Optimal Monetary Policy under Model Uncertainty: a Structural-Bayesian Approach*

Alexander Kriwoluzky,[†] Christian Stoltenberg[‡]

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Abstract

This paper studies the optimal conduct of monetary policy in several micro-founded macro models with nominal rigidities if the decision-maker faces uncertainty about the true structure of the economy. We employ bayesian methods to assess the relevant sources of uncertainty within (parameter uncertainty) and across models (specification uncertainty) using EU 11 data. We propose a novel method to determine the robustly-optimal policy rule as the maximum of the utility of the individual agent across models where each of the models is weighted with its corresponding posterior odd. We require the optimal rule to be implementable , i.e. implying a low probability of a binding zero bound on interest rates. The robustly-optimal policy rule is characterized by a super-inertial reaction on the past interest rate and a muted reaction on current inflation.

JEL classification: E32, C11, C51, C52, E52, E58.
Keywords: Optimal monetary policy, model uncertainty, bayesian model estimation.

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[†]Humboldt University Berlin, Department of Economics, D-10178 Berlin, Germany, email: kriwoluz@wiwi.hu-berlin.de, fax: +49 30 2093-5934, tel: +49/30/2093-5935.

[‡]Humboldt University Berlin, Department of Economics, D-10178 Berlin, Germany, email: stoltenb@wiwi.hu-berlin.de, fax: +49 30 2093-5934, tel: +49/30/2093-5935.

I have myself said several times that the Governing Council of the ECB has no intention of being the prisoner of a single system of equations. We both (Greenspan and Trichet) highly praise robustness. There is no substitute for a comprehensive analysis of the risks to price stability that pays due attention to all relevant information.

Jean-Claude Trichet, President of the ECB, 2005

Introduction

What is a robustly-optimal stabilizing device for monetary policy? Following Brainard (1967) numerous researchers consider the performance of monetary policy under parameter uncertainty and across various macroeconomic models.[1] Not surprisingly, the policy recommendations are sensitive with respect to the specification of uncertainty. Optimal policy can be either cautious or more aggressive compared to the case where the true model is known. On the other hand the quantitative evaluation of micro-founded DSGE models employing bayesian methods has made substantial progress (Smets and Wouters, 2003 and An and Schorfheide, 2006). We propose a method that combines both lines of research.

First, we estimate the structural parameters of a variety of micro-founded macroeconomic models with nominal rigidities (Woodford, 2003) employing bayesian model estimation techniques. Then we determine the robustly optimal monetary policy using the uncertainty in and across models that is in line with data on EU 11 countries. To be more precise, we assess the relevant parameter uncertainty using the posterior distribution of the estimated structural parameters. In addition, each of the models under consideration is weighted with their ability to explain the data in an efficient way. I.e. a model that is better in explaining the data using fewer parameters is perceived to be more likely the true model. The robustly-optimal rule is the rule that takes into account both - possible high losses in a certain model and how likely theses losses are. We find that the robustly-optimal rule is characterized by a super-inertial reaction on the past interest rate and a low but positive reaction on current inflation and output.

[1]For example McCallum (1998), Soderstrom (2002), Onatski and Stock, 2002 or Hansen and Sargent (2003).

Our benchmark model is a standard cashless New Keynesian economy with staggered price setting without indexation. Subsequently, we introduce indexation, habit formation and a demand for cash. This bottom-up approach allows us to evaluate quantitatively the explanatory power gain as well as the effect of each of these components on the optimal policy decision separately. We use the marginal densities of the models as a measure of their likelihood to be the true model. Obviously, confronting these models with the data and taking into account the model's performance plays an important role: otherwise the choice of possible extensions would be arbitrary. This approach marks a difference to the related literature that starts with a larger model (see Levin, Onatski, Williams and Williams, 2005 or Smets and Wouters, 2003) and then proceeds to include some extensions.

Notably, the objective of the central bank – given by a quadratic approximation to the utility of the individual agent in a particular model – can vary substantially across the parameter and model space. First we determine the optimal policy in a particular model with and without parameter uncertainty. In the cashless models price stability is the predominant aim of stabilization policy. If we require optimal policy to lead to low probability of a binding zero bound on interest, i.e. the difference between the real interest rate and the zero bound should represent at least 2 standard deviations, the picture changes: optimal monetary policy is characterized by low reaction coefficients on inflation and output. However, the requirement comes at substantial welfare costs. In case the model features a demand for cash, this introduces a conflicting aim, the stabilization of the nominal interest rate. Optimal interest policy balances these two effects and does not react as strong on inflation as in the other four models. In this case, imposing the zero bound on interest does not change the results.

We continue our analysis by determining the robustly-optimal rule as the one that minimizes households's expected utility over the model space. Remarkably, the optimal robust rule is characterized by stabilization of the nominal interest rate as the main principle – implying a super-inertial reaction on past interest rates and a muted reaction on inflation, even if the zero bound on interest is not imposed. Focussing on smoothing interest rates over time instead of price stability avoids possible high welfare losses in the model with transaction frictions.

Related Literature Our paper is related to Levin, Onatski, Williams and Williams (2005). They estimate a medium-scale New Keynesian Model with staggered price setting using US data and determine the optimal monetary policy in that model across the posterior distribution of the estimated parameters. Their optimal simple interest rate feedback rule is shown to be robust with respect to parameter uncertainty in structural parameters including the coefficients of the shock processes. However, as a side aspect, they find the optimal rule to be not robust to different extensions of the model including a demand for cash and different price and wage setting algorithms. The main differences to our approach are first that our estimation strategy allows us to quantify the importance of each model component for explaining the data and for the optimal conduct of monetary policy separately. As our main novel feature we analyze the robustly-optimal policy across models using the posterior odds of all models under consideration. I.e. we give policy recommendations in the presence of specification uncertainty. Additionally, we analyze the welfare effects of the zero bound restriction on interest rates.

The remainder of the paper is organized as follows. In the next section we describe briefly the models that enter our analysis. In the section 3 we describe the estimation procedure and results. Then, the optimal policy rule is derived in each model. In section 4 we compute the optimal policy across all model variants. The last section concludes.

1 The economic environment

We consider a standard sticky price cashless economy as our benchmark model (Woodford, 2003). It consists of a continuum of infinitely lived households indexed with $j \in [0, 1]$. It is assumed that households have identical initial asset endowments and identical preferences. Household j acts as a monopolistic supplier of labor services l_j. Lower (upper) case letters denote real (nominal) variables. At the beginning of period t, households' financial wealth comprises a portfolio of state contingent claims on other households yielding a (random) payment Z_{jt}, and one period nominally non-state contingent government bonds B_{jt-1} carried over from the previous period. Assuming complete financial markets let $q_{t,t+1}$ denote the period t price of one unit of currency in a particular state of period $t+1$ normalized by the probability of occurrence of that state, conditional on the information available in period t. Then, the price of a random payoff Z_{t+1} in period $t+1$ is given by $E_t[q_{t,t+1}Z_{jt+1}]$. The budget constraint of the representative household reads

$$B_{jt} + E_t[q_{t,t+1}Z_{jt+1}] + P_t c_{jt} \leq R_{t-1}B_{jt-1} + Z_{jt} + P_t w_{jt} l_{jt} + \int_0^1 D_{jit} di - P_t T_t, \qquad (1)$$

where c_t denotes a Dixit-Stiglitz aggregate of consumption with elasticity of substitution θ, P_t the aggregate price level, w_{jt} the real wage rate for labor services l_{jt} of type j, T_t a lump-sum tax, R_t the gross nominal interest rate on government bonds, and D_{it} dividends of monopolistically competitive firms. Further, households have to fulfill the no-Ponzi game condition, $\lim_{i\to\infty} E_t q_{t,t+i}(B_{jt+i}+Z_{jt+1+i}) \geq 0$. The objective of the representative household is

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^t \{u(c_{jt}) - v(l_{jt})\}, \quad \beta \in (0,1), \qquad (2)$$

where β denotes the subjective discount factor. The instantaneous utility function is assumed to be non-decreasing in consumption and real balances, decreasing in labor time, strictly concave, twice continuously differentiable, and to fulfill the Inada conditions.

Households are wage-setters supplying differentiated types of labor l_j which are transformed into aggregate labor l_t with $l_t^{(\epsilon_t-1)/\epsilon_t} = \int_0^1 l_{jt}^{(\epsilon_t-1)/\epsilon_t} dj$. We assume that the elasticity of substitution between different types of labor, $\epsilon_t > 1$, varies exogenously over time. The time variation in this markup parameter introduces a so called cost-push shock into the model that gives rise to a stabilization problem for the central bank. Cost minimization implies that the demand for differentiated labor services l_{jt}, is given by $l_{jt} = (w_{jt}/w_t)^{-\epsilon_t} l_t$, where the aggregate real wage rate w_t is given by $w_t^{1-\epsilon_t} = \int_0^1 w_{jt}^{1-\epsilon_t} dj$. Maximizing (2) subject to (1) and the no-Ponzi game condition for given initial values Z_0, B_{t_0-1}, and $R_{t_0-1} \geq 0$ leads to the following first order conditions for consumption, the real wage rate for labor type j, government bonds, and contingent claims:

$$\lambda_{jt} = u_c(c_{jt}), \quad v_l(l_{jt}) = w_{jt}\lambda_{jt}/\mu_t, \tag{3}$$

$$q_{t,t+1} = \frac{\beta\lambda_{jt+1}}{\pi_{t+1}\lambda_{jt}}, \quad \lambda_{jt} = \beta R_t E_t \frac{\lambda_{jt+1}}{\pi_{t+1}} \tag{4}$$

where λ_{jt} denotes a Lagrange multiplier, π_t the inflation rate $\pi_t = P_t/P_{t-1}$, and $\mu_t = \epsilon_t/(\epsilon_t - 1)$ the stochastic wage mark-up with mean $\bar{\mu}^w > 1$. The first order condition for contingent claims holds for each state in period $t+1$, and determines the price of one unit of currency for a particular state at time $t+1$ normalized by the conditional probability of occurrence of that state in units of currency in period t. Arbitrage-freeness between government bonds and contingent claims requires $R_t = 1/E_t q_{t,t+1}$. The optimum is further characterized by the budget constraint (1) holding with equality and by the transversality condition $\lim_{i\to\infty} E_t\beta^i\lambda_{jt+i}(B_{jt+i} + Z_{jt+1+i})/P_{jt+i} = 0$.

The final consumption good Y_t is an aggregate of differentiated goods produced by monopolistically competitive firms indexed with $i \in [0,1]$ and defined as $y_t^{\frac{\theta-1}{\theta}} = \int_0^1 y_{it}^{\frac{\theta-1}{\theta}} di$, with $\theta > 1$. Let P_{it} and P_t denote the price of good i set by firm i and the price index for the final good. The demand for each differentiated good is $y_{it}^d = (P_{it}/P_t)^{-\theta} y_t$, with $P_t^{1-\theta} = \int_0^1 P_{it}^{1-\theta} di$. A firm i produces good y_i using a technology that is linear in the labor bundle $l_{it} = [\int_0^1 l_{jit}^{(\epsilon_t-1)/\epsilon_t} dj]^{\epsilon_t/(\epsilon_t-1)}$: $y_{it} = a_t l_{it}$, where $l_t = \int_0^1 l_{it} di$ and a_t is a productivity shock with mean 1.

Labor demand satisfies: $mc_{it} = w_t/a_t$, where $mc_{it} = mc_t$ denotes real marginal costs independent of the quantity that is produced by the firm.

We allow for a nominal rigidity in form of a staggered price setting as developed by Calvo (1983). Each period firms may reset their prices with the probability $1-\alpha$ independently of the time elapsed since the last price setting. The fraction $\alpha \in [0,1)$ of firms are assumed to keep their previous period's prices, $P_{it} = P_{it-1}$, i.e. indexation is absent. Firms are assumed to maximize their market value, which equals the expected sum of discounted dividends $E_t \sum_{T=t}^{\infty} q_{t,T} D_{iT}$, where $D_{it} \equiv P_{it} y_{it}(1-\tau) - P_t mc_t y_{it}$ and we used that firms also have access to contingent claims. Here, τ denotes an exogenous sales tax introduced to offset the inefficiency of steady state output due to markup pricing (Rotemberg and Woodford, 1999). In each period a measure $1-\alpha$ of randomly selected firms set new prices $\widetilde{P}_{it}$ as the solution to $\max_{\widetilde{P}_{it}} E_t \sum_{T=t}^{\infty} \alpha^{T-t} q_{t,T}(\widetilde{P}_{it} y_{iT}(1-\tau) - P_T mc_T y_{iT})$, s.t. $y_{iT} = (\widetilde{P}_{it})^{-\theta} P_T^{\theta} y_T$. The first order condition for the price of re-optimizing producers is for $\alpha > 0$ given by

$$\frac{\widetilde{P}_{it}}{P_t} = \frac{\theta}{\theta - 1} \frac{F_t}{K_t}, \tag{5}$$

where K_t and F_t are given by the following expressions:

$$F_t = E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} u_c(c_T) y_T \left(\frac{P_T}{P_t}\right)^{\theta} mc_T \tag{6}$$

and

$$K_t = E_t \sum_{T=t}^{\infty} (\alpha\beta)^{T-t} u_c(c_T)(1-\tau) y_T \left(\frac{P_T}{P_t}\right)^{\theta-1}. \tag{7}$$

Aggregate output is given by $y_t = a_t l_t / \Delta_t$, where $\Delta_t = \int_0^1 (P_{it}/P_t)^{-\theta} di \geq 1$ and thus $\Delta_t = (1-\alpha)(\widetilde{P}_t/P_t)^{-\theta} + \alpha \pi_t^{\theta} \Delta_{t-1}$. The dispersion measure Δ_t captures the welfare decreasing effects of staggered price setting. If prices are flexible, $\alpha = 0$, then the first order condition for the optimal price of the differentiated good reads: $mc_t = (1-\tau)\frac{\theta-1}{\theta}$.

The public sector consists of a fiscal and a monetary authority. The central bank as the monetary authority is assumed to control the short-term interest rate R_t via a simple feedback contingent on past interest rates, inflation and output:

$$R_t = f(a(L)R_{t-1}, b(L)\pi_t, c(L)y_t). \tag{8}$$

The fiscal authority issues risk-free one period bonds, has to finance exogenous government expenditures $P_t G_t$, receives lump-sum taxes from households and tax-income from an exogenous given constant sales tax τ, such that the consolidated budget constraint reads: $R_{t-1}B_{t-1} + P_tG_t = +B_t + P_tT_t + \int_0^1 P_{it}y_{it}\tau di$. The exogenous government expenditures G_t evolve around a mean $\bar{G}$, which is restricted to be a constant fraction of output, $\bar{G} = \bar{y}(1-sc)$. We assume that tax policy guarantees government solvency, i.e., ensures $\lim_{i\to\infty} (B_{t+i})\prod_{v=1}^{i} R_{t+v}^{-1} = 0$. Due to the existence of the lump-sum tax, we consider only the demand effect of government expenditures and focus exclusively on optimal monetary policy.

We collect the exogenous disturbances in the vector $\xi_t = [a_t, G_t, \mu_t]$. It is assumed that the percentage deviation of each of the elements of the vector from their means evolve according to autonomous AR(1)-processes with autocorrelation coefficients $\rho_\zeta, \rho_a, \rho_G, \rho_\mu \in [0,1)$. The innovations are assumed to be i.i.d..

The recursive equilibrium is defined as follows:

Definition 1 *Given initial values $P_{t_0-1} > 0$ and $\Delta_{t_0-1} \geq 0$, a monetary policy and a ricardian fiscal policy $T_t\ \forall\, t \geq t_0$, a sales tax τ, a rational expectations equilibrium (REE) for $R_t \geq 1$, is a set of sequences $\{y_t,\ c_t,\ l_t,\ mc_t,\ \Delta_t,\ P_t,\ \widetilde{P}_{it},\ m_t, R_t\}_{t=t_0}^{\infty}$ satisfying the firms' first order condition $mc_t = w_t/a_t$, (5) with $\widetilde{P}_{it} = \widetilde{P}_t$, and $P_t^{1-\theta} = \alpha P_{t-1}^{1-\theta} + (1-\alpha)\widetilde{P}_t^{1-\theta}$, the households' first order conditions $u_c(y_t - G_t, \zeta_t)w_t = v_l(l_t)\mu_t$, $u_c(y_t - G_t, \zeta_t)/P_t = \beta R_t E_t u_c(y_{t+1} - G_{t+1}, \zeta_{t+1})/P_{t+1}$, the aggregate resource constraint $y_t = a_t l_t/\Delta_t$, clearing of the goods market $c_t + G_t = y_t$ and the transversality condition, for $\{\xi_t\}_{t=t_0}^{\infty}$.*

In the next step, we seek to estimate the model employing Bayesian methods. To do so we log-linearizing the model around the deterministic steady state under zero inflation. Correspondingly, the dynamics in the benchmark economy are described by the following two structural equations:

$$\sigma(E_t\widehat{y}_{t+1} - E_t\widehat{y}_{t+1}^n) = \sigma(\widehat{y}_t - \widehat{y}_t^n) + \widehat{R}_t - E_t\widehat{\pi}_{t+1} - \widehat{R}_t^n \tag{9}$$

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \kappa(\widehat{y}_t - \widehat{y}_t^n), \tag{10}$$

where $\sigma = -u_{cc}c/(u_c sc)$, $\omega = v_{ll}l/v_l$ and $\kappa = (1-\alpha)(1-\alpha\beta)(\omega+\sigma)/\alpha$. Further $\widehat{z}_t$ denotes the percent deviation of a generic variable z_t from its steady state value z. The natural rates of output and interest, i.e the values for output and real interest under flexible prices, are given by the following expressions

$$\widehat{y}_t^n = \frac{(1+\omega)\widehat{a}_t + \sigma g_t - \widehat{\mu}_t}{\omega + \sigma}, \tag{11}$$

$g_t = (G_t - G)/y$ and

$$\widehat{R}_t^n = \sigma[(g_t - \widehat{y}_t^n) - E_t(g_{t+1} - \widehat{y}_{t+1}^n)]. \tag{12}$$

The model is closed by a simple feedback rule for the nominal interest rate

$$\widehat{R}_t = \rho_R \widehat{R}_{t-1} + \phi_\pi \widehat{\pi}_t + \phi_y \widehat{y}_t. \tag{13}$$

Hours worked are determined with the following equation:

$$\widehat{l}_t = \widehat{y}_t - \widehat{a}_t. \tag{14}$$

In the following we analyze the optimal monetary policy under model uncertainty. To be more precisely, we look for steady state invariant monetary policies that maximize households' utility. The objective of optimal policy will be given by a quadratic approximation of households' utility, which is constrained by the structural equations (9)-(10). Given this environment and various sources of model uncertainty we seek to find the coefficients ρ_R, ϕ_π and ϕ_y of the feedback rule that maximize welfare up to second order. I.e. we set up the familiar Linear-Quadratic (LQ) framework. Building on Woodford (2003), the purely quadratic approximation to (2) in our benchmark model is given by[2]:

$$\frac{u}{1-\beta} - \frac{u_c y \theta(\omega+\sigma)}{2\kappa} \sum_{t=0}^{\infty} \beta^t \{var_0(\widehat{\pi}_t) + \lambda_x var_0(\widehat{y}_t - \widehat{y}_t^e)\}, \tag{15}$$

[2]Throughout we assume that the steady state is rendered efficient by an appropriate setting of the sales tax rate

where $\lambda_x = \kappa/\theta$ and the efficient rate of output is given by

$$\widehat{y}_t^e = \widehat{y}_t^n + \widehat{\mu}_t/(\omega + \sigma). \tag{16}$$

For now we consider four extensions of the benchmark. First we allow for an internal habit (e.g. Boivin and Giannoni, 2003; Woodford, 2003) in households' total consumption, we allow the prices not reconsidered to be indexed with past inflation (e.g. Smets and Wouters, 2003), we introduce a transaction friction by letting real money balances entering households utility (Sidrauski, 1967) and we combine all variants. These changes will modify the structural equations as well as the relevant stabilization aims of the monetary authority.

In the first case the marginal utility of consumption λ_{jt} is no longer a function of current consumption. Instead, the amount consumed the period before affects households' behavior:

$$\lambda_{jt} = u_c(c_t - \eta c_{t-1}) - \beta\eta E_t u_c(c_{t+1} - \eta c_t), \quad 0 \leq \eta \leq 1. \tag{17}$$

The natural rates of output and interest satisfy

$$\begin{aligned} [\omega + \varphi(1 + \beta\eta^2)]\widehat{y}_t^n - \varphi\eta\widehat{y}_{t-1}^n - \varphi\eta\beta E_t\widehat{y}_{t+1}^n &= \varphi(1 + \beta\eta^2)g_t - \varphi\eta g_{t-1} - \varphi\eta\beta E_t g_{t+1}... \\ &+ (1 + \omega)\widehat{a}_t - \widehat{\mu}_t \end{aligned} \tag{18}$$

$$\widehat{R}_t^n = \widehat{\lambda}_t^n - E_t\widehat{\lambda}_{t+1}^n, \tag{19}$$

where $\varphi = \sigma/(1 - \beta\eta)$ and

$$\widehat{\lambda}_t^n = -\varphi[(\widehat{y}_t^n - g_t) - \eta(\widehat{y}_{t-1}^n - g_{t-1})] + \varphi\beta\eta E_t[(\widehat{y}_{t+1}^n - g_{t+1}) - \eta(\widehat{y}_t^n - g_t)] \tag{20}$$

Because of that the Euler equation and the New Keynesian Philips curve are modified to:

$$\begin{aligned} \varphi[x_t - \eta x_{t-1}] - \varphi\beta\eta E_t[x_{t+1} - \eta x_t] &= E_t\widehat{\pi}_{t+1} + \widehat{R}_t^n - \widehat{R}_t... \\ &+ E_t\varphi[x_{t+1} - \eta x_t] - \varphi\beta\eta E_t[x_{t+2} - \eta x_{t+1}] \end{aligned} \tag{21}$$

$$\widehat{\pi}_t = \kappa_h[(x_t - \delta^* x_{t-1}) - \beta\delta^* E_t(x_{t+1} - \delta^* x_t)] + \beta E_t \widehat{\pi}_{t+1} \tag{22}$$

where $x_t = \widehat{y}_t - \widehat{y}_t^n$ and $\kappa_h = \eta\varphi\kappa/[\delta^*(\omega + \sigma)]^{-1}$. The parameter δ^*, $0 \leq \delta^* \leq \eta$, is the smaller root of the following quadratic equation[3]

$$\eta\varphi(1 + \beta\delta^2) = [\omega + \varphi(1 + \beta\eta^2)]\delta. \tag{23}$$

Approximating households' utility to second order results in the following expression (Woodford, 2003)

$$\frac{u^h}{1-\beta} - \frac{(1-\beta\eta)\eta\varphi u_c^h y^h \theta}{2\kappa_h \delta^*} \sum_{t=0}^{\infty} \beta^t \{var_0(\widehat{\pi}_t) + \lambda_x^h var_0(\widehat{y}_t - \widehat{y}_t^e - \delta^*(\widehat{y}_{t-1} - \widehat{y}_{t-1}^e))\}, \tag{24}$$

where $\lambda_x^h = \kappa_h/\theta$ and the efficient rate of output is characterized by

$$\begin{aligned}
[\omega + \varphi(1 + \beta\eta^2)]\widehat{y}_t^e - \varphi\eta\widehat{y}_{t-1}^e - \varphi\eta\beta E_t \widehat{y}_{t+1}^e &= \varphi(1 + \beta\eta^2)g_t - \varphi\eta g_{t-1} - \varphi\eta\beta E_t g_{t+1} ... \\
&+ (1 + \omega)\widehat{a}_t \qquad (25)
\end{aligned}$$

, which differs from (18) in setting the time-varying distortions that do not result from sticky prices $\widehat{\mu}_t$ to zero.

Another possible extension of the benchmark is indexation of the prices that are not reconsidered. To be more precise we assume this fraction α of all prices adjusts according to the following simple rule:

$$\log P_{it} = \log P_{it-1} + \gamma \log \pi_{t-1}, \tag{26}$$

with $0 \leq \gamma \leq 1$ as the degree of indexation. This implies that the dispersion measure evolves according to

$$\Delta_t = (1-\alpha)(\frac{\widetilde{P}_t}{P_t})^{-\theta} + \alpha\pi_{t-1}^{-\theta\gamma}\Delta_{t-1}\pi_t^{\theta}.$$

Correspondingly, the economy with indexation is characterized by a modified aggregate supply

[3]This the root that is assigned to past values of the natural rate of output in its stationary solution (see (18)).

curve

$$\widehat{\pi}_t - \gamma\widehat{\pi}_{t-1} = \beta E_t(\widehat{\pi}_{t+1} - \gamma\widehat{\pi}_t) + \kappa(\widehat{y}_t - \widehat{y}_t^n), \tag{27}$$

(9) and (13). A second order approximation of households' utility results in (Woodford, 2003)

$$\frac{u}{1-\beta} - \frac{u_c y\theta(\omega+\sigma)}{2\kappa}\sum_{t=0}^{\infty}\beta^t\{var_0(\widehat{\pi}_t - \gamma\widehat{\pi}_{t-1}) + \lambda_x var_0(\widehat{y}_t - \widehat{y}_t^e)\}, \tag{28}$$

where λ_x and the efficient rate of output are defined as in the benchmark economy.

Finally, we consider an economy that combines both extensions – habit formation and indexation. In this case the euler equation is given by (21) and the New Keynesian Phillips curve reads:

$$\widehat{\pi}_t - \gamma\widehat{\pi}_{t-1} = \beta E_t(\widehat{\pi}_{t+1} - \gamma\widehat{\pi}_t) + \kappa_h[(x_t - \delta^* x_{t-1}) - \beta\delta^* E_t(x_{t+1} - \delta^* x_t)] \tag{29}$$

and the the quadratic approximation to households' utility is given by

$$\frac{u^h}{1-\beta} - \frac{(1-\beta\eta)\eta\varphi u_c^h y^h\theta}{2\kappa_h\delta^*}\sum_{t=0}^{\infty}\beta^t\{var_0(\widehat{\pi}_t - \gamma\widehat{\pi}_{t-1}) + \lambda_x^h var_0(\widehat{y}_t - \widehat{y}_t^e - \delta^*(\widehat{y}_{t-1} - \widehat{y}_{t-1}^e))\}. \tag{30}$$

While the extensions discusses above modify preferences or technology they do not add any new distortion to our economy. The primary aim – in principle – is to stabilize inflation, still. In order to allow for the possibility of conflicting aims we introduce a transaction friction by letting real money balances entering households' utility in a separable way.[4] To be more precise, an euler equation or demand equation for real money balances enters the set of equilibrium conditions:

$$\frac{z(m_t)}{\lambda_t} = \frac{R_t - 1}{R_t}. \tag{31}$$

[4]However, we assume that this does not lead to a optimal state steady that corresponds to Friedman's rule. I.e. the approximation point is still characterized by flexible prices and zero inflation (see Paustian and Stoltenberg, 2006). Note that our specification of utility is consistent with recent findings by Andrés, López-Salido and Vallés (2006) for the Euro area. They estimate the role of money for the business cycle of the Euro area and the US and find that preferences are separable between consumption and real money balances.

We assume that $z(m_{jt})$ implies satiation in real money balances at a finite positive level. The derivatives z_m, z_{mm} have finite limiting values as m approaches the satiation level from below. In particular, the limiting value of z_{mm} from below is negative (see Woodford, 2003a, Assumption 6.1). Log-linearizing (31) results in:

$$\widehat{m}_t = -\frac{1}{\sigma_m(R-1)}\widehat{R}_t + \frac{\varphi}{\sigma_m}(\tilde{y}_t - \tilde{g}), \tag{32}$$

where $\sigma_m = -z_{mm}m/z_m$ and $\tilde{x}_t = (\widehat{x}_t - \eta\widehat{x}_{t-1}) - \beta\eta E_t(\widehat{x}_{t+1} - \eta\widehat{x}_t)$. The utility of the representative household can be approximated as:

$$\frac{u^h}{1-\beta} - \frac{(1-\beta\eta)\eta\varphi u_c^h y^h \theta}{2\kappa_h\delta^*}\sum_{t=0}^{\infty}\beta^t\{var_0(\widehat{\pi}_t - \gamma\widehat{\pi}_{t-1}) + \lambda_x^h var_0(\widehat{y}_t - \widehat{y}_t^e - \delta^*(\widehat{y}_{t-1} - \widehat{y}_{t-1}^e)) + \lambda_R var_0(\widehat{R}_t)\}, \tag{33}$$

with

$$\lambda_R = \frac{\lambda_x^h\beta\delta^*}{v\sigma_m(1-\beta)(1-\beta\eta)\eta\varphi},$$

and $v = y/m$.

2 Data and estimation results

We treat the variables employment, output and consumer price inflation as observables. The data consists of linearly-de-trended quarterly values of these variables for the EU 11 countries from 1970-1999 [5]. Since the data set provides data on employment but not on hours worked we follow Smets and Wouters (2003) and use the following auxiliary relationship between (unobserved) hours worked $\widehat{l_t}$ and number of people employed $\widehat{E}_t$:

$$\widehat{E}_t = \beta E_t\widehat{E}_{t+1} + \frac{(1-\eta_e)(1-\eta_e\beta)}{\eta_e}(\widehat{l}_t - \widehat{E}_t),$$

[5] For a description how this data is constructed see Smets and Wouters, 2003 and Fagan, Henry and Mestre, 2001

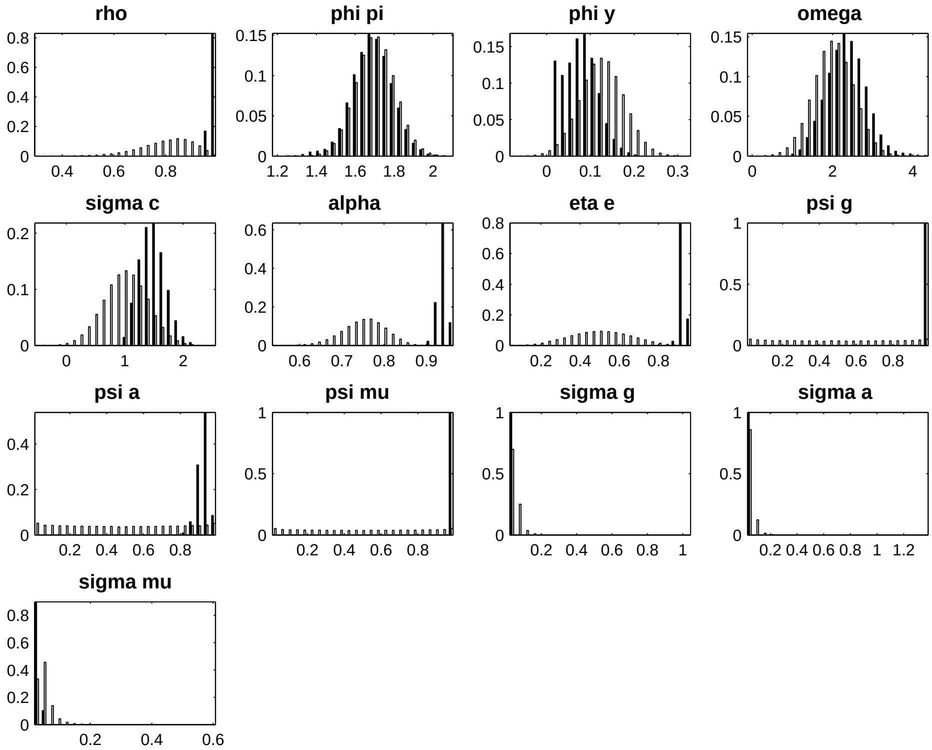

Figure 1: Prior vs. posterior in model 1

where we assume that in any given period only a fraction η_e of firms is able to adjust employment to its desired labor input.

We do not estimate the parameters $\beta = 0.99$, the fraction $c/y = 0.8$ and $\theta = 6$. We assume the disturbances $\widehat{\mu}_t$, g_t and $\widehat{a}_t$ to follow a stationary $AR(1)$ processes. Table 1 gives an overview about the priors and Table 2 displays the posterior estimates of the structural parameters. Graph 1 to 5 show histograms of prior and posterior distributions.

The figures indicate that all parameters accept some policy parameters and σ_m are identified. The Frisch elasticity also seems not to be identified, however a closer look reveals that its mean of the posterior is moved to the right and the posterior distribution does differ from the prior.

	Priordistribution		
Parameter	distribution	mean	std
ρ	beta	0.8	0.1
$\phi_p i$	normal	1.7	0.1
ϕ_y	normal	0.125	0.05
ω	normal	2	0.5
σ_c	normal	1	0.375
α	beta	0.75	0.05
η	beta	0.7	0.1
γ	beta	0.75	0.15
η_e	beta	0.5	0.15
σ_m	normal	1.25	0.375
ψ_g	beta	0.5	0.3
ψ_a	beta	0.5	0.1
ψ_μ	beta	0.5	0.3
σ_g	invgamma	0.04	4
σ_a	invgamma	0.04	4
σ_μ	invgamma	0.04	4

Table 1: Prior distribution of the structural parameters

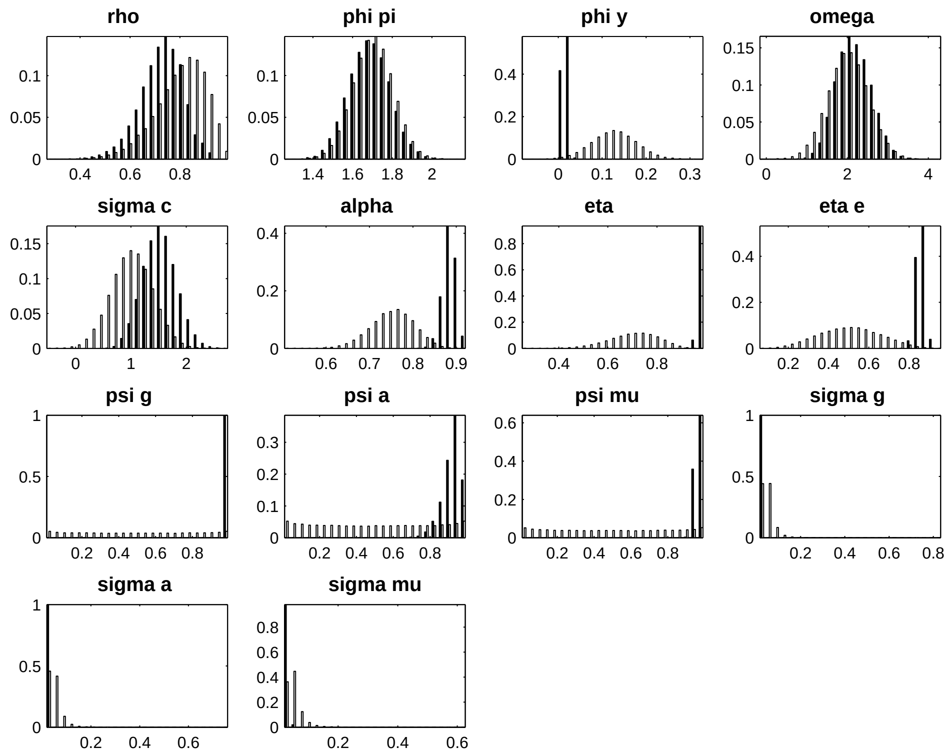

Figure 2: Prior vs. posterior in model 2

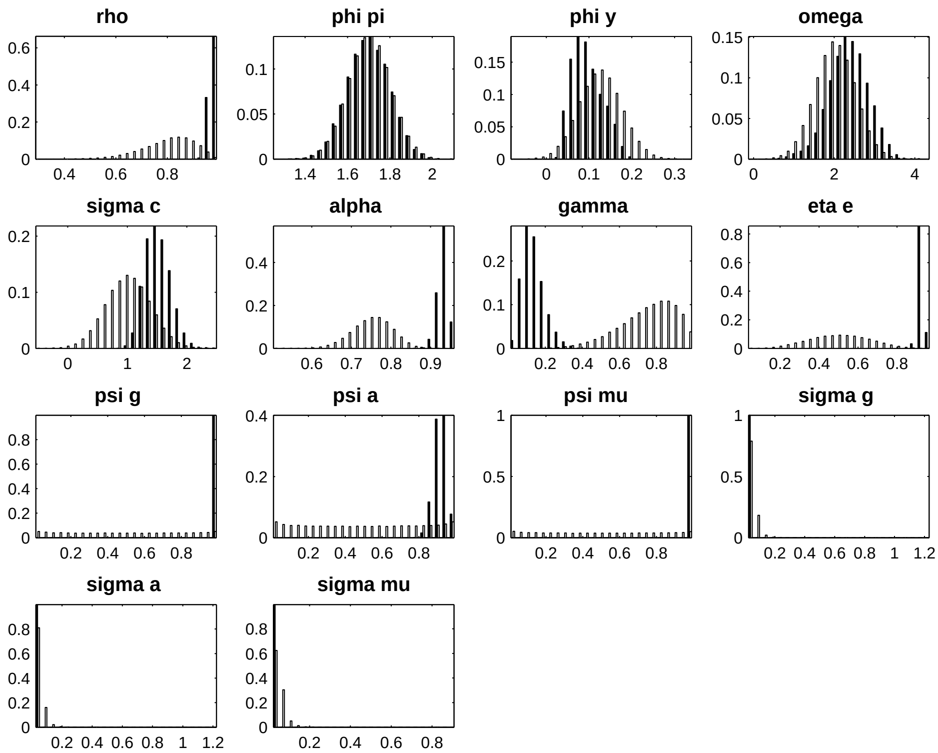

Figure 3: Prior vs. posterior in model 3

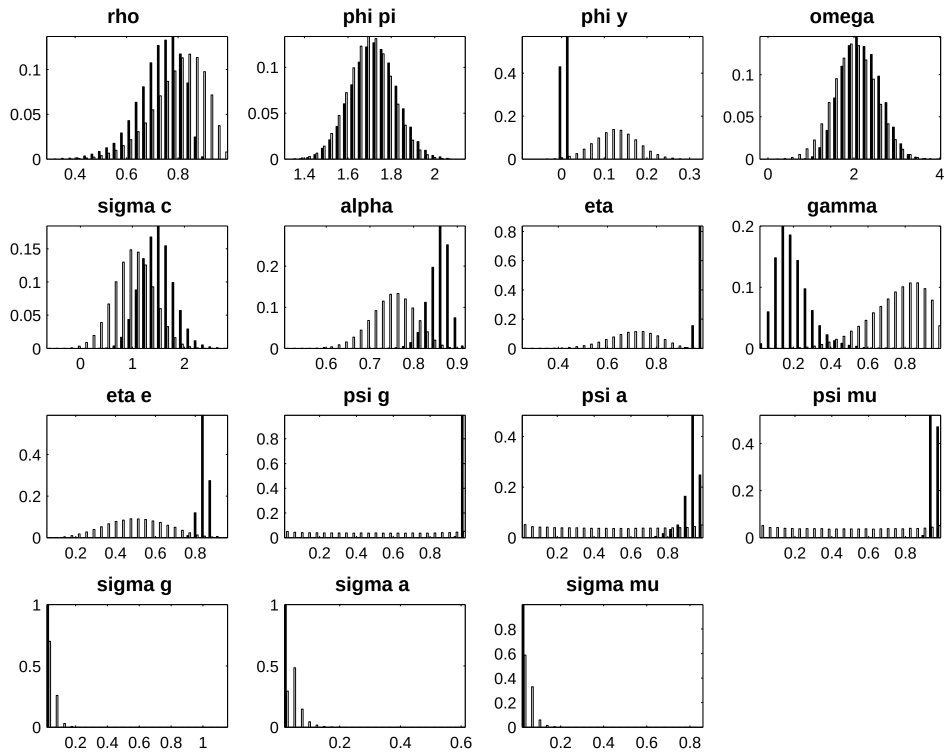

Figure 4: Prior vs. posterior in model 4

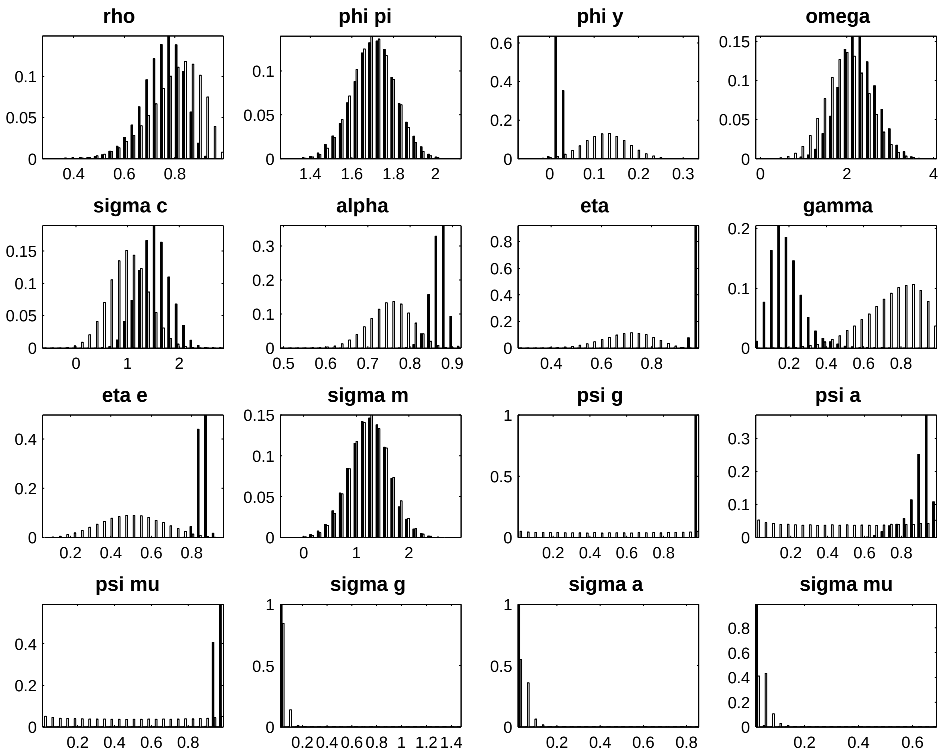

Figure 5: Prior vs. posterior in model 5

		$Model_1$		$Model_2$		$Model_3$		$Model_4$		$Model_5$	
Parameter	distribution	mean	std	mean	std	mean	std	mean	std	mean	std
ρ	beta	0.980	0.010	0.727	0.085	0.973	0.010	0.729	0.087	0.750	0.082
ϕ_π	normal	1.689	0.104	1.686	0.102	1.700	0.100	1.722	0.105	1.707	0.105
ϕ_y	normal	0.078	0.037	0.016	0.006	0.097	0.035	0.008	0.005	0.021	0.009
ω	normal	2.366	0.488	2.179	0.421	2.380	0.491	2.174	0.440	2.255	0.433
σ_c	normal	1.483	0.217	1.530	0.302	1.518	0.215	1.483	0.312	1.528	0.308
α	beta	0.938	0.010	0.884	0.015	0.931	0.013	0.860	0.023	0.869	0.018
η	beta	-	-	0.981	0.009	-	-	0.98	0.014	0.980	0.010
γ	beta	-	-	-	-	0.138	0.058	0.201	0.094	0.192	0.088
η_e	beta	0.924	0.010	0.858	0.020	0.922	0.012	0.846	0.022	0.855	0.019
σ_m	normal	-	-	-	-	-	-	-	-	1.23	0.378
ψ_g	beta	0.988	0.006	0.997	0.004	0.986	0.005	0.989	0.007	0.998	0.001
ψ_a	beta	0.925	0.028	0.918	0.048	0.916	0.032	0.932	0.043	0.899	0.065
ψ_μ	beta	0.996	0.003	0.964	0.014	0.998	0.002	0.958	0.014	0.963	0.016
σ_g	invgamma	0.020	0.003	0.010	0.001	0.021	0.003	0.010	0.001	0.010	0.001
σ_a	invgamma	0.031	0.006	0.018	0.003	0.032	0.008	0.019	0.003	0.019	0.003
σ_μ	invgamma	0.029	0.007	0.025	0.005	0.026	0.006	0.026	0.005	0.026	0.006

Table 2: Posterior estimates of the structural parameters in each model

3 Optimal policy

In this section we describe how we studied the optimal conduct of monetary policy: first with and without parameter uncertainty in a particular model, second with and without parameter uncertainty across models.

Throughout the paper we express the welfare costs as the loss in certainty equivalent consumption at the marginal posterior mean: First we compute the loss of a certain policy μ and a parameter vector Θ to derive overall utility:

$$U(c(\Theta), l(\Theta), m(\Theta), \Theta) - L(\mu, \Theta).$$

Since we want to express utility as reduction in certainty consumption equivalents we set this expression equal to:

$$U(c(\Theta) * (1 - \varrho), l(\Theta), m(\Theta), \Theta)$$

and solve for ϱ. To do so we used the following simple utility function

$$U(c, l, \Theta) = \frac{c^{1-\sigma_c}}{1-\sigma_c} - a_1 \frac{l^{1+\omega}}{1+\omega} + a_2 \frac{m^{1-\sigma_m}}{1-\sigma_m}.$$

We calibrate a_1 such that households work 1/3 of their available time at posterior mean. Furthermore, we set the parameter a_2 such that at a nominal interest rate of $R = 1.083$ the annual ratio of $M1$ over nominal GDP equals 0.17. This value is consistent with postwar Euro and U.S. data and similar to the one used by Schmitt-Grohé and Uribe (2006).

3.1 Optimal policy within a particular model

The optimal policy problem without parameter uncertainty can be stated as follows.

Given a parameter vector Θ, find a set of coefficients ρ_r, ϕ_π, ϕ_y of the simple feedback rule to maximize households'utility approximated by:

$$\frac{u}{1-\beta} - \frac{u_c y\theta(\omega+\sigma)}{2\kappa}\sum_{t=0}^{\infty}\beta^t\{var(\widehat{\pi}_t) + \frac{\kappa}{\theta}var(\widehat{y}_t - \widehat{y}_t^e)\}, \quad (34)$$

s.t. the euler equation and the aggregate supply curve. Natural choices for a particular Θ are the prior or posterior mean or the mean that results of the estimation.

Under parameter uncertainty, the task is to find coefficients of the simple feedback rule such that $E_\Theta L$ is minimized s.t. the euler equation and the aggregate supply curve. Suppose we use n draws of the posterior distribution, then the model is solved n-times for each combination of ρ_r, ϕ_π and ϕ_y.

To be completed

3.2 Optimal policy across models

In this section we determine the optimal simple interest rate rule across all models. Thereby each model is weighted with its corresponding marginal density. In the first subsection we compute the optimal rule across models at the posterior mean of the estimated parameters. Then we determine the optimal rule across all models and along the whole parameter space, i.e. we combine the effects of parameter and structural uncertainty.

The optimal policy problem without parameter uncertainty can be stated as follows: Given a particular $\bar{\Theta}$ (e.g. the marginal posterior mean), find coefficients of the simple feedback rule such that

$$E_{\mathcal{M}}L = p(Y|M1)W(L_{M1}(\bar{\Theta}_1)) + ... + p(Y|Mn)W(L_{Mn}(\bar{\Theta}_{M_n})) \quad (35)$$

is minimized, where $\mathcal{M} = \{M_1, ...M_n\}$ and $\sum_{i=1}^n p(M_i) = 1$. Thereby $W(L_i)$ is a function of the loss in model i. Possible functional forms are

- Strictly Bayesian: $W(L_i) = L_i$
- Loss-Averse:$W(L_i) = L_i^j, \quad j < 1$
- Most likely model: $W(L_i) = L_i/L_i^*$,

where L_i^* is the lowest loss for a given realization $\bar{\Theta}_i$ in model i.

The optimization problem for the robustly-optimal policy rule across the model and parameter space can be formulated as follows.

Find coefficients of the simple feedback rule such that

$$E_{\mathcal{M},\Theta}L = p(Y|M_1)E_{\Theta_{M_1}}W(L_{M_1}(\Theta_{M_1})) + ... + p(Y|M_n)E_{\Theta_{M_n}}W(L_{M_n}(\Theta_{M_n})) \quad (36)$$

is minimized, where $\mathcal{M} = \{M_1, ...M_n\}$ and $\sum_{i=1}^{n} p(M_i) = 1$. Again, the function $W(L_i)$ can take on the forms stated in the previous section.

4 Results

4.1 Optimal policy without parameter uncertainty

Throughout our analysis we consider two different scenarios, either we impose the optimal policy to be implementable or not. Consider first the case where we do not impose this additional requirement. The optimal coefficients for the simple rule and the corresponding business cycle costs are displayed in Table 3:[6]

Coefficients	$Model_1$	$Model_2$	$Model_3$	$Model_4$	$Model_5$
ρ_R	0.9955	1.051	1.0022	1.0505	1.2116
ϕ_π	3	3	3	3	0.03854
ϕ_y	0.0003	0.0163	0.0018	0.0175	0.0003
Business Cycle Costs	4.0657 %	0.989275 %	5.973975 %	0.79655%	2.98965 %

Table 3: The optimal rule and business cycle costs at the posterior mean - no zero bound restriction

Remarkably, all rules except the first one feature an super-inertial reaction on past interest. Besides that all rules are characterized by a positive short run reaction on inflation and the

[6] We find that the optimal simple rules achieve business cycle costs close to those obtained by the timeless perspective.

Weights	$Model_1$	$Model_2$	$Model_3$	$Model_4$	$Model_5$
λ_x	0.0033	0.2085	0.0041	0.2968	0.2673
λ_r	-	-	-	-	2.774

Table 4: The weights λ_x and λ_r at the posterior mean

response to output plays a minor role.[7]

Not surprisingly, the welfare costs of business cycles are the larger the larger the estimated degree staggered pricing α is. $Model_5$ adds the interest variability as an additional and conflicting aim to price stabilization. Note that our estimates imply that price stability is no longer the predominant aim of stabilization policy (see Table 4). Correspondingly, the optimal rule in $Model_5$ reacts only weakly to an increase in inflation.

Coefficients	$Model_1$	$Model_2$	$Model_3$	$Model_4$	$Model_5$
ρ_R	1.1433	1.1671	1.2528	1.2321	1.2116
ϕ_π	0.0482	0.0394	0.0644	0.0371	0.038
ϕ_y	0.0001	0.0002	0.0005	0.0002	0.0003
Business Cycle Costs	5.81%	3.31%	7.22%	2.669975 %	2.98965 %

Table 5: The optimal small rule and business cycle costs at the posterior mean with zero bound requirement.

Notably, imposing the zero bound on interest rate comes at a cost which can be substantial. In $Model_1$ this corresponds to a 2 percent drop in certainty equivalent consumption (see Tables 3 and 5). The reason for it is that households' utility is not a decreasing function of the variability of interest. Indeed, price stability is the predominant aim in theses models which is in conflict to the zero bound requirement. Imposing the zero bound in $Model_5$ however has literally no effect.

Is there a need for a robustly-optimal rule? To put it differently, what would happen if we optimize in one but the true model is a different one? The results of this exercise can be found in Tables 6 and 7. Not surprisingly, the possible losses are lower in case of implementable

[7] These results correspond to the one obtained by Schmitt-Grohé and Uribe (2006).

policies and the highest if the optimal rules from the first 4 models are applied in case of a transaction friction. Correspondingly, applying a robustly-optimal rule can be rewarding if policy is not constrained by the zero bound requirement.

Best rules of	In: $Model_1$	$Model_2$	$Model_3$	$Model_4$	$Model_5$
$Model_1$	0	0.0097	0.00035	0.0055	15.234325
$Model_2$	0.060675	0	0.032425	2.5E-05	15.11195
$Model_3$	0.0002	0.00785	0	0.0045	15.213025
$Model_4$	0.070675	5E-05	0.0381	0	15.132375
$Model_5$	1.9825	2.38435	1.347575	1.852675	0

Table 6: Difference in performance of the best rule in each model across models – no zero bound restriction.

Best rules of	In: $Model_1$	$Model_2$	$Model_3$	$Model_4$	$Model_5$
$Model_1$	0	zb	zb	zb	zb
$Model_2$	0.1424	0	0.0453	zb	zb
$Model_3$	0.1116	zb	0	zb	zb
$Model_4$	0.2944	0.1257	0.1247	0.0000	0.0048
$Model_5$	0.2450	0.0693	0.0995	zb	0

*indicates that policy violates the zero-bound restriction.

Table 7: Difference in performance of the best rule in each model across models – with zero bound restriction.

The optimal robust policy rule across models is first characterized by an super-inertia reaction on past inflation. This is not surprising since the optimal policy rules in each model featured similar coefficients. The low reaction coefficient on inflation may not have been expected: in almost 80% (see Table 8) of the relevant model space is a high reaction coefficient on inflation optimal, lowering it even led to substantial welfare losses (see last line of Table 6). Nevertheless welfare cost of a high reaction coefficient on inflation in $Model5$ are of a sizeable magnitude. Therefore optimal policy is characterized by alleviating this risk. Correspondingly, imposing a zero bound on interest rate does not lead to different results.

	$Model_1$	$Model_2$	$Model_3$	$Model_4$	$Model_5$
$PosteriorOdd$	0.2040	0.1989	0.2024	0.1974	0.1973

Table 8: Posterior odds

Coefficients	*No Zero Bound*	*Zero Bound*
ρ_R	1.181	1.1907
ϕ_π	0.141	0.0339
ϕ_y	0.0006	0.0001

Table 9: The optimal robust policy rule at the posterior mean across models

The relative performance of the robustly-optimal rule to the best rule in a particular setup for the different models are displayed in Tables .

	$Model_1$	$Model_2$	$Model_3$	$Model_4$	$Model_5$
Business Cycle Costs	0.25	0.09	0.11	0.01	0.01

Table 10: Relative performance of the robustly-optimal rule – with zero bound requirement.

	$Model_1$	$Model_2$	$Model_3$	$Model_4$	$Model_5$
Business Cycle Costs	1.23	1.69	0.85	1.31	0.71

Table 11: Relative performance of the robustly-optimal rule – no zero bound requirement.

Summing up the gains of following a robustly-optimal rule are substantial and corresponds to a permanent 2 percent increase in certainty equivalent consumption on average – if policy is not constrained by the zero bound. Imposing the zero bound the gains of pursuing a robustly-optimal rule evaporate.

4.2 Optimal policy under parameter uncertainty

to be added.

5 Conclusion

This paper studies the optimal conduct of monetary policy in several micro-founded macro models with nominal rigidities if the decision-maker faces uncertainty about the true structure of the economy. We employ bayesian methods to assess the relevant sources of uncertainty within (parameter uncertainty) and across models (specification uncertainty) using EU 11 data. We propose a novel method to determine the robustly-optimal policy rule as the maximum of the utility of the individual agent across models where each of the models is weighted with its corresponding posterior odd.

We repeat the exercise twice: once we require the optimal rule to be implementable , i.e. implying a low probability of a binding zero bound on interest rates, and once we do not. In the latter scenario optimal policy is characterized by a super-inertial reaction on the past interest rate and a muted reaction on current inflation, except in the model featuring a demand for cash. In this model optimal policy requires a low reaction coefficient on inflation. Even though this model has only 20% posterior odds optimal robust policy rule across models is alleviating the negative welfare effects that a muted coefficient on inflation has in it and is quite similar to the optimal rule there.

If we impose the zero bound on interest rates optimal policy in all models is again characterized by a super-inertial reaction on the past interest rate, but also by a lower coefficient on inflation and output. Imposing the zero bound have substantial negative welfare effects. However, since interest rate stability enters implicitly the central bank's objective in all models, the difference in the optimal policy rules vanishes. The optimal policy rule across models is therefore not different from the single ones.

To be added optimal robust rule.

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