Population, Pensions, and Endogenous Economic Growth[1]

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Abstract:

We study the effect of a declining labor force on the incentives to engage in labor-saving technical change. As labor becomes scarcer it becomes more expensive and innovation investments that increase labor productivity become more profitable. We incorporate this channel in a dynamic general equilibrium model with overlapping generations that is able to match the empirically observed distribution of income and wealth. We calibrate the model for the US and study the effect of a decline in population growth on per-capita income growth. The net effect on the growth rate is found to be positive and quantitatively significant. The endogeneity of growth is also found to be important for the welfare analysis of public pension reforms.

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1 Introduction

The demographic transition in the US has raised the concern that the reduction of the labor force share leads to a decrease in the savings rate and, therefore, to a diminished rate of economic growth. We argue that this line of reasoning neglects the stimulating effect of a declining labor force on the incentives to engage in labor-saving technical change. As labor becomes scarcer relative to capital, the relative price of labor increases. In turn, the profitability of innovation investments that raise labor productivity is higher.[1]

The study of our argument requires an analytical framework where both the economy's savings rate and its growth rate are endogenous. To provide for such a framework we combine a household sector comprising heterogeneous and overlapping generations with a production sector à la Hellwig and Irmen (2001) and Irmen (2005) where technical change is endogenous. Each generation can possibly turn 90 years and receives pensions through a pay-as-you-go system.

We use this setup to analyze the consequences of the actual and predicted decline in the population growth rate for the US economy. Our results apply to the time span from 1950 to 2400.[2] The focus is on the analysis of the demographic transition.

As a benchmark, we calibrate the demographic transition in a neoclassical framework with exogenous productivity growth at rate 1.8% per annum and a constant replacement ratio of 50%. The contribution rate to the pension system adjusts to keep the system's budget balanced. The decline in the share of the working age population from 88% in 1950 to 67% in 2400 induces a rise in the amount of capital per unit of efficient labor of roughly 30% over this time span.

When productivity growth becomes endogenous there is a positive feedback from an increase in the capital intensity to innovation incentives. Labor is scarcer and more expensive. Therefore, more innovation investments in productivity enhancing technologies are undertaken and the speed of productivity growth rises. In our calibrations, we find this effect to be of considerable size. Compared to the benchmark, productivity growth rises from 1.8% to 2.4% in the new steady state. Faster productivity growth and the fact that real resources have to be channeled into innovation investments imply that the amount of capital per unit of efficient labor is lower here than in the benchmark steady state.

Along the transition, the evolution of productivity is not monotonous. For instance, the calibrated growth rate is found to increase during the 1960s, it declines during the 1970s, and increases again during the 1990s.

[1] This intuition is often attributed to Hicks (1932).

[2] Projections for the years 1950 to 2050 are taken from United Nations (2002). We use the method suggested by Krüger and Ludwig (2006) to forecast the US population until 2400.

We use these results to assess the consequences of three different pension reform proposals. The first scenario has a constant replacement ratio, the second a constant contribution rate after 2010, the third keeps the contribution rate constant and allows for a later retirement age for agents born after 2010. We find that the differential effects of these three scenarios on the long-run growth rate of productivity are not very strong. Though, steady-state productivity growth is highest in the second scenario.

Finally, we follow Hunag et al. (1997) and de Nardi et al. (1999) and compare the welfare effects of these pension reform proposals. We find that welfare rises relative to the benchmark for all generations entering the labor force after 1990 if the contribution rate is kept constant.

The paper is organized as follows. Section 2 presents the model and establishes the stationary equilibrium. Section 3 describes our calibration strategy. Our main findings appear in Section 4. Here, we present and discuss our findings concerning the demographic transition. Section 5 concludes.

2 The model

We analyze the effects of a demographic transition on savings and economic growth in a model with overlapping generations and heterogeneous agents. Workers build up savings for old age. Firms invest in a productivity-enhancing technology and hire workers such that profits are maximized. The government collects taxes and social security contributions and runs a balanced budget. Prices are expressed in units of a final good that can be consumed or invested.

2.1 Demographics and timing

A period, t, corresponds to one year. At each t, a new generation of households is born. Newborns have a real life age of 20 denoted by $s = 1$. All generations retire at age 65 ($s = R = 46$) and live up to a maximum age of 90 ($s = J = 75$). At t, all agents of age s survive until age $s + 1$ with probability $\phi_{t,s}$ where $\phi_{t,0} = 1$ and $\phi_{t,J} = 0$.

Let $N_t(s)$ denote the number of agents of age s at t. The population $N_t(s)$ of the periods $t = 0, 1, ..., 450$ is calibrated as the actual and predicted US population of the years 1950 to 2400.

2.2 Households

Each household comprises one representative worker. Households maximize intertemporal utility at the beginning of age 1 in period t:

$$\max \sum_{s=1}^{J} \beta^{s-1} \left(\Pi_{j=1}^{s} \phi_{t+j-1,j-1} \right) \; u(c_{t+s-1}(s), l_{t+s-1}(s)), \tag{1}$$

where instantaneous utility $u(c, l)$ is a function of consumption c and labor supply l:

$$u(c,l) = \frac{\left(c^{\gamma}(1-l)^{1-\gamma}\right)^{1-\theta}}{1-\theta}, \quad \theta > 0, \quad \gamma \in (0,1); \tag{2}$$

here, $\beta > 0$ denotes the discount factor.

Households are heterogeneous with regard to their age, s, their individual labor efficiency, $e(s,j)$, and their wealth, ω. We stipulate that an agent's efficiency $e(s,j) = \bar{y}_s \epsilon_j$ depends on its age, $s \in \mathcal{S} \equiv \{1, 2, ..., 75\}$ and its efficiency type, $\epsilon_j \in \mathcal{E} \equiv \{\varepsilon_1, \varepsilon_2\}$. We choose the age-efficiency profile, $\{\bar{y}_s\}$, in accordance with the US wage profile. The permanent efficiency types ϵ_1 and ϵ_2 are meant to capture differences in education and ability. We use Γ to denote the unique invariant distribution of $e_j \in \mathcal{E}$.

The net wage income in period t of an s-year old household with efficiency type j is given by $(1 - \tau_w - \tau_b)\, w_t\, e(s,j)\, l_t(s)$, where w_t denotes the wage rate per efficiency unit in period t. The wage income is taxed at rate τ_w. Furthermore, the worker has to pay contributions to the pension system at rate τ_b. A retired worker receives pensions $b(s,j)$ that depend on his efficiency type j. Clearly, $b(s,j) = 0$ for $s < R$.

Households are born without assets at the beginning of age $s = 1$, hence $\omega_t(1) = 0$. Parents do not leave bequests to their children and all accidental bequests are confiscated by the government. The household earns interest r_t on his wealth $\omega \in \mathbb{R}$. Capital income is taxed at rate τ_r. In addition, households receive lump-sum transfers tr_t from the government. As a result, the budget constraint at t of an s-year old household with productivity type j, and wealth ω_t is:

$$b_t(s,j) + (1 - \tau_w - \tau_b) w_t e(s,j) l_t(s) + [1 + (1 - \tau_r) r_t]\, \omega_t(s) + tr_t = c_t(s) + \omega_{t+1}(s+1). \tag{3}$$

2.3 Firms

Firms belong either to the final-good or to the intermediate-good sector. Both sectors are competitive. Innovation activity occurs in the intermediate sector and leads, in a deterministic way, to labor-augmenting technical change. The setup follows Irmen (2005).

2.3.1 Final Goods

The measure of all final-good firms is equal to one. At each t, firms produce output, Y_t, according to the following constant-returns-to-scale production function:

$$Y_t = K_t^{\alpha} X_t^{1-\alpha}. \tag{4}$$

Hence, profits are

$$Y_t - r_t K_t - p_t X_t - \delta K_t, \tag{5}$$

where p_t is the price of the intermediate good X_t at t, and δ is the rate at which capital depreciates. Firms pay capital service payments $r_t K_t$ to the capital owners and $p_t X_t$ to the intermediate-goods producers. The individual firm takes prices $\{r_t, p_t\}$ as given. Profit maximization gives rise to the first-order conditions:

$$\frac{\partial Y_t}{\partial K_t} = r_t + \delta = \alpha k_t^{\alpha-1}, \tag{6}$$

$$\frac{\partial Y_t}{\partial X_t} = p_t = (1-\alpha) k_t^{\alpha}, \tag{7}$$

where $k_t \equiv K_t / X_t$ denotes the capital intensity in the final-good sector in period t.

2.3.2 Intermediate Goods

The set of all intermediate-good firms is represented by the set $\mathbb{R}_+$ of nonnegative real numbers.

Technology All firms have the same technology and face a capacity limit of 1. The output of the intermediate good x_t is given by

$$x_t = \min\left\{1, a_t\, l_t^d\right\}, \tag{8}$$

where a_t and l_t^d denote the firm's labor productivity and its labor input, respectively. Firms hire effective labor, l_t^d, defined as the product of the efficiency factor $e(s, j)$ and the 'number' of working hours $l_t(s)$. Labor is assumed to be divisible between firms.

The individual firm's labor productivity at t depends on both the economy-wide labor productivity A_{t-1} of period $t-1$ and its individual productivity growth rate q_t according to:

$$a_t = A_{t-1}(1 + q_t); \tag{9}$$

In order to achieve a productivity growth at rate q_t, the firm must invest $i(q_t)$ units of the final good in period t. This input requirement function is given by:

$$i(q) = v_0 q^v, \quad \text{with} \quad v > 1. \tag{10}$$

If the firm decides not to innovate it can produce with the technology A_{t-1} that became available in period $t-1$. Following Hellwig and Irmen (2001), the innovation reflected in a_t is proprietary knowledge of the firm only in period t; afterwards it becomes common knowledge. The evolution of A_t over time is described below.

Profit Maximization and Zero-Profits We assume that a firm's innovation investment depreciates completely after one year. Therefore, interest rate costs are $(1 + r_t)i(q_t)$. The profit maximization problem of an intermediate-good firm is static. More precisely, a firm chooses a plan $\{q_t, l_t^d, x_t\}$ to maximize profits in period t:

$$\pi_t = p_t\, x_t - w_t\, l_t^d - (1 + r_t)\, i(q_t), \tag{11}$$

As shown in Hellwig and Irmen (2001), the firm produces at the capacity limit $x_t = 1$ if it innovates. The innovation $q_t > 0$ introduces a positive scale effect so that the firm wants to apply the innovation to as large an output as possible. In the equilibria that we consider, $q_t > 0$ holds in all periods t. Hence $x_t = 1$ such that (8) delivers

$$l_t^d = \frac{1}{A_{t-1}(1+q_t)}. \tag{12}$$

The profit-maximizing q_t must minimize unit cost, i. e.,

$$q_t \in \arg\min_{q \geq 0} \left[\frac{w_t}{A_{t-1}(1+q)} + (1+r_t)\, i(q) \right]. \tag{13}$$

With (10), the first-order condition

$$\frac{w_t}{A_{t-1}(1+q_t^*)^2} = (1+r_t)\, i_q(q_t^*) = (1+r_t)\, v_0\, v\, q_t^{v-1} \tag{14}$$

is also sufficient for an interior solution $q_t > 0$.

2.3.3 Consolidating the Production Sector

The set of all active firms in the intermediate-good sector has measure n_t. All active firms are equal and choose the same innovation rate q_t^* generating the aggregate investment demand $n_t i(q_t^*)$ and labor demand $n_t l_t^d$. In equilibrium, the number of active firms n_t is determined by free entry so that profits are zero:

$$\pi_t\left(q_t^*; p_t, w_t, r_t, A_{t-1}\right) = 0, \tag{15}$$

Since all innovation is publicly available after one period and since all active firms are equal, $A_t = a_t$ holds in equilibrium:

$$A_t = A_{t-1}\,(1+q_t^*), \tag{16}$$

Combining the no-profit condition with the first-order condition for investment we get:

$$(1-\alpha)k_t^{\alpha} = (1+r_t)\left[i(q_t) + (1+q_t)i_q(q_t)\right]. \tag{17}$$

2.4 Government

The government collects income taxes T_t in order to finance its expenditures on government consumption G_t and transfers Tr_t. In addition, it confiscates all accidental bequests Beq_t. The government budget is balanced in every period t:

$$G_t + Tr_t = T_t + Beq_t. \tag{18}$$

In view of the tax rates τ_w and τ_r, the government's tax revenue is:

$$T_t = \tau_w w_t L_t + \tau_r r_t \Omega_t, \tag{19}$$

where Ω_t is aggregate wealth at t.

Government spending is a constant fraction of output:

$$G_t = \bar{g} Y_t$$

2.5 Social security

The social security system is a pay-as-you-go system. The social security authority collects contributions from the workers in order to finance its pension payments to the retired agents. Pensions are a constant fraction of net labor income of the productivity type j:

$$b_t(s,j) = \begin{cases} 0 & s < R \\ \zeta(1-\tau_w-\tau_b) w_t \epsilon_j \bar{l}_t & s \geq R, \end{cases} \tag{20}$$

where $\bar{l}_t$ denotes the average hourly labor supply of the workers in period t.

In equilibrium, the social security budget is balanced and will be defined below. We will distinguish 3 different scenarios during the transition:

1. constant replacement ratio: $\zeta = \frac{b_t}{(1-\tau_w-\tau_b) w_t \bar{l}_t}$,
2. constant contribution rate τ_b,
3. constant contribution rate and later retirement at age 70, i. e. $R = 51$.

2.6 Stationary equilibrium

In the stationary equilibrium, individual behavior is consistent with the aggregate behavior of the economy, firms maximize profits, households maximize intertemporal utility, and factor and goods' markets are in equilibrium. To express the equilibrium in terms of stationary variables only, we have to divide aggregate quantities by X_t and individual variables and prices by A_t. Therefore, we define the following stationary aggregate variables:

$$k_t \equiv \frac{K_t}{X_t}, \;\; \tilde{Beq}_t \equiv \frac{Beq_t}{X_t}, \quad \tilde{T}_t = \frac{T_t}{X_t}, \;\; \tilde{G}_t = \frac{G_t}{X_t}, \;\; \tilde{\Omega}_t = \frac{\Omega_t}{X_t}, \;\; \tilde{C}_t = \frac{C_t}{X_t}, \;\; \tilde{Y}_t = \frac{Y_t}{X_t},$$

and stationary individual variables:

$$\tilde{c}_t \equiv \frac{c_t}{A_t}, \;\; \tilde{w}_t \equiv \frac{w_t}{A_t}, \;\; \tilde{b}_t \equiv \frac{b_t}{A_t}, \;\; \tilde{\omega}_t \equiv \frac{\omega_t}{A_t}, \;\; \tilde{tr}_t \equiv \frac{tr_t}{A_t}.$$

Let $\tilde{f}_t(\tilde{\omega}, s, j)$ denote the distribution of individual wealth $\tilde{\omega}$, age s, and the efficiency type j in the period t.

A *stationary equilibrium* for a government policy $\{\tau_r, \tau_w, \tau_b, \bar{g}, \zeta, tr\}$ and initial measures $\tilde{f}_0(\tilde{\omega}, s, j)$ in period 0 corresponds to a price system, an allocation, and a sequence of aggregate productivity indicators $\{A_t\}$ that satisfy the following conditions:

1. Population grows at the rate $\lambda_t = \frac{N_{t+1}}{N_t} - 1$.
2. The number of intermediate-good firms, n_t, is equal to the total output of intermediates as each firms produces one unit of output, $x_t = 1$:

   $$n_t = X_t.$$

3. Labor market equilibrium: Aggregate labor supply, L_t, is equal to the aggregate labor demand of all intermediate goods firms:

   $$L_t = n_t l_t^d. \tag{21}$$

4. Capital market equilibrium: aggregate wealth is equal to aggregate capital plus aggregate innovation investment:

   $$\Omega_t = K_t + n_t i_t(q_t)$$

5. Households maximize the intertemporal utility (1) subject to the budget constraint:

$$\tilde{b}_t(s,j)+(1-\tau_w-\tau_b)\tilde{w}_t e(s,j) l_t(\tilde{\omega},s,j)+\left[1+(1-\tau_r)r_t\right]\tilde{\omega}_t+\tilde{tr}_t = \tilde{c}_t(s)+\tilde{\omega}_{t+1}(1+q_{t+1}),$$

This gives rise to the first-order conditions:

$$\begin{aligned}
\frac{1-\gamma}{\gamma}\frac{\tilde{c}_t(s)}{1-l_t(s)} &= (1-\tau_w-\tau_b)\tilde{w}_t e(s,j), && (22)\\
\gamma \tilde{c}_t(s)^{\gamma(1-\theta)-1}(1-l_t(s))^{(1-\gamma)(1-\theta)} &= \beta(1+q_{t+1})^{\gamma(1-\theta)-1}\phi_{t,s}\left[1+(1-\tau_r)r_{t+1}\right] && (23)\\
&\cdot\quad \gamma\tilde{c}_{t+1}(s+1)^{\gamma(1-\theta)-1}(1-l_{t+1}(s+1))^{(1-\gamma)(1-\theta)}.
\end{aligned}$$

Individual labor supply $l_t(\tilde{\omega},s,j)$, consumption $c_t(\tilde{\omega},s,j)$ and optimal next period assets $\tilde{\omega}'_t(\tilde{\omega},s,j)$ of period t are functions of the individual state variables $\tilde{\omega}$, j, and s, and also depend on the period t.

6. Firms maximize profits satisfying (6), (7) and (14). In equilibrium, firm profits are zero. Using (16), the zero profit (15) condition can be rewritten as follows:

$$\tilde{w}_t = (1-\alpha)k_t^{\alpha} - (1+r)i(q_t). \qquad (24)$$

7. Aggregate variables are equal to the sum of the individual variables:

$$\begin{aligned}
L_t &= \sum_{s=1}^{R-1}\sum_j \int_{\tilde{\omega}} e(s,j) l_t(\tilde{\omega},s,j)\ f_t(\tilde{\omega},s,j)\ d\tilde{\omega},\\
\tilde{\Omega}_t &= k_t + i(q_t) = \frac{1}{L_t}\sum_{s=1}^{T}\sum_j \int_{\tilde{\omega}} \tilde{\omega} f_t(\tilde{\omega},s,j)\ d\tilde{\omega},\\
\tilde{B}eq_{t+1} &= \frac{A_t}{X_{t+1}}\sum_{s=1}^{T}\sum_j \int_{\tilde{\omega}} (1-\phi_{t+1,s+1})(1+r_{t+1}(1-\tau_r))\tilde{\omega}'_t(\tilde{\omega},s,j)\ f_t(\tilde{\omega},s,j)\ d\tilde{\omega}\\
&= \frac{1}{L_{t+1}}\frac{1}{1+q_{t+1}}\sum_{s=1}^{T}\sum_j \int_{\tilde{\omega}} (1-\phi_{t+1,s+1})(1+r_{t+1}(1-\tau_r))\tilde{\omega}'_t(\tilde{\omega},s,j)\ f_t(\tilde{\omega},s,j)\ d\tilde{\omega}\\
\tilde{C}_t &= \frac{1}{L_t}\sum_{s=1}^{T}\sum_j \int_{\tilde{\omega}} c_t(\tilde{\omega},s,j)\ f_t(\tilde{\omega},s,j)\ d\tilde{\omega},\\
\tilde{T}_t &= \tau_w\tilde{w}_t + \tau_r r_t(k_t+i(q_t)).
\end{aligned}$$

8. The government budget is balanced:

$$\bar{g}k_t^{\alpha} + \tilde{t}r_t \frac{N_t}{L_t} = \tilde{T}_t + \tilde{B}eq_t.$$

9. The budget of the social security system is balanced:

$$\frac{1}{L_t}\sum_{s=R}^{T}\sum_{j}\int_{\omega} \tilde{b}_t(s,j)\, f_t(\tilde{\omega},s,j)\; d\tilde{\omega} = \tau_b \tilde{w}_t,$$

10. The goods market clears.

11. The cross-sectional measure f_t evolves as

$$f_{t+1}(\tilde{\mathcal{W}} \times \mathcal{S} \times \mathcal{E}) = \int \sum_{s,j} P_t\left((\tilde{\omega},s,j), \tilde{\mathcal{W}} \times \mathcal{S} \times \mathcal{E}\right)\; f_t(\tilde{\omega},s,j) d\tilde{\omega}$$

with

$$P_t\left((\tilde{\omega},s,j), \tilde{\mathcal{W}} \times \mathcal{S} \times \mathcal{E}\right) = \begin{cases} \phi_t(s) & \text{if } \tilde{\omega}_t'(\tilde{\omega},s,j) \in \tilde{\mathcal{W}}, \\ & j \in \mathcal{E}, s+1 \in \mathcal{S} \\ 0 & else, \end{cases}$$

and for the newborns

$$f_{t+1}(\tilde{\mathcal{W}} \times 1 \times \mathcal{E}) = N_{t+1}(1) \cdot \begin{cases} \Gamma(\mathcal{E}) & \text{if } 0 \in \tilde{\mathcal{W}} \\ 0 & else. \end{cases}$$

2.7 Building intuition: The two main forces

This section provides an intuitive account of the two main effects that drive our calibration results on the link between the endogenous growth rate of per-capita output and the exogenous population growth rate. First, a lower population growth rate reduces the labor force share and, therefore, the savings rate. This diminishes the economy's growth rate. The second effect has the opposite sign. A lower population growth rate strengthens the incentive to engage in labor-saving technical change since labor becomes scarcer and more expensive. As a consequence, the growth rate of labor productivity rises and, thus, per-capita income grows faster.[3]

To see these effects most clearly we first consider the link between k_t and q_t. It is provided by the four equilibrium conditions of the production sector, i. e., the first-order conditions for K_t, L_t, and q_t and the zero-profit conditions of intermediate good firms. To simplify assume that $\delta = 1$. From equations (6) and (17) we find that

$$\frac{1-\alpha}{\alpha} k_t = i(q_t) + (1+q_t) i_q(q_t). \tag{25}$$

Whenever $q_t > 0$, this equation defines a function $q_t^* = q(k_t)$ with $q' > 0$. Hence, in a steady state we have

$$q^* = q(k^*) \quad \text{with} \quad q' > 0. \tag{26}$$

Next, we turn to the equation of motion for k_t, which gives rise to the steady-state value k^*. To simplify, abstract from the complicated OLG-structure and assume a constant savings rate s. Then, in efficiency units of period t the evolution of k_t is given by

$$k_{t+1} + i(q_{t+1}(k_{t+1})) = \frac{s}{(1+q_{t+1}(k_{t+1}))(1+\lambda)}\, k_t^{\alpha}. \tag{27}$$

This is essentially an extension of Solow's famous equation in discrete time. Compared to the neoclassical growth model with exogenous technical change two new features arise. First, the growth rate of labor productivity, q_{t+1}, is not costless but requires resources in efficiency units equal to $i(q_{t+1})$. Second, q_{t+1} is not exogenous but depends on k_{t+1} through the link (25) provided by the equilibrium conditions of the production sector.

Consider a unique steady state which satisfies (27), i. e.,

$$(1+q(k^*))\,(k^* + i(q(k^*))) = \frac{s}{1+\lambda}\,(k^*)^{\alpha}. \tag{28}$$

An application of the implicit function theorem reveals that this equation defines a function $k^* = k(s, \lambda)$ with

[3] For a detailed discussion of the latter point and the derivations below, see Irmen (2005).

$$\frac{\partial k^*}{\partial s} > 0, \quad \frac{\partial k^*}{\partial \lambda} < 0. \tag{29}$$

Hence, ceteris paribus, the capital intensity per unit of efficient labor increases when the savings rate rises and when the population growth rate declines. In view of (26) these results carry over to q^*.

The point of our analysis is that the comparative static results of (29) are misleading in a context with heterogenous agents of different ages. In such a setting a lower population growth rate reduces the labor force share and, therefore, lowers the savings rate. We can account for this fact and replace s in (28) by $s(\lambda)$ with $s' > 0$. It follows that $k^* = k(s(\lambda), \lambda)$ such that a decline in λ generates two opposing effects on k^*, i. e.,

$$\frac{dk^*}{d\lambda} = \frac{\partial k^*}{\partial s}\frac{ds}{d\lambda} + \frac{\partial k^*}{\partial \lambda}. \tag{30}$$

Our calibrations for the US economy suggests that $dk^*/d\lambda < 0$ such that a decline in the population growth rate raises the steady-state capital intensity k^* (compare the two steady states in the years 1950 and 2400 with the population growth rates 1.1% and 0.0%, respectively, in Figure 4).

In view of the link between per-capita income growth and population growth we have with (28) that $q^* = q\left(k(s(\lambda), \lambda)\right)$ and

$$\frac{dq^*}{d\lambda} = \frac{dq}{dk}\frac{dk^*}{d\lambda} < 0. \tag{31}$$

Hence, a decline in the population growth rate on per-capita income growth is positive since k^* rises. Our calibrations deliver this result, too (see, Figure 5). Figures 4 and 5 also suggest that the evolution of k and q are not monotonous along the transition to the new steady state. To develop an intuition for these features we proceed with calibrations of the economy.

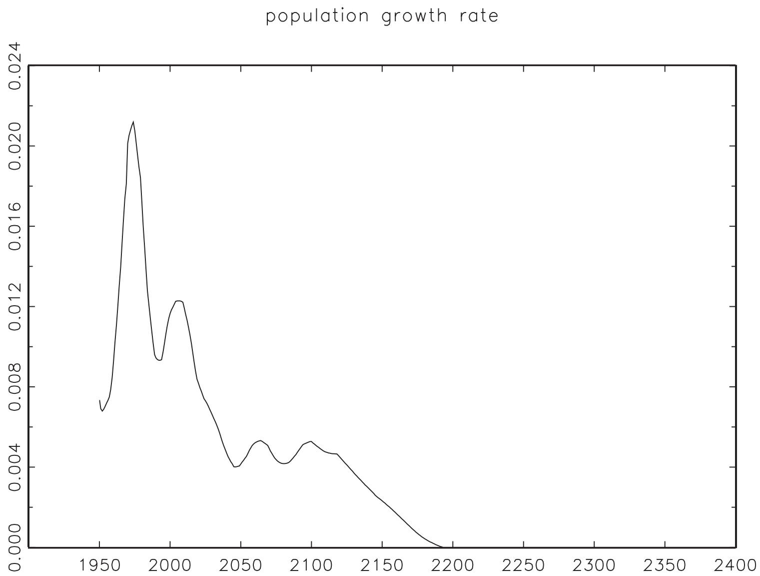
3 Calibration

In this section, we describe our calibration of the model parameters.

3.1 Demographics

Our projection of the US population demographics for 1950-2050 is taken from United Nations (2002). We use the method of Krueger and Ludwig (2006) to forecast the US population development until 2400. The projections are based on the assumptions that life time expectancy increases at constant rates until the year 2100 and that the number of newborns is constant after 2200. Figure 1 illustrates the time profile of the population growth rate 1950-2400.

Figure 1: Demographics



3.2 Endowments and preferences

We choose the coefficient of risk aversion $\theta = 2.0$ and the discount factor $\beta = 0.99$. For our choice of β, the capital-output coefficient amounts to 2.2 in the benchmark case. The parameter γ of the utility function is calibrated so that the average labor supply of the workers is approximately 0.30. For the model in Section 2, $\gamma = 0.32$.

The s-year old household of type j has the productivity $e(s,j) = \bar{y}_s \epsilon_j$. The age-efficiency profile $\{\bar{y}_s\}_{s=1}^{45}$ is taken from Hansen (1993), interpolated to in-between years and normalized to one. The set of the equally distributed productivity types $\{\epsilon_1, \epsilon_2\} = \{0.57, 1.43\}$ is taken from Storesletten et al. (2004). With this calibration, we are able to replicate the empirical distribution of US wages. In our model, the Gini coefficient of the wage distribution is equal to 0.301 which compares favorably with empirical values reported by Díaz-Giménez et al. (1997).

3.3 Production

Following Prescott (1986), the capital income share α and the depreciation rate δ are set equal to 0.36 and 0.08, respectively. A central parameter of our model is the elasticity of the growth rate of productivity q with respect to innovation investment i, $1/v$. With an increasing value of $1/v$, the endogenous growth rate q becomes more sensitive to the exogenous variables of our model, in particular the population growth rate or government policies parameters. We estimate a value of $v = 1.07$ when we regress the log of the GDP per capita growth rate on the log of the R&D investment expenditure divided by GDP. We use annual time series data for the US economy from 1973-2000. Data on business enterprise R&D expenditure in current national prices are from OECD (1997, 2002a), and the price deflator and GDP data are from OECD (2002b).[4] The parameter v_0 is set equal to 0.983 implying an average productivity growth equal to 1.80% over the years 1990-2000 during the demographic transition.

3.4 Government

The government share $\bar{g} = G/Y$ is set equal to the average ratio of government consumption in GDP, $\bar{g} = 0.195$ in the US during 1959-93 according to the Economic Report of the President (1994). The tax rates $\tau_w = 24.8\%$ and $\tau_r = 42.9\%$ are computed as the average values of the effective US tax rates over the time period 1965-88 that are reported by Mendoza et al. (1994). The pension net replacement rate $\zeta = 0.50$ is taken from İmohoroğlu et al. (1995). Government transfers tr and the social security contribution rate τ_b are computed with the help of the equilibrium condition on the government budget (18) and the social security budget (20).

[4] In our model, total investment expenditure, $n_t i_t$, divided by total output, Y_t, is equal to i_t/k_t^{α}. Therefore, in our regression, we make the identification assumptions that 1) capital intensity k is approximately constant in the US during 1973-90, and 2) the productivity growth rate is approximately equal to the GDP per capita growth rate. In our regression, $R^2 = 0.42$ and the standard deviation of the parameter $1/v$ amounts to 0.21. We would like to thank Klaus Wälde and Ulrich Woitek for providing us with the data.

3.5 Computation of the transition dynamics

In order to compute the transition dynamics presented in the next section, we assume that the US economy attains a new steady state with zero population growth in the year 2400.[5] In addition, we assume an arbitrary steady state with 1.1% population growth in the year 1950. Given that we simulate the economy starting in 1950, the initialization is found to have a small effect on the transition after 2000.

In a first step, we make an initial guess of the time path of $\{k_t, \bar{l}_t, \tau_{b,t}, tr_t, q_t\}_{t=1950}^{t=2400}$. We iterate backwards in time and compute the household decisions of the newborn generation in each period $t = 2400, 2399, \ldots, 1875$.[6] In each period, we aggregate savings of all generations and compute new values for $\{\underline{k}_t^i, \underline{\bar{l}}_t^i, \underline{\tau}_{b,t}^i, \underline{tr}_t^i, \underline{q}_t^i\}$. We update the time path using the Gauss-Seidel-Quasi-Newton algorithm presented by Ludwig (2006). In particular, we use the derivatives of the final steady state equilibrium conditions with respect to k, $\bar{l}$, τ_b, tr, and q in order to initialize the Jacobi matrix in the Broyden algorithm. Moreover, we solve the model blockwise by first solving the transition for $\{k_t^1\}_{t=1950}^{2400}$ given the initial guess $\{\bar{l}_t^0, \tau_{b,t}^0, tr_t^0, q_t^0\}$. In the next step, we solve the transition for $\{\bar{l}_t^1\}$ given the remaining variable values $\{k_t^1, \tau_{b,t}^0, tr_t^0, q_t^0\}$ and so forth. When we were trying to solve the model for all endogenous aggregate variables simultaneously, the program did not converge. Therefore, computational time is considerable and amounts to approximately 3 hours on a Pentium III, 860 MHz, for an accuracy chosen to be equal to 10^{-5} for both the individual decision rules and the time path of the endogenous aggregate variables.

4 Demographic transition

We consider the demographic transition in the US starting in the year 1950. We assume that the economy reaches its new long-run equilibrium in 2400. First, we analyze the behavior of savings, the labor force, and the factor prices in the model where exogenous growth at a constant rate is costless. Then, the production side coincides with the neoclassical growth model of Solow (1956). We undertake the analysis for a constant pension replacement ratio ζ and refer to this model as the benchmark case. Second, we show how the demographic transition affects the growth rate of productivity. Third, we consider the pension system and its effects on savings and growth. We close with a welfare analysis of some popular pension reform proposals.

[5] In fact, the deviation of the steady-state capital intensity in 2400 from the computed capital intensity at the end of the transition in period 2399 is less than 0.01%.

[6] In order to compute the individual household problem, we have to solve a non-linear equation system in two variables which are the capital stocks of the 21-year old households with productivities ϵ_1 and ϵ_2, respectively. The methods are described in more detail in Heer and Maussner (2005).

4.1 Transition in the model with exogenous growth

The transition from 1950 until 2400 is graphed in Figures 2 and 3 for the benchmark case. Here, growth is exogenous and the increase in productivity does not cause any social or private costs. Furthermore, the replacement ratio of pensions remains constant at $\zeta = 50\%$, while the social security contribution rate τ_b adjusts in order to balance the budget of the social security system. As the population ages,[7] the share of the working age population falls from 88% in 1950 to 67% in 2400. Therefore, the contribution rate to the social security system τ_b has to increase from 5% to 14%.

The dynamics of the capital intensity k closely mimics the one of the labor force share; when the labor force declines, capital per worker increases. However, swings in the capital intensity k are more pronounced than those in the labor force, for example in the first decade of 2000. During this periods, the capital intensity in the final-good production, k, also increases because, in an economy with a larger labor force, savings for retirement are also relatively high. The net savings rate of the economy is graphed in the lower left diagram of Figure 3. For example, both the net savings rate and the labor force display a temporary rise in the years 2000 and 2050.

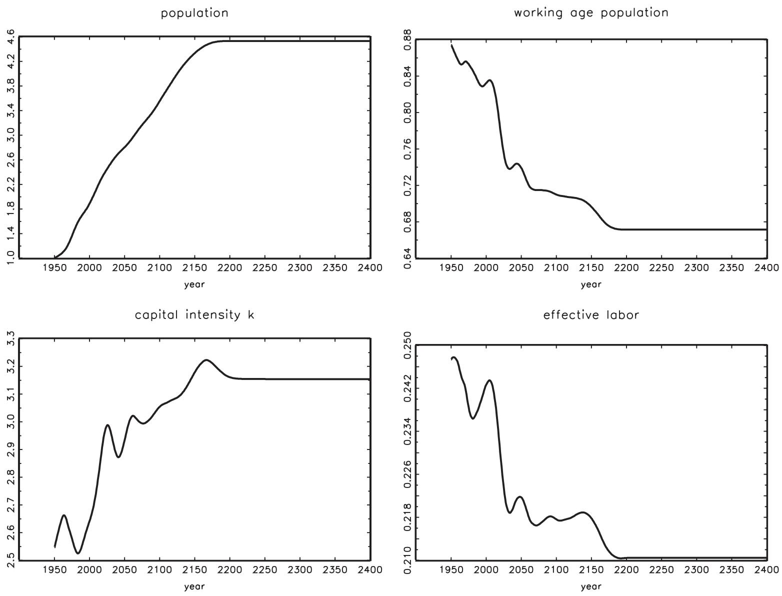
In the new steady state with constant population, both the capital stock per capita $\tilde{K}/N$ and the output per capita $\tilde{Y}/N$ (both measured in efficiency units and not illustrated) are much smaller than the corresponding values in 1950 where we started the computation assuming a steady state with population growth equal to 1.1%. The per capita capital stock and output decline by 5.1% and 8.2%, respectively. Even though a decline in the fertility rate results in a substantial capital deepening and higher output per worker,[8] the effect of the decline in the work force share in the total population dominates. Notice that all endogenous variables of the model like the capital stock per capita or the labor supply in efficiency units per capita stabilize around the year 2200.

Factor prices are the mirror images of the capital intensity k. While the wage rate w increases with a higher k, the real interest rate r falls. As the share of the working-age population falls, tax revenues also decline. In order to balance the budget, government transfers are cut, especially during the first decades of 21st century. In the new steady state in 2400, government transfers are reduced by approximately 1% of total consumption compared to 1950.

[7] The size of the population in the year 1950 is normalized to one.

[8] Furthermore, we have a positive composition effect in the labor force as the share of high-productive old-age workers in the total labor force increases.

Figure 2: Transition in the model with exogenous growth, constant pension replacement ratio ζ (scenario 1), I

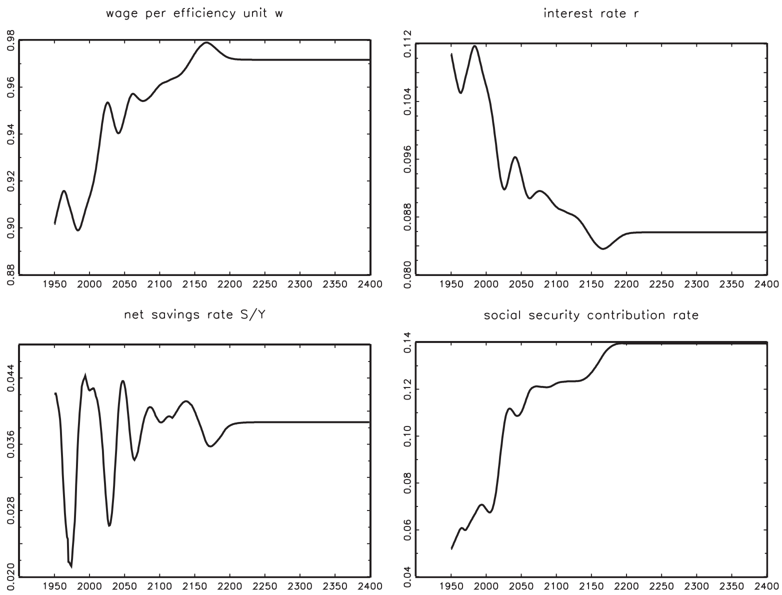


4.2 Endogenous growth

The transition of the endogenous variables in the endogenous-growth model is very akin to the one in the exogenous growth model. In particular, the capital intensity increases from the present over the next 100-200 years even though it is subject to swings. The labor force declines. Figure 4 graphs the dynamics of the capital intensity for both the benchmark case with exogenous growth and the endogenous growth case. Notice that the long-run capital intensity is smaller in the endogenous growth model. In this model, the growth rate will be higher, as we will see below, and, similar as in the Solow model with exogenous technological growth, the capital intensity is smaller for higher growth rates.

As a consequence of the declining labor force and higher capital intensity, the productivity growth rate q increases over the years 2000-2200. As depicted by the solid line in Figure 5 the growth rate rises from an average rate of 1.8% during 1991-2000 to 2.3% in 2100. The long-run average in the constant-population economy amounts to 2.4% after 2200. As one of our main results, the demographic transition is associated with an economically significant increase of the growth rate by 0.6 percentage points. Notice further that our model is able to roughly replicate the most recent pattern of the growth rate in the US. In particular, the growth rate increases during the 60s, declines during the 70s, and increases again during the 90s.

Figure 3: Transition in the model with exogenous growth, constant pension replacement ratio ζ (scenario 1), II

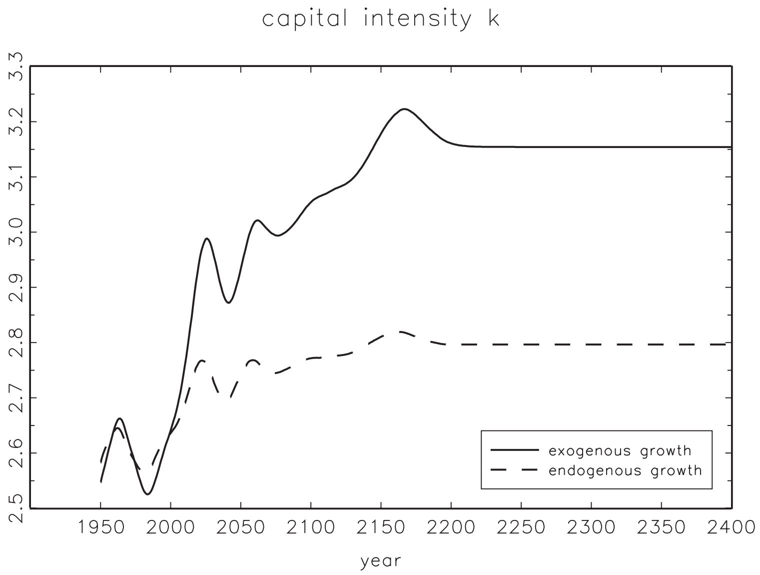


We would like to mention the following caveats of our simple analysis:

1. In our model, the labor force participation rate is constant. Over the last three decades, we have been observing an increase of the labor force participation rate in the US from 58.9% in 1966 to 67.1% in 1998. Since 1998, it has declined to 66.0% in 2005 basically due to cyclical effects (see McEwen et al., 2005). However, it seems plausible that the labor force participation also depends on the demographic transition and the design of the public pension system. We, however, assume that labor force participation is exogenous.

2. Due to the globalization and market liberalization in many emerging countries, notably China and India, the global labor force has increased over recent years. However, the population ageing is already a predominant phenomenon in many countries including China, Japan, or Europe. In an open economy, these developments are likely to affect wages and interest rates in a way that possibly mitigate the effects emphasized in our paper. In a related study with exogenous growth, Krueger and Ludwig (2006) analyze the demographic transition in a multi-country framework. They, however, find that the decline in the labor force in Europe and Japan even magnifies the effects the dynamics of capital and labor compared to those in the closed US economy. Hence, in our model of the US economy, wages would even be higher and interest rates lower if we considered an open rather than a closed economy.

Figure 4: Transition of k in the model with exogenous and endogenous growth, scenario (1)



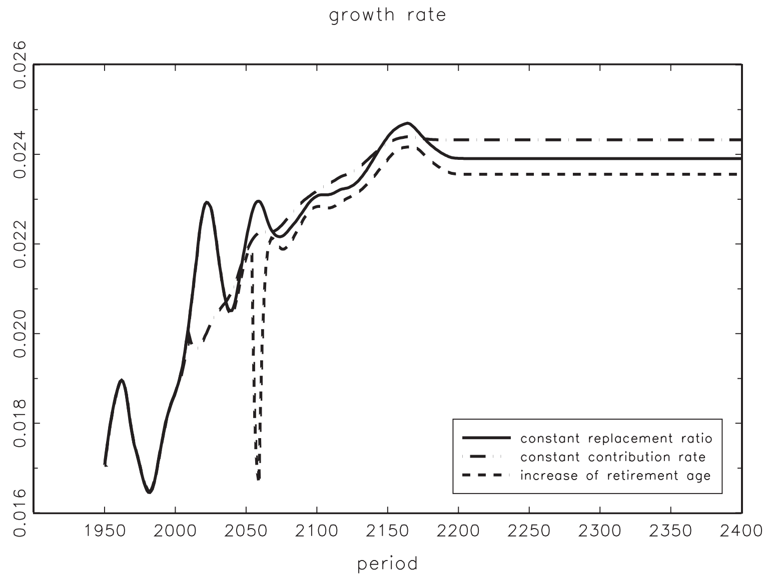
4.3 Pension reform proposals

The pension system affects savings and hence the growth rate. In the famous 'AK'-model, an increase in the capital stock also results in a higher growth rate. Therefore, if we switch from a pay-as-you-go system to a more defined contribution system, the capital stock and ultimately the growth rate increases. In the previous section, we found out that the demographic transition and the decline in the population growth rate increases productivity growth. In the following, we will consider how the design of the pension system affects our previous results. We consider the three scenarios already introduced in Section 2:

1. constant replacement ratio $\zeta = \frac{b_t}{(1-\tau_w-\tau_b)w_t\bar{l}_t}$ (solid line),
2. constant contribution rate τ_b after 2010 (dotted line),
3. constant contribution rate and later retirement at age 70, i. e. $R = 51$, for those agents born in 2010 and afterwards (broken line).

Figure 6 illustrates the dynamics of the US economy for different pension reform scenarios 1-3 in the case of endogenous growth. In the scenarios 1 and 3 (2), the contribution rate

Figure 5: Transition in the model with endogenous growth, constant contribution



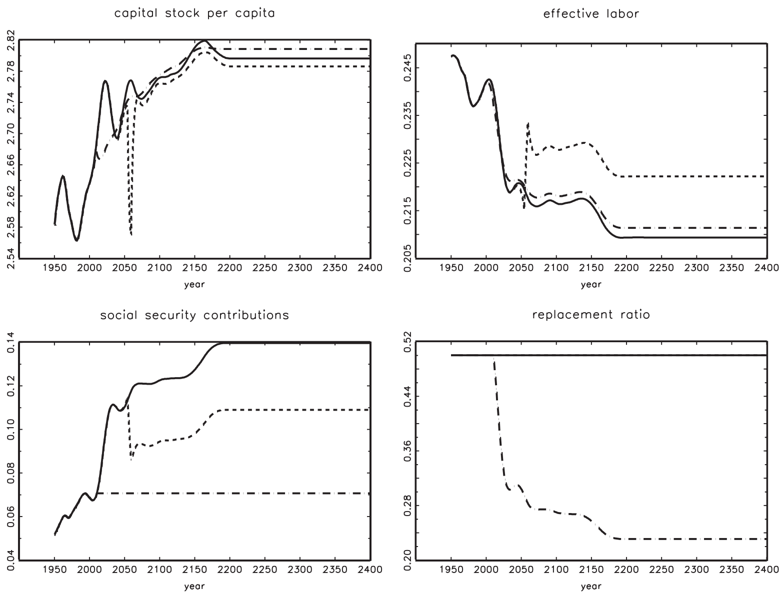
τ_b (the replacement ratio of the pensions ζ) adjusts in order to balance the social security budget. Consequently, pensions fall in the scenario 2 as ζ is reduced from 50% to 23%. In the scenario 3, agents retire at age 71 if they are born after 2010. Therefore, the burden on the pension system declines significantly after 2050 when the dependency ratio falls. The social security contribution rates in the new steady states amount to 14%, 11%, and 7% in the scenarios 1, 2, and 3, respectively.

The lower contribution rates in the scenarios 2 and 3 also increase the incentives to supply labor as the net wage rate increases.[9] Therefore, aggregate labor increases. In scenario 2, households also accumulate higher savings for old age, while in scenario 3 households accumulate savings over a longer working life. As a consequence, savings are also much higher in the scenarios 2 and 3. As both savings and the labor force increase, the net effect on capital intensity is ambiguous. As can be seen from the upper left picture in Figure 6, the capital intensity is higher (lower) in the scenario 2 (3) than in scenario 1.

The productivity growth rate q behaves exactly the same way during the transition as the capital intensity k. Therefore, the long-run change in productivity growth q is largest in scenario 2 that keeps the contribution rate τ_b constant after 2010. As our second main result, we find that while the effect of the demographics on productivity growth is large, the pension system only has a small quantitative effect on the growth rate in the new steady states. For the three scenarios, the productivity growth rates amount to 2.39%,

[9] In our model, the substitution effect dominates the income effect.

Figure 6: Transition dynamics for different public pension reform scenarios, endogenous growth



2.43%, 2.36%, respectively. Only during the transition do we observe big differences in the productivity growth rates, e.g. in scenario 3 when the public pension system switches to a higher retirement age and the factor labor is becoming abundant.

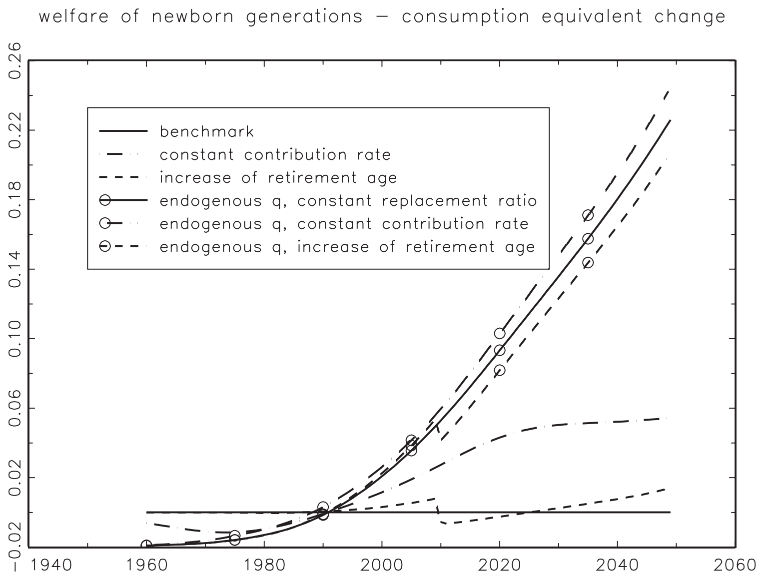
4.4 Welfare effects

Huang et al. (1997) and de Nardi et al. (1999) compare the equilibrium and welfare effects of different public pension reform proposals. Different from our analysis, the productivity growth rate is assumed to be constant and exogenous in these studies. In this section, we study if the negligence of growth rate effects of the social security system is important. As we found out in the previous Section, the magnitude of the growth effect is small in the steady state. However, if we compare the complete transition and the long-run, small deviations may accumulate to sizeable economic effects. In order to make our results comparable to these former studies, we also compute the transition dynamics and welfare effects for the case of exogenous growth.

Figure 7 displays the welfare of the newborn generations that results from our computations for the cases of exogenous growth and endogenous growth for the three different scenarios, respectively. The upper curve represent the expected lifetime utility of the newborn generations in the case of endogenous growth, while the lower three curves consider the case of exogenous growth. In order to provide an interpretable measure of welfare effects, we express the deviations in utility from the benchmark (exogenous growth, scenario 1) in terms of consumption equivalent changes.[10]

Obviously, the welfare effects are large and public pension reform matters. For example, if we consider the generation that is born in the year 2050, the compensation in consumption that is necessary in order to make the average newborn indifferent between the reference economy (constant growth, constant replacement ratio, as represented by the lower solid line) and the economy with constant replacement ratio τ_b and exogenous growth (the lower dotted line) amounts to -5.5% of total consumption. If growth is endogenous, this value (in absolute numbers) falls to -3.5% (the difference between the upper solid and the upper dotted curve in the year 2050).

Figure 7: Welfare of newborn generation



Our results with regard to the welfare effects of the public pension reform can be summarized as follows:

1. Welfare relative to the benchmark increases for all generations born (or rather entering the labor force) after 1990 if the contributions rate τ_b is kept constant irrespective of the assumption of exogenous or endogenous growth.

[10]The consumption equivalent change is equal to the percentage change of the life-time consumption for the average newborn that is necessary in order to make her/him indifferent to the benchmark. It is computed from the average expected life-time utility. Similarly, de Nardi et al. (1999) use wealth equivalent changes.

2. In the case of endogenous growth, all generations born after 2010 suffer from an increase in the retirement age, while this is not true for the exogenous growth economy.

3. Quantitative effects of a pension reform are smaller in the case of endogenous growth.

At this point, let us mention a careful reminder. We have not tried to conduct a fully-fledged thorough welfare analysis of public pension reforms. In our model, we neglect, among others, public debt and a pension system where benefits are tied to previous contributions. Our point, however, is more subtle. In fact, we would like to emphasize our finding that welfare results may depend on the assumption of endogenous or exogenous growth and that, therefore, welfare studies of public pension reforms that neglect endogenous growth should be considered very carefully.

5 Conclusion

to be written

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