rate controlled by the central bank, see Smets and Wouters (2007). The household also owns representative shares of all firms in the economy. Ψ_t denotes real dividend income per member of the family arising from these firms' profits:

$$\Psi_t = \frac{1}{\nu}\left\{\Psi_t^C + \nu(1-u_t^{(1)})\Psi_t^{(1)} + (1-\nu)(1-u_t^{(2)})\Psi_t^{(2)}\right\}. \tag{6}$$

Here $\Psi_t^C, \Psi_t^{(1)}$ and $\Psi_t^{(2)}$ are the profits arising in the differentiating industry and in the labor good industry; see Section 2.2. k_t is the amount of physical capital held per member of the family. The family chooses the capacity utilization rate z_t and leases its effective capital, $z_t k_{t-1}$, to wholesale firms in a perfectly competitive capital market. The real rental rate of capital is r_t^k. The increasing, convex function $\psi(z_t)$ denotes the resource cost, in units of consumption goods, of setting the utilization rate to z_t. We assume that

$$\psi(z_t) = \gamma_{z,1}(z_t - 1) + \frac{\gamma_{z,2}}{2}(z_t - 1)^2,$$

where $\gamma_{z,1}$ and $\gamma_{z,2}$ are such that $\psi(1) = 0$, $\psi'(1) = r^k$, $\psi''(1) > 0$, as in Christiano, Eichenbaum, and Evans (2005).[9]

Capital accumulation is subject to capital adjustment costs summarized by the function $S(\cdot)$.

$$k_t = k_{t-1}(1-\delta) + \left[1 - S\left(\frac{i_t}{i_{t-1}}\right)\right]\epsilon_t^I i_t. \tag{7}$$

Here δ is the monthly rate of capital depreciation and ϵ_t^I is a serially correlated "investment shock" with unit steady state. As in Christiano, Eichenbaum, and Evans (2005) we assume that $S(1) = 0$, $S'(1) = 0$ and $S''(1) > 0$. Specifically, the functional form we use is

$$S\left(\frac{i_t}{i_{t-1}}\right) = \frac{\kappa^I}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2, \ \kappa^I > 0.$$

[9] Here as in the remainder of the paper, endogenous variables which do not carry a time index refer to steady state values. For example r^k consequently denotes the steady state value of r_t^k.

2.1.2 A Representative Family's First-order Conditions

The family maximizes (3) by choosing capital, k_t, investment, i_t, consumption, $c_t^{(1)}$, capacity utilization, z_t, and bond-holdings, D_t, subject to (5) and (7). The first-order condition for investment is

$$q_t^k S_t'(\cdot) \frac{\epsilon_t^I i_t}{i_{t-1}} - \beta E_t \left\{ q_{t+1}^k \frac{\lambda_{t+1}}{\lambda_t} S_{t+1}'(\cdot) \left(\frac{i_{t+1}}{i_t} \right) \frac{i_{t+1}}{i_t} \right\} + 1 = q_t^k (1 - S_t) \epsilon_t^I, \tag{8}$$

where $S_t := S\left(\frac{i_t}{i_{t-1}}\right)$ and q_t^k is the shadow value of installed capital (measured in consumption units). $\lambda_t = \frac{\partial \mathrm{U}(c_t^{(1)}, \ldots)}{\partial c_t^{(1)}} = \left(c_t^{(1)} - \varrho^{(1)} c_{t-1}^{(1)}\right)^{-\sigma}$ is the marginal family utility of additional consumption for each of the family members; see again Appendix 6.1. The first-order condition for capital can be written as

$$q_t^k = E_t \left\{ \beta \frac{\lambda_{t+1}}{\lambda_t} \left[q_{t+1}^k (1 - \delta) + r_{t+1}^k z_{t+1} - \psi(z_{t+1}) \right] \right\}. \tag{9}$$

Capacity utilization is chosen so as to equate the real return on capital and the marginal cost of capacity utilization

$$r_t^k = \psi'(z_t). \tag{10}$$

Bonds are chosen so as to intertemporally equate marginal utilities of consumption:

$$1 = E_t \left\{ \beta \frac{\lambda_{t+1}}{\lambda_t} \frac{R_t \epsilon_t^b}{\Pi_{t+1}} \right\}. \tag{11}$$

Finally, the optimal consumption plan satisfies the two transversality conditions

$$\lim_{j \to \infty} E_t \left\{ \beta^j \frac{\lambda_{t+j}}{\lambda_t} k_{t+j} \right\} = 0, \ \forall t. \tag{12}$$

and

$$\lim_{j \to \infty} E_t \left\{ \beta^j \frac{\lambda_{t+j}}{\lambda_t} \frac{D_{t+j}}{P_{t+j}} \right\} = 0, \ \forall t. \tag{13}$$

We turn to describe preferences and budget constraints of liquidity-constrained consumers.

2.1.3 Liquidity-constrained Consumers

Liquidity-constrained consumers, are not insured by a family. They do not have access to liquidity providing services and financial markets. They also cannot insure each other against the risk of becoming unemployed. Liquidity-constrained consumers seek to maximize expected utility (1), which is reproduced here for type $o = 2$ workers for convenience:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \mathbf{u}^{(2)}(c_{i,t}^{(2)}, c_{t-1}^{(2)}, h_{i,t}^{(2)}) \right\}, \tag{14}$$

where period utility is of the form (2). Due to being prevented from saving and borrowing, liquidity-constrained consumers always consume their entire after-tax income. Depending on parameter $\chi \in \{0, 1\}$, they contribute to lump-sum taxes. That is, their budget constraint is given by

$$c_{i,t}^{(2)} = \begin{cases} c_{e,i,t}^{(2)} = w_{i,t}^{(2)} h_{i,t}^{(2)} - \chi t_t & \text{if employed} \\ c_{u,i,t}^{(2)} = b^{(2)} - \chi t_t & \text{if unemployed,} \end{cases} \tag{15}$$

$c_{e,i,t}^{(2)}$ marks consumption of liquidity-constrained consumer i if he is employed. $c_{u,i,t}^{(2)}$ is the consumption level if he is unemployed instead. $b^{(2)}$ are real unemployment benefits paid to unemployed liquidity-constrained workers.

2.2 Firms

There are three sectors of production. One sector produces a homogenous intermediate good, which we shall call the "labor good". Firms in this sector need to find exactly one worker in order to produce. They take hours worked as their sole input into production. Searching for a worker is a costly and time-consuming process due to matching frictions. Once a firm and a worker have met, they Nash-bargain over wages and hours on a period by period basis.

Labor goods are sold to a wholesale sector in a perfectly competitive market. Firms in the wholesale sector take intermediate labor goods and physical capital as inputs, and produce differentiated goods using a constant-returns-to-scale production technology. Subject to price-setting impediments à la Calvo (1983), they sell to a final retail sector under monopolistic competition.[10]

Retailers bundle differentiated goods into a homogenous consumption/investment basket, y_t. They sell this final good to consumers and to the government at price P_t. We next turn to a detailed description of the respective sectors.

[10] Following the literature (see e.g. Trigari, 2006) we part the markup pricing decision from the labor demand decision. For an application which operates with tempoarily firm-specific labor and a matching market in the price-setting sector, see Kuester (2006).

2.2.1 Retail Firms

The retail sector operates in perfectly competitive factor markets. It takes inputs $y_{j,t}$ of wholesale good type $j \in [0,1]$ and aggregates all varieties into the final homogenous consumption and investment good, y_t, according to

$$y_t = \left(\int_0^1 y_{j,t}^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}}, \epsilon > 1. \tag{16}$$

The cost-minimizing expenditure, P_t, to produce one unit of the final good is given by

$$P_t = \left(\int_0^1 P_{j,t}^{1-\epsilon} dj \right)^{\frac{1}{1-\epsilon}}. \tag{17}$$

Here $P_{j,t}$ marks the price of good $y_{j,t}$. P_t coincides with the consumer/GDP price index. The demand function for each single good $y_{j,t}$ is given by

$$y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\epsilon} y_t. \tag{18}$$

2.2.2 Wholesale Firms

Firms in the wholesale sector have unit mass and are indexed by $j \in [0,1]$. Firm j produces variety j of a differentiated good according to

$$y_{j,t} = k_{j,t}^{\alpha} l_{j,t}^{1-\alpha}, \ \alpha \in (0,1). \tag{19}$$

Here $k_{j,t}$ denotes demand for capital by wholesale firm j. Capital is homogenous and instantaneously transferable across firms. Demand for capital is satisfied out of the utilization, z_t, of the capital stock formed up to $t-1$, νk_{t-1}, where k_{t-1} denotes the amount of capital to which each member of the family is entitled. Total supply of capital in period t is therefore $\nu z_t k_{t-1}$. $l_{j,t}$ denotes demand for the intermediate labor good which a wholesale firm j can acquire in a perfectly competitive market at real price x_t^L. Real period profits of firm j, $\Psi_{j,t}^C$, are given by

$$\Psi_{j,t}^C = \frac{P_{j,t}}{P_t} y_{j,t} - \left(k_{j,t} r_t^k + l_{j,t} x_t^L \right) - (\epsilon_t^C - 1) \left(k_{j,t} r_t^k + l_{j,t} x_t^L \right).$$

The first term are wholesale firm revenues, the second term are real payments for capital and for the labor good. The final term represents overhead costs: ϵ_t^C is a wholesale sector "cost-push" shock.[11]

[11] In the literature this shock frequently is also labeled a "price-markup" shock.

We follow Yun (1996) in assuming that in each period a random fraction $\omega \in [0,1)$ of firms cannot reoptimize their price. As in Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2003), firms which cannot reoptimize their price index to realized inflation, $\Pi_{t-1} := \frac{P_{t-1}}{P_{t-2}}$. The degree of indexation is measured by parameter $\gamma_p \in [0,1]$. Those firms which reoptimize their price in period t face the problem of maximizing the value of their enterprise by choosing their sales price, $P_{j,t}$, taking into account the pricing frictions, demand function (18) and production function (19). Realizing that for any given demand, the optimal factor input choice leads to marginal cost which are independent of the production level, the price-setting problem simplifies to

$$\max_{P_{j,t}} E_t \left\{ \sum_{s=0}^{\infty} \omega^s \beta_{t,t+s} \left[\frac{P_{j,t}}{P_{t+j}} \prod_{l=0}^{s-1} \left((\Pi_{t+l-1})^{\gamma_p} \, \Pi^{1-\gamma_p} \right) - mc_{t+s} \right] y_{j,t+s} \right\}. \tag{20}$$

Here Π is the gross inflation rate in steady state and mc_t are real marginal cost[12]

$$mc_t = \epsilon_t^C (r_t^k)^{\alpha} (x_t^L)^{1-\alpha} \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)}. \tag{21}$$

$\beta_{t,t+s} := \beta^s \frac{\lambda_{t+s}}{\lambda_t}$ is the equilibrium stochastic discount factor. The typical reoptimizing wholesale firm's first order condition for price-setting is:

$$E_t \left\{ \sum_{s=0}^{\infty} \omega^s \beta_{t,t+s} \left[\frac{P_t^*}{P_{t+s}} \prod_{l=0}^{s-1} \left((\Pi_{t+l-1})^{\gamma_p} \, \Pi^{1-\gamma_p} \right) - \frac{\epsilon}{\epsilon - 1} mc_{t+s} \right] y_{j,t+s} \right\} = 0, \tag{22}$$

where P_t^* marks the optimal price. The demand functions of individual firms for the labor good and capital are, respectively,

$$l_{j,t} = mc_t \frac{1-\alpha}{x_t^L \, \epsilon_t^C} y_{j,t} \tag{23}$$

and

$$k_{j,t} = mc_t \frac{\alpha}{r_t^k \, \epsilon_t^C} y_{j,t}. \tag{24}$$

Total real profits of the wholesale (Calvo) sector are $\Psi_t^C = \int_0^1 \Psi_{j,t}^C dj$, where

$$\Psi_{j,t}^C = \left\{ \frac{P_{j,t}}{P_t} - mc_t \right\} y_{j,t} \tag{25}$$

[12] Due to the assumption of competitive factor markets and constant returns to scale, all firms in the wholesale sector will face the same level of marginal cost. Especially, in this specification marginal cost are independent of each firm's output. Similar to Altig, Christiano, Eichenbaum, and Linde (2005) one could also make firms' marginal cost depend on the level of own output by assuming firmspecific production factors, say capital. This would introduce an internal motive to keep prices constant and so require less Calvo stickiness; see also Eichenbaum and Fisher (2004), Woodford (2005). For the case of real rigidities due to temporarily firmspecific labor in a matching model, see Kuester (2006).

denotes the period profits of firm j.[13] These profits flow to the families of asset-holding workers.

2.2.3 Labor Good Firms

The labor good is homogenous. Each firm in this sector consists of one and only one worker matched with an entrepreneur. In period t there is thus a mass $\nu(1-u_t^{(1)})$ of labor firms with workers of type $o=1$ and a mass $(1-\nu)(1-u_t^{(2)})$ with workers of type $o=2$. Match i can produce amount $l_{i,t}$ of the labor good according to

$$l_{i,t} = A_t^{(o)} h_{i,t}^{(o)}. \tag{26}$$

Labor productivity $A_t^{(o)}$ depends on which type of the worker is employed at the respective firm. Throughout the paper we associate liquidity-constrained workers with those who are relatively less productive and low-skilled. In particular, we assume that the productivity levels of low- and high-skilled workers are linked by a constant factor of proportionality:

$$A_t^{(2)} = aA_t^{(1)},\ a \in (0,1).$$

In equilibrium, labor good demand by the wholesale sector must match the labor good sector's supply:

$$l_t := \int_0^1 l_{j,t} dj = \int_0^{\nu(1-u_t^{(1)})} l_{i,t} di + \int_0^{(1-\nu)(1-u_t^{(2)})} l_{i,t} di. \tag{27}$$

2.3 Labor Market

We now turn to the specification of the labor market in our model. We first describe the matching technology and then focus on the bargaining and vacancy posting decisions for each class of worker $o \in \{1,2\}$.

[13] Real profits in equilibrium thus depend on the distribution of $P_{j,t}$:

$$\Psi_t^C = \left[1 - mc_t \int_0^1 \left(\frac{P_{j,t}}{P_t}\right)^{-\epsilon} dj\right] y_t.$$

2.3.1 Matching Firms and Workers

There is a separate matching market for the two types of workers. The matching process in each market is governed by a standard Cobb-Douglas matching technology

$$m_t^{(o)} = \sigma_m^{(o)} (u_t^{(o)})^{\xi^{(o)}} (v_t^{(o)})^{1-\xi^{(o)}}, \ \sigma_m^{(o)} > 0, \xi^{(o)} \in (0,1). \tag{28}$$

Here $m_t^{(o)}$ is the number of new matches of worker-type o, $v_t^{(o)}$ is the number of vacancies of type o. With probability $q_t^{(o)} = \frac{m_t^{(o)}}{v_t^{(o)}}$ a searching firm finds a worker in period t. With probability $s_t^{(o)} = \frac{m_t^{(o)}}{u_t^{(o)}}$ a worker of type o will find a job. For future reference, we define "market tightness" in sector o from the firms' point of view as $\theta_t^{(o)} = \frac{v_t^{(o)}}{u_t^{(o)}}$.

In the US, most of the variation of employment over the business cycle is explained by variations in vacancy posting, see Hall (2005), while the separation rate is rather stable. We therefore assume that separations occur with a constant, exogenous probability $\vartheta^{(o)} \in (0,1)$ in each period. Note that separation rates/job destruction rates can differ between the two skill groups. New matches in t, m_t, become productive for the first time in $t+1$. As a consequence of these assumptions, the employment rate $n_t^{(o)} := 1 - u_t^{(o)}$ evolves according to

$$n_t^{(o)} = (1 - \vartheta^{(o)}) n_{t-1}^{(o)} + m_{t-1}^{(o)}. \tag{29}$$

2.3.2 Bargaining of Asset-holding Families

Due to both a skill-dependent fixed cost $\kappa^{(1)}$ of posting a vacancy and the time-consuming matching process, formed matches entail economic rents. Firms and workers bargain about their share of the overall match surplus. Since the family perfectly insures its members against unemployment risks, individual members would not take up work voluntarily. We follow den Haan, Ramey, and Watson (2000) in assuming that the family takes the labor supply decision for its workers. We start by describing the gain of a representative family from having a marginal member i more in employment. This is (see Appendix 6.2 for a derivation)

$$\begin{aligned} \Delta_t^{(1)} &= \lambda_t \left(w_{i,t}^{(1)} h_{i,t}^{(1)} - b^{(1)} \right) - \kappa^L \frac{\left(h_{i,t}^{(1)}\right)^{1+\varphi}}{1+\varphi} \\ &\quad + (1 - s_t^{(1)} - \vartheta^{(1)}) E_t \left\{ \beta \Delta_{t+1}^{(1)} \right\}. \end{aligned} \tag{30}$$

The first term describes the difference between the net wage earned by the marginal member when employed and non-employment income, $b^{(1)}$. This difference is converted to marginal utility units by λ_t. The second term describes the increase in the disutility of work. The final term pertains to the continuation value.

We assume that firms cease to exist when they separate from a worker. The market value, $J_t^{(1)}$, of a representative firm with a worker of type $o = 1$ is given by

$$J_t^{(1)} = x_t^L A_t^1 h_{i,t}^{(1)} - w_{i,t}^{(1)} h_{i,t}^{(1)} + (1 - \vartheta^{(1)}) E_t \left\{ \beta_{t,t+1} J_{t+1}^{(1)} \right\}. \tag{31}$$

Each period, wages and hours worked are determined by means of Nash-bargaining about the match surplus:

$$\arg \max_{w_{i,t}^{(1)}, h_{i,t}^{(1)}} \left(\Delta_t^{(1)} \right)^{\mu^{(1)}} \left(J_t^{(1)} \right)^{1-\mu^{(1)}} \tag{32}$$

where $\mu^{(1)} \in (0, 1)$ denotes the family's bargaining power. In a symmetric equilibrium, all firms with workers pertaining to asset-holding families hire the same amount of hours, $h_t^{(1)}$, and pay the same wage, $w_t^{(1)}$. The associated first-order condition for the wage choice is

$$\frac{\mu^{(1)} \lambda_t}{\Delta_t^{(1)}} = \frac{1 - \mu^{(1)}}{J_t^{(1)}}. \tag{33}$$

The associated first-order condition for the hour choice in this case is given by:

$$\frac{\mu^{(1)} \left(\lambda_t w_t - \kappa^L \left(h_t^{(1)} \right)^{\varphi} \right)}{\Delta_t^{(1)}} = \frac{(1 - \mu^{(1)})(w_t^{(1)} - x_t^L A_t^1)}{J_t^{(1)}}. \tag{34}$$

Rearranging (33) and (34) leads to

$$\frac{\kappa^L \left(h_t^{(1)} \right)^{\varphi}}{\lambda_t} = x_t^L A_t^1, \tag{35}$$

which states that the marginal rate of substitution between consumption and hours worked at the family level corrected for taxes is equal to the marginal product of labor. Again, the resulting period profits of the representative firm with a worker of type 1, $\Psi_t^{(1)} = x_t^L A_t^1 h_t^{(1)} - w_t^{(1)} h_t^{(1)}$, accrue to the family.

Using the vacancy posting condition derived below in (37), one can show that the following wage equation obtains.

$$w_t^{(1)} h_t^{(1)} \quad = \quad \mu^{(1)} \left(A_t^{(1)} h_t^{(1)} x_t^L + \theta_t^{(1)} \kappa^{(1)} \right) + (1 - \mu^{(1)}) \left(b^{(1)} + \tfrac{1}{1+\varphi} mrs_t^{(1)} h_t^{(1)} \right) \tag{36}$$

where $mrs_t^{(1)} := \kappa^L \frac{\left(h_t^{(1)}\right)^\varphi}{\lambda_t^{(1)}}$ is the marginal rate of substitution of the family. Wage income is therefore a convex combination of productivity plus the savings from not having to re-post a vacant position and the outside option defined as the monetary benefit from staying at home plus the value of leisure expressed in terms of marginal consumption. The better the bargaining position of the worker is, captured here by a higher $\mu^{(1)}$, the closer is the wage rate to the marginal productivity of the worker and therefore to the (standard) competitive model.

2.3.3 Vacancy Posting for Workers of Type 1

In order to stand a chance of finding a worker associated with a family, firms need to post a vacancy. Vacancy posting costs are resource costs. They enter the economy's resource constraint as pure waste. Closing our description of the labor market for asset-holding workers, we assume free entry into the vacancy posting market as is standard in the literature. This guarantees that expected profits of new entrants, once properly discounted, are zero, i.e. the vacancy posting cost equals discounted expected profits in equilibrium

$$\kappa^{(1)} = E_t \left\{ \beta_{t,t+1} q_t^{(1)} J_{t+1}^{(1)} \right\}, \tag{37}$$

where $q_t^{(1)}$ is the probability of finding a worker of type 1 once a vacancy has been posted.

2.3.4 Bargaining of Liquidity-Constrained Workers

Liquidity-constrained workers do not live in a family which provides for full consumption insurance. Once matched, they bargain directly and individually with the firm, taking their own consumption when unemployed as the threshold. The utility difference of a representative liquidity-constrained worker can be written as

$$\begin{aligned} \Delta_t^{(2)} &= \left(\mathrm{u}^{(2)}(c_{e,t}^{(2)}, c_{t-1}^{(2)}, h_t^{(2)}) - \mathrm{u}^{(2)}(c_{u,t}^{(2)}, c_{t-1}^{(2)}, 0) \right) + (1 - s_t^{(2)} - \vartheta^{(2)}) E_t \left\{ \beta \Delta_{t+1}^{(2)} \right\} \\ &\equiv \Delta \mathrm{u}(c_{e,t}^{(2)}, c_{u,t}^{(2)}, c_{t-1}^{(2)}, h_t^{(2)}) + (1 - s_t^{(2)} - \vartheta^{(2)}) E \left\{ \beta \Delta_{t+1}^{(2)} \right\}. \end{aligned} \tag{38}$$

Where $c_{e,t}^{(2)}$ and $c_{u,t}^{(2)}$, defined in (15), are the consumption levels of the representative low-skilled worker when employed and unemployed, respectively. We take last month's average consumption of the liquidity-constrained population, $c_{t-1}^{(2)} = (1-u_{t-1}^{(2)})c_{e,t-1}^{(2)} + u_{t-1}^{(2)} c_{u,t-1}^{(2)}$, as the reference point for habit formation. A type 2 firm discounts future profit streams using the capital market's pricing kernel, which is the one of the typical asset-holding family. To shorten notation, denote period profits of a firm associated with a liquidity-constrained worker by

$$\Psi_t^{(2)} = x_t^L A_t^{(2)} h_t^{(2)} - w_t^{(2)} h_t^{(2)}, \tag{39}$$

where $A_t^{(2)}$ is the time-varying productivity level of the liquidity-constrained types. The market value of a firm with a low-skilled worker is given by

$$J_t^{(2)} = \Psi_t^{(2)} + (1 - \vartheta^{(2)}) E_t \left\{ \beta_{t,t+1} J_{t+1}^{(2)} \right\}, \tag{40}$$

The Nash bargaining defines how the joint match surplus is split among worker and firm. A type 2 worker and its employer set their real wage rate, $w_t^{(2)}$, and hours worked, $h_t^{(2)}$, as

$$\arg \max_{w_t^{(2)}, h_t^{(2)}} (\Delta_t^{(2)})^{\mu^{(2)}} (J_t^{(2)})^{1-\mu^{(2)}}, \tag{41}$$

where $\mu^{(2)} \in (0,1)$ is the bargaining power of low-skilled workers. The wage-setting first-order condition is

$$\frac{\mu^{(2)} \lambda_t^{(2)}}{\Delta_t^{(2)}} = \frac{(1 - \mu^{(2)})}{J_t^{(2)}}, \tag{42}$$

where $\lambda_t^{(2)} := (c_{e,t}^{(2)} - \varrho^{(2)} c_{t-1}^{(2)})^{-\sigma}$. The first-order condition with respect to the hour choice yields

$$\frac{\mu^{(2)} \left(\lambda_t^{(2)} w_t^{(2)}) - \kappa^L \left(h_t^{(2)} \right)^{\varphi} \right)}{\Delta_t^{(2)}} = \frac{(1 - \mu^{(2)})(w_t^{(2)} - x_t^L A_t^{(2)})}{J_t^{(2)}}. \tag{43}$$

Rearranging gives that also in the low-skilled sector, the marginal product of labor equals the marginal disutility of work.

The liquidity-constrained worker and the firm do not agree about the evaluation of their future joint surplus since the discounting kernels of the owner of the firm (the asset-holding family) and the liquidity-constrained worker differ. We cannot, therefore, derive a closed form expression for the wage in terms of current variables only. The current wage depends positively on both current and future profits which in turn depend on labor productivity, similar to (36). Since the liquidity-constrained worker is not insured against unemployment by a family, however, his attitudes towards risk as represented by the curvature of the utility function play a direct and important role in the wage bargaining process.

The Mortensen and Pissarides (1994) class of models with efficient bargaining need to rely on a small match surplus in order to endogenously generate fluctuations in unemployment rates as shown in Costain and Reiter (2005), Hagedorn and Manovskii (2005) and Jung (2005).[14]

2.3.5 Vacancy Posting for Workers of Type 2

Vacancy posting proceeds as for asset-holding workers. In order to have a chance, $q_t^{(2)}$, to find a worker, firms looking to recruit low-skilled workers need to post a vacancy at real cost $\kappa^{(2)} > 0$. We assume that hiring low-skilled workers requires less recruitment effort in real terms. Consequently, vacancy posting costs are lower than for asset-holding workers: $\kappa^{(2)} \leq \kappa^{(1)}$. To close the description of the labor market, we assume free entry into the vacancy posting market for liquidity-constrained workers. So real costs of posting a vacancy for a low-skilled worker, $\kappa^{(2)}$, are equated to the price of the claim to a firm of this type in t+1:

$$\kappa^{(2)} = E_t \left\{ \beta_{t,t+1} q_t^{(2)} J_{t+1}^{(2)} \right\}. \tag{44}$$

2.4 Government

We next turn to the behavior of the monetary and fiscal authorities.

2.4.1 Monetary Policy

The monetary authority is assumed to control the nominal one-month risk-free interest rate on nominal bonds, R_t, apart from the risk premium shock. The empirical literature (see, e.g. Clarida, Galí, and Gertler, 2000) finds that simple generalized Taylor-type rules of the form

$$\begin{aligned} \log(R)_t \quad = \quad & \log(\overline{\Pi}/\beta)(1-\gamma_R) + \log(R)_{t-1}(\gamma_R) + (1-\gamma_R)\left\{\gamma_\pi \log(\Pi_t/\overline{\Pi}) + \gamma_y \log(y_t/y)\right\} \\ & + \gamma_{\Delta_y} \log(y_t/y_{t-1}) + \log(\epsilon_t^{money}), \end{aligned} \tag{45}$$

once linearized are a good representation of monetary policy in recent decades. ϵ_t^{money} is an iid log-normal shock to the monetary policy stance.

2.4.2 Fiscal Policy

"Government spending", g_t, is exogenous and follows the following AR(1) process

$$g_t = (1-\rho_g)\overline{g} + \rho_g g_{t-1} + g\epsilon_t^g, \ \rho_g \in [0,1). \tag{46}$$

[14] Costain and Reiter (2005) criticize the implication that the unemployment rate for this particular calibration is very sensitive to a change in the outside option. Alleviating the criticism, in the liquidity-constrained worker's case the semi-elasticity of the unemployment rate with respect to a change in benefits is a bit smaller due to the curvature of the utility function. For a discussion of this point see e.g. Jung (2006).

$\overline{g}$ is the long-run target for government expenditures, ϵ_t^g is a Gaussian shock to fiscal policy with zero mean. In the calibration below, we set g_t equal to the sum of government expenditures and the net export component of GDP. The government budget constraint is given by

$$\begin{aligned} &\nu t_t + (1-\nu)\chi t_t \\ &+\tfrac{D_t}{P_t} + (\epsilon_t^C - 1)\left(\nu k_{t-1} z_t r_t^k + x_t^L l_t\right) \quad = \quad \nu u_t^{(1)} b^{(1)} + (1-\nu) u_t^{(2)} b^{(2)} + \tfrac{D_{t-1}}{P_t} R_{t-1} + g_t. \end{aligned} \tag{47}$$

The government generates revenue from lump-sum taxes; see the first row of (47). It also earns income from new debt issues, $\frac{D_t}{P_t}$. The final term on the left-hand side of (47) clarifies the "tax" nature of the cost-push shock. On the expenditure-side appear unemployment benefits (the terms involving $b^{(1)}, b^{(2)}$), debt repayment and coupon as well as the government expenditure. Let t_t^{tot} be total tax revenue in period t:

$$t_t^{tot} = \nu t_t + (1-\nu)\chi t_t. \tag{48}$$

Following Schmitt-Grohé and Uribe (2004) we assume that total tax revenues, t_t^{tot}, adjust in order to assure that real debt, D_{t-1}/P_{t-1}, does not deviate too much from the long-run debt target, $\overline{d}$, of the fiscal authority.

2.5 Market Clearing

2.5.1 Retail good

The aggregate retail good is used for consumption by the two types of consumers, for investment by asset-holding households and for government spending. In addition vacancy posting activity in the two labor markets requires ressources. Total demand is

$$\begin{aligned} y_t \quad = \quad & c_t + \nu\left[i_t + \psi(z_t) k_{t-1}\right] + g_t \\ + \quad & \nu \kappa^{(1)} v_t^{(1)} + (1-\nu)\kappa^{(2)} v_t^{(2)}, \end{aligned} \tag{49}$$

where aggregate consumption, c_t, is given by

$$c_t = \nu c_t^{(1)} + (1-\nu)[(1-u_t^{(2)}) c_{e,t}^{(2)} + u_t^{(2)} c_{u,t}^{(2)}]. \tag{50}$$

Market clearing in the retail market requires that above demand of retail goods equals total supply, which is given by

$$y_t = \left[\int_0^1 \left(y_{j,t}^d\right)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}. \tag{51}$$

2.6 Wholesale goods

For each firm j in the wholesale sector, its supply

$$y_{j,t} = (k_{j,t})^{\alpha} (l_{j,t})^{1-\alpha}, \tag{52}$$

must be matched by the corresponding demand

$$y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\epsilon} y_t \tag{53}$$

in order to clear the wholesale market.

2.7 Labor good

Total demand for the labor good is

$$l_t = \int_0^1 l_{j,t} dj, \tag{54}$$

where $l_{j,t}$ is given by (23). Market clearing in the labor good market requires that this demand equals the supply of the labor good which is given by

$$l_t = \nu(1 - u_t^{(1)}) \left(l_t^{(1)} \right) + (1 - \nu)(1 - u_t^{(2)}) \left(l_t^{(2)} \right), \text{ where} \tag{55}$$

$$l_t^{(o)} = A_t^{(o)} h_t^{(o)}, \ o \in \{1, 2\}. \tag{56}$$

2.8 Capital rental market

Total demand for capital is given by

$$k_t = \int_0^1 k_{j,t} dj, \tag{57}$$

where $k_{j,t}$ is given by (24). Total supply of capital is given by

$$k_t = \nu z_t k_{t-1}. \tag{58}$$

In equilibrium both quantities are equated.

3 Estimation of the Model

For the welfare analysis not least the structure and relative size of shocks will be important. We conduct a Bayesian estimation exercise in which we use quarterly data for the US from 1964:q1 to 2005:q4. The start of the sample is dictated by the first date the index of average weekly hours which we employ is available. We treat output per capita, consumption per capita, investment per capita, the nominal interest rate, GDP inflation and total hours worked as observable variables. Consumption is measured as the sum of consumption of non-durable goods and services. Consequently, investment is the sum of fixed private investment and consumption of durable goods. All data are obtained from the Federal Reserve Bank of St. Louis' database FRED II. Data on total compensation per capita, vacancies (measured by the help-wanted index) and unemployment rates are used for validation purposes. For the exact sources of the data, the definitions linking the data to our model variables and key properties of the data, we refer to Appendix 6.3.

The volatility of many aggregate real variables has decreased considerably since the early 1980s.[15] We therefore use the data until 1983:q4 only as a burn-in sample and let the observation sample start in 1984:q1. Calibrated steady state values are computed using the latter part of the sample. Some of the labor market series, for example the help-wanted index and the unemployment rate, exhibit low-frequency movements. Our model is not designed to explain these longer-run trends in the labor market. Consequently, we follow Shimer (2005) in detrending our data with a very low-frequency HP-filter with a weight of 100,000. Appendix 6.3 compares the raw data and their filtered business-cycle component.

Agents in our model economy take decisions at a monthly frequency. The data we employ, however, are quarterly. The estimation algorithm deals with this frequency mismatch; see Appendix 6.4. Autocorrelation coefficients, the Calvo frequency ω and the interest rate smoothing coefficient have been rescaled to quarterly terms for better comparability with the literature. In the tables and subsequent text, a superscript q signals this, e.g. $\omega \equiv (\omega^q)^{1/3}$.

[15] Kim and Nelson (1999) locate the break date in the amplitude of US GDP growth rates and the volatility of shocks to US GDP growth rates at 1984:q1 (their posterior mode). The same break date is found by McConnell and Perez-Quiros (2000). Stock and Watson (2002) document that this evidence is not limited to real GDP growth but can be found in a great number of US macroeconomic time series. This is also documented in Appendix 6.3.2 where we report the standard deviations of the series for two subsamples, the pre-break sample 1964:q1 to 1983:q4 and the post-break sample 1984:q1 to 2005:q4. The change in standard deviations is noticeable.

The remainder of this section is structured as follows. We briefly discuss our calibration and the value of fixed parameters in the next subsection. Subsection 3.2 discusses the shape of our prior. Subsection 3.3 reports the parameter values at the mode of the posterior distribution and gives approximate standard deviations. Subsection 3.4 provides performance statistics for the model in terms of data density, root-mean-squared errors and the implied standard deviation of model variables. In subsection 3.5 we extract the implications which the model carries for the labor market data: unemployment rates for the different skill groups, vacancies and wages and compare this to the available data.

3.1 Calibration

The estimation procedure seeks to be as parsimonious as possible. Those parameters which are well identified by long-run averages or outside information are calibrated in advance.[16] Table 1 summarizes our choices and gives the targets we match. In the following we elaborate on some of the parameters chosen.

With respect to parameters related to the labor market we follow Shimer (2005) in choosing a constant destruction rate of 3% on a monthly basis. We normalize $\sigma_m^{(1)}$ and $\sigma_m^{(2)}$, the scaling parameters in the respective matching functions, so as to match the steady state probability of finding a worker, $q^{(1)}$ and $q^{(2)}$, to the value used in den Haan, Ramey, and Watson (2000) and Hagedorn and Manovskii (2005). The choice of q has a bearing on the amount spent on vacancy posting in equilibrium. In the latter authors' setup, vacancy posting costs are lump-sum "tax" costs which do not figure in the economy's resource constraint. In our model this choice has some bearing on the resources available in equilibrium. Resource costs associated with vacancy posting, however, are small in our calibration; see also Table 2.

In setting the parameters which govern preferences and technology we follow standard procedures. Through its influence on the vacancy to unemployment ratio and therefore, ultimately, on the unemployment rate, the share of profits that firms in the labor good sector obtain after bargaining is a key determinant of labor market dynamics; see Hagedorn and Manovskii (2005) and Jung (2005). Two parameters, the vacancy posting costs, $\kappa^{(1)}$ and $\kappa^{(2)}$ and the bargaining power of the worker, $\mu^{(1)}$ and $\mu^{(2)}$ have a noticeable repercussion on both the steady state and the cyclical behavior of this profit-share. The vacancy posting cost directly determines the level

[16] In general equilibrium all steady state values may depend on a variety of parameters. For expositional purposes, we phrase the description in terms of the target on which the particular parameter at hand has most influence.

Table 1: Calibration with Liquidity-Constrained Consumers

Param.	Value	Meaning	Target/Reference
Preferences and Constraints			
β	0.998	time-discount factor	Match annual real rate of 3 percent.
σ	1.500	risk-aversion	Smets and Wouters (2007).
φ	2.000	inverse of Frisch elasticity of labor supply	Domeij and Flodén (2006).
κ^L	237.55	scaling parameter disutility of work	Avg. hours worked per employee = 1/3.
$\varrho^{(2)}$	0	habit persistence liq. constrained	no consumption habits.
$1-\nu$	0.16	share of liq. constrained	Gruber (2001).
Production			
a	0.50	relative technology	Relative wage income of 50 percent.
α	0.33	capital elasticity of production	Capital share of 31 percent.
δ	0.008	depreciation rate (monthly)	Investment/GDP ratio of 29 percent.
ϵ	21	markup of 5 percent	Small aggregate profits.
γ_z	0.0106	steady state marg. capacity util. cost	Full capacity utilization $z = 1$.
$\Psi^{(1)}/y$	0.008	profit share (unconstr. labor)	small profits.
$\Psi^{(2)}/y$	0.008	profit share (liquidity-constr. labor)	small profits.
Labor Market			
$\xi^{(1)}$	0.5	elasticity of matches w.r.t. unempl.	Petrongolo and Pissarides (2001).
$\xi^{(2)}$	0.5	elasticity of matches w.r.t. unempl.	Petrongolo and Pissarides (2001).
$\sigma_m^{(1)}$	0.63	efficiency of matching	$q^{(1)} = 0.7$, den Haan et al. (2000).
$\sigma_m^{(2)}$	0.43	efficiency of matching	$q^{(2)} = 0.7$, den Haan et al. (2000).
$\vartheta^{(1)}$	0.03	monthly rate of separation	Shimer (2005).
$\vartheta^{(2)}$	0.03	monthly rate of separation	Shimer (2005).
$\mu^{(1)}$	0.10	bargaining power asset holders	Fluctuations in unemployment.
$\mu^{(2)}$	0.19	bargaining power liq.-constr.	Fluctuations in unemployment.
Government			
$\frac{g}{y}$	16	"government spending"/GDP ratio	Consumption/GDP ratio of 59 percent.
χ	0.00	taxation of liqu.-constr. workers	They are not taxed.

Notes: Calibration strategy. For the figures reported, the data span is 1984:1 to 2005:4. The reported investment GDP is calculated as the ratio of fixed private investment plus durable consumption to GDP. The consumption output ratio consequently is computed as the ratio of non-durable consumption plus services to GDP.

of the profit-share in steady state and the bargaining power of the worker governs its cyclical properties. We assign a profit share $\Psi^{(o)}/y = 0.008$ in both labor sectors and a low bargaining power in both groups. The bargaining power was chosen so as to generate sufficient volatility in the respective unemployment rates. The unemployment fluctuation mechanism rests on a low steady-state profit-share, i.e. a high outside option for the worker, but a high implicit bargaining power of firms.

A notable share of the US population is liquidity-constrained. For example Gruber (2001) puts the share, $1-\nu$, of the population whose savings cannot cover the cost of an unemployment spell to at least 16%. Consequently, we set the size of the liquidity-constrained group of workers to $1-\nu = 0.16$.[17]

As mentioned above, we make two simplifying assumptions. First, we assume that the group without the financial means to cover a shortfall of income in an unemployment spell does not (or cannot) hold liquid assets at all, so called liquidity-constrained consumers. This assumption seems hardly tenable for agents which generate sizeable wage income. We therefore, second, equate the liquidity-constrained group with a group of agents who are low-skilled. Naturally, these workers are found at the lower end of the wage distribution.

The most natural candidate for representing unemployment fluctuations among low-skilled workers is the unemployment rate among high-school drop-outs. The Bureau of Labor Statistics series for the unemployment rate for high-school drop-outs, however, is only available from 1992:q1 onwards. This sample seems too short to corroborate the empirical fit of the extended model. Consequently, we select the unemployment rate of those who are 16 to 24 years old as representing unemployment among low-skilled, liquidity-constrained consumers.[18] This series is available from 1964:q1 onwards. Unemployment rates for low-skilled workers are higher than for higher-skilled ones. We set the average unemployment rate among the low-skilled (and by assumption liquidity-constrained) workers to 10 percent which lies in between the mean of the two above-mentioned series. For the properties of the data see Appendix 6.3. Our calibration ensures that the average unemployment rate in the economy matches the one in the data, namely $u = 5.8\%$. As is well known, see Mortensen and Pissarides (1999), in order to match the fact that low-skilled workers have higher unemployment rates than the average, one can either assume that they have a larger bargaining power or (possibly in the light of the relatively higher value of their home-

[17] Gruber (2001) obtains this number by using total wealth as the relevant pool of assets. If only liquid assets are taken into account the share rises significantly. This number therefore represents a lower bound for the share of liquidity constrained workers.

[18] Note that, in favor of our interpretation, high-school drop-outs and workers at the lower range of the skill-distribution are overrepresented in this series. Those who seek higher education enter the relevant labor force only at around age 20 or older. Apart from the length of the sample, our results are not affected by this choice.

production) a higher replacement rate. In this section, we set the liquidity-constrained workers' bargaining power and their replacement rate such that the steady state unemployment rate u^2 of 10 percent is matched and, using the one degree of freedom we have, such that we match the volatility of unemployment.

In order to calibrate our model to an economy-wide steady state unemployment rate of $u = 5.8\%$ we adjust the outside option, $b^{(1)}$, using the steady state wage equation (36). In doing so, we interpret replacement income as reflecting both (observable) unemployment benefits and self-insurance by means of (unobservable) home production or production in the shadow economy. Since unemployment benefits are part of the overall lump-sum transfers the family receives from the government, combining both elements in "unemployment benefits" is an innocent modeling device.

Of key importance is the relative skill/productivity level of the two groups, a. The OECD employment outlook reports compensation before taxes in the US for different income deciles. We use this information to calibrate the relative productivity as follows. The low-skilled, liquidity-constrained consumers range at the lower end of the income distribution. We approximate their average income by a weighted average of the income at the 10th percentile and the 20th income percentile. The average income in the asset-holding group is approximated by the 58th percentile (16%+84%/2) of the wage distribution. Dividing these two numbers and averaging over the years we obtain an average wage in the low-skilled, liquidity-constrained population of around 50% relative to the asset-holding population.[19] Consequently, we set the relative technology to $a = 0.5$.

We simplify our analysis by assuming that liquidity-constrained consumers are not subject to taxation, $\chi = 0$, which may be a good first-order approximation for low-income households. Lump-sum taxes thus fall entirely on the asset-holding households, which ensures that Ricardian equivalence holds. As such, the nature of the tax rule is irrelevant for the allocations in our model as long as the tax rule is debt-stabilizing which our calibration guarantees.

Consumers in different skill groups share the same preferences. The inverse of the intertemporal elasticity of substitution is set to $\sigma = 1.5$. One of the more controversial parameters concerns the Frisch elasticity, $1/\varphi$. Most of the Real Business Cycle (RBC) literature assumes that labor supply is rather elastic. Typically a value of at most $\varphi = 1$ is chosen. Since our model does not only feature an intrinsic margin of employment (hours choice) but also an extrinsic (hiring) margin we can rely on a Frisch elasticity more in line with micro-evidence. We use the value of 0.5 estimated by Domeij and Flodén (2006) which implies a value of $\varphi = 2$.[20] Consumers'

[19] In computing this number, wer use the entire span of data at our hands. For the income deciles, we do, however, only have data from 1973 to 2000.

preferences differ only in their consumption habits. Liquidity-constrained consumers are highly dependent on their current income. They therefore do not develop consumption habits at all, so $\varrho^2 = 0$.

Table 2: Steady State with Liquidity-Constrained Consumer

Variable	Value	Economic Meaning
$v^{(1)}$	0.04	vacancies (as share of labor force).
$v^{(2)}$	0.04	vacancies (as share of labor force).
$s^{(1)}$	0.57	probability of finding a job.
$s^{(2)}$	0.27	probability of finding a job.
$\theta^{(1)}$	0.81	market tightness.
$\theta^{(2)}$	0.39	market tightness.
$b^{(1)}/(w^{(1)}h^{(1)})$	0.64	replacement rate (including home production).
$b^{(2)}/(w^{(2)}h^{(2)})$	0.70	replacement rate (including home production).
$\mu^{(2)}$	0.19	bargaining power of liq.-constrained workers.
$w^{(2)}h^{(2)}/(w^{(1)}h^{(1)})$	0.49	relative wage income.
y/l	5.32	labor (good) productivity.
$k\nu/y$	29.73	capital output ratio.
i/y	0.29	share of investment in GDP.
c/y	0.59	share of consumption in GDP.
$(\nu\kappa^{(1)}v^{(1)} + (1-\nu)(\kappa^{(2)}v^{(2)}))/y$	0.01	share of vacancy posting costs in GDP.
whn/y	0.63	labor share all.
$r^k ks/y$	0.31	capital share.
Ψ^C/y	0.05	profit share (Calvo sector).
$\Psi^{(1)}/y$	0.01	profit share (asset-holding labor sector).
$\Psi^{(2)}/y$	0.01	profit share (asset-holding labor sector).

Notes: Selected features of the steady state when allowing for liquidity-constrained consumers. All values refer to a monthly frequency. The reported investment GDP ratio is calculated as the ratio of fixed private investment plus durable consumption to GDP. The consumption output ratio consequently is computed as the ratio of non-durable consumption plus services to GDP.

The implied steady state, which we report in Table 2 features a lower probability of finding a job for a low skilled, liquidity-constrained worker than for a higher-skilled worker; compare

[20] For a given level of unemployment benefits, $b^{(1)}$ and $b^{(2)}$, a change in the Frisch elasticity alters the surplus of a match. The choice of the Frisch elasticity can thus have a considerable impact on the dynamics of the model. For intuition confer e.g. wage equation (36). Given that we understand part of the benefits, $b^{(1)}$ and $b^{(2)}$, as representing unobservable home production, the mechanism we highlight is not affected by our choice of the Frisch elasticity; see Jung (2005).

$s^{(1)}$ and s^2. The steady state replacement rate for the low-skilled workers is slightly higher, too, which makes sense if part of this replacement is interpreted in terms of home-production, which typically tends to be associated with lower-skilled work. The replacement rate $\frac{b}{wh}$ is 0.64 for the consumer's in the family and higher for the low-skilled workers, reflecting the official unemployment benefits plus any value from home production. Our labor market is one in which a bit more than half of the higher skilled workers looking for work find a job in each month, $s^{(1)} = 0.57$ and about 27% of the low-skilled workers. The share of vacancy posting costs in GDP is small at one percent.

3.2 Priors

The first four columns in Table 3 present summary statistics of the prior distribution for each of the estimated parameters. We report the mean of the prior distribution, the standard deviation and the class of density which we use to model the prior. The marginal priors are assumed to be independent.

The standard deviations of the innovations to the structural shocks $A_t^{(1)}$, ϵ_t^C, ϵ_t^g, ϵ_t^I and ϵ_t^b are assumed to follow inverse gamma distributions. With regard to the standard deviation of the innovation to the monetary policy shock ϵ_t^{money} we entertain a prior mean of 0.015.[21] For the other shocks these means are selected by experimentation with the model.

We fix the autocorrelation of the technology shock to 0.9 at quarterly frequency. The persistence parameters of the other AR(1) shock-processes are assumed to follow beta distributions. The auto-correlations of all other shock processes carry a prior mean of 0.85 (at a quarterly frequency) with a standard deviation of 0.1. The cost-push shock is modelled as either iid or as highly serially correlated in the literature (cp. e.g. Smets and Wouters (2003) and Smets and Wouters (2007)). We opt for the middle-ground and set a mean correlation of 0.5.

The central bank's behavioral equation is parameterized similar to Smets and Wouters (2007). The smoothing coefficient ρ_m^q is beta with mean 0.70 and standard deviation 0.15. The reaction coefficients to inflation and output follow the proposal of Taylor (1993). For the reaction to output growth we choose a prior mean of zero.

The habit persistence parameter, $\varrho^{(1)}$, is assigned a beta prior with mean 0.7 as in Smets and

[21] The mean we choose corresponds to 1.5 basis points at a monthly rate.

Wouters (2005) and a similar standard deviation of 0.15.

The Calvo probability, ω^q, *a priori* lies around 0.75 at the quarterly frequency, implying an average duration of a year in line with the results by Galí and Gertler (1999).[22] The degree of indexation of these prices to lagged inflation, γ_p, is assigned a mean of 0.4, close to the estimates of Smets and Wouters (2005). We give it a beta prior distribution with a wide standard deviation of 0.2.

Turning to the investment adjustment costs and capacity utilization costs, we use the posterior mode estimates of Smets and Wouters (2005) as our priors. The scaling parameter of investment adjustment costs, κ^I, is assigned a gamma prior with a mean equal to 5 and a standard deviation of 0.5. The curvature of capacity utilization costs, $\gamma_{z,2}$, has a gamma prior with mean 0.3 and a standard deviation of 0.25.

3.3 Posterior Mode of Model Parameters

This subsection discusses the parameter estimates at the posterior mode and the steady state implied by the estimates. We wish to emphasize again that the model runs at a monthly frequency. All persistence parameters of shocks and the Calvo frequency have been brought to a quarterly frequency to allow for comparisons with the literature. In columns 5 to 7, Table 3 displays the posterior mode estimate, an approximate standard deviation and a resulting "t-statistic".

The estimates in the monetary policy reaction function are in line with the literature, albeit at the higher end for the response of interest rates to inflation. Habits, $\varrho^{(1)}$, are estimated to a value well below 0.7. This fits well with the low estimates of habits that Smets and Wouters (2005) obtain on a similar sample for the US at a quarterly frequency.

The estimated value of the Calvo parameter, ω^q, of roughly 0.77 implies an average price duration of more than a year. While this is well above the average duration of half a year obtained in

[22] A duration of six months (half a year/two quarters) as in the micro-data of Bils and Klenow (2004) would correspond to $\omega^q = 0.5$. Recently, there has been an active literature how to reconcile these macro results with the evidence of higher nominal flexibility in micro-data. For example assuming that production factors are firmspecific as in Altig, Christiano, Eichenbaum, and Linde (2005) and Eichenbaum and Fisher (2004) helps to dampen the response of inflation to marginal costs. The only feature working in this direction which our model entertains is variable capacity utilization. As highlighted by Smets and Wouters (2005), however, the data do not support this feature as economically very important once one moves to general equilibrium. Most prominently, for the bargaining framework when the labor good and the retail good are produced in different sectors, the literatue so far has not come up with a way to build rigidity into x_t^L, the price of the labor good. In particular, wage rigidity does not help to achieve this, see Krause and Lubik (2005).

Table 3: Estimated Parameters at the Posterior Mode

Parameter	prior			posterior		"t-stat"
	mean	std	distr.	mode	std	
Parameters of Structural Model						
γ_R	0.700	0.150	beta	0.847	0.031	27.270
γ_π	1.500	1.000	gamm	3.836	0.757	5.067
γ_y	0.500	0.200	gamm	0.586	0.183	3.202
$\gamma_{\Delta y}$	0.000	0.100	norm	0.041	0.009	4.361
$\varrho^{(1)}$	0.700	0.150	beta	0.315	0.083	3.805
ω^q	0.750	0.150	beta	0.790	0.035	22.856
γ_p	0.400	0.200	beta	0.120	0.115	1.045
κ^I	5.000	0.500	gamm	4.659	0.492	9.478
$\gamma_{z,2}$	0.300	0.025	gamm	0.296	0.025	11.887
Correlation of Shocks						
ρ_g^q	0.850	0.100	beta	0.844	0.044	18.973
ρ_I^q	0.850	0.100	beta	0.660	0.067	9.867
ρ_b^q	0.850	0.100	beta	0.807	0.043	18.614
ρ_C^q	0.500	0.200	beta	0.526	0.083	6.318
Standard Deviation of Shocks						
σ^b	0.200	Inf	invg	0.060	0.009	6.685
σ^A	0.100	Inf	invg	0.475	0.035	13.407
σ^C	5.000	Inf	invg	1.707	0.483	3.536
σ^g	0.500	Inf	invg	1.831	0.139	13.143
σ^m	0.015	Inf	invg	0.032	0.003	10.355
σ^I	0.100	Inf	invg	1.596	0.282	5.666

Notes: Estimates of the posterior mode. The standard deviation is obtained by a Gaussian approximation at the posterior mode. "t-stat" refers to the mode estimate divided by the posterior marginal standard deviation.

Bils and Klenow (2004), this high value is in line with much of the empirical macroeconomic literature. As a comparison, for the US Smets and Wouters (2005) estimate an average duration of more than two and a half years.

Our estimate of the price indexation parameter, γ_p, is more on the low side.

The investment adjustment costs as captured by κ^I move slightly below their prior mode, yet overall remain close to their priors. The technology shock has a posterior correlation of 0.93 in quarterly terms and is thus close to the value of 0.95 typically used in the RBC literature. All other shocks are also highly correlated.

While the posterior mode of the standard deviation of each of the innovations is hard to interpret, we remark that none of these is considerably larger than 1 percent.

Having described the parameter estimates and the steady state of the model, we next look deeper into the empirical fit of our model.

3.4 Performance Measures

In this section we evaluate the performance of our estimation procedure. We start by reporting the marginal data density of the model and the root-mean-squared forecast error (RMSE) for the observable variables. Implicitly these are the prime metrics the estimation procedure seeks to optimize. We benchmark our results against a Bayesian vector auto-regression (BVAR) with flat priors as a non-structural description of the data. Unless stated otherwise all results pertaining to the model refer to the parameters evaluated at the posterior mode.

Table 4 shows the marginal data density of the model using a Laplace approximation around the posterior mode. For completeness, we additionally report the log likelihood and the log prior evaluated at the posterior mode.

Table 4: Log Marginal Data Densities

VAR(1)		VAR(2)		VAR(2)		Model		
Exact	Laplace	Exact	Laplace	Exact	Laplace	Laplace	Likelihood	Prior
-247.47	-248.32	-238.23	-240.50	-250.32	-254.75	-317.17	270.1883	8.5078

Notes: Marginal data density of Bayesian VARs with one to three lags under flat priors, using the Laplace approximation and the exact formula each. The model marginal data density is computed using the Laplace approximation. Also reported are the log likelihood and the log prior evaluated at the posterior mode

As an empirical benchmark we report the marginal data density for Bayesian VARs up to a lag of order three. The data density of the model unfortunately is quite far away from the data density of the non-structural competitors.[23] Yet as the next section will highlight, the model at hand still captures labor market fluctuations very well.

Table 5 contrasts the in-sample root-mean-squared forecast errors of the observable time-series when using the model (column two) with those obtained in a VAR(1) (column three).

Table 5: Model RMSE and Second Moments Compared to Data

Variable	RMSE (model)	RMSE (VAR)	std (model)	std (data)	std (VAR)
$\widehat{y}_t$	0.55	0.46	2.10	1.61	1.56
$\widehat{c}_t$	0.41	0.33	1.59	1.15	1.10
$\widehat{R}_t$	0.12	0.11	0.47	0.38	0.38
$\widehat{\Pi}_t$	0.23	0.20	0.39	0.24	0.24
$\widehat{i}_t$	1.79	1.33	6.41	6.39	5.93
$\widehat{h}_t + \widehat{n}_t$	0.44	0.30	1.52	1.20	1.18

Notes: The table compares root mean squared forecast errors of the model (in sample) at the posterior mode to those of a VAR(1) with flat priors at the posterior mode (second and third column). All values are computed from 1984q1 to 2005q4. The fourth to sixth columns compare the standard deviations of the observable variables implied by the model (column four) to those obtained from the data directly (column five) and to standard deviations implied by the Bayesian VAR (again at the posterior mode where applicable). All variables are in logs, HP(100.000) filtered and multiplied by 100 in order to express them in percent deviation from trend. All data are in quarterly terms. From top to bottom:
log output per capita, log consumption per capita log gross nominal interest rate, log gross GDP inflation rate, log investment per capita and log total hours worked. Investment includes durable consumption. Consequently, consumption is computed as non-durable consumption plus consumption of services.

Columns four to six of Table 5 compare the unconditional standard deviations of the observable variables as implied by the model to those of the data.

We next turn to independent evidence with regard to whether the model captures labor market fluctuations.

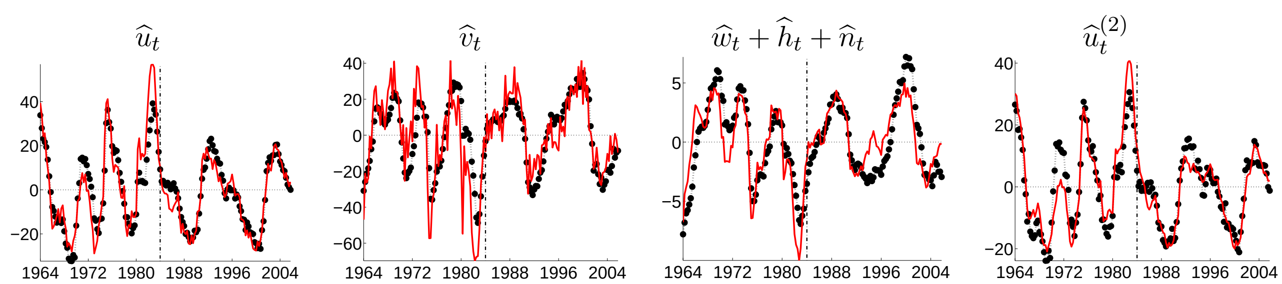
3.5 Independent Evidence and Evaluation

The information of labor market related time series so far has not been exploited in the estimation of the model parameters and the shock process. In particular, we reserved data for nation-wide unemployment, $\widehat{u}_t$, low-skilled unemployment, $\widehat{u}_t^{(2)}$, aggregate vacancies, $\widehat{v}_t$, and total wages, $\widehat{w}_t + \widehat{h}_t + \widehat{n}_t$, for validation purposes. Towards this aim, Figure 1 plots the actual data for these variables – together with total wage income – against the Kalman-smoothed estimates

[23] Like in the estimation of the model, the BVARs feature a long burn-in sample, from 1964:q1 to 1983:q4.

evaluated at the parameters of the posterior mode. Dots mark the actual observations and red solid lines depict the counterfactual time series implied by the model. All information refers to quarterly aggregates as discussed in Appendix 6.4.3. A vertical dashed line marks the start of our observation sample. Given that the estimation was not designed to match these variables, success in replicating the respective fluctuations in these series presents striking independent evidence in favor of the model.

Figure 1: Smoothed Endogenous Variables Not Used in Estimation



Notes: The graphs show the actual data (black dotted line marked by larger dots) against the (Kalman-smoothed) estimates originating from the model (red solid line) when parameters are evaluated at the posterior mode. These are used as independent evidence of the model's fit. The data is of quarterly frequency. All four series are HP(100,000) filtered log series and scaled by 100 to represent percent deviations. From left to right: nationwide unemployment rate, vacancies, real total wages, low-skilled unemployment rate. A vertical dashed line marks the beginning of the sample period in 1984:q1.

The model replicates both the variability of the unemployment and vacancy data and and its cyclical behaviour very well as can be inferred from the first, second and fourth panels of Figure 1. This highlights that the description of the labor market, which underlies the Mortensen and Pissarides (1994) type matching function, fits in well with the facts when embedded in a general equilibrium structure. The key mechanism of the model which endogenously generates labor market fluctuations relies on a low bargaining power of workers paired with a small steady state profit share in the labor good sector. Small profits in steady state mean that a given real fluctuation in profits causes large percentage fluctuations. A low bargaining power of workers causes wages to vary less pro-cyclically than in the benchmark RBC model or in a New Keynesian model without wage rigidity. These two features together imply that any increase in revenue translates into a noticeable increase in percentage profits. Since expected profits are the driving force of vacancy posting activity in our model, large percentage fluctuations in profits induce strong fluctuations in employment as found in the data.

The model also captures the cycles in the actual data for total wages (third panel of Figure 1). By replicating employment fluctuations, however, the model does not allow for enough volatility in wages per hour during our observation period. In other words, the wages implied by the model are slightly too rigid. Interestingly, for the sample running from 1964:q1 to 1983:q4 the model *does* match the volatility of wage rates even without using any information on wages in our estimation. During this period wage rates were less volatile than output, with a relative standard deviation of 0.86. Stock and Watson (2002), among others, have highlighted that in the "great moderation" of the 1980s the standard deviation of a great many US time-series has decreased, as is the case e.g. for output. The standard deviation of wages in contrast has increased; confer Tables 8 and 9 in Appendix 6.3.2. In our data this is noticeable in a more than 50% increase in the standard deviation of the wage rate relative to output – rendering wages more volatile than output in the latter part of the sample. Consequently, the model captures wage fluctuations better in the earlier half of the sample.

4 Welfare Criteria

We evaluate the welfare costs of business cycles in terms of consumption equivalents. In particular, we ask how much of steady state consumption would a consumer be willing to give up in order to swap the actual allocation against the steady state allocation, i.e. the allocation prevailing in the absence of business cycle shocks.

4.1 Asset-holding households

Welfare of asset-holding households is given by

$$W_t^{(1)} = \frac{\left(c_t^{(1)} - \varrho^{(1)} c_{t-1}^{(1)}\right)^{1-\sigma}}{1-\sigma} - \kappa^L (1 - u_t^{(1)}) \frac{\left(h_t^{(1)}\right)^{1+\varphi}}{1+\varphi} + \beta E_t \left\{ W_{t+1}^{(1)} \right\}.$$

Assigning $\lambda_t^{\text{equiv},1}$ times steady state consumption and the steady state allocation of unemployment and hours worked, welfare is

$$\overline{W}_t^{(1)} = \left(\lambda_t^{equiv,1}\right)^{1-\sigma} \frac{1}{1-\beta} \left[\frac{\left(c - \varrho^{(1)} c\right)^{1-\sigma}}{1-\sigma} \right] - \frac{1}{1-\beta} \left[\kappa^L (1 - u^{(1)}) \frac{\left(h^{(1)}\right)^{1+\varphi}}{1+\varphi} \right]$$

Our measure of welfare defines the consumption-equivalent by equating actual welfare with counterfactual welfare in steady state:

$$W_t^{(1)} \equiv \overline{W}_t^{(1)} \Rightarrow \lambda_t^{equiv,1}.$$

4.2 Liquidity-constrained workers

For liquidity-constrained consumers the consumption-equivalents are computed under the assumption that the consumer would need to give up the same percentage of consumption relative to steady state when unemployed and when employed.

4.2.1 Welfare of unemployed and employed workers

$$W_{u,t}^{(2)} = \frac{\left(c_{u,t}^{(2)} - \varrho^{(2)} c_{t-1}^{(2)}\right)^{1-\sigma}}{1-\sigma} + \beta s_t^{(2)} E_t \left\{ W_{e,t+1}^{(2)} \right\} + \beta (1 - s_t^{(2)}) E_t \left\{ W_{u,t+1}^{(2)} \right\}.$$

$$W_{e,t}^{(2)} = \frac{\left(c_{e,t}^{(2)} - \varrho^{(2)} c_{t-1}^{(2)}\right)^{1-\sigma}}{1-\sigma} - \kappa^L \frac{\left(h_t^{(2)}\right)^{1+\varphi}}{1+\varphi} + \beta (1 - \vartheta^{(2)}) E_t \left\{ W_{e,t+1}^{(2)} \right\} + \beta \vartheta^{(2)} E_t \left\{ W_{u,t+1}^{(2)} \right\}.$$

Taking steady state welfare evaluated at welfare equivalent consumption:

$$\begin{bmatrix} \overline{W}_u^{(2)} \\ \overline{W}_e^{(2)} \end{bmatrix} = \frac{1}{[1-\beta(1-s^{(2)})]\,[1-\beta(1-\vartheta^{(2)})]-\beta^2\vartheta^{(2)}s^{(2)}} \begin{bmatrix} 1-\beta(1-\vartheta^{(2)}) & \beta s^{(2)} \\ \beta\vartheta^{(2)} & 1-\beta(1-s^{(2)}) \end{bmatrix}$$
$$\cdot \left[\left(\lambda_t^{equiv,2,x}\right)^{1-\sigma} \begin{bmatrix} \frac{\left(c_u^{(2)}-\varrho^{(2)}c^{(2)}\right)^{1-\sigma}}{1-\sigma} \\ \frac{\left(c_e^{(2)}-\varrho^{(2)}c^{(2)}\right)^{1-\sigma}}{1-\sigma} \end{bmatrix} - \begin{bmatrix} 0 \\ \kappa^L \frac{\left(h^{(2)}\right)^{1+\varphi}}{1+\varphi} \end{bmatrix} \right].$$

Here the superscript x in $\lambda_t^{equiv,2,x}$ shows that we entertain three different concepts of welfare measures for low-skilled households. One of these asks for the willingness to pay by a currently unemployed worker:

$$W_{u,t}^{(2)} \equiv \overline{W}_u^{(2)} \Rightarrow \lambda_t^{equiv,2,u}.$$

Similarly, one can ask for the willingness to pay by a currently employed household:

$$W_{e,t}^{(2)} \equiv \overline{W}_e^{(2)} \Rightarrow \lambda_t^{equiv,2,e}.$$

Alternatively, one can target the average liquidity-constrained household:

$$u_t^{(2)}W_{u,t}^{(2)} + (1-u_t^{(2)})W_{e,t}^{(2)} \equiv u^{(2)}\overline{W}_u^{(2)} + (1-u^{(2)})\overline{W}_e^{(2)} \Rightarrow \lambda_t^{equiv,2}.$$

5 Conclusion

This paper points out that the basic Mortensen and Pissarides (1994) search and matching model embedded into a new Keynesian general equilibrium structure fits the labor market very well. We estimate the model via Bayesian methods on six time-series with six structural shocks. We argue that the model reasonably approximates the data although non-structural competitors produces significantly lower forecast errors. In particular, the paper demonstrates that the model can reproduce the dynamics of labor market related time series such as unemployment rates and vacancies within tight bounds.

Therefore, in our view, the model can now be used to derive welfare implications. In particular the introduction of a low-skilled sector, which is designed to approximate the heterogeneity of consumers in a simple fashion, might lead to new insights about the cost of business cycles. The standard model relies on a perfect insurance assumption to insure that individual wages are not a function of individual capital holdings. In this context it is well known since Lucas (1987) that the welfare costs of business cycles are close to negligible. However, once the assumption of perfect insurance is relaxed, the cost for at least a fraction of the population - namely the low-skilled low income groups - might be more severe. Our paper presented a rich economic structure that describes the data to a reasonable extent. We are currently working on evaluating the welfare costs of business cycles in our model framework.

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6 Appendix

6.1 Derivation of Family Utility Function $\mathtt{U}$

In each period the family optimally provides consumption allocations to its members. Consumption allocations are optimal if all members receive the same marginal utilities with respect to consumption. The optimality condition is given by

$$(c_{e,i,t}^{(1)} - \varrho^{(1)} c_{t-1}^{(1)})^{-\sigma} = (c_{u,t}^{(1)} - \varrho^{(1)} c_{t-1}^{(1)})^{-\sigma}, \tag{59}$$

where $c_{e,i,t}^{(1)}$ denotes consumption of an employed member i and $c_{u,t}^{(1)}$ denotes the consumption of unemployed members. Therefore, since marginal utility of consumption does not depend on hours worked, we have

$$c_{e,i,t}^{(1)} = c_{u,t}^{(1)} = c_t^{(1)}.$$

Consequently, inner-period family utility can be expressed as

$$\mathtt{U}(c_t^{(1)}, c_{t-1}^{(1)}, \{h_{i,t}^{(1)}\}, u_t^{(1)}, \epsilon_t^L) = \frac{\left(c_t^{(1)} - \varrho^{(1)} c_{t-1}^{(1)}\right)^{1-\sigma}}{1-\sigma} - \epsilon_t^L \int_0^{1-u_t^1} \frac{\left(h_{i,t}^{(1)}\right)^{1+\varphi}}{1+\varphi} di. \tag{60}$$

6.2 Derivation of Utility Difference

The utility difference for the family can be derived by computing the value of a marginal family member in employment. Towards that aim, we compute the change in (3) subject to an optimal allocation of ressources among family members (60), budget constraint (5) and the employment flow constraint (29) which states that the fraction of employed members in the family evolves according to

$$n_t^{(1)} = (1 - \vartheta^{(1)}) n_{t-1}^{(1)} + s_{t-1}^{(1)} u_{t-1}^{(1)}.$$

The utility difference, Δ_t, for family utility (60) is then easily computed as

$$\begin{aligned} \frac{\partial \mathtt{W}_t^{(1)}}{\partial(-u_t^{(1)})} :\ &= \Delta_t \\ &= \epsilon_t^{\mathtt{W}} \left[\left(c_t^{(1)} - \varrho^{(1)} c_{t-1}^{(1)}\right)^{-\sigma} \left(w_{i,t}^{(1)} h_{i,t}^{(1)} (1-\tau^L) - b^{(1)}\right) \right. \\ &\quad \left. - \epsilon_t^L \frac{\left(h_{i,t}^{(1)}\right)^{1+\varphi}}{1+\varphi} \right] + (1 - s_t^{(1)} - \vartheta^{(1)}) E_t \left\{ \beta \Delta_{t+1}^{(1)} \right\}. \end{aligned} \tag{61}$$

6.3 Data

6.3.1 Source and Definition of Data

Table 6: Description of Raw Data

	Mnemonic	Data description
Consumption of services	PCESV	Personal Consumption Expenditures: Services quarterly, seasonally adjusted annual rates billions of dollars.
Consumption of non-durables	PCND	Personal Consumption Expenditures: Nondurable Goods quarterly, seasonally adjusted annual rates billions of dollars.
Consumption of durables	PCDG	Personal Consumption Expenditures: Durable Goods quarterly, seasonally adjusted annual rates billions of dollars.
Fixed investment	FPI	Fixed Private Investment quarterly, seasonally adjusted annual rates billions of dollars.
GDP deflator	GDPDEF	Gross Domestic Product: Implicit Price Deflator quarterly, seasonally adjusted annual rate index 2000=100.
Interest rate	FEDFUNDS	Effective Federal Funds Rate monthly average, % p.a. quarterly average of monthly figures (own aggregation).
Labor force	CNP16OV	Civilian Noninstitutional Population thousands, quarterly average of monthly figures (own aggregation).
Output	GDP	Gross Domestic Product quarterly, seasonally adjusted annual rates billions of dollars.
Total hours worked	AWHNONAG	Average Weekly Hours: Total Private Industies (index) monthly, seasonally adjusted, quarterly average of monthly figures (own aggregation).
Total wages	WASCUR	Compensation of Employees: Wages and Salary Accruals, billions of dollars, quarterly, seasonally adjusted at annual rate.
Unemployment rate	UNRATE	Civilian Unemployment Rate monthly, seasonally adjusted, quarterly average of monthly figures (own aggregation).
Unempl. rate low-skilled	LNS14027659	Unempl. Rate - Less than High School Diploma, 25 yrs and over monthly, seasonally adjusted, source: Bureau of Labor Statistics, quarterly average of monthly figures (own aggregation).
Vacancies	HELPWANT	Index of Help-Wanted Advertising base year 1987=100, seasonally adjusted quarterly average of monthly figures (own aggregation).
Youth unemployment rate	LNS14024887	Unemployment Rate - 16-24 yrs, monthly, seasonally adjusted, source: Bureau of Labor Statistics. quarterly average of monthly figures (own aggregation).

Notes: All data are obtained from the Federal Reserve Bank of St. Louis database FRED unless explicitly stated otherwise.

Table 7: Data Used for Estimation and Verification

Variable	Formula
Consumption per capita	$c_t = (\text{PCESV}+\text{PCND})_t/(4 \times \text{GDPDEF}_t \cdot \text{CNP16OV}_t)$.
Investment per capita	$i_t = (\text{PCDG}+\text{FPI})_t/(4 \times \text{GDPDEF}_t \cdot \text{CNP16OV}_t)$.
Output per capita	$y_t = \text{GDP}_t/(4 \times \text{GDPDEF}_t \cdot \text{CNP16OV}_t)$.
Quarterly federal funds rate	$R_t = 1 + \text{FEDFUNDS}_t/400$.
Quarterly inflation rate	$\Pi_t = \text{GDPDEF}_t/\text{GDPDEF}_{t-1}$.
Total hours worked	$h_t \cdot n_t = \text{AWHNONAG}_t \cdot (1 - \text{UNRATE}_t/100)$.
Total wages	$w_t \cdot h_t \cdot n_t = \text{WASCUR}_t/(4 \times \text{GDPDEF}_t \cdot \text{CNP16OV}_t)$.
Unemployment rate	$u_t = \text{UNRATE}_t/100$.
Unemp. rate liquidity-constr.	$u_t^{(2)} = \text{LNS14024887}_t/100$.
Vacancies	$v_t = \text{HELPWANT}_t$.

Notes: Mnemonics in the formulae refer to the definitions in Table 6.

6.3.2 Moments and Plots of the Data

Table 8: Moments of Data (1984:q1 to 2005:q4)

Variable	Meaning	std	std to y	corr with y	mean ratio to y
$\widehat{y}_t$	output	1.61	1	1	1
$\widehat{c}_t$	consumption	1.15	0.71	0.73	0.59
$\widehat{i}_t$	investment	6.39	3.98	0.81	0.24
		std	std to y	corr with y	mean
$\widehat{R}_t$	nom. interest rate (gross)	0.38	0.24	0.59	1.013
$\widehat{\pi}_t$	GDP inflation rate (gross)	0.24	0.15	-0.04	1.006
$\widehat{h}_t + \widehat{n}_t$	total hours worked	1.20	0.75	0.86	—
$\widehat{w}_t + \widehat{h}_t + \widehat{n}_t$	total wage income	3.05	1.90	0.86	—
$\widehat{w}_t$	wage per hour	2.33	1.45	0.68	—
$\widehat{u}_t$	unemployment rate (all)	13.89	8.64	-0.92	5.79
$\widehat{u}_t^{(2)}$	u. rate 16-24 (proxy low-skill)	10.02	6.23	-0.91	11.94
$\widehat{u}_t^{(2)}$	u. rate high-school drop-out	12.41	7.72	-0.90	8.47
$\widehat{v}_t$	vacancies (help-wanted index)	18.81	11.70	0.82	—

Notes: The table reports summary statistics of the data. The third to fifth column show statistics for HP(100.000) filtered data. The third column reports the standard deviation of the series, the fourth its standard deviation relative to GDP. The fifth column shows the cross-correlation with GDP. For GDP components, the final column reports the mean share in GDP. For all other series, the mean of the series in levels is shown. The computations are performed on the sample from 1984:q1 to 2005:q4. The data is HP-filtered from 1964:q1 to 2005:q4. The reported investment GDP ratio is calculated as the ratio of fixed private investment plus durable consumption to GDP. The consumption output ratio consequently is computed as the ratio of non-durable consumption plus services to GDP.

Table 9: Moments of Data (1964:q1 to 1983:q4)

Variable	Meaning	std	std to y	corr with y	mean ratio to y
$\widehat{y}_t$	output	2.92	1	1	1
$\widehat{c}_t$	consumption	1.94	0.66	0.91	0.54
$\widehat{i}_t$	investment	6.42	2.20	0.80	0.24
		std	std to y	corr with y	mean
$\widehat{R}_t$	nom. interest rate (gross)	0.68	0.23	0.04	1.019
$\widehat{\pi}_t$	GDP inflation rate (gross)	0.45	0.15	0.15	1.014
$\widehat{h}_t + \widehat{n}_t$	total hours worked	1.69	0.58	0.92	—
$\widehat{w}_t + \widehat{h}_t + \widehat{n}_t$	total wage income	3.64	1.25	0.90	—
$\widehat{w}_t$	wage per hour	2.52	0.86	0.68	—
$\widehat{u}_t$	unemployment rate (all)	19.06	6.52	-0.92	5.90
$\widehat{u}_t^{(2)}$	u. rate 16-24 (proxy low-skill)	15.22	5.21	-0.92	12.31
$\widehat{u}_t^{(2)}$	u. rate high-school drop-out	—	—	—	—
$\widehat{v}_t$	vacancies (help-wanted index)	20.08	6.87	0.89	—

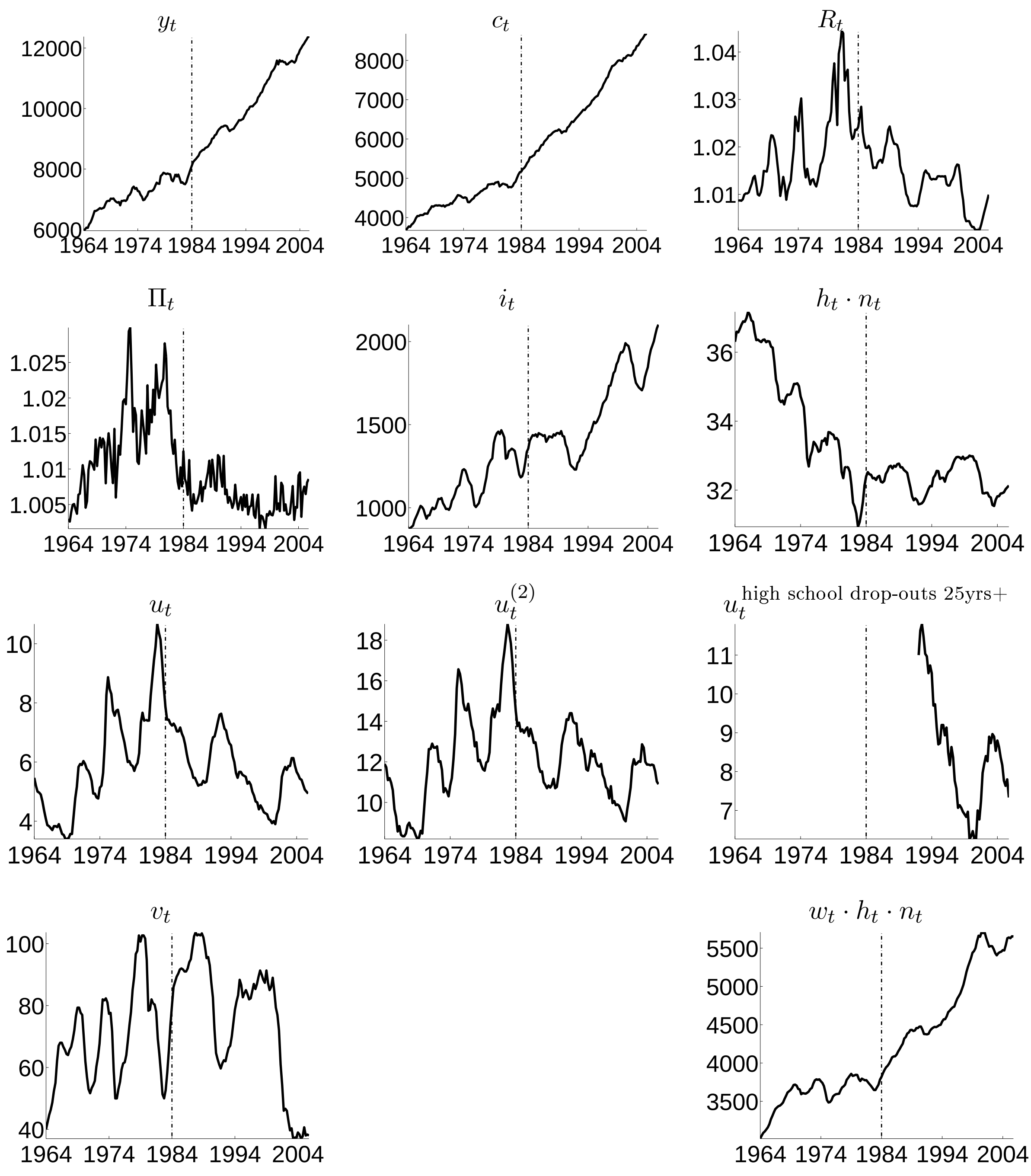
Notes: The table reports summary statistics of the data. The third to fifth column show statistics for HP(100.000) filtered data. The third column reports the standard deviation of the series, the fourth its standard deviation relative to GDP. The fifth column shows the cross-correlation with GDP. For GDP components, the final column reports the mean share in GDP. For all other series, the mean of the series in levels is shown. The computations are performed on the sample from 1964:q1 to 1983:q4. The data is HP-filtered from 1964:q1 to 2005:q4. The reported investment GDP ratio is calculated as the ratio of fixed private investment plus durable consumption to GDP. The consumption output ratio consequently is computed as the ratio of non-durable consumption plus services to GDP.

Table 10: Moments of Data (1964:q1 to 2005:q4)

Variable	Meaning	std	std to y	corr with y	mean ratio to y
$\widehat{y}_t$	output	2.34	1	1	1
$\widehat{c}_t$	consumption	1.59	0.68	0.86	0.56
$\widehat{i}_t$	investment	6.39	2.73	0.77	0.24
		std	std to y	corr with y	mean
$\widehat{R}_t$	nom. interest rate (gross)	0.54	0.23	0.16	1.016
$\widehat{\pi}_t$	GDP inflation rate (gross)	0.36	0.15	0.08	1.010
$\widehat{h}_t + \widehat{n}_t$	total hours worked	1.46	0.62	0.90	—
$\widehat{w}_t + \widehat{h}_t + \widehat{n}_t$	total wage income	3.34	1.43	0.87	—
$\widehat{w}_t$	wage per hour	2.41	1.03	0.66	—
$\widehat{u}_t$	unemployment rate (all)	16.62	7.09	-0.91	5.90
$\widehat{u}_t^{(2)}$	u. rate 16-24 (proxy low-skill)	12.84	5.48	-0.92	12.12
$\widehat{u}_t^{(2)}$	u. rate high-school drop-out	12.41	5.29	-0.90	8.47
$\widehat{v}_t$	vacancies (help-wanted index)	19.48	8.31	0.84	—

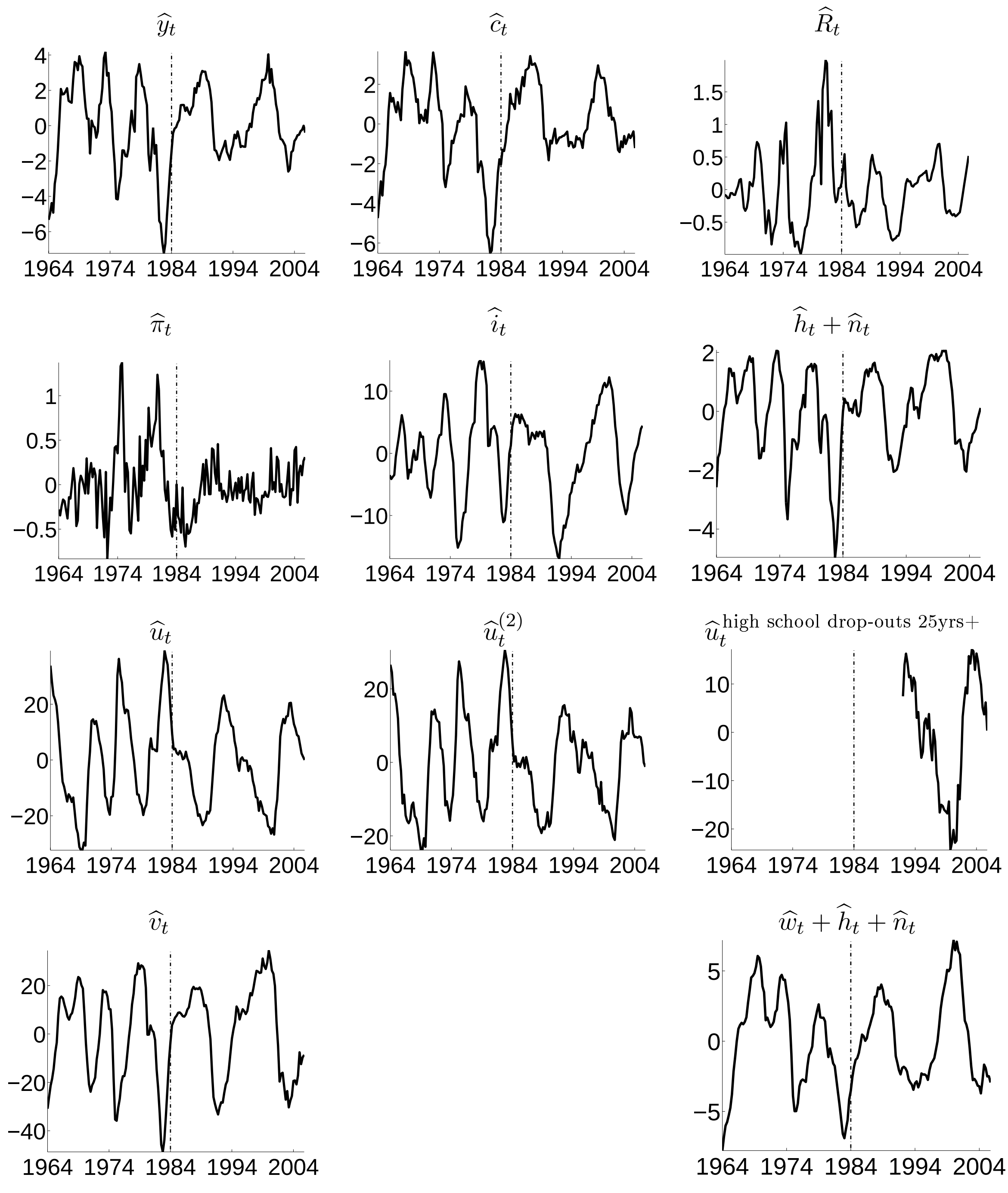
Notes: The table reports summary statistics of the data. The third to fifth column show statistics for HP(100.000) filtered data. The third column reports the standard deviation of the series, the fourth its standard deviation relative to GDP. The fifth column shows the cross-correlation with GDP. For GDP components, the final column reports the mean share in GDP. For all other series, the mean of the series in levels is shown. The computations are performed on the sample from 1964:q1 to 2005:q4. The data is HP-filtered from 1964:1 to 2005:4. The reported investment GDP ratio is calculated as the ratio of fixed private investment plus durable consumption to GDP. The consumption output ratio consequently is computed as the ratio of non-durable consumption plus services to GDP.

Figure 2: Plots of the Raw Data



Notes: The graphs plot the raw data in levels. A vertical dashed line marks the start of the observation sample in 1984:q1.

Figure 3: Plots of HP(100,000) Filtered Data



Notes: The graphs plot the business cycle component of the data in Figure 2. The data have been logged, HP(100,000)-filtered and multiplied by 100. A vertical dashed line marks the start of the observation sample in 1984:q1.

6.4 Algorithm

For the estimation of our model we apply Bayesian techniques as in Smets and Wouters (2005). The estimation methodology consists of five steps. In step one we solve the linearized rational expectations model for a given set of parameters. A non-standard feature is that the economy is modeled at a monthly frequency in order to achieve consistency of the employment stock and flow data but estimated using data at a quarterly frequency. In step two we thus derive a state-equation in quarterly terms and a measurement equation which links the seven observable variables to the vector of state variables. In step three the likelihood function is evaluated using the Kalman filter. Step four involves combining this likelihood function with a prior distribution over the parameters to form the posterior density function. The final step consists of numerically deriving the posterior distribution of the parameters. We compute the posterior mode and the Laplace approximation to the marginal data density. This appendix summarizes the approach taken.

6.4.1 Observation and State Equation

We start by deriving our observation equation and the state equation for the Kalman filter. Using these, the Kalman filter is standard; see e.g. Hamilton, 1994, ch. 13.

Observation Equation

For a particular set of parameters, the equilibrium law of motion of the linearized model takes the form

$$\mathbf{y}_t = F_{\mathbf{x}}\mathbf{x}_t + F_{\mathbf{u}}\mathbf{u}_t, \tag{62}$$

where $\mathbf{y}_t$ ($n_{\mathbf{y}}$ x 1) collects the endogenous variables of the model in percent deviation from steady state, $\mathbf{x}_t$ ($n_{\mathbf{x}}$ x 1) collects the endogenous states of the model, $n_{\mathbf{x}} < n_{\mathbf{y}}$, and $\mathbf{u}_t$ (n_u x 1) collects the innovations of the model. Here a time index t refers to one month. The conformable matrizes $F_{\mathbf{x}}$ and $F_{\mathbf{u}}$ are functions of the model parameters. The endogenous states of the monthly model in equilibrium evolve according to

$$\mathbf{x}_t = A_{\mathbf{x}}\mathbf{x}_{t-1} + B_{\mathbf{u}}\mathbf{u}_t,\ \mathbf{u}_t \overset{iid}{\sim} N(0, \Omega), \tag{63}$$

where $A_{\mathbf{x}}$ and $B_{\mathbf{u}}$ are again functions of the model parameters. By assumption, the econometrician can observe data only at a quarterly frequency. Let $n_{\mathbf{z}}$ be the number of series which are observable each quarter. Let t_q be the month at the end of an arbitrary quarter, q, and let $\mathbf{z}_q$ denote the observation of the vector $\mathbf{z}$ in quarter q. The observable variables are defined as

$$\mathbf{z}_q = W_0 \mathbf{y}_{t_q} + W_1 \mathbf{y}_{t_q-1} + W_2 \mathbf{y}_{t_q-2} + \nu_q. \tag{64}$$

Here $\mathbf{z}_q$ is ($n_{\mathbf{z}}$ x 1), and $\nu_q \overset{iid}{\sim} N(\mathbf{0}, \Sigma)$ is a conformable measurement error which is orthogonal to the structural innovations, $\mathbf{u}_t$, contemporaneously and at all lags and leads. Conformable matrices W_0, W_1 and W_2 are of dimension (n_z x $n_{\mathbf{y}}$). They appropriately weight monthly endogenous variables for the quarterly observations. For the construction of the weighting matrices using the precise empirical exercise of our paper see Appendix 6.4.3. Rewriting (64) in matrix form yields

$$\mathbf{z}_q = W \begin{pmatrix} \mathbf{y}_{t_q} \\ \mathbf{y}_{t_q-1} \\ \mathbf{y}_{t_q-2} \end{pmatrix} + \nu_q, \tag{65}$$

where weighting matrix $W = [W_0, W_1, W_2]$. Define the state vector $\tilde{\mathbf{x}}_t$ by stacking endogenous states in t and the innovations,

$$\tilde{\mathbf{x}}_t \equiv \begin{pmatrix} \mathbf{x}_t \\ \mathbf{u}_t \end{pmatrix}.$$

Using this definition, we can rewrite (62) as

$$\mathbf{y}_t = F \tilde{\mathbf{x}}_t, \tag{66}$$

where $F = [F_{\mathbf{x}}, F_{\mathbf{u}}]$. As a results, the vector of observable variables in (65), $\mathbf{z}_q$, can be expressed as

$$\mathbf{z}_q = W_{\tilde{\mathbf{x}}} \begin{pmatrix} \tilde{\mathbf{x}}_{t_q} \\ \tilde{\mathbf{x}}_{t_q-1} \\ \tilde{\mathbf{x}}_{t_q-2} \end{pmatrix} + \nu_q, \tag{67}$$

where weighting matrix $W_{\tilde{\mathbf{x}}} = [W_0 F, W_1 F, W_2 F]$. Define $\tilde{\mathbf{x}}_q^{\text{long}} \equiv \tilde{\mathbf{x}}_{t_q}^{\text{long}} \equiv \begin{pmatrix} \tilde{\mathbf{x}}_{t_q} \\ \tilde{\mathbf{x}}_{t_q-1} \\ \tilde{\mathbf{x}}_{t_q-2} \end{pmatrix}$. Using this notation, the following equation (67) is our observation equation:

$$\mathbf{z}_q = W_{\tilde{\mathbf{x}}} \tilde{\mathbf{x}}_q^{\text{long}} + \nu_q. \tag{68}$$

State Equation

Stack $\mathbf{x}_t$ in (63) and $\mathbf{u}_t$, so

$$\tilde{\mathbf{x}}_{t_q} = A_{\tilde{\mathbf{x}}}\tilde{\mathbf{x}}_{t_q-1} + B_{\tilde{\mathbf{u}}}\mathbf{u}_{t_q}. \tag{69}$$

Here $A_{\tilde{\mathbf{x}}} = \begin{bmatrix} A_{\mathbf{x}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$ and $B_{\tilde{\mathbf{u}}} = \begin{bmatrix} B_{\mathbf{u}} \\ I_{n_{\mathbf{u}}} \end{bmatrix}$. Here $I_{n_{\mathbf{u}}}$ denotes an identity matrix of dimension $n_{\mathbf{u}}$. Stacking further in order to produce a law of motion for $\tilde{\mathbf{x}}_{t_q}^{\text{long}}$ gives

$$\tilde{\mathbf{x}}_{t_q}^{\text{long}} = A_{\tilde{\mathbf{x}}^{\text{long}}}\tilde{\mathbf{x}}_{t_q-1}^{\text{long}} + B_{\tilde{\mathbf{u}}^{\text{long}}}\mathbf{u}_{t_q}, \tag{70}$$

where $A_{\tilde{\mathbf{x}}^{\text{long}}} = \begin{bmatrix} A_{\tilde{\mathbf{x}}} & \mathbf{0} & \mathbf{0} \\ I_{n_{\mathbf{x}}+n_{\mathbf{u}}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & I_{n_{\mathbf{x}}+n_{\mathbf{u}}} & \mathbf{0} \end{bmatrix}$ and $B_{\tilde{\mathbf{u}}^{\text{long}}} = \begin{bmatrix} B_{\tilde{\mathbf{u}}} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}$. Now substitute (70) iteratively in (70) to obtain

$$\begin{aligned} \tilde{\mathbf{x}}_{t_q}^{\text{long}} = \; & A_{\tilde{\mathbf{x}}^{\text{long}}}^{3}\tilde{\mathbf{x}}_{t_q-3}^{\text{long}} \\ & + A_{\tilde{\mathbf{x}}^{\text{long}}}^{2} B_{\tilde{\mathbf{u}}^{\text{long}}}\mathbf{u}_{t_q-2} + A_{\tilde{\mathbf{x}}^{\text{long}}} B_{\tilde{\mathbf{u}}^{\text{long}}}\mathbf{u}_{t_q-1} + B_{\tilde{\mathbf{u}}^{\text{long}}}\mathbf{u}_{t_q}. \end{aligned} \tag{71}$$

Collecting terms in (71) we have that

$$\tilde{\mathbf{x}}_{t_q}^{\text{long}} = A_{\tilde{\mathbf{x}}^{\text{long}}}^{3}\tilde{\mathbf{x}}_{t_q-3}^{\text{long}} + \epsilon_q, \tag{72}$$

where

$$\epsilon_q \equiv A_{\tilde{\mathbf{x}}^{\text{long}}}^{2} B_{\tilde{\mathbf{u}}^{\text{long}}}\mathbf{u}_{t_q-2} + A_{\tilde{\mathbf{x}}^{\text{long}}} B_{\tilde{\mathbf{u}}^{\text{long}}}\mathbf{u}_{t_q-1} + B_{\tilde{\mathbf{u}}^{\text{long}}}\mathbf{u}_{t_q}.$$

Since $\mathbf{u}_t \overset{iid}{\sim} N(\mathbf{0}, \Omega)$, from a quarterly perspective $\epsilon_q \overset{iid}{\sim} N(\mathbf{0}, \Xi)$ with

$$\begin{aligned} \Xi = \; & A_{\tilde{\mathbf{x}}^{\text{long}}}^{2} B_{\tilde{\mathbf{u}}^{\text{long}}}\Omega B_{\tilde{\mathbf{u}}^{\text{long}}}{}' A_{\tilde{\mathbf{x}}^{\text{long}}}^{2}{}' \\ & + A_{\tilde{\mathbf{x}}^{\text{long}}} B_{\tilde{\mathbf{u}}^{\text{long}}}\Omega B_{\tilde{\mathbf{u}}^{\text{long}}}{}' A_{\tilde{\mathbf{x}}^{\text{long}}}{}' \\ & + B_{\tilde{\mathbf{u}}^{\text{long}}}\Omega B_{\tilde{\mathbf{u}}^{\text{long}}}{}'. \end{aligned} \tag{73}$$

Our observation equation at a quarterly frequency is therefore given by

$$\tilde{\mathbf{x}}_{q}^{\text{long}} = A_{\tilde{\mathbf{x}}^{\text{long}}}^{3}\tilde{\mathbf{x}}_{q-1}^{\text{long}} + \epsilon_q, \tag{74}$$

with $\epsilon_q \overset{iid}{\sim} N(\mathbf{0}, \Xi)$ and Ξ given by (73).

6.4.2 Bayesian Estimation

Let $\boldsymbol{\theta}$ denote the vector of parameters we estimate. $\boldsymbol{\theta}$ contains all the model parameters, including the second moments in variance-covariance matrix Ω. $\boldsymbol{\theta}$ also contains the standard deviation of the measurement error in quarterly terms as summarized by variance-covariance matrix Σ. Let the prior be given by $p(\boldsymbol{\theta})$. Marginal priors are assumed to be independent. Conditional on the parameters, $\boldsymbol{\theta}$, and an initial state, $\tilde{\mathbf{x}}_0^{\text{long}} = 0$, the endogenous variables are normally distributed; compare (68) and (74). Let $\mathcal{L}(\{\mathbf{z}_q\};\boldsymbol{\theta})$ denote the Gaussion likelihood of sample $\{\mathbf{z}_q\}$ given $\boldsymbol{\theta}$. The likelihood is evaluated using a standard Kalman filter with the observation and state equation defined in Appendix 6.4.1. The posterior is given by

$$p(\boldsymbol{\theta}|\{\mathbf{z}_q\}) = \frac{p(\boldsymbol{\theta})\mathcal{L}(\{\mathbf{z}_q\};\boldsymbol{\theta})}{p(\{\mathbf{z}_q\})} \propto p(\theta)\mathcal{L}(\{\mathbf{z}_q\};\theta).$$

The marginal data density is given by

$$p(\{\mathbf{z}_q\}) = \int p(\boldsymbol{\theta})\mathcal{L}(\{\mathbf{z}_q\};\boldsymbol{\theta})d\boldsymbol{\theta}.$$

In our paper we compute the marginal data density by means of a Laplace approximation around the posterior mode.

6.4.3 Aggregation

This subsection discusses the time-aggregation conducted by matrix $W_{\tilde{\mathbf{x}}}$ in equation (67). In the following let x_q^{quart} denote the observation in quarter q of a quarterly transformation of the monthly variables x_{t_q}, x_{t_q-1} and x_{t_q-2}. x_q^{quart} is one of the elements of the vector of observable variables, z_q. Hats, in $\widehat{x}_q^{\text{quart}}$ for example, denote percent deviations of a variable from its steady state. Quarterly gross interest rates and gross inflation rates are not annualized and thus computed as the product of their monthly counterparts. Consequently, log-linearized, we have

$$\widehat{R}_q^{\text{quart}} = \widehat{R}_{t_q} + \widehat{R}_{t_q-1} + \widehat{R}_{t_q-2}.$$

In each of the matrices W_0, W_1, and W_2, in the row pertaining to $\widehat{R}_q^{\text{quart}}$ the weight in the column belonging to $\widehat{R}_{t_q}$ is therefore unity. Similarly,

$$\widehat{\Pi}_q^{\text{quart}} = \widehat{\Pi}_{t_q} + \widehat{\Pi}_{t_q-1} + \widehat{\Pi}_{t_q-2}.$$

As an example for a *stock variable*, the percent deviation of the average unemployment rate over the quarter is (up to a first-order approximation)

$$
\begin{aligned}
\widehat{u}_q^{\text{quart}} &= \frac{u_q^{\text{quart}} - u^{\text{quart}}}{u^{\text{quart}}} \\
&= \frac{\frac{1}{3}\left(u_{t_q} + u_{t_q-1} + u_{t_q-2}\right) - \frac{1}{3}(u+u+u)}{\frac{1}{3}(u+u+u)} \\
&= \frac{\frac{1}{3}\left((u_{t_q} - u) + (u_{t_q-1} - u) + (u_{t_q-2} - u)\right)}{\frac{1}{3}(u+u+u)} \\
&= \frac{(u_{t_q} - u) + (u_{t_q-1} - u) + (u_{t_q-2} - u)}{(u+u+u)} \\
&= \frac{(u_{t_q} - u) + (u_{t_q-1} - u) + (u_{t_q-2} - u)}{3u} \\
&= \frac{1}{3}\left(\frac{u_{t_q} - u}{u} + \frac{u_{t_q-1} - u}{u} + \frac{u_{t_q-2} - u}{u}\right) \\
&= \frac{1}{3}\left(\widehat{u}_{t_q} + \widehat{u}_{t_q-1} + \widehat{u}_{t_q-2}\right).
\end{aligned}
$$

Here u^{quart} refers to the steady state of the average unemployment rate of a quarter, and u to the steady state unemployment rate in an arbitrary month. Naturally, these two values coincide. In each of the matrices W_0, W_1, and W_2, in the row pertaining to $\widehat{u}_q^{\text{quart}}$ the weight in the column belonging to $\widehat{u}_{t_q}$ are therefore equal to $\frac{1}{3}$. Following the same lines, the percentage deviation of average stock variables over the quarter is in general computed as the mean of the monthly deviation. Consequently, the average vacancy posting activity is given by

$$
\widehat{v}_q^{\text{quart}} = \frac{1}{3}\left(\widehat{v}_{t_q} + \widehat{v}_{t_q-1} + \widehat{v}_{t_q-2}\right)
$$

and the average wage rate over a quarter is

$$
\widehat{w}_q^{\text{quart}} = \frac{1}{3}\left(\widehat{w}_{t_q} + \widehat{w}_{t_q-1} + \widehat{w}_{t_q-2}\right).
$$

As an example for a *flow variable*, the percent deviation of quarterly GDP from steady state (up to a first-order approximation) is

$$
\begin{aligned}
\widehat{y}_q^{\text{quart}} &= \frac{y_q^{\text{quart}} - y^{\text{quart}}}{y^{\text{quart}}} \\
&= \frac{\left(y_{t_q} + y_{t_q-1} + y_{t_q-2}\right) - 3y}{3y} \\
&= \frac{(y_{t_q} - y) + (y_{t_q-1} - y) + (y_{t_q-2} - y)}{3y} \\
&= \frac{1}{3}\left(\frac{y_{t_q} - y}{y} + \frac{y_{t_q-1} - y}{y} + \frac{y_{t_q-2} - y}{y}\right) \\
&= \frac{1}{3}\left(\widehat{y}_{t_q} + \widehat{y}_{t_q-1} + \widehat{y}_{t_q-2}\right).
\end{aligned}
$$

In the row pertaining to $\widehat{y}_q^{\text{quart}}$, the weights on $\widehat{y}_{t_q}$ are therefore equal to $\frac{1}{3}$ in each of the matrices W_0, W_1, and W_2. Percent deviations of quarterly flow variables are hence also computed as the mean of the corresponding monthly observations. Consequently, the deviation of quarterly consumption from steady state is

$$\widehat{c}_q^{\text{quart}} \quad = \quad \frac{1}{3}\left(\widehat{c}_{t_q} + \widehat{c}_{t_q-1} + \widehat{c}_{t_q-2}\right),$$

the deviation of quarterly investment from steady state is

$$\widehat{i}_q^{\text{quart}} \quad = \quad \frac{1}{3}\left(\widehat{i}_{t_q} + \widehat{i}_{t_q-1} + \widehat{i}_{t_q-2}\right)$$

and, finally, the deviation of total hours worked in quarter q is given by

$$\widehat{(h \cdot n)}_q^{\text{quart}} \quad = \quad \frac{1}{3}\left(\widehat{(h \cdot n)}_{t_q} + \widehat{(h \cdot n)}_{t_q-1} + \widehat{(h \cdot n)}_{t_q-2}\right).$$

6.5 List of Symbols

6.5.1 Roman Lower Case

a : ratio of labor productivity in labor good firms of type 2 to those of type 1. $a \in (0,1)$.

$b^{(o)}$: real unemployment benefits in group o.

$c_{i,t}^{(o)}$: consumption of individual i which belongs to group o.

$c_t^{(o)}$: consumption per capita in group o.

$c_{e,i,t}^{(1)}$: consumption of an employed asset-holding consumer i.

$c_{e,t}^{(2)}$: consumption of an employed liquidity-constrained consumer.

$c_{u,i,t}^{(1)}$: consumption of an unemployed asset-holding consumer i.

$c_{u,t}^{(2)}$: consumption of an unemployed liquidity-constrained consumer.

$\overline{d}$: target level for real government debt.

$\overline{g}$: target level for real government expenditure.

$h_{i,t}^{(o)}$: hours worked by individual i which belongs to group o.

i_t : investment per capita in asset-holding family.

k_t : capital stock at the end of t per capita of the asset-holding family.

$k_{j,t}$: demand for capital by firm j in wholesale sector.

l_t : total supply of the labor good.

$l_{j,t}$: demand for the labor good by firm j in wholesale sector.

$l_{i,t}$: production of labor good by firm i in labor good sector.

$m_t^{(o)}$: new matches of firms and workers of type o.

mc_t : real marginal cost in wholesale sector.

$n_t^{(o)}$: employment rate in group o.

o : index for type of consumer. $o = 1$: asset-holding consumer. $o = 2$: liquidity-constrained consumer.

$q_t^{(o)}$: probability of finding a worker of type o.

q_t^k : shadow value of installed capital.

r_t^k : real rental rate of capital.

$s_t^{(o)}$: probability of finding a job of type o.

t_t : per capita lump-sum taxes.

t_t^{tot} : per capita total tax revenue.

u_t : economy-wide unemployment rate.

$u_t^{(o)}$: unemployment rate in group o.

$v_t^{(o)}$: vacancies for worker of type o.

$w_{i,t}^{(o)}$: real wage per hour earned by individual i in group o.

x_t^L : real price of the labor good.

y_t : GDP.

$y_{j,t}$: output of variety j of the differentiated good.

z_t : capacity utilization. Steady state: $z = 1$.

6.5.2 Roman Upper Case

$A_t^{(o)}$: labor productivity in labor good firms of type o.

D_t : nominal amount of risk-free bonds (government debt) held by asset-holding family per member of the family.

$J_t^{(o)}$: value of firm with a matched worker of type o.

P_t : consumption/GDP price index.

P_t^* : optimal price set by resetting wholesale firms.

$P_{j,t}$: price of one unit of variety j of differentiated good.

$\overline{\Pi}$: the central bank's inflation target.

R_t : nominal gross rate of return on bonds from t to $t+1$.

$S(\cdot)$: capital adjustment costs.

6.5.3 Greek Lower Case

α : elasticity of production with respect to capital in wholesale sector ($y_{j,t} = k_{j,t}^{\alpha} l_{j,t}^{1-\alpha}$). $\in (0,1)$.

β : time discount factor $\in (0,1)$.

$\beta_{t,t+s}$: stochastic real discount factor between t and $t+s$.

γ_D : response of tax revenue to deviations of government debt from debt target.

γ_g : response of tax revenue to deviations of government expenditure from target.

γ_p : degree of inflation indexation for firms which cannot update $\in [0,1]$.

γ_π : interest rate response to inflation in Taylor rule.

γ_R : interest rate response to lagged interest rate.

γ_u : interest rate response to unemployment in Taylor rule.

γ_y : interest rate response to output in Taylor rule.

$\gamma_{z,1}$: linear response of utilization cost to excess utilization.

$\gamma_{z,1}$: quadratic response of utilization cost to excess utilization.

δ : monthly rate of capital depreciation.

ϵ : elasticity of demand for wholesale good, $\epsilon > 1$.

ϵ_t^C : "cost-push shock" with steady state value of unity.

ϵ_t^g : "goverment spending shock" with steady state value of zero.

ϵ_t^I : "investment shock" with steady state value of unity.

ϵ_t^L : shock to disutility of work ("labor supply shock"). Steady state $\epsilon^L>0$.

ϵ_t^{money} : "monetary policy shock" with steady state value of unity.

$\epsilon_t^{\mathtt{W}}$: shock to intertemporal substitution with steady state value of unity.

$\theta_t^{(o)}$: market tightness from the viewpoint of type o firms.

$\boldsymbol{\theta}$: vector of parameters (symbol used in estimation algorithm section).

$\vartheta^{(o)}$: monthly destruction rate of jobs of type o. $\vartheta^{(o)} \in (0,1)$.

κ^I : parameter governing slope of investment adjustment costs, $\kappa^I > 0$.

$\kappa^{(o)}$: real costs of posting a vacancy for a worker of type o.

λ_t : marginal period utility of consumption of the family (and thus of asset-holders).

$\lambda_t^{(2)}$: marginal utilityof a liquidity-constrained worker of additional consumption when employed.

$\mu^{(o)}$: bargaining power of workers of type o. $\mu^{(o)} \in (0,1)$.

ν : share of asset holding households in the economy. $\nu \in (0,1]$.

$\xi^{(o)}$: elasticity of matches w.r.t. unemployment for type o workers. $\xi^{(o)} \in (0,1)$.

$\varrho^{(o)}$: degree of habit persistence for group o. $\varrho^{(o)} \in [0,1)$.

σ : degree of risk aversion. $\sigma > 0$.

$\sigma_m^{(o)}$: efficiency of matching for type o workers. $\sigma_m > 0$.

τ^L : labor tax rate.

φ : inverse of labor supply elasticity. $\varphi > 0$.

χ : $\chi = 1(0)$ indicates whether liquidity-constrained consumers do (do not) pay lump-sum taxes.

$\psi(\cdot)$: capital utilization costs.

ω : probability that a wholesale firm cannot update its price, $\in [0,1)$.

6.5.4 Greek Upper Case

$\Delta_t^{(o)}$: utility difference employment versus unemployment in group o.

Π_t : gross inflation rate from $t-1$ to t.

Ψ_t : dividends per capita accruing to asset-holding family.

Ψ_t^C : dividends per capita accruing to asset-holding family from entire wholesale sector.

$\Psi_{j,t}^C$: dividends per capita accruing to asset-holding family from firm j in wholesale sector.

$\Psi_t^{(o)}$: dividends accruing to asset-holding family from a typical labor firm of type o.

6.5.5 Typewriter Style

$\mathtt{u}^{(o)}(\cdot)$: period utility of an individual consumer of type o.

$\mathtt{U}(\cdot)$: aggregate period utility function of the representative family for optimal allocation of consumption among members.

$\mathtt{W}_t^{(1)}$: aggregate utility (welfare) of the representative asset-holding family.