The design of pension pay out options when the health status during retirement is uncertain

Mathias Kifmann*

February 2007

Abstract

This paper examines the optimal design of pension plans when the health status during retirement is uncertain. Assuming that the health status affects both life expectancy and the marginal utility of consumption, we find that choice between a lump-sum payment and an annuity can be welfare-enhancing if the health status is not observable. This result holds if the marginal utility of consumption is larger in the health state with a lower life expectancy. Whether the first best or only a second best can be implemented depends on the correlation of the marginal utility of consumption and life expectancy.

JEL-**classification:** G23, H55, D82.

Keywords: pensions, lump-sum withdrawal, annuities, longevity.

*Universität Augsburg, Universitätsstr. 16, 86159 Augsburg, Germany; Email: Mathias.Kifmann@wiwi.uni-augsburg.de. This research is part of the project "Lebensarbeitszeit, Humankapitalbildung und Transfersystem" funded by the Deutsche Forschungsgemeinschaft under grant BR 740/15-1.

1 Introduction

In a seminal paper, Yaari (1965) showed that individuals who maximize expected utility should annuitize all of their savings. Recently, Davidoff, Brown, and Diamond (2005) extended his analysis and showed that this result also holds under weaker conditions. Nevertheless, full annuitization remains the exception rather than the rule. The literature has therefore tried to explain why individuals only partially annuitize their wealth or choose not to annuitize at all.[1]

This paper deals with a particular deviation from Yaari's result. Frequently, individuals have a choice between a lump-sum payment and an annuity upon entering retirement. For example, this is the case in many private pension plans in the US. Also publicly regulated programmes such as Chile's funded pension system, the Swiss occupational pension scheme or state-subsidized supplementary private pensions in Germany ('Riester pensions') allow such a choice.

An important question is whether the possibility to select a lump sum can be desirable. In a standard model, this option can only reduce welfare since individuals with a low life expectancy will opt for the lump sum. This reduces the redistribution from short-living to long-living individuals which is optimal ex ante when life expectancy is still uniform (Brugiavini, 1993, and Sheshinski, 2004).

The standard model, however, assumes that the utility function is independent of life expectancy. This is questionable as life expectancy is closely related to the health status which is likely to have an impact on the utility function. In this paper, we show that considering the links between life expectancy, health and utility can make a lump-sum option valuable. Concretely, we find that rational individuals might prefer a choice between a lump-sum payment and an annuity if the health status during retirement is uncertain and unobservable. This is the case if the marginal utility of consumption and life expectancy are negatively correlated.[2]

[1] Possible explanations include inferior returns to annuities (Friedman and Warshawsky, 1990, Mitchell, Poterba, Warshawsky, and Brown, 1999, Milevsky and Young, 2002), bequest motives (Hurd, 1989, Kotlikoff and Summers, 1981, Bernheim, 1991), incomplete markets (Yagi and Nishigaki, 1993, Davidoff, Brown, and Diamond, 2005), within couple-risk sharing (Brown and Poterba, 2000), and pre-existing annuities from public pensions (Bernheim, 1991).

[2] A similar result has been obtained by Diamond (2003) in an optimal income tax framework. He finds that a lump-sum option is optimal if life expectancy and productivity are positively correlated.

The paper is structured as follows. In Section 2, we present the basic model and derive conditions under which a choice option in a pension plan is optimal. Section 3 extends the basic model in various directions. In Section 4, we discuss the implications for public pensions. Section 5 concludes and points out directions for further research.

2 The basic model

2.1 Health status, marginal utility and life expectancy

Individuals are initially identical. They have wealth W which they can invest in a pension plan before entering retirement. Retirement is reached with probability $\delta < 1$. Upon retirement, the health status $h = g, b$ of individuals is revealed. With probability π the 'good' state g arises, with probability $1 - \pi$ the health status is 'bad'. The health status has implications both for life expectancy and the marginal utility of consumption:

- *Health status and life expectancy*

  Individuals with health status b will only live one period after retirement (period 1). With status g, one can live up to two periods. The survival probability for period 2 is $0 < \rho < 1$. Individuals possess information on their life expectancy.[3]

- *Health status and utility*

  Utility is state-dependent. In state g, utility in each period is $u(c_t)$ where c_t is consumption in period $t = 1, 2$. In stage b, utility is $\alpha u(c_t)$, with $u'(c_t) > 0$, $u''(c_t) < 0, \lim_{c_t \to 0} u'(c_t) = \infty$ and $\alpha > 0$.

For simplicity, consumption before period 1 is not modelled since it does not affect the structure of the optimal pension plan. Only the amount invested in the annuity may vary. Furthermore, we assume that the interest rate is zero. Individuals do not discount the future and have no bequest motive.

[3]For evidence on this hypothesis, see Hurd and McGarry (1995).

Expected utility is given by

$$EU = \delta\left(\pi\left(u(c_1^g) + \rho u(c_2^g)\right) + (1-\pi)\alpha u(c_1^b)\right). \tag{1}$$

We leave open whether $\alpha \gtreqless 1$, i.e. whether marginal utility of consumption is higher in state b or g. In particular, we do not find $\alpha > 1$ implausible.[4] Knowledge of nearby early death may make consumption more valuable. For example, individuals may want to spend money on an expensive trip they always dreamed of.[5]

Figure 1 shows the different states of nature and the corresponding utilities. Three risks which individuals would like to insure through a pension plan can be identified:

(i) the risk to reach retirement,

(ii) the risk that marginal utility differs between the health states,

(iii) the longeveity risk in state g.

We assume that pensions plans are actuarially fair and maximize expected utility of individuals. This can be interpreted as the outcome of competition on the market for pension plans. Alternatively, this assumption can be justified by a public pension scheme set up to meet this objective (see Section 4). Furthermore, we rule out that individuals can draw loans on future pension payments and guarantee repayments through life insurance. Therefore, it is not possible to borrow against future income.

[4] Viscusi and Evans (1990) find evidence for a lower marginal utility when the health status declines. However, this study is based on chemical workers and not on elderly. In a further study using survey data on adults approaching middle age, Evans and Viscusi (1991) could not identify an effect of health on the marginal utility of consumption.

[5] In our set-up, $\alpha > 1$ implies that individuals are actually better off in state b. However, we can also write utility in the bad state as $\alpha u(c) - \kappa$ which is compatible with higher marginal utility of consumption but lower total utility. Since only the marginal utility of consumption is important in the following, we stick to our simpler version.

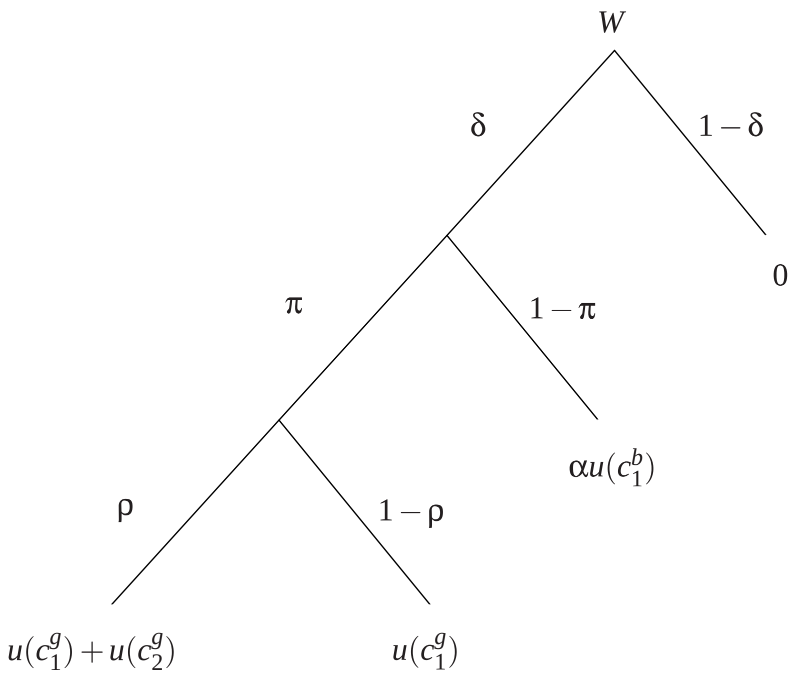

Figure 1: States of nature and utilities

2.2 Observable health status

If the health status is observable, pension plans can make their payments dependent on the health status and the age of the individual. We therefore solve the following problem

$$\begin{aligned} \max_{c_1^g, c_2^g, c_1^b} EU &= \delta\left(\pi\left(u(c_1^g)+\rho u(c_2^g)\right)+(1-\pi)\alpha u(c_1^b)\right) \\ \text{s.t.} \quad W &= \delta\left(\pi c_1^g+\pi\rho c_2^g+(1-\pi)c_1^b\right). \end{aligned} \tag{2}$$

From the first-order conditions, we obtain that the marginal utility of consumption must be the same in all states and periods, i.e.

$$u'(c_1^{g*}) = u'(c_2^{g*}) = \alpha u'(c_1^{*b}). \tag{3}$$

This implies

$$c_1^{g*} = c_2^{g*} = c^{g*}, \quad c_1^{b*} \gtrless c^{g*} \;\Leftrightarrow\; \alpha \gtrless 1.$$

Thus, a constant annuity is optimal in the good health state. The payment in the bad health state is larger if marginal utility of consumption is higher.

Proposition 2.1. *If the health status is observable, then it is optimal to pay out c_1^{b*} in the bad health state and to provide an annuity c^{g*} in the good health state. The payment in the bad health state is larger than the annuity if and only if marginal utility of consumption is higher in that state.*

2.3 Unobservable health status

If the health status is not observable, the optimal pension plan must be incentive-compatible, i.e. no type should have an advantage by claiming to be the other type. For b-types, the incentive constraint is

$$\alpha u(c_1^b) \geq \alpha u(c_1^g) \quad \Leftrightarrow \quad c_1^b \geq c_1^g. \qquad \text{(ICB)}$$

Clearly, the one period payments for b-types cannot be smaller than the first-period payments for g-types. Note that the first best violates (ICB) if $c_1^g > c_1^b$ which corresponds to $\alpha < 1$, i.e. a lower marginal utility of consumption in the bad state of health.

Incentive compatibility for g-types could be ensured if pension plans are able to punish g-types in period 2 if they claimed a one-period payment since only g-types can be alive in this period. However, it is doubtful whether courts would enforce it. We therefore do not consider this possibility.[6]

If g-types pretend to be b-types, they exchange a one-period payment for a payment stream over two periods. This raises the question how they finance their consumption in period 2. Their preferred method is to annuitize the one-period payment. In this section, we assume that they are able to do so, e.g. because pensions plan are not able to monitor further annuity purchases.[7] The price g-types must pay for an annuity will be ρ per unit consumption in period 2 since only g-types will demand annuities. g-types will therefore buy annuities up to the point where $u'(\hat{c}_1^g) = u'(\hat{c}_2^g)$. If g-types claim to be b-types and receive c_1^b in period 1,

[6] See also Section 3.4 where we consider that b-types may also live up to period 2.
[7] In Section 3.1, we allow pensions plans to prohibit to buy further annuities.

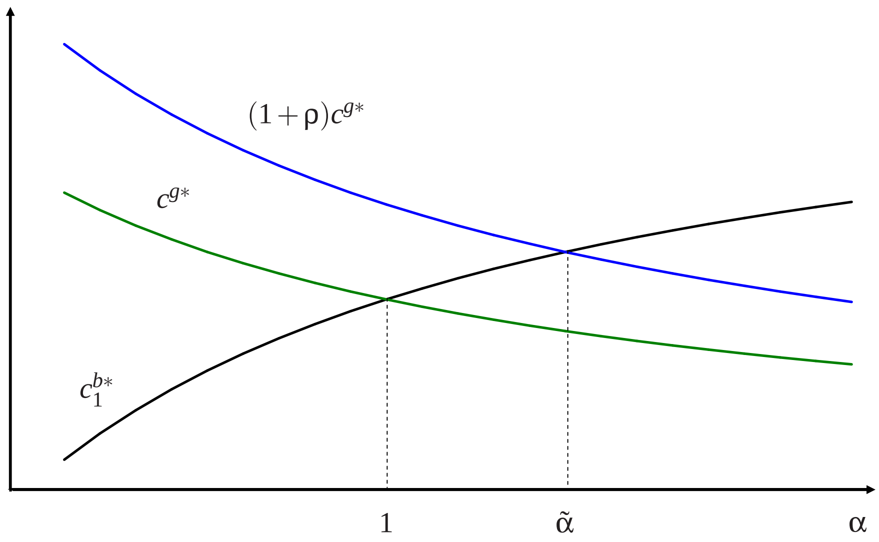

Figure 2: First-best consumption and incentive compatiblity

their consumption is therefore given by $\hat{c}_1^g = \hat{c}_2^g = c_1^b/(1+\rho)$ yielding expected utility in period 1

$$EU^g(t=1) = u(\hat{c}_1^g) + \rho u(\hat{c}_2^g) = (1+\rho)u(c_1^b/(1+\rho)).$$

Therefore the incentive constraint for g-types is

$$u(c_1^g) + \rho u(c_2^g) \geq (1+\rho)u(c_1^b/(1+\rho)). \tag{ICG1}$$

The first-best solution violates (ICG1) if $c_1^{b*} > (1+\rho)c^{g*}$, i.e. if the payment for b-types is larger than the present value of the annuity for g-types. This is the case if α exceeds a critical value $\tilde{\alpha} > 1$. For example, if $u(c) = \ln(c)$, we have $c_1^{b*} = \alpha c^{g*}$ in the first best which yields a critical value $\tilde{\alpha} = 1+\rho$.

Figure 2 illustrates the conflict between the first best and incentive compatibility. It shows first-best consumption c^{g*} and c_1^{b*} and the present value $(1+\rho)c^{g*}$ as functions of α. If $\alpha < 1$, then $c_1^{b*} < c^{g*}$ and the incentive-constraint for b-types is violated. $\alpha > \tilde{\alpha}$ implies $c_1^{b*} > (1+\rho)c^{g*}$ and g-types have the incentive to claim c_1^{b*} and convert this into an annuity. We therefore find that the first best is incentive-compatible only if $\alpha \in [1;\tilde{\alpha}]$. In this case, the first best can be implemented by giving individuals a *choice* between a lump-sum payment c_1^{b*} and an annuity c_t^{g*}. Individuals will self-select since $c_1^{b*} \geq c^{g*}$ and $(1+\rho)c^{g*} \geq c_1^{b*}$. However, if $\alpha < 1$ or $\alpha > \tilde{\alpha}$, only a second-best solution is possible. We consider both cases in the following.

Second-best solution for $\alpha < 1$

If $\alpha < 1$, then marginal utility of consumption and life expectancy are positively correlated. The first best is not compatible with the incentive constraint for *b*-types (ICB). In the second-best solution, this constraint will therefore be binding. To determine the second best, we solve the problem

$$\max EU = \delta\left(\pi\left(u(c_1^g) + \rho u(c_2^g)\right) + (1-\pi)\alpha u(c_1^b)\right)$$

s.t.

$$\begin{aligned} W &= \delta\left(\pi c_1^g + \pi\rho c_2^g + (1-\pi)c_1^b\right) \\ \hat{c}_1^b &= \hat{c}_1^g = c_1 \end{aligned} \tag{4}$$

where (4) is the incentive constraint (ICB) with equality sign. Substituting (4) yields the Lagrangean

$$\mathcal{L} = \delta\left(\pi\left(u(c_1) + \rho u(c_2^g)\right) + (1-\pi)\alpha u(c_1)\right) + \lambda\left\{W - \delta\left(c_1 + \pi\rho c_2^g\right)\right\}$$

with the first-order conditions

$$\frac{\partial\mathcal{L}}{\partial c_1} = \delta(\pi + (1-\pi)\alpha)u'(c_1) - \lambda\delta = 0 \tag{5}$$

$$\frac{\partial\mathcal{L}}{\partial c_2^g} = \delta\pi\rho u'(c_2^g) - \lambda\delta\pi\rho = 0 \tag{6}$$

$$\frac{\partial\mathcal{L}}{\partial\lambda} = W - \delta\left(c_1 + \pi\rho c_2^g\right) = 0. \tag{7}$$

We obtain

$$(\pi + (1-\pi)\alpha)u'(c_1) = u'(c_2^g). \tag{8}$$

Since $(\pi + (1-\pi)\alpha) < 1$, this implies $c_1 < c_2^g$, i.e. the annuity rises over time.[8] Thus, the incentive for *b*-types to claim to be *g*-types leads to a distorted annuity for *g*-types. Ex ante, of course, all individuals are worse off. The second best can be implemented by specifying the payments c_1 and c_2. No choice option in

[8]Note that we ruled out borrowing against future payments.

necessary. Compared to first best, it can be shown that $c_1^{b*} < c_1 < c^{g*}$.[9] A priori, it is not clear whether $c_2^g \gtrless c^{g*}$.[10]

Second-best solution for $\alpha > \tilde{\alpha}$

In this case, marginal utility of consumption and life expectancy are strongly negatively correlated. The first best violates the incentive constraint for *g*-types (ICG1). The second best can be found by solving the problem

$$\max EU = \delta\left(\pi\left(u(c_1^g) + \rho u(c_2^g)\right) + (1-\pi)\alpha u(c_1^b)\right)$$

s.t.

$$W = \delta\left(\pi c_1^g + \pi\rho c_2^g + (1-\pi)c_1^b\right) \tag{9}$$

$$u(c_1^g) + \rho u(c_2^g) = (1+\rho)u(c_1^b/(1+\rho)). \tag{10}$$

where (10) is the incentive constraint (ICG1) with equality sign. The Lagrangean is

$$\begin{aligned}\mathcal{L} &= \delta\left(\pi\left(u(c_1^g) + \rho u(c_2^g)\right) + (1-\pi)\alpha u(c_1^b)\right) \\ &\quad +\lambda\left\{W - \delta\left(\pi c_1^g + \pi\rho c_2^g + (1-\pi)c_1^b\right)\right\} \\ &\quad +\mu\left\{u(c_1^g) + \rho u(c_2^g) - (1+\rho)u(c_1^b/(1+\rho))\right\}\end{aligned}$$

with the first-order conditions

$$\frac{\partial\mathcal{L}}{\partial c_1^g} = \delta\pi u'(c_1^g) - \lambda\delta\pi + \mu u'(c_1^g) = 0 \tag{11}$$

$$\frac{\partial\mathcal{L}}{\partial c_2^g} = \delta\pi\rho u'(c_2^g) - \lambda\delta\pi\rho + \mu\rho u'(c_2^g) = 0 \tag{12}$$

$$\frac{\partial\mathcal{L}}{\partial c_1^b} = \delta(1-\pi)\alpha u'(c_1^b) - \lambda\delta(1-\pi) - \mu u'(c_1^b/(1+\rho)) = 0 \tag{13}$$

$$\frac{\partial\mathcal{L}}{\partial\lambda} = W - \delta\left(\pi c_1^g + \pi\rho c_2^g + (1-\pi)c_1^b\right) = 0 \tag{14}$$

$$\frac{\partial\mathcal{L}}{\partial\mu} = u(c_1^g) + \rho u(c_2^g) - (1+\rho)u(c_1^b/(1+\rho)) = 0. \tag{15}$$

[9] Equation (8) implies $\alpha u'(c_1) < u'(c_2^g)$. Taking into account the budget constraint, the first-best condition (3) and the incentive constraint (4), this is only possible if $c_1 > c_1^{b*}$. Furthermore, $c_1 \geq c^{g*}$ is not compatible with the budget constraint since $c_2^g > c_1$.

[10] For constant relative risk aversion γ, it can be shown that $c_2^g > c^{g*}$ if and only if $\gamma < 1$.

From (11) und (12) we obtain

$$u'(c_1^g) = u'(c_2^g) = \frac{\lambda\delta\pi}{\delta\pi + \mu} \quad \Rightarrow \quad c_1^g = c_2^g = c^g \tag{16}$$

and a standard annuity is optimal for g-types. Substituting into (10) yields

$$(1+\rho)u(c^g) = (1+\rho)u(c_1^b/(1+\rho)) \quad \Rightarrow \quad c_1^b = (1+\rho)c^g > c^g, \tag{17}$$

i.e. the present value of the lump-sum payment for b-types and the annuity for g-types are the same. In the first best, in contrast, $\alpha > \tilde{\alpha}$ implies that the present value of the annuity is smaller than the payment for b-types (see Figure 2). Thus, the annuity level for people in good health must be larger in the second best and the payment for b-types must be smaller. Substituting (16) and (17) into the budget constraint (9), we obtain

$$c_1^b = \frac{W}{\delta} \quad \text{and} \quad c_1^g = c_2^g = \frac{W}{\delta(1+\rho)}$$

as the optimal solution which can be implemented by giving individuals a choice between a lump-sum payment and an annuity.[11]

Proposition 2.2. *If the health status is unobservable, then the first best can only be implemented if life expectancy and marginal utility are weakly negatively correlated (*$1 \leq \alpha \leq \tilde{\alpha}$*). Otherwise a second-best solution prevails. It is optimal to give individuals a choice between a lump-sum payment and an annuity if life expectancy and marginal utility are negatively correlated (*$\alpha \geq 1$*). Otherwise, an annuity which increases over time is preferable.*

[11] It is also possible to implement the second best by paying out a lump sum c_1^b which is then annuitized by g-types. In Section 3.1 where we allow pension plans to prohibit the purchase of further annuities, however, this solution is inferior to a choice between a lump-sum payment and an annuity (see footnote 12).

3 Extensions

In this section, we extend the basic model in various directions to check whether choice between a lump-sum payment and an annuity at retirement remains optimal. We consider the following extensions:

1. In Subsection 3.1, we allow pension plans to monitor the purchase of further annuities.
2. Imperfect correlation between marginal utility of consumption and life expectancy is considered in Subsection 3.2.
3. The possibility of moral hazard due to a state-guaranteed minimum income is examined in Subsection 3.3
4. Subsection 3.4 allows for a positive survival probability to period 2 for individuals in bad health.

3.1 Monitoring of annuities possible

If annuity purchases can be monitored, pension plans can make it more difficult for g-types to pretend to be b-types by prohibiting annuitization of the lump-sum payment. Then the only way for g-types to transfer income to period 2 is to save. If they chose the lump-sum payment, they will therefore solve the problem

$$\max EU^g = u(c_1^g) + \rho u(c_2^g) \quad \text{s.t.} \quad c_1^g + c_2^g = c_1^b. \tag{18}$$

At the optimum

$$u'(\hat{c}_1^g(c_1^b)) = \rho u'(\hat{c}_2^g(c_1^b)) \tag{19}$$

and therefore $\hat{c}_1^g(c_1^b) > \hat{c}_2^g(c_1^b)$ as $\rho < 1$. With probability $1-\rho$, individuals will leave unintended bequests $c_2^g(c_1^b)$.

The new incentive constraint for g-types is

$$u(c_1^g) + \rho u(c_2^g) \geq u(\hat{c}_1^g(c_1^b)) + \rho u(\hat{c}_1^g(c_1^b)). \tag{ICG2}$$

Clearly, the RHS of (ICG2) will be smaller than the RHS of (ICG1) for a given value of c_1^b. Thus, the corresponding critical value of $\tilde{\alpha}_{\text{mon}}$ will be higher than the critical value $\tilde{\alpha}$ without monitoring and the first best can implemented for a larger range of α. For example, if $u(c) = \ln(c)$, we obtain

$$(1+\rho)\ln(c^g) \geq \ln\left(\frac{c^b}{1+\rho}\right) + \rho \ln\left(\frac{\rho c^b}{1+\rho}\right).$$

With the first-best condition $c^b = \alpha c^g$, this yields a critical value

$$\tilde{\alpha}_{\text{mon}} = (1+\rho)\rho^{-\frac{\rho}{1+\rho}} > 1+\rho = \tilde{\alpha}.$$

If $\alpha > \tilde{\alpha}_{\text{mon}}$, then we obtain a similar result as in section 2.3. The second best can be implemented by a choice option between a lump-sum payment and a constant annuity.[12] However, the present value of annuity for g-types is lower than lump-sum payment for b-types.

Proposition 3.1. *If pensions plans can monitor further annuity purchases, then the first best can be implemented for higher levels of the marginal utility of consumption in the bad health state.*

3.2 Heterogenous marginal utility

Now we relax the assumption that marginal utility is unique given life expectancy. In each health state, marginal utility can take different values. In state b, utility is $\alpha u(c_1^b)$ with $\alpha \in [\alpha_1, \alpha_2]$. In state g, utility is $\beta u(c_1^g) + \rho\beta u(c_2^g)$ with $\beta \in [\beta_1, \beta_2]$. Expected utility is given by

$$EU = \delta\left(\pi\bar{\beta}\left(u(c_1^g) + \rho u(c_2^g)\right) + (1-\pi)\bar{\alpha}u(c_1^b)\right) \tag{20}$$

where $\bar{\alpha}$ and $\bar{\beta}$ are the average values of α and β.

In the first best, we obtain that the marginal utility of consumption should be equalized across all states, i.e.

$$\forall \alpha, \beta \quad \alpha^b u'(c_1^b(\alpha)) = \beta^g u'(c_t^g(\beta)), \quad t = 1, 2.$$

The optimal pay outs are therefore increasing in α and β. This implies that the first best requires knowledge of these parameters which is highly unlikely. Thus, even

[12] As opposed to the basic model, only paying out a lump-sum payment c_1^b cannot implement the second best since g-types are not allowed to annuitize.

if the health status is observable, only a second-best solution can be implemented. Normalizing $\bar{\beta} = 1$, we obtain the following condition

$$u'(c_1^g) = u'(c_2^g) = \bar{\alpha} u'(c_1^b)$$

which states that marginal utility of consumption should be equalized on average across health states.

When the health status is not observable, the incentive constraints are

$$\forall \alpha \quad \alpha u(c_1^b) \geq \alpha u(c_1^g)$$

and

$$\forall \beta \quad \beta u(c_1^g) + \rho \beta u(c_2^g) \geq (1+\rho) \beta u(c_1^b/(1+\rho)).$$

Note that α and β have no impact on the incentive constraints. Thus, they are identical to (ICB) and (ICG1) and we can use the results from above by interpreting α as the average $\bar{\alpha}$. Thus, although the first best cannot be implemented, giving individuals a choice between a lump-sum payment and an annuity is still optimal if $\bar{\alpha} \geq 1$. For $1 \leq \bar{\alpha} \leq \tilde{\alpha}$, we obtain a second-best solution, otherwise a third best arises.

Proposition 3.2. *If marginal utility of consumption and life expectancy are only imperfectly correlated and neither marginal utility of consumption nor the health status are observable, then it is optimal to give individuals a choice between a lump-sum payment and an annuity if life expectancy and marginal utility are negatively correlated. Otherwise, an annuity which increases over time is preferable.*

3.3 Minimum income and moral hazard

If society grants a minimum income to its citizens, an important concern with respect to lump-sum withdrawals is moral hazard. Individuals may then have the incentive to take the lump sum, spend it regardless of their life expectancy on immediate consumption and rely on public transfers if they live longer. To rule this out, the government may therefore require that individuals only buy pensions plans which guarantee a payment that is at least as high as guaranteed minimum income in each period.

In the following, we examine the consequences of this policy for the second-best pension plan. We denote minimum income by m and assume that in the first best, individuals do not qualify for public assistance, i.e. $c_1^{g*}, c_2^{g*}, c_1^{b*} > m$. If the health status is not observable, the incentive constraint (ICB) for b-types and therefore the results for $\alpha < 1$ remain unchanged. However, the incentive constraint for g-types needs to modified. Since the contract for b-types cannot be lump sum, it consists of a payment c_1^b in period 1 and m in period 2 (which is never paid in the first best). If g-types pretend to be b-types, they therefore can also claim a payment m in period 2. The present value of the payment for b-types is therefore $(c_1^b + \rho m)/(1+\rho)$ and the incentive constraint is

$$u(c_1^g) + \rho u(c_2^g) \geq (1+\rho) u((c_1^b + \rho m)/(1+\rho)). \tag{21}$$

For a given value of c_1^b, it is therefore more attractive for b-types to claim to be g-types. This lowers the critical value $\tilde{\alpha}$. For logarithmic utility, we obtain

$$\tilde{\alpha}_m = 1 + \rho - \frac{\rho m}{c^{g*}} < 1 + \rho = \tilde{\alpha}.$$[13]

As above, the first best is incentive-compatible if $1 \leq \alpha \leq \tilde{\alpha}_m$. In this case, individuals can be given a choice between the payment stream c_1^{b*}, m and c_1^{g*}, c_2^{g*}. Since $\alpha > 1$ implies $c_1^{b*} > c^{g*}$, we can interpret $c_1^{b*} - c^{g*}$ as a partial lump-sum withdrawal which results in a reduction $c^{g*} - m$ in the second-period payment.

To determine the optimal pension plan for $\alpha > \tilde{\alpha}_m$, we solve the problem

$$\max EU = \delta \left(\pi \left(u(c_1^g) + \rho u(c_2^g) \right) + (1-\pi) \alpha u(c_1^b) \right)$$

s.t.

$$W = \delta \left(\pi c_1^g + \pi \rho c_2^g + (1-\pi) c_1^b \right)$$
$$u(c_1^g) + \rho u(c_2^g) = (1+\rho) u((c_1^b + \rho m)/(1+\rho)). \tag{22}$$

[13] Note that $\tilde{\alpha}_m > 1$ since we assumed $c^{g*} > m$.

Setting up the Lagrangean

$$\begin{aligned}\mathcal{L} &= \delta\left(\pi\left(u(c_1^g)+\rho u(c_2^g)\right)+(1-\pi)\alpha u(c_1^b)\right)\\ &+\lambda\left\{W-\delta\left(\pi c_1^g+\pi\rho c_2^g+(1-\pi)c_1^b\right)\right\}\\ &+\mu\left\{u(c_1^g)+\rho u(c_2^g)-(1+\rho)u((c_1^b+\rho m)/(1+\rho))\right\}\end{aligned}$$

yields the first-order conditions

$$\frac{\partial\mathcal{L}}{\partial c_1^g} = \delta\pi u'(c_1^g)-\lambda\delta\pi+\mu u'(c_1^g)=0 \tag{23}$$

$$\frac{\partial\mathcal{L}}{\partial c_2^g} = \delta\pi\rho u'(c_2^g)-\lambda\delta\pi\rho+\mu\rho u'(c_2^g)=0 \tag{24}$$

$$\frac{\partial\mathcal{L}}{\partial c_1^b} = \delta(1-\pi)\alpha u'(c_1^b)-\lambda\delta(1-\pi)-\mu u'((c_1^b+\rho m)/(1+\rho))=0 \tag{25}$$

$$\frac{\partial\mathcal{L}}{\partial\lambda} = W-\delta\left(\pi c_1^g+\pi\rho c_2^g+(1-\pi)c_1^b\right)=0 \tag{26}$$

$$\frac{\partial\mathcal{L}}{\partial\mu} = u(c_1^g)+\rho u(c_2^g)-(1+\rho)u(c_1^b/(1+\rho))=0. \tag{27}$$

From (23) und (24) we obtain as in the basic model

$$u'(c_1^g)=u'(c_2^g)=\frac{\lambda\delta\pi}{\delta\pi+\mu} \quad\Rightarrow\quad c_1^g=c_2^g=c^g.$$

Inserting into condition (22) yields

$$(1+\rho)u(c^g)=(1+\rho)u((c_1^b+\rho m)/(1+\rho)) \;\Rightarrow\; c_1^b=(1+\rho)c^g-\rho m. \tag{28}$$

Thus, the present value of the payment for b-types is smaller than the annuity for g-types. Furthermore, we must have $c_1^b>c^g$.[14] Therefore, it is optimal to give individuals a choice between a partial lump-sum withdrawal $c_1^b-c^g$ with a reduction c^g-m in the second-period payment and a constant annuity paying c^g.

[14]By (28), $c_1^b\le c^g$ implies $m\ge c^g$. Since we assumed for the first best $c^{g*}>m$, this implies $c^g<c^{g*}$ and, by the budget constraint, $c_1^b>c_1^{b*}>m$. Thus, $c_1^b\le c^g$ is only possible if $c^g>m$ which is incompatible with (28). Therefore, we must have $c_1^b>c^g$.

Proposition 3.3. *If the government requires that individuals only buy pensions plans which guarantee a payment that is at least as high as minimum income in each period and the health status is not observable, then it becomes more difficult to implement the first best. If marginal utility of consumption is higher in the bad state, it is optimal to give individuals a choice between a partial lump-sum withdrawal and a constant annuity.*

3.4 Positive survival probability for individuals in bad health

So far, we maintained the assumption that individuals in state b will not live more than one period. Now we allow both types to survive to the second period. The respective survival probabilities are ρ^b and ρ^g with $0 < \rho^b < \rho^g < 1$, i.e. b-types have a lower life expectancy. Expected utility is

$$EU = \delta\left(\pi\left(u(c_1^g) + \rho^g u(c_2^g)\right) + (1-\pi)\left(\alpha u(c_1^b) + \rho^b \alpha u(c_2^b)\right)\right). \qquad (29)$$

It is straightforward to show that the first-best solution requires annuities c^g and c^b with

$$c^b \gtrless c^g \quad \Leftrightarrow \quad \alpha \gtrless 1$$

If the health status is not observable, then it is impossible to implement the first best unless $\alpha = 1$. If $\alpha < 1$, then b-types pretend to be g-types and vice versa. Consequently, one incentive constraint will be binding for $\alpha < 1$, the other for $\alpha > 1$.

In the following, we assume that pension plans can monitor and thus prohibit further annuity purchases. They will always do so since this would add another constraint in the design of the optimal contract. For simplicity, we also allow pension plans to monitor and rule out savings (see footnote 17).

Second-best solution for $\alpha < 1$

In this case, the incentive-constraint for *b*-types

$$u(c_1^b) + \rho^b u(c_2^b) \geq u(c_1^g) + \rho^b u(c_2^g) \tag{30}$$

will be binding. Setting up the Lagrangean

$$\begin{aligned}\mathcal{L} \;=\; & \delta\left(\pi\left(u(c_1^g) + \rho^g u(c_2^g)\right) + (1-\pi)\left(\alpha u(c_1^b) + \rho^b \alpha u(c_2^b)\right)\right) \\ & + \lambda\left\{W - \delta\left(\pi c_1^g + \pi\rho^g c_2^g + (1-\pi)c_1^b + (1-\pi)\rho^b c_2^b\right)\right\} \\ & + \mu\left\{u(c_1^b) + \rho^b u(c_2^b) - u(c_1^g) - \rho^b u(c_2^g)\right\}\end{aligned}$$

yields the first-order conditions

$$\frac{\partial \mathcal{L}}{\partial c_1^g} = \delta\pi u'(c_1^g) - \lambda\delta\pi - \mu u'(c_1^g) = 0 \tag{31}$$

$$\frac{\partial \mathcal{L}}{\partial c_2^g} = \delta\pi\rho^g u'(c_2^g) - \lambda\delta\pi\rho^g - \mu\rho^b u'(c_2^g) = 0 \tag{32}$$

$$\frac{\partial \mathcal{L}}{\partial c_1^b} = \delta(1-\pi)\alpha u'(c_1^b) - \lambda\delta(1-\pi) + \mu u'(c_1^b) = 0 \tag{33}$$

$$\frac{\partial \mathcal{L}}{\partial c_2^b} = \delta(1-\pi)\rho^b \alpha u'(c_2^b) - \lambda\delta(1-\pi)\rho^b + \mu\rho^b u'(c_2^b) = 0 \tag{34}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = W - \delta\left(\pi c_1^g + \pi\rho c_2^g + (1-\pi)c_1^b\right) = 0 \tag{35}$$

$$\frac{\partial \mathcal{L}}{\partial \mu} = u(c_1^b) + \rho^b u(c_2^b) - u(c_1^g) - \rho^b u(c_2^g) = 0. \tag{36}$$

We obtain $c_1^b = c_2^b$ from (33) and (34). Conditions (31) and (32) yield

$$u'(c_1^g) = \frac{\lambda\pi}{\delta\pi - \mu} \quad \text{and} \quad u'(c_2^g) = \frac{\lambda\pi}{\delta\pi - \mu\frac{\rho^b}{\rho^g}}.$$[15]

Since $\rho^g > \rho^b$, this implies

$$u'(c_2^g) < u'(c_1^g) \quad \Rightarrow \quad c_2^g > c_1^g.$$

Thus, the incentive for *b*-types to pretend to be *g*-types is countered by an increasing annuity with his less attractive for *b*-types. From the binding incentive

[15] The assumption $\lim_{c_t \to 0} u'(c_t) = \infty$ ensures positive marginal utilities.

constraint (30), we obtain the solution

$$c_2^g > c_1^b = c_2^b > c_1^g.$$

Thus, it is optimal to give individuals a choice between a constant and an increasing annuity in period 1.

Second-best solution for $\alpha > 1$

In this case, the incentive-constraint for g-types

$$u(c_1^g) + \rho^g u(c_2^g) \geq u(c_1^b) + \rho^g u(c_2^b). \tag{37}$$

will be binding. The Lagrange function is

$$\begin{aligned}
\mathcal{L} \quad = \quad & \delta\left(\pi\left(u(c_1^g) + \rho^g u(c_2^g)\right) + (1-\pi)\left(\alpha u(c_1^b) + \rho^b \alpha u(c_2^b)\right)\right) \\
& +\lambda\left\{W - \delta\left(\pi c_1^g + \pi\rho^g c_2^g + (1-\pi)c_1^b + (1-\pi)\rho^b c_2^b\right)\right\} \\
& +\mu\left\{u(c_1^g) + \rho^g u(c_2^g) - u(c_1^b) - \rho^g u(c_2^b)\right\}.
\end{aligned}$$

The first-order conditions are

$$\frac{\partial \mathcal{L}}{\partial c_1^g} = \delta\pi u'(c_1^g) - \lambda\delta\pi + \mu u'(c_1^g) = 0 \tag{38}$$

$$\frac{\partial \mathcal{L}}{\partial c_2^g} = \delta\pi\rho^g u'(c_2^g) - \lambda\delta\pi\rho^g + \mu\rho^g u'(c_2^g) = 0 \tag{39}$$

$$\frac{\partial \mathcal{L}}{\partial c_1^b} = \delta(1-\pi)\alpha u'(c_1^b) - \lambda\delta(1-\pi) - \mu u'(c_1^b) = 0 \tag{40}$$

$$\frac{\partial \mathcal{L}}{\partial c_2^b} = \delta(1-\pi)\rho^b \alpha u'(c_2^b) - \lambda\delta(1-\pi)\rho^b - \mu\rho^g u'(c_2^b) = 0 \tag{41}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = W - \delta\left(\pi c_1^g + \pi\rho c_2^g + (1-\pi)c_1^b\right) = 0 \tag{42}$$

$$\frac{\partial \mathcal{L}}{\partial \mu} = u(c_1^g) + \rho^g u(c_2^g) - u(c_1^b) - \rho^g u(c_2^b) = 0 \tag{43}$$

We obtain $c_1^g = c_2^g$ from (38) and (39). Conditions (40) and (41) lead to

$$u'(c_1^b) = \frac{\lambda(1-\pi)}{\delta(1-\pi)\alpha - \mu} \quad \text{and} \quad u'(c_2^b) = \frac{\lambda(1-\pi)}{\delta(1-\pi)\alpha - \mu\frac{\rho^g}{\rho^b}}.$$[16]

[16]The assumption $\lim_{c_t \to 0} u'(c_t) = \infty$ ensures positive marginal utilities.

$\rho^g > \rho^b$ implies

$$u'(c_1^b) < u'(c_2^b) \quad \Leftrightarrow \quad c_1^b > c_2^b.^{17}$$

Finally, using the binding incentive constraint (37) yields

$$c_1^b > c_1^g = c_2^g > c_2^b.$$

This solution can be implemented by allowing individuals a choice between a partial lump-sum withdrawal $c_1^b - c^g$ with a reduction $c^g - c_2^b$ in the second-period payment and a constant annuity c^g.

Proposition 3.4. *If individuals in bad health have a positive probability to survive to period 2 and a lower life expectancy, the first best cannot be implemented if the health status is not observable. Assuming that pension plans can rule out further annuity purchases and savings, it is optimal to give individuals a choice between a partial lump-sum withdrawal and a constant annuity if life expectancy and marginal utility are negatively correlated (*$\alpha > 1$*). Otherwise, choice between a constant and an increasing annuity is preferable.*

4 Implications for public pensions

So far, we have left open whether the pension plan is private or public. Since we assumed that individuals are ex ante identical, however, there seems to be little justification for public intervention. This changes if we allow for ex ante differences in the probability δ of reaching retirement and the probability π to be in good health. A private market for pensions plans will then discriminate according to these probabilities. Individuals with high δ will be offered lower consumption during retirement. With respect to π it depends on whether an individual in good or bad health is more expensive.[18] This discrimination may well be against public

[17] We assumed that pension plans can prohibit savings. This is necessary if the optimal solution implies $u'(c_1^h) < \rho^h u'(c_2^h)$, $h = g, b$. If savings cannot be monitored then the additional constraint $u'(c_1^h) \geq \rho^h u'(c_2^h)$ needs to be imposed.

[18] If the health status is not observable and annuity purchases cannot be monitored, the lump-sum payment for b-types cannot be larger than the present value of the annuity for g-types. However, this restriction does not hold if pension plans can rule out further annuity purchases.

objectives and speaks in favor of a government pension plan which provides uniform conditions for retirement. Consider, e.g., a utilitarian welfare function. With differences in δ and π, average welfare is given by

$$\mathcal{W} = \bar{\delta}\left(\bar{\pi}\left(u(c_1^g) + \rho u(c_2^g)\right) + (1-\bar{\pi})\alpha u(c_1^b)\right) \tag{44}$$

where $\bar{\delta}$ and $\bar{\pi}$ are the average probabilities to reach retirement and to be in good health, respectively. The per capita resource constraint is

$$W = \bar{\delta}\left(\bar{\pi}c_1^g + \bar{\pi}\rho c_2^g + (1-\bar{\pi})c_1^b\right). \tag{45}$$

Maximizing (44) subject to (45) yields the optimal public pension plan. As above, choice between a lump sum and an annuity can be welfare-maximizing if the health status is not observable.

If public pensions are financed on a pay-as-you-go basis, a further aspect must be considered. If budget balance is required, lump-sum withdrawals require adjustments of the contribution or the replacement rate and lead to intergenerational transfers. This can be avoided by allowing the pension system to run a deficit. Since the system is neutral in present value terms, the deficit will be balanced by lower payments in later periods.

5 Conclusion

This paper examined the optimal design of pension plans when the health status during retirement is uncertain. In contrast to standard models, we assumed that the health status affects both life expectancy and the marginal utility of consumption. A simple model demonstrated that choice between a lump-sum payment and an annuity can be welfare-enhancing if the health status is not observable. This result holds if the marginal utility of consumption and life expectancy are negatively correlated. This result proved robust in several extensions. For example, we allowed marginal utility of consumption to be imperfectly correlated with the health status and considered that the maximum life-span does not depend on the health status. In the latter case, the possibility of a partial lump-sum withdrawal proved to be optimal if marginal utility of consumption and life expectancy are negatively correlated.

A limitation of the analysis is that we assumed a uniform retirement age. However, health and life expectancy can be expected to have an impact on the retirement age as well. For example, McGarry (2004) finds that the less healthy are likely to retire earlier. Similarly, Hurd, Smith, and Zissimopoulos (2004) observe that those with very low subjective probabilities of survival choose a lower retirement age. An interesting question for future research is whether early retirement and lump sum payments are substitutes if individuals value consumption higher when their health state is bad.

Finally, the paper raises the empirical question on how health affects utility and life expectancy of the elderly. In particular, it would be interesting to investigate the correlation of marginal utility of consumption and life expectancy in old age. As the paper shows this correlation is crucial for the optimal design of pension plans.

References

Bernheim, B. (1991). "How Strong Are Bequest Motives? Evidence Based on Estimates of the Demand for Life Insurance and Annuities," *Journal of Political Economy* 99, 899–927.

Brown, J., and J. Poterba (2000). "Joint Life Annuities and Annuity Demand by Married Couples," *Journal of Risk and Insurance* 67, 527–553.

Brugiavini, A. (1993). "Uncertain Resolution and the Timing of Annuity Purchases," *Journal of Public Economics* 50, 31–62.

Davidoff, T., J. Brown, and P. Diamond (2005). "Annuities and Individual Welfare," *American Economic Review* 95, 1573–1590.

Diamond, P. (2003). *Taxation, Incomplete Markets, and Social Security*. Munich Lectures in Economics MIT Press: Cambridge, Massachusetts.

Evans, W., and W. Viscusi (1991). "Estimation of State-Dependent Utility Functions Using Survey Data," *Review of Economics and Statistics* 73, 94–104.

Friedman, B., and M. Warshawsky (1990). "The Cost of Annuities: Implication for Saving Behavior and Bequests," *Quarterly Journal of Economics* 105, 135–154.

Hurd, M. (1989). "Mortality Risk and Bequest," *Econometrica* 57, 779–813.

Hurd, M., and K. McGarry (1995). "Evaluation of the Subjective Probabilities of Survival in the Health and Retirement Study," *Journal of Human Resources* 30, S268–S292.

Hurd, M., J. Smith, and J. Zissimopoulos (2004). "The Effects of Subjective Survival on Retirement and Social Security Claiming," *Journal of Applied Econometrics* 19, 761–775.

Kotlikoff, L., and L. Summers (1981). "The Role of Intergenerational Transfers in Aggregate Capital Accumulation," *Journal of Political Economy* 89, 706–732.

McGarry, K. (2004). “Health and Retirement: Do Changes in Health Affect Retirement Expectations?,” *Journal of Human Resources* 39, 624–648.

Milevsky, M., and V. Young (2002). “Optimal Asset Allocation and the Real Option to Delay Annuitization: It’s Not Now-or-Never,” Discussion Paper PI-0211, Pensions Institute, Birkbeck College, University of London.

Mitchell, O., J. Poterba, M. Warshawsky, and J. Brown (1999). “New Evidence on the Money’s Worth of Individual Annuities,” *American Economic Review* 89, 1299–1318.

Sheshinski, E. (2004). “Annuities and Retirement,” in C.D. Aliprantis, K.J. Arrow, P. Hammond, F. Kubler, H-M. Wu, and N.C. N. Yannelis (ed.), *Assets, Beliefs, and Equilibria in Economic Dynamics: Essays in Honor of Mordecai Kurz* pp. 27–54 Springer: Berlin.

Viscusi, W., and W. Evans (1990). “Utility Functions that Depend on Health Status: Estimates and Economic Implications,” *American Economic Review* 80, 353–374.

Yaari, M. (1965). “Uncertain Lifetime, Life Insurance, and the Theory of the Consumer,” *Review of Economic Studies* 32, 137–150.

Yagi, T., and Y. Nishigaki (1993). “The Inefficiency of Private Constant Annuities,” *Journal of Risk and Insurance* 60, 385–412.