Towards a Foundation of Comprehensive Income Taxation

Martin Barbie*, Claudia Hermeling†, and Ashok Kaul‡

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Abstract

Comprehensive income taxation is the leading principle underlying most real world income tax systems. The macroeconomics and public finance literature offers no theoretical foundation of this principle. We develop a fairly standard dynamic Ramsey taxation framework with physical and human capital that allows us to assess under which circumstances it is optimal to aggregate labor and capital income to one tax base and then to apply a single tax schedule to this "comprehensive income". We do not restrict attention to steady states. No-Arbitrage conditions point towards comprehensive income taxation. However, changes in the value of human capital over time and the treatment of depreciation of human capital interfere with this approach. We also show that the consumption value of effective labor investments leads to deviations from production efficiency. Finally, the consequences for optimal taxation are considered when human capital has a direct consumption value.

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*Barbie: Department of Economics, University of Karlsruhe, Kollegium IV am Schloss, D-76128 Karlsruhe, Germany; email: barbie@wior.uni-karlsruhe.de.

†Centre for European Economic Research (ZEW), L7,1, 68161 Mannheim, Germany; email: hermeling@zew.de

‡Kaul: Gutenberg University Mainz, Department of Economics - FB 03, D-55099 Mainz, Germany; email: ashok.kaul@uni-mainz.de.

1 Introduction

Most existing income tax systems were originally designed to reflect the structure recommended by Schanz (1896), Haig (1921) and Simons (1938). They propose to define an income tax base that includes all forms of income. All expenses incurred in earning income are subtracted from the base. The tax liability is then calculated by applying one tax schedule to this comprehensive measure of income. Therefore, the Schanz-Haig-Simons structure came to be known as a comprehensive income tax. Comprehensive income taxation lacks theoretical foundation. This paper offers a partial foundation within the framework of the dynamic Ramsey taxation literature.[1]

The theoretical literature in this area has largely focused on measuring the many types of efficiency costs that arise from the current tax treatment of income from physical capital, like the distortion of overall savings and investment incentives. These efficiency losses are a major consideration in the design of the tax structure. Therefore, while existing income taxes still maintain elements of a comprehensive income tax, there have been many tax reform proposals that would exempt income from physical capital. Examples include the report of the Meade Committee (1978), Bradford (1986), Hall and Rabushka (1995), and McLure and Zodrow (1996). In fact, in many countries an increasing fraction of revenue over time has been collected through value-added taxes and payroll taxes, both of which exempt capital income from taxation.

Interestingly these proposals would not give human capital investments the same treatment they advocate for physical capital investment. In fact, several Scandinavian countries have for example moved away from equal tax rates on wage and capital income by maintaining a progressive tax on wage income but imposing a flat tax at a relatively low rate on capital income (the so-called "dual income tax", see Nielsen and Sørensen (1997)). Judd (1998, 1999) argues for a symmetric treatment of physical and human capital, a type of "level playing field" argument. He bases this recommendation (see also Judd (2001)) on the Diamond-Mirrlees result against intermediate goods taxation. Judd (1998) suggests continuing to tax workers' incomes but immediately expensing all human capital expenditures. His proposal is to extend the deductibility of foregone earnings costs to all direct education and training outlays.

This paper develops a fairly standard two-sector growth model with human capital [see Jones, Manuelli and Rossi (1993, 1997) for a similar model that also uses a Ramsey taxation framework] and endogenous government expenditures [see Bull (1993), Judd (1999)], where human capital is a market good [see Milesi-

[1] Nerlove, Razin, Sadka and von Weizsäcker (1993) consider the effects of comprehensive income taxation on growth but do not have an optimal taxation framework. See also the corrigendum on that paper by Echevarría and Iza (1997).

Ferretti and Roubini (1998a,b)]. Using a dynamic Ramsey taxation approach we show that a general arbitrage argument based on social costs implies that - without invoking a steady-state assumption - physical and human capital should basically be treated symmetrically.[2] Consequently, the (net) capital returns and wage income should be subject to the same tax rate, as comprehensive income taxation requires.

Two complications arise in the application of this result. First, in practice, deducting human capital depreciation expenses would be very difficult [see Steuerle (1996)]. Second, changes in the market price of human capital (relative to the market price of physical capital or consumption) have to be accounted for when calculating a comprehensive measure of income [see Kaplow (1996) for an informal discussion]. We show that these problems lead to deviations from comprehensive income taxation as an optimal tax system and provide back-of-the-envelope calculations that these deviations may be quantitatively important.

Further, we consider the tax treatment of goods and services that are fully effective as (labor-augmenting) investment goods but have an additional consumption component. The optimal tax code has to balance two competing forces. On the one hand, production efficiency points towards full deductibility of investments from the consumption and income tax base. On the other hand, any pure consumption component ought to be covered by a consumption tax. We show how the optimal tax code solves this tradeoff. First, it fully taxes the consumption value of effective labor investments. Furthermore, it sacrifices production efficiency by only allowing to deduct a fraction of the investment expenditures from the income tax base. This is true although the investment is fully productive.

Another issue that may make it desirable to deviate from comprehensive income taxation is the consumption value of human capital [see Judd (2001)]. We provide a formal analysis that shows how human capital entering the utility function changes the optimal tax code and which economic forces lead to a differential treatment of physical and human capital. We show that there is no a priori case for discriminating against human capital investment and favoring investments in physical capital.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 solves the optimal Ramsey taxation problem when a capital income tax, a wage tax and a consumption tax is available but the changes in the value of human capital are not taken into consideration when designing the tax code. Section 4 shows that the results are robust to restrictions in government's asset holdings (with balanced budget as a special case). Section 5 considers the case where a

[2]There is a vast literature on the effects of taxation on human capital accumulation. Most papers do not employ a Ramsey taxation approach. See Trostel (1993) among others for an early and influential contribution. Judd (1999) and Jones, Manuelli, Rossi (1997) are exceptions.

fraction of the output-good investment in the effective labor production has a consumption value. Section 6 then shows that comprehensive income taxation is optimal if the change in the market value of human capital is accounted for. Section 7 provides a first approach to modeling the consumption value of human capital. Section 8 concludes. All proofs are given in the appendix.

2 The Model

We consider a deterministic discrete-time infinite-horizon economy. A large number of identical households own all factors of production (human capital and physical capital) that they rent to firms at perfectly competitive rates. The economy has two production sectors, one for the production of output (the final good, which is a consumption and a physical-capital good) and one for the production of human capital. A government imposes flat-rate taxes on factor income, intermediate goods and consumption.

Production of the Consumption-Capital Good The production technology is given by a constant returns to scale production function $F(k_t^m, e_t^m,)$. It describes how output is created with the input of a fraction of the physical capital stock $k_t^m = s_t k_t$ with $0 \leq s_t \leq 1$ and effective labor e_t^m. It is assumed that F is a positive function, twice continuously differentiable with strictly decreasing (but everywhere positive) marginal products of all factors. There is a representative firm in this sector which rents capital and effective labor from the household, takes prices (r_t^m, w_t^m) as given and maximizes profits $F(k_t^m, e_t^m) - r_t^m k_t^m - w_t^m e_t^m$ in each period. r_t^m denotes the capital rental rate and w_t^m is the wage rate. Profit maximization implies

$$r_t^m = F_k(k_t^m, e_t^m),$$
$$w_t^m = F_e(k_t^m, e_t^m).$$

Production of Human Capital New human capital $G(.)$ is created as a market activity.[3] Our focus is therefore on schools, universities, colleges and other educational institutions. It is created with a constant returns to scale technology $G(k_t^h, e_t^h)$. Inputs are physical capital $k_t^h = (1 - s_t)k_t$, namely the fraction of the capital stock which is not used in the other production sector, and effective labor e_t^h used in the human capital sector. There is a representative firm which takes prices (r_t^h, w_t^h, q_t^I) as given and maximizes profits $q_t^I G(k_t^h, e_t^h) - r_t^h(1 - s_t)k_t - w_t^h e_t^h$ in

[3]For a comprehensive discussion about the incentives for the formation of human capital see Jones and Manuelli (1999).

each period. The profit is measured in units of the consumption good. r_t^h denotes return on investments of physical capital and w_t^h is the wage rate in the human capital sector. q_t^I is the market price of one unit of newly installed human capital in units of the consumption good in period t. We briefly call q_t^I the price of human capital investment. Profit maximization implies:

$$r_t^h = q_t^I G_k(k_t^h, e_t^h),$$
$$w_t^h = q_t^I G_e(k_t^h, e_t^h).$$

Capital Accumulation The representative agent owns the physical capital stock k_t and the human capital stock h_t. Capital is accumulated taking depreciation into account. Both stocks depreciate at rates $\delta^k \geq 0$ and $\delta^h \geq 0$ respectively:

$$k_{t+1} = (1 - \delta^k)k_t + x_t^k, \tag{1}$$
$$h_{t+1} = (1 - \delta^h)h_t + G(.), \tag{2}$$

where x_t^k is the portion of output invested in the physical capital good.

Production of Effective Labor Units of effective labor for the use in the production of output as well as in the production of human capital are produced with a constant returns to scale technology:

$$e_t^m = M^m(h_t, n_t^m, x_t^m) =: M^m(t) \quad \text{and} \quad e_t^h = M^h(h_t, n_t^h, x_t^h) =: M^h(t).$$

Effective labor is produced with human capital h_t, the consumption goods x_t^m and x_t^h respectively and hours devoted to the production of effective labor n_t^m and n_t^h respectively. The technologies M^m and M^h are assumed to be homogeneous of degree one in the stock of human capital h and the market good x. M^m and M^h are positive functions, twice continuously differentiable with strictly decreasing (but everywhere positive) marginal products of all factors. They are modelled as a general version of the effective labor supply functions introduced in Heckman (1976) and Lucas (1988). We capture that the accumulation of human capital involves both direct and indirect costs to individuals. The direct costs x_t^h are cash expenses, such as expenditures for working clothes, small offices and computers at home, and other out-of-pocket costs. The indirect costs are the income that people lose when people spend time for n_t^h rather than working in the other production sector (n_t^m).

The Representative Agent's Problem The representative agent is endowed with one unit of time and has preferences over consumption, a private good provided by the government (briefly called "government expenditures") and leisure.

The utility function is time-separable and given in each period by $u(c_t, g_t, n_t)$ where c_t is consumption, g_t public expenditures, $n_t := n_t^m + n_t^h$ with $0 \leq n_t \leq 1$ is time spent on working. Working time is made up of the number of hours allocated to market activities, n_t^m, and number of hours allocated to human capital formation, n_t^h. u is increasing, strictly concave, and three times continuously differentiable in c, g and n. Future utility is discounted by the factor β, $(0 < \beta < 1)$:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, g_t, n_t). \tag{3}$$

The agent buys units of new human capital $G(k_t^h, e_t^h) = h_{t+1} - (1 - \delta^h)h_t$ from the representative firm in the human capital sector and supplies effective units of labor for each production sector.
The agent's problem is to maximize utility by optimally choosing consumption, c_t, the number of hours allocated to output production, n_t^m, the number of hours allocated to human capital formation, n_t^h, the physical capital stock, k_{t+1}, the fraction, s_t of physical capital employed in the output production (versus the human capital production), the human capital stock, h_{t+1}, bond holdings, b_{t+1}, and the amounts of consumption goods used in production of effective labor x_t^m and x_t^h (given an initial stock of physical and human capital and bonds in the first period, k_0, h_0, b_0). The problem is constrained by the budget constraints, $\forall t$:

$$\begin{aligned}(1+\tau_t^c)c_t + k_{t+1} + b_{t+1} + q_t^I \overbrace{(h_{t+1} - (1-\delta^h)h_t)}^{\text{new human capital}} \\ +(1+\tau_t^{xm})x_t^m + (1+\tau_t^{xh})x_t^h \\ \leq (1-\tau_t^w)w_t^m M^m(h_t, n_t^m, x_t^m) + (1-\tau_t^w)w_t^h M^h(h_t, n_t^h, x_t^h) \\ +((1-\tau_t^{km})(r_t^m - \delta^k) + 1)s_t k_t + ((1-\tau_t^{kh})(r_t^h - \delta^k) + 1)(1-s_t)k_t \\ +R_t b_t.\end{aligned} \tag{4}$$

τ_t^c is the consumption tax, τ_t^{xm} and τ_t^{xh} taxes or subsidies on the portion of output invested in effective labor, τ_t^w the tax on labor income, and τ_t^{km} and τ_t^{kh} flat-rate taxes on capital income (net of deprecation δ^k). The outstanding stock of (one-period) government debt at time t is denoted by b_t. Bond holders receive the gross rate of return, R_t which is w.l.o.g untaxed. The agent takes prices and taxes as given and maximizes utility (3) subject to the budget constraint (4) in each period. Since investments in physical capital and bonds are perfect substitutes for households, to rule out any arbitrage between the return to capital and government debt, returns must be such that:

$$R_t = 1 + (1-\tau_t^{kh})(r_t^h - \delta^k) = 1 + (1-\tau_t^{km})(r_t^m - \delta^k).$$

Government Government expenditures g_t are financed by issuing bonds each period and by collecting taxes. The government's budget constraint for period t is given by:

$$\begin{aligned} g_t = & \tau_t^c c_t + \tau_t^{km}(r_t^m - \delta^k)k_t^m + \tau_t^{kh}(r_t^h - \delta^k)k_t^h + b_{t+1} - b_t R_t \\ & + \tau_t^w w_t^m M^m(t) + \tau_t^w w^h(t) M^h(t) + \tau_t^{xm} x_t^m + \tau_t^{xh} x_t^h. \end{aligned}$$

Feasibility The resource constraints for the consumption good and for human capital are given by the following feasibility constraints:

$$c_t + g_t + k_{t+1} + x_t^m + x_t^h \leq F(s_t k_t, M^m(h_t, n_t^m, x_t^m)) + (1 - \delta^k)k_t, \tag{5}$$

$$h_{t+1} \leq (1 - \delta^h)h_t + G((1 - s_t)k_t, M^h(h_t, n_t^h, x_t^h)). \tag{6}$$

We call an allocation $\{k_t, s_t, h_t, c_t, n_t^h, n_t^m, x_t^m, x_t^h, g_t\}_{t=0}^{\infty}$ *feasible* if it fulfills the feasibility constraints (5) and (6) in all periods.

Competitive Equilibrium A *competitive equilibrium* consists of a feasible allocation $\{k_t, s_t, h_t, c_t, n_t^h, n_t^m, x_t^m, x_t^h, g_t\}_{t=0}^{\infty}$ a strictly positive and bounded price system $\{w_t^m, w_t^h, r_t^m, r_t^h, R_t, q_t^I\}_{t=0}^{\infty}$, and a government policy $\{g_t, \tau_t^{kh}, \tau_t^{km}, \tau_t^w, \tau_t^c, \tau_t^{xh}, \tau_t^{xm}, b_t\}_{t=0}^{\infty}$ such that

(i) Given the price system and the government policy:
The allocation solves the firms' and the household's maximization problems in each period.

(ii) Given the price system and the feasible allocation:
The government policy satisfies the government budget constraint in each period.

(iii) Markets clear.

3 A Partial Foundation of Comprehensive Income Taxation

In this section we set up and solve the optimal taxation problem of the government. Two properties are crucial and will be reconsidered in the next section. First, depreciation of human capital is not taken into account in the tax code. Second, changes in the market value of human capital (measured in units of the consumption good) are untaxed. Real world tax systems are rather close to this description of the tax code. It is therefore interesting to see how this tax code shapes the optimal tax structure.

Ramsey Problem Given k_0, b_0 ,h_0, the *Ramsey problem* for the government is to choose a competitive equilibrium which maximizes the utility of the representative agent.
Hence the government maximizes the utility of the agent taking into account the agent's first order conditions, the agent's budget constraint, the firm's first order conditions and the resource constraints of the economy. The government's budget constraint is automatically satisfied due to Walras' Law.
Since we will adopt the primal approach to Ramsey taxation, we summarize the household side by two equations: first, the Euler equation for human capital:

$$\frac{u_c(t)q_t^I}{(1+\tau_t^c)} = \beta u_c(t+1)\left[\frac{(1-\delta^h)q_{t+1}^I}{(1+\tau_{t+1}^c)} - \frac{u_n(t+1)}{u_c(t+1)}\left[\frac{M_h^h(t+1)}{M_n^h(t+1)} + \frac{M_h^m(t+1)}{M_n^m(t+1)}\right]\right] \quad (7)$$

and second the *implementability constraint*:

$$\sum_{t=0}^{\infty} \beta^t u_c(t)c_t = A_0 \quad (8)$$

where $A_0 := \frac{u_c(0)}{1+\tau_0^c}[k_0+(1-\tau_0^{km})(F_k(0)-\delta^k)s_0k_0+(1-\tau_0^{kh})(q_t^I G_k(0)-\delta^k)(1-s_0)k_0+R_0b_0 + h_0(1-\delta^h)q_0^I] - \frac{u_n(0)}{G_n(0)}h_0[G_n(0)\frac{M_h^m(0)}{M_n^m(0)} + G_h(0)]$ denotes wealth at time zero. The government can now be thought of as directly choosing an allocation and the consumption tax τ_t^c. The government maximizes the utility of the representative household subject to the implementability constraint (8), the feasibility constraints (5)-(6) and the Euler equation for human capital (7). The Lagrangian for this optimization problem is:

$$\begin{aligned} L = \sum_{t=0}^{\infty} &\beta^t[u(c_t, g_t, n_t) + \alpha u_c(t)c_t \\ &- \gamma_t[-u_c(t)\frac{q_t^I}{1+\tau_t^c} + \beta u_c(t+1)\cdot \\ &\quad [(1-\delta^h)\frac{q_{t+1}^I}{1+\tau_{t+1}^c} - \frac{u_n(t+1)}{u_c(t+1)}[\frac{M_h^m(t+1)}{M_n^m(t+1)} + \frac{M_h^h(t+1)}{M_n^h(t+1)}]]] \\ &+ \lambda_t[-c_t - g_t - k_{t+1} - x_t^m - x_t^h + F(k_t s_t, M(x_t^m, h_t, n_t^m)) + (1-\delta^k)k_t] \\ &+ \mu_t[-h_{t+1} + (1-\delta^h)h_t + G((1-s_t)k_t, M^h(x_t^h, h_t, n_t^h))]] - \alpha A_0. \end{aligned}$$

where $\alpha, \beta^t\gamma_t, \beta^t\lambda_t$ and $\beta^t\mu_t$ are Lagrange multipliers associated with the constraints. To avoid the well known problem of arbitrary lump sum taxation of k_0 and b_0 we additionally impose (arbitrary) bounds on τ_0^{km}, τ_0^{kh}, and R_0. In the following we will focus only on the periods $t > 0$.[4]

[4]The period 0 choices of course affect the value of the Ramsey problem but otherwise do not restrict the choices and conditions in all other periods.

Proposition 1 *The following tax code is optimal for all periods $t > 0$:*

1. *Equality of capital taxes across sectors: $\tau_t^{km} = \tau_t^{kh}$.*
2. *Zero Tax on Intermediate Goods: $\tau_t^{xm} = \tau_t^{xh} = -\tau_t^w$.*
3. *The optimal income tax structure is given by:*

$$\frac{\tau_{t+1}^k}{\tau_{t+1}^w} = 1 + \frac{1 - \frac{q_{t+1}^I}{q_t^I}(1-\delta^h)}{F_k(t+1) - \delta^k}.$$

The first result, that capital taxes should be equal across sectors, is an immediate and straightforward consequence of arbitrage considerations.
The second result is derived from production efficiency. It says that investment expenditures on intermediate goods, i.e. goods that are used in the production of efficiency units of labor, should be fully deductible from the *wage income tax base.* Zero taxes on intermediate goods are in contrast to the current tax treatment of such expenses. Under the current US income tax system individuals receive essentially no preferential tax treatment for their direct cash expenses, although the government tends to subsidize some of those costs.
Also, the second result requires that these costs should be exempted from *consumption taxes.* However, at the base of any effort to implement this exemption lies a significant administrative problem in defining what constitutes a legitimate investment in effective labor. The reason for this is that many forms of investment may have a direct consumption value. We consider this in section 5.
The third result directly addresses the question of comprehensive income taxation. It is derived from a comparison of the Euler equations of physical capital and human capital:

$$\begin{aligned} &1 + (1-\tau_{t+1}^k)(F_k(t+1) - \delta^k) \\ =&\frac{1}{q_t^I}[q_{t+1}^I(1-\delta^h) + q_{t+1}^I(1-\tau_{t+1}^w)(G_e(t+1)M_h^h(t+1) + F_e(t+1)M_h^m(t+1))]. \end{aligned}$$

This is a standard arbitrage condition for the household which says that the after-tax return (in $t+1$) of one unit of physical capital (invested in t) has to be equal to the after-tax return of one unit of human capital. If the household invest one unit of the physical capital good (which has a price of 1) in the production sectors the return in period $t+1$ is equal to the value of the capital unit $1-\delta^k$ plus the return from the production sectors $F_k(t+1)$ or $q_{t+1}^I G_k(t+1)$ which is equal to $F_k(t+1)$ in equilibrium. Capital taxes are levied on $F_k(t+1) - \delta^k$. If the household invest one unit of the human capital good (which has the price q_t^I in t) the return in period $t+1$ is equal to the value of the capital unit $q_{t+1}^I(1-\delta^h)$ plus the return from both

production sectors ($F_e M_h^m(t+1)$ and $q_{t+1}^I G_e(t+1) M_h^h(t+1)$). The wage rates are given by $F_e = w^m(t+1)$ and $q_{t+1}^I G_e(t+1) = w^h(t+1)$ and the wage tax is levied on the wage income.
The solution of the Ramsey problem requires that the (before tax) returns on capital for both capital stocks have to be equal to avoid tax-based distortions of the investment decisions:

$$\begin{aligned} & 1 + F_k(t+1) - \delta^k \\ = & \frac{1}{q_t^I}[q_{t+1}^I(1-\delta^h) + q_{t+1}^I G_e(t+1) M_h^h(t+1) + F_e(t+1) M_h^m(t+1)]. \end{aligned}$$

Because the tax system does not take into account the difference in the prices q_t^I and q_{t+1}^I and because the depreciation of human capital cannot be set up against the tax liabilities comprehensive income taxation, i.e. $\tau_t^{km} = \tau_t^{kh} = \tau_t^w$, will not be optimal in general.
Comprehensive income taxation is optimal if and only if $1 = (1-\delta^h)\frac{q_{t+1}^I}{q_t^I}$ holds. Setting $\tau_{t+1}^{km} > \tau_{t+1}^{wh}$ is optimal if and only if $1 > \frac{q_{t+1}^I}{q_t^I}(1-\delta^h)$ and vice versa. Is the deviation from comprehensive income taxation significant? For a back-of-the-envelope calculation consider the case $q_{t+1}^I = q_t^I$ which will e.g. be true on a balanced growth path. Suppose $\delta^h = 0.01$, a value at the lower end of what is typically specified as the depreciation for human capital [see Driffill and Rosen (1983) for such a choice and Gomez (2003) for a recent summary of the literature]. Set the user cost of capital net of depreciation $F_k - \delta^k = 0.07$. Then it follows:

$$\frac{\tau_{t+1}^k}{\tau_{t+1}^w} = 1 + \frac{\delta^h}{F_k(t+1) - \delta^k} = 1 + \frac{1}{7}.$$

Since human capital depreciations are not tax deductible, the capital income tax rate should be higher by more than 14% than the labor tax rate. If the depreciation rate for human capital is increased to $\delta^h = 0.05$, the "correction factor" is 5/7, i.e. more than 71%. This shows that deviations from comprehensive income taxation can be quantitatively significant if the tax system does not tax labor income net of human capital depreciations, as it is done in the case of physical capital.

4 Debt Limits, Balanced Budget, and Steady States

4.1 The Optimal Tax Code with Debt Limits

So far we have considered an extreme case where the government can accumulate large asset holdings. In this section, we introduce incomplete asset markets by restricting bond issues and holdings of the government.[5] The Ramsey problem in the presence of exogenous limits on the debt in each period reads:

$$
\begin{aligned}
L = \sum_{t=0}^{\infty} & \beta^t [u(c_t, g_t, n_t) \\
& - \gamma_t [-u_c(t) \frac{q_t^I}{1+\tau_t^c} + \beta u_c(t+1) \cdot \\
& \quad [(1-\delta^h) \frac{q_{t+1}^I}{1+\tau_{t+1}^c} - \frac{u_n(t+1)}{u_c(t+1)} [\frac{M_h^m(t+1)}{M_n^m(t+1)} + \frac{M_h^h(t+1)}{M_n^h(t+1)}]] \\
& + \lambda_t [-c_t - g_t - k_{t+1} - x_t^m - x_t^h + f(k_t s_t, M^m(x_t^m, h_t, n_t^m)) + (1-\delta^k) k_t] \\
& + \mu_t [-h_{t+1} + (1-\delta^h) h_t + G((1-s_t) k_t, M^h(x_t^h, h_t, n_t^h)] \\
& + \nu_t [b_{t+1} - M_t \cdot c_t]]. \\
& + \sum_{t=1}^{\infty} \beta^t \alpha_t [u_c(t) c_t + \frac{u_c(t)}{1+\tau_t^c} [k_{t+1} + b_{t+1} + q_t^I h_{t+1}] - \frac{1}{\beta} \frac{u_c(t-1)}{1+\tau_{t-1}^c} [k_t + b_t + q_{t-1}^I h_t]] \\
& + \alpha_0 A_0^D .
\end{aligned} \tag{9}
$$

$A_0^D := [u_c(0) c_0 + \frac{u_c(0)}{1+\tau_0^c} [k_1 + b_1 + q_0^h h_1] - \frac{u_c(0)}{1+\tau_0^c} [k_0 + (1-\tau_0^{km})(F_k(0) - \delta^k) s_0 k_0 + (1-\tau_0^{kh})(q_0^I G_k(0)) - \delta^k)(1-s_0) k_0 + R_0 b_0 - h_0 \frac{u_n(0)}{G_n(0)} [G_n(0) \frac{M_h^m(0)}{M_n^m(0)} + G_h(0)]]$ denotes the budget constraint of the representative agent in period 0 given k_0, b_0 and h_0. $\{M_t\}_{t=0}^{\infty}$ is a sequence of exogenously given constants. Multiplying by c_t guarantees that the bound $|M_t c_t|$ increases with the same (endogenous) growth rate as consumption. We have $\nu_t \geq 0$ for an upper bound which describes a debt limit for the government and $\nu_t \leq 0$ for an lower bound, i.e. a bound on asset holding of the government. The second case is important in the long run as a restriction of the optimal tax code since it avoids confiscatory levels of taxes in a transition period that tend to characterize optimal (Ramsey) tax codes. The first case has some relevance if tax reforms are considered in times where Maastricht Treaty limits on debt and deficits are binding.
Because we do not want to exclude the possibility that the debt limits are set equal to zero ($M_t \equiv 0$) we have to replace the implementability constraint by the budget constraint of the representative agent in each period where the prices and taxes are replaced using first order conditions.

Proposition 2 *Suppose there are debt limits or bounds on the government's asset holdings. Then the tax code of Proposition 1 is still optimal for all periods $t > 0$.*

[5] For recent work on debt limits in the Ramsey taxation literature see for instance Aiyagari et al. (2002)

As a corollary, we obtain the properties of the optimal tax policy with balanced budget. The absence of government bonds can be identified with the case of $M_t \equiv 0$. Therefore, the optimal tax code of Proposition 1 also holds with a balanced budget, even outside of a steady state.

4.2 Debt Limits and the Existence of a Steady State

The paper focuses on out-of-steady state results. One reason for this lies in the fact that well-known steady state optimal zero tax results - as the one by Jones, Manuelli and Rossi (1997) for example - crucially rely on the accumulation of large asset holdings in the transition towards a steady state. This section demonstrates that debt limits - as analyzed in the previous section - are inconsistent with the existence of a steady state for a large class of utility functions. We now study the effect of debt limits on the long run behavior of the economy. Some regularity assumptions are needed:

Assumption 1 *Consider a utility function* $u(c_t, g_t, n_t) = g_t^Z \Psi(\frac{c_t}{g_t}, n_t^m + n_t^h)$ *where* Ψ *is a positive, twice continuously differentiable function and* $Z > 0$. *Suppose* $u_g(t) > 0$ *with* $u_g(c_t, 0, n_t) = \infty$. *Suppose further that* $\Psi\left(\frac{c}{g}, n\right) \leq A + B\left(\frac{c}{g}\right)^Z$ *for* $\frac{c}{g} \in [W, \infty)$ *and all* $n \in [0, 1]$, *where* A, B *and* W *are nonnegative constants. Finally, the function* Ψ *is assumed to satisfy:*[6] $\frac{c}{g}\frac{\Psi_{12}(\frac{c}{g},n)}{\Psi_1(\frac{c}{g},n)} = 1 + \frac{c}{g}\frac{\Psi_{11}(\frac{c}{g},n)}{\Psi_1(\frac{c}{g},n)}$.

The last assumption is an assumption about Ψ introduced in Bull (1993). This assumption is automatically satisfied whenever Ψ is homogenous of some degree in $\frac{c}{g}$ and is consistent with utility functions of the form $u(c, n, g) = \frac{((c^\epsilon g^{1-\epsilon})^{1-\theta} \exp[(1-\theta)\cdot\omega(n)] - 1}{1-\theta}$ with $0 < \epsilon < 1$, $0 < \theta < 1$, $\omega\prime(n) < 0$ and $\omega\prime\prime(l) \leq 0$ which are very similar[7] to the ones required for the existence of a balanced growth path in a standard Ramsey model with endogenous growth [see Barro and Sala-i-Martin (2004)]. The assumption is also consistent with utility functions which have constant elasticities: $u(c, n, g) = (\frac{[(c^\rho g^{1-\rho})(1-n)^\gamma]^\epsilon}{\epsilon})$ with $0 < \rho < 1$, $0 < \epsilon < 1$ and $0 < \gamma < 1$ and $u(c, n, g) = \rho \ln c + (1-\rho)\ln g + \gamma \ln(1-n)$ for $\epsilon = 1$ which are frequently used in macroeconomic computations [see also Kydland and Prescott (1982), Chari, Christiano and Kehoe (1991a), Christiano and Eichenbaum (1992) and Devereux and Love (1994)]. Our class of utility functions also contains the ones used by Doménech and García (2002) and Davies, Zeng, and Zhang (2000) in their endogenous growth models with human capital.

[6] The subscript indicates the argument being differentiated, with multiple subscripts indicating multiple differentiations.

[7] The only difference is the existence of government expenditure in the utility function.

The other assumptions are only of a technical nature and ensure that Ψ is well behaved. Note that the asymptotic boundness assumption holds automatically whenever $Z \geq 1$ and Ψ is concave in its first argument.
We now consider the long-run behavior of the economy by focusing on a balanced growth path. Recall that $Y_t = F(k_t s_t, M^m(t))$ denotes the output of the economy in period t and define $\kappa = \frac{Y_{t+1}}{Y_t} - 1$ as the endogenous growth rate of the economy. Along a balanced growth path $\kappa_t = \kappa$ will be constant over time. Furthermore, time allocated to labor, n_t^h and n_t^m, has to be constant over time.

Proposition 3
Let Assumption 1 hold.

(a) *No solution to the Ramsey problem with a balanced budget and a non-negative growth rate will lead to a balanced growth path.*

(b) *If the debt limits go to zero, i.e. if $(M_0, M_1, M_2, ...) \rightarrow (0, 0, 0, ...)$, then the solution of the Ramsey problem with debt limits converges (pointwise) to the solution of the Ramsey problem with balanced budget.*

This proposition shows that the optimal Ramsey solution with balanced budget will not converge to a balanced growth path. If the debt limits are sufficiently tight, it shows by a continuity argument, that the solution of the Ramsey problem with debt limits also converges to a situation without balanced growth path. The reason for this is that a balanced growth path implies zero tax rates. In this case, government expenditures can only be financed by interest income from asset holdings. However, if the admissible wealth to GDP ratio is sufficiently small, then only a small government sector is feasible. In this case it will eventually be better to raise taxes in the long run and finance higher government spending. But this cannot be consistent with a balanced growth path since tax rates are positive. The two results in the proposition are an additional justification to concentrate on out-of-steady state results.
In summary, section 4 has shown that the optimal tax code of the previous section does not change even if we heavily restrict the government's ability to make use of bond markets. Furthermore, sufficiently tight limits on asset holdings and government debt imply an optimal Ramsey allocation that does not converge to a steady state.

5 Consumption Value of Investment Goods

A wide range of goods and services are investment goods and *additionally* serve as a consumption good. As an example consider a laptop computer that is used on a

daily basis by an employee in the office but is also used at home by the employee in the evenings to play computer games. Also, books and software are examples of investment goods that often have an additional direct consumption value, just like regular consumption goods [see Davies, Zeng and Zhang (2000) for an extensive discussion].
We consider this in the setup of section 3 by introducing the investments x_t^m and x_t^h in the utility function with $\xi \in (0,1)$ being the fraction of the investment expenditures that generates direct consumption value:

$$\sum_{t=0}^{\infty} \beta^t u(c_t + \xi(x_t^h + x_t^m), g_t, n_t).$$

The implementability constraint is now given by

$$\sum_{t=0}^{\infty} \beta^t (u_c(t)(c_t + \xi(x_t^h + x_t^m))) = \hat{A}_0,$$

and the Euler equation for human capital does not change. $\hat{A}_0$ denotes period zero wealth.

Proposition 4 *The following tax code is optimal for all $t > 0$:*

1. *Equality of capital taxes across sectors: $\tau_t^{km} = \tau_t^{kh}$.*
2. *Non-zero Tax on Intermediate Goods:*

$$\tau_t^{xm} = \tau_t^{xh} = -\tau_t^w(1-\xi) + \xi\tau_t^c.$$

3. *The optimal income tax structure is given by:*

$$\frac{\tau_{t+1}^k}{\tau_{t+1}^w} = 1 + \frac{1 - \frac{q_{t+1}^I}{q_t^I}(1-\delta^h)}{F_k(t+1) - \delta^k}.$$

The first and the third result do not change in comparison to section 3. The second result is an important departure from the Diamond and Mirrlees (1971) principle of production efficiency. The optimal tax code has to trade off two distortions. Distortions on the production side by not leaving the previously (i.e. in the previous sections) pure intermediate goods untaxed have to be balanced against distortions on the consumption side (affecting the implementability constraint) by possibly treating pure consumption goods different from consumption-investment goods. Interestingly, production efficiency is sacrificed in the optimal tax code and the consumption component of the consumption-investment good is fully taxed. Let us interpret the result for the case of physical capital investment and τ^{xm}. The argument for investments in effective labor for the human capital sector is analogous, leading to the same result for τ_t^{xh}. When investment in effective labor has no consumption value ($\xi = 0$), the investment cost for intermediate goods used in the production of effective labor can be fully deducted from the labor income tax base. This result is changed in two respects. First, the fraction of investment that has a consumption value is taxed at the consumption tax rate. Second, only the fraction $1 - \xi$ of the investment can be deducted from the labor tax base. This is true although the total expenditure enters production (not only the fraction $1-\xi$). Consider the extreme case $\xi = 1$, in which case the good under consideration is a consumption investment good. In this case the investment is fully taxed at the consumption tax rate and no deduction of the investment is possible. How can this be understood?

Consider the following arbitrage argument from a social planner's perspective. First, a consumer can use one unit of the consumption-capital good in period t for consumption. A second possible use of the unit of the consumption-capital good is to invest it effective labor in the physical capital production sector by buying one unit of x_t^m. From the perspective of the social planer, this yields a return of ξ in units of consumption and a return equal to $F_e(t)M_x^m(t)$ in terms of additional production of the consumption-capital good, which could be used directly for consumption in period t. From the perspective of the social planer, these two activities must yield the same social return and thus we must have that $\xi + F_e(t)M_x^m(t) = 1$ or $F_e(t)M_x^m(t) = 1 - \xi$. The social planner wants to invest more in x_t^m than in the pure investment good case with $\xi = 0$ (the marginal product of x_t^m is *lower* than 1) since the investment good yields an additional consumption value.

Now consider the consumer's decision. If the consumer buys a unit of the consumption good directly, he has to pay the consumer price for the consumption good, 1 plus the consumption tax, τ_t^c. Alternatively, he can obtain one unit of consumption by buying $1 - \xi$ units of the consumption good directly (which costs him $(1 + \tau_t^c)(1 - \xi)$) and indirectly enjoy ξ units of consumption by investing in one unit of x_t^m. This has a direct cost of $1 + \tau_t^{xm}$. In addition, he receives a higher net wage of $(1 - \tau_t^w)w_t M_x^m(t) = (1 - \tau_t^w)F_e(t)M_x^m(t)$, which reduces the direct cost of investing in x_t^m. From a consumer's perspective it is therefore optimal to invest in x_t^m until $1 + \tau_t^c = (1 + \tau_t^c)(1 - \xi) + 1 + \tau_t^{xm} - (1 - \tau_t^w)F_e(t)M_x^m(t)$. If we now want a consumer to choose the socially optimal amount of x_t^m, that is to choose x_t^m such that $1 - \xi = F_e(t)M_x^m(t)$, we have to set taxes in a way that $1 + \tau_t^c = (1 + \tau_t^c)(1 - \xi) + 1 + \tau_t^{xm} - (1 - \tau_t^w)(1 - \xi)$ holds. This is equivalent to the second condition in Proposition 4. Consider e.g. the choice $\tau_t^{xm} < -\tau_t^w(1-\xi) + \tau_t^c\xi$. Then the consumer invests in x_t^m until $1 - \xi > F_e(t)M_x^m(t)$. Now, each (marginal) unit invested in the physical production sector yields ξ units of direct consumption but less than $1-\xi$ units of additional output. Therefore, there is too much investment in x_t^m. The social planer can increase welfare by increasing τ_t^{xm}. Intuitively, investment goods that are close to consumption goods in terms of their direct utility-generating effects will be purchased by consumers even if their social production value is low. But in this case, the full investment expenditures should not be a allowed to be deducted from the labor tax base. Only the social marginal production value should be deductible.

6 Taxation of Changes in the Value of Human Capital as a Theoretical Foundation of Comprehensive Income Taxation

As we have seen in section 3, the principle of comprehensive income taxation can only be shown to be optimal in a knife edge case. In this section, we reveal how the incorporation of (changes in) the market value of human capital in a comprehensive measure of income leads to a theoretical but arguably unrealistic foundation of the principle of comprehensive income taxation in a general equilibrium setup. To be more precise, we show that taxing the change in the market value of human capital at the labor income tax rate yields the optimality of comprehensive income taxation.

The new budget constraint of the representative agent is:

$$\begin{aligned}
(1+\tau_t^c)c_t + k_{t+1} + b_{t+1} + q_t^I \overbrace{(h_{t+1}-(1-\delta^h)h_t)}^{\text{new human capital}} \\
+(1+\tau_t^{xm})x_t^m + (1+\tau_t^{xh})x_t^h \\
+\tau_t^w((1-\delta^h)q_t^I - q_{t-1}^I)h_t \\
\leq (1-\tau_t^w)w_t^m M^m(h_t, n_t^m, x_t^m) + (1-\tau_t^w)w_t^h M^h(h_t, n_t^h, x_t^h) \\
+((1-\tau_t^{km})(r_t^m-\delta^k)+1)k_t^m + ((1-\tau_t^{kh})(w_t^h-\delta^k)+1)k_t^h \\
+R_t b_t.
\end{aligned}$$

Notice the difference to the previous section (see equation (4)). In the third row, the change in the value of human capital (measured in units of the consumption good), $((1-\delta^h)q_t^I - q_{t-1}^I)h_t$, is taxed at the labor income tax rate, τ_t^w. Now the first order condition of the consumer's maximization problem with respect to human capital is given by:

$$\begin{aligned}
h_{t+1}: \quad & -\gamma_t^H q_t^I + \beta\gamma_{t+1}^H[(1-\delta^h)q_{t+1}^I - \tau_{t+1}^w(q_{t+1}^I(1-\delta^h) - q_t^I) \\
& + (1-\tau_{t+1}^w)[w_{t+1}^m M_h^m(t+1) + w_{t+1}^h M_h^h(t+1)] = 0
\end{aligned}$$

where the $\beta^t \gamma_t^H$ again denote the Lagrange multipliers associated with the budget constraints. The only change takes place in the human capital Euler equation:

$$u_c(t)\frac{q_t^I}{(1+\tau_t^c)} = \beta u_c(t+1)[\frac{1}{(1+\tau_{t+1}^c)} - \frac{u_n(t+1)}{u_c(t+1)}\frac{1}{G_e(t+1)M_n^h(t+1)}[1-\delta^h - \frac{q_t^I}{q_{t+1}^I}]$$
$$-\frac{u_n(t+1)}{u_c(t+1)}\left[\frac{M_h^h(t+1)}{M_n^h(t+1)} + \frac{M_h^m(t+1)}{M_n^m(t+1)}\right]]$$

The other first order conditions correspond to the equations (14)- (19) in section 2.

The Ramsey Problem is now given as:

$$\begin{aligned} L = \sum_{t=0}^{\infty} \beta^t [&u(c_t, g_t, n_t) + \alpha u_c(t) c_t \\ &- \gamma_t(-u_c(t)\frac{q_t^I}{(1+\tau_t^c)} \\ &\quad + \beta u_c(t+1)[\frac{1}{(1+\tau_{t+1}^c)} - \frac{u_n(t+1)}{u_c(t+1)}\frac{1}{G_e(t+1)M_n^h(t+1)}[1-\delta^h - \frac{q_t^I}{q_{t+1}^I}] \\ &\quad - \frac{u_n(t+1)}{u_c(t+1)}\left[\frac{M_h^h(t+1)}{M_n^h(t+1)} + \frac{M_h^m(t+1)}{M_n^m(t+1)}\right]]) \\ &+ \lambda_t[-c_t - g_t - k_{t+1} - x_t^m - x_t^h + F(k_t s_t, M(x_t^m, h_t, n_t^m)) + (1-\delta^k)k_t] \\ &+ \mu_t[-h_{t+1} + (1-\delta^h)h_t + G((1-s_t)k_t, M^h(x_t^h, h_t, n_t^h))]]] - \alpha \bar{A}_0, \end{aligned}$$

where $\bar{A}_0$ denotes the wealth in period 0 which differs from A_0 in section 3.

Proposition 5 *Suppose that changes in the market value of human capital are taxed at the labor income tax rate τ^w. Then the following tax code is optimal for all $t > 0$:*

1. *Equality of capital taxes across sectors: $\tau_t^{km} = \tau_t^{kh}$.*
2. *Zero Tax on Intermediate Goods: $\tau_t^{xm} = \tau_t^{xh} = -\tau_t^w$.*
3. *Comprehensive income taxation: $\tau_{t+1}^{km} = \tau_{t+1}^w$ is optimal.*

The first two results are as before. The third result, however, isolates formally the reason why real world tax codes should generally (see section 3) not follow the principle of comprehensive income taxation. They do not consider changes in the market value of human capital and in fact it would be difficult to do so.

7 Consumption Value of Human Capital

Education or human capital are investment goods since they increase labor productivity. But human capital may also generate direct utility or have a consumption value [see for example Shaffer (1961), Haveman and Wolfe (1984), Gullason (1989), Judd (1999) and Judd (2001), section 4]. This aspect can be taken into consideration in our analysis by directly incorporating human capital as an argument in the utility function. There are different ways of formalizing this, for example by specifying leisure as "quality time" [see Heckman (1976) and Milesi-Ferretti and Roubini (1998b)]. Alternatively, one could view human capital - besides its role in the production process - as a (durable) consumption good that is produced using capital and effective labor. The latter view is in the spirit of Becker (1965) and rather close to the home production literature [see Greenwood and Hercowitz (1991)].[8] As an example, consider foreign language skills that are produced by employing a computer (physical capital), a teacher (human capital), a learning software (output goods) and raw time .

This section addresses the question which consequences a consumption-investment-good view of human capital has for optimal income taxation. We extend the analysis in the previous section, i.e. we assume that changes in the value of human capital can be taxed. This approach is chosen in order to model physical and human capital as similar as possible, the only essential difference being the fact that the stock of human capital directly generates utility in the analysis of this section. This allows us to evaluate a popular argument: the tax system should tax the consumption component of human capital - just like it raises taxes on pure consumption goods. Therefore, the tax rate on labor income should be higher than the tax rate of physical capital. Alternatively, the (intermediate) input goods to the production of human capital should not be treated as input goods to the final good production, i.e. should not be left untaxed. This section shows that these arguments are flawed.

We now introduce the stock of human capital in the utility function with $u_h > 0$ and $u_{hh} < 0$:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, g_t, n_t, h_t).$$

The implementability constraint is now given by

$$\sum_{t=0}^{\infty} \beta^t (u_c(t)c_t + u_h(t)h_t)) = \tilde{A}_0,$$

[8] The difference to the home production literature is that human capital is a market good here whereas the utility-generating activity involving human capital is a non-market good in the home production literature, see Gronau (1997) for an overview.

where $\tilde{A}_0$ is the wealth in period 0.

From the first order conditions of the representative agent follows the Euler equation for human capital:

$$\frac{(1+\tau^c_{t+1})u_c(t)}{(1+\tau^c_t)u_c(t+1)}$$
$$= \beta\left[1+(1-\tau^w_{t+1})\frac{q^I_{t+1}}{q^I_t}\left[1-\delta^h+G_h(t+1)+G_n(t+1)\frac{M^m_h(t+1)}{M^m_n(t+1)}-\frac{q^I_t}{q^I_{t+1}}\right]\right]$$
$$+\beta\frac{u_h(t+1)}{u_c(t+1)}\frac{(1+\tau^c_{t+1})}{q^I_t}. \tag{10}$$

Rewriting the Euler equation for human capital, the Ramsey problem is now:

$$L = \sum_{t=0}^{\infty}\beta^t[u(c_t,g_t,n_t,h_t)+\alpha(u_c\,(t)\,c_t+u_h\,(t)\,h_t)$$
$$-\gamma_t(-u_c(t)\frac{q^I_t}{(1+\tau^c_t)}+\beta u_c(t+1)[\frac{1}{(1+\tau^c_{t+1})}-\frac{u_n(t+1)}{u_c(t+1)}\frac{1}{G_e(t+1)M^h_n(t+1)}$$
$$\cdot[1-\delta^h-\frac{q^I_t}{q^I_{t+1}}]$$
$$-\frac{u_n(t+1)}{u_c(t+1)}\left[\frac{M^h_h(t+1)}{M^h_n(t+1)}+\frac{M^m_h(t+1)}{M^m_n(t+1)}\right]]+\beta u_h(t+1))$$
$$+\lambda_t[-c_t-g_t-k_{t+1}-x^m_t-x^{xh}_t-x^h_t+F(k_ts_t,M(x^m_t,h_t,n^m_t))+(1-\delta^k)k_t]$$
$$+\mu_t[-h_{t+1}+(1-\delta^h)h_t+G(x^h_t,(1-s_t)k_t,M^h(x^{xh}_t,h_t,n^h_t))]]]-\alpha\tilde{A}_0.$$

Notice the changes of the Ramsey problem in comparison to the previous section. The first-order conditions with respect to c_t and g_t have an additional term $h_tu_{cn}(t)$; the first-order conditions with respect to n^m_t and n^h_t have an additional term $u_{hn}(t)$; and the first-order condition with respect to h_{t+1} has additional terms. In general, we do not obtain clear-cut results about the optimal tax code when human capital enters the utility function directly. However, some examples with specific functional forms reveal interesting properties. Consider the following preferences that are widely used in the macroeconomics literature:[9]

1. CES form:
$$u(c,g,n,h) = \frac{1}{1-\theta}[\theta c^\alpha+(1-\theta)(h^\phi(1-n)^\gamma)^\alpha]^{\frac{1-\theta}{\alpha}}+w(g) \tag{11}$$
This utility function nests as special cases: (1) $\phi=\gamma=1$, $\alpha=0$, namely the Cobb Douglas case as well as (2) log-utility, if additionally $\theta=1$.

[9]Apart from government expenditures that usually do not enter.

2. Weakly separable quality-time form:

$$u(c, g, l, h) = v(c, (1-n)h) + w(g). \tag{12}$$

Proposition 6 *Suppose preferences are of the form (11) or (12). Then the optimal tax code requires comprehensive income taxation for all* $t > 0$*:* $\tau_{t+1}^{km} = \tau_{t+1}^{w}$.

We now consider additively separable preferences with human capital entering in the constant-elasticity-of-substitution form, where $f(.)$, $v(.)$ and $w(.)$ are increasing, strictly concave, and twice continuously differentiable functions:

$$u(c_t, g_t, l_t, h_t) = f(c_t) + v(1-n_t) + \frac{h^{1-\sigma}}{1-\sigma} + w(g_t). \tag{13}$$

Judd (1999) (equation (34)) considers the special case of separable utility with consumption and labor entering in isoelastic form, just as human capital.

Proposition 7 *Suppose preferences are of the form (13). Then the optimal tax code requires for all* $t > 0$*:*

1. $\tau_{t+1}^{w} < \tau_{t+1}^{km} \Leftrightarrow \sigma < 1$,
2. $\tau_{t+1}^{w} > \tau_{t+1}^{km} \Leftrightarrow \sigma > 1$,
3. $\tau_{t+1}^{w} = \tau_{t+1}^{km} \Leftrightarrow \sigma = 1$.

Comprehensive income taxation is thus only optimal for the special case of log-utility. In particular, the result shows that it may be optimal - depending on the parameter σ - to tax (physical) capital income higher than labor income although capital and labor are modeled very similar *and* households derive additional utility from human capital. This shows that general equilibrium effects - entering through the implementability constraint - lead at first sight to quite unexpected results about the design of an optimal tax system.

To understand the logic behind the results in this section recall the nature of Ramsey taxation. A commodity is taxed according to the distortion this taxation causes from the perspective of the social planer. The distortion effects are summarized in the implementability constraint. If human capital enters now directly into the utility function, one has to check for the effects of such a change on the implementabilty constraint.[10] E.g. in the case of Proposition 7, 2., due to the additive separability and the isoelastic form of utility from human capital, the additional affect of human capital on the implementability constraint is zero and

[10] These effects are summarized in the variables $x_1(t+1)$ and $x_2(t+1)$ in Lemma 1 in the Appendix and depend on the form of the utility function.

therefore comprehensive income taxation still applies.
It is also helpful to compare the situation here with section 5. There, investment in effective labor also yields direct utility since it augments the final-good consumption. Therefore, it enters the implementability constraint in the same way as consumption (up to the constant ξ) and therefore it should be treated from the planer's perspective in the same way as consumption (up to the constant ξ). This results in the optimal scheme of Proposition 4. The same logic also applies to this section, but since human capital is a new commodity, distinct from final good consumption in the utility function, the effects it causes on the implementability constraint are not necessarily the same as consumption of the final good causes.
The analysis in this section is not meant to have immediate consequences for tax reform proposals. It is a first step towards a comprehensive analysis that takes the conceptual differences between human capital and physical capital seriously.
Proposition 7 can be compared to Theorem 14 in Judd (1999) although Judd's set-up is slightly different in some important aspects: Judd (1999) models the accumulation of human capital as home production, introduces a direct tax on the stock of human capital instead of taxing the inputs to human capital accumulation as in our set-up, and does not introduce a consumption tax. He derives closed form solutions for the absolute labor and human capital tax whereas we only consider the relative tax rates. But our result that the labor income tax is strictly smaller than the capital income tax if the elasticity of demand for human capital is less than one (see 1. of Proposition 7) does not conflict with the result of Judd (1999) that the capital tax is on average zero and that the labor income tax is strictly positive. The reason is that our tax system is different from the one considered by Judd (1999). In his set-up, only the capital tax enters the margin between intertemporal consumption which has to be undistorted on average in the optimum. In our set-up, both the consumption tax and the capital income tax enter this margin and Judd's result thus only shows that the disturbing effects of these taxes have to offset each other on average. In particular, the capital tax does not have to be zero on average.
Since the setup in this section allows to tax the changes on the value of human capital, it would be interesting to analyze how the consumption value of human capital changes the optimal code if we consider the more realistic setup of section 3. It can be shown that the results in this section carry over to that setup in the sense that deviations from comprehensive income taxation due to the nontaxation of changes in the value of human capital do not interact with those coming from human capital in the utility function. E.g. in the case of an additive utility function with logarithmic human capital as in Proposition 7, 3., we would obtain the same deviation from comprehensive income taxation as in Proposition 7, 3.
Summarizing this section, we have shown that the effects on the optimal tax code

depend on the specification of preferences in the case that the stock of human capital generates direct utility. However, there is no a priori reason to expect that physical capital should be taxed lower than wage income. In particular, there is no clear-cut case for departing from comprehensive income taxation.

8 Conclusion

This paper offered a theoretical but difficult to implement foundation of comprehensive income taxation. In fact, in section 6 we showed that comprehensive income taxation is optimal if the change in the market value of human capital (including depreciation) is accounted for. Real world tax systems do not take this into account. In section 3 we found out that in this case the optimal tax code requires possibly large deviations from comprehensive income taxation. As we saw in section 4 our results are robust to restrictions in government's asset holdings (with balanced budget as a special case).

Section 5 considers the case where a fraction of the output-good investment in effective labor has a consumption value. This is an empirically highly relevant situation. The main result of that section is that the optimal tax code sacrifices production efficiency, i.e. taxes intermediate goods as far as they provide a direct consumption value.

An interesting departure from the standard human capital investment approach is to incorporate consumption benefits derived from the stock of human capital as a kind of durable consumption good. Section 7 provides a first approach to modeling the consumption value of human capital. The main result is that the effects of a positive consumption value of human capital on the optimal tax code is not clear. In particular, there is no a priori case for discriminating against human capital investment and favoring investments in physical capital.

A possible extension of our analysis is to integrate our approach in the literature on cash-flow taxation. Consider a cash-flow tax on human capital. Cash flows in period t are the (net) returns r_t^m and r_t^h from the firm to the representative agent, the cash flows for labor income, $w_t^h M^h(h_t, n_t^h, x_t^h) + w_t^m M^m(h_t, n_t^m, x_t^m)$, and expenditures for new human capital $q_t^I(h_{t+1} - (1-\delta^h)h_t)$. Interestingly, the consumption tax and the labor tax would then not be independent instruments any more. Therefore, the human capital Euler equation cannot be adjusted independent of the static labor-leisure choice. It is beyond the scope of this paper to offer an analysis of this issue. In any case, a cash-flow tax is not an equivalent or superior alternative to the tax code we have considered.

Proofs

In this section we will use the short notation: $G_n(t) := G_e(t)M_n^h(t)$, $G_x(t) := G_e(t)M_x^h(t)$ and $G_h(t) := G_e(t)M_h^h(t)$.

Representative Agent's Problem in Section 3: A solution to the the optimization problem of the representative agent has to fulfill the following first order conditions:

$$1 + (1 - \tau_t^{km})(F_k(t) - \delta^k) = 1 + (1 - \tau_t^{kh})(q_t^I G_k(t) - \delta^k) = R_t, \tag{14}$$

$$(1 - \tau_t^w) q_t^I G_e(t) = -\frac{u_n(t)(1 + \tau_t^c)}{u_c(t) M_n^h(t)}, \tag{15}$$

$$(1 - \tau_t^w) F_e(t) = -\frac{u_n(t)(1 + \tau_t^c)}{u_c(t) M_n^m(t)}, \tag{16}$$

$$\beta(1 + (1 - \tau_{t+1}^{km})(F_k(t+1) - \delta^k)) = \frac{u_c(t)(1 + \tau_{t+1}^c)}{u_c(t+1)(1 + \tau_t^c)}, \tag{17}$$

$$1 + \tau_t^{xm} = (1 - \tau_t^w) F_e(t) M_x^m(t), \tag{18}$$

$$1 + \tau_t^{xh} = (1 - \tau_t^w) q_t^I G_x(t), \tag{19}$$

$$\frac{u_c(t)(1 + \tau_{t+1}^c)}{u_c(t+1)(1 + \tau_t^c)} = \beta \frac{q_{t+1}^I}{q_t^I} [(1 - \delta^h) + (1 - \tau_{t+1}^w) \left[G_h(t+1) + G_n(t+1) \frac{M_h^m(t+1)}{M_n^m(t+1)} \right]]. \tag{20}$$

From (15) and (20) we derive the Euler equation for human capital:

$$u_c(t) \frac{q_t^I}{(1 + \tau_t^c)} = \beta u_c(t+1) \left[(1 - \delta^h) \frac{q_{t+1}^I}{(1 + \tau_{t+1}^c)} - \frac{u_n(t+1)}{u_c(t+1)} \left[\frac{M_h^h(t+1)}{M_n^h(t+1)} + \frac{M_h^m(t+1)}{M_n^m(t+1)} \right] \right].$$

By taking the infinite sum over the budget constraints in all periods, using the homogeneity of G, M^h and M^m and equation (20) we obtain the *implementability constraint*: $\sum_{t=0}^{\infty} \beta^t u_c(t) c_t = A_0$ where $A_0 := \frac{u_c(0)}{1+\tau_0^c}[k_0 + (1 - \tau_0^{km})(F_k(0) - \delta^k) s_0 k_0 + (1 - \tau_0^{kh})(q_t^I G_k(0) - \delta^k)(1 - s_0) k_0 + R_0 b_0 + h_0 (1 - \delta^h) q_0^I] - \frac{u_n(0)}{G_n(0)} h_0 [G_n(0) \frac{M_h^m(0)}{M_n^m(0)} + G_h(0)]$ denotes the wealth at period zero.

Proof of Proposition 1

Given any allocation, the consumption taxes $\{\tau_t^c\}_{t=0}^{\infty}$ can always be chosen so that the human capital Euler equation (7) holds for all $t \geq 0$. As can easily be seen from (7), for any given allocation and given some τ_t^c, we can always choose τ_{t+1}^c in a way that (7) holds. Since $\{\tau_t^c\}_{t=0}^{\infty}$ enters neither the objective function nor any other constraint in our Ramsey problem, this will indeed be optimal to do and thus hold in any solution to the Ramsey problem. Thus, independent of the particular form of the optimal allocation, the constraints (7) for $t \geq 0$ will automatically be satisfied given the right choice of $\{\tau_t^c\}_{t=0}^{\infty}$, which results in $\gamma_t \equiv 0$ for all $t \geq 0$. Using this result ($\gamma_t = 0$) the other first order conditions for $t > 0$ to the Ramsey problem can be calculated as:

$$\begin{aligned}
c_t :\quad & \lambda_t = u_c(t) + \alpha[u_c(t) + c_t u_{cc}(t)], & (21)\\
g_t :\quad & \lambda_t = u_g(t) + \alpha c_t u_{cg}(t), & (22)\\
k_{t+1} :\quad & \lambda_t = \beta\lambda_{t+1}[F_k(t+1)s_{t+1} + 1 - \delta^k] + \beta\mu_{t+1}(1 - s_{t+1})G_k(t+1), & (23)\\
n_t^m :\quad & u_n(t) + \alpha u_{cn}(t)c_t + \lambda_t F_e(t)M_n^m(t) = 0, & (24)\\
n_t^h :\quad & u_n(t) + \alpha u_{cn}(t)c_t + \mu_t G_n(t) = 0, & (25)\\
x_t^m :\quad & \lambda_t[-1 + F_e(t)M_x^m(t)] = 0, & (26)\\
x_t^h :\quad & \mu_t G_x(t) = \lambda_t, & (27)\\
s_t :\quad & \lambda_t F_k(t)k_t - \mu_t G_k(t)k_t = 0, & (28)\\
h_{t+1} :\quad & \beta\lambda_{t+1}F_e(t+1)M_h^m(t+1) - \mu_t + \beta\mu_{t+1}[1 - \delta^k + G_h(t+1)] = 0. & (29)
\end{aligned}$$

Subtracting equation (24) from equation (25) yields

$$\lambda_t F_e(t)M_n^m(t) = \mu_t G_n(t) \tag{30}$$

and with equation (27) we have:

$$F_e(t)M_n^m(t) = \frac{M_n^h(t)}{M_x^h(t)}. \tag{31}$$

From the first order conditions (15) and (16) of the representative agent it follows $(1-\tau_t^w)q_t^I G_n(t) = (1-\tau_t^w)F_e(t)M_n^m(t)$ and with equation (19) $(1+\tau_t^{xh})\frac{M_n^h(t)}{M_x^h(t)} = (1-\tau_t^w)F_e(t)M_n^m(t)$. Comparing this equation to (31) yields $\tau_t^{xh} = -\tau_t^w$. By comparing the first order condition of the representative agent (18) to the first order condition of the government (26) it follows $\tau_t^{xm} = -\tau_t^w$ since $\lambda_t \neq 0$. From equation (19) we get $q_t^I = \frac{1}{G_x(t)}$. The first order condition of the government (28) together with (30) yields $F_k(t)\frac{G_n(t)}{F_e(t)M_n^m(t)} = G_k(t)$. With (31) and $q_t^I = \frac{1}{G_x(t)}$ it follows: $F_k(t) = q_t^I G_k(t)$. Comparing this equation to the first order condition of the representative agent (14) yields $\tau_t^{km} = \tau_t^{kh}$. It follows from the first order conditions (23) and (28):

$$k_{t+1} :\quad \frac{\lambda_t}{\lambda_{t+1}} = \beta[F_k(t+1) + 1 - \delta^k]. \tag{32}$$

Use equation (27) to replace the Lagrange multiplier μ in (29):

$$\frac{\lambda_t}{\lambda_{t+1}} = \beta[\frac{G_x(t)}{G_x(t+1)}[1-\delta^h + G_h(t+1)] + G_x(t)F_e(t+1)M_h^m(t+1)].$$

With (31) and $q_t^I = \frac{1}{G_x(t)}$ this can be rewritten as $\frac{\lambda_t}{\lambda_{t+1}} = \beta\frac{q_{t+1}^I}{q_t^I}[1-\delta^h + G_h(t+1) + G_n(t+1)\frac{M_h^m(t+1)}{M_n^m(t+1)}]$ and with equation (32): $1+F_k(t+1)-\delta^k = \frac{q_{t+1}^I}{q_t^I}[1-\delta^h + G_h(t+1)+G_n(t+1)\frac{M_h^m(t+1)}{M_n^m(t+1)}]$ which is equivalent to

$$\frac{q_{t+1}^I}{q_t^I}[1-\delta^h] = 1+F_k(t+1)-\delta^k - \frac{q_{t+1}^I}{q_t^I}[G_h(t+1)+G_n(t+1)\frac{M_h^m(t+1)}{M_n^m(t+1)}]. \quad (33)$$

From the first order conditions (17) and (20) of the representative agent it follows (with $\tau_t^k := \tau_t^{km} = \tau_t^{kh}$.)

$$\begin{aligned} &1+(1-\tau_{t+1}^k)(F_k(t+1)-\delta^k) \\ &= \frac{q_{t+1}^I}{q_t^I}\cdot[(1-\delta^h)+(1-\tau_{t+1}^w)[G_h(t+1)+G_n(t+1)\frac{M_h^m(t+1)}{M_n^m(t+1)}]]. \end{aligned} \quad (34)$$

From (33) and (34) it follows directly that $\tau_{t+1}^k = \tau_{t+1}^w$ iff $1 = \frac{q_{t+1}^I}{q_t^I}(1-\delta^h)$, $\tau_{t+1}^k > \tau_{t+1}^w$ iff $1 > \frac{q_{t+1}^I}{q_t^I}(1-\delta^h)$ and $\tau_{t+1}^k < \tau_{t+1}^w$ iff $1 < \frac{q_{t+1}^I}{q_t^I}(1-\delta^h)$. Using (33) to rearrange (34) yields:

$$\frac{\tau_{t+1}^k}{\tau_{t+1}^w} = \frac{1+F_k(t+1)-\delta^k - \frac{q_{t+1}^I}{q_t^I}(1-\delta^h)}{F_k(t+1)-\delta^k} = 1 + \frac{1-\frac{q_{t+1}^I}{q_t^I}(1-\delta^h)}{F_k(t+1)-\delta^k}.$$

Proof of Proposition 2

Because we do not want to exclude the possibility that the debt limits are set equal to zero we have to replace the implementability constraint in the Ramsey problem by the budget constraints of the representative agent in each period where the prices and taxes are replaced using first order conditions:

$$u_c(t)c_t + \frac{u_c(t)}{1+\tau_t^c}[k_{t+1}+q_t^I h_{t+1}] - \frac{1}{\beta}\frac{u_c(t-1)}{1+\tau_{t-1}^c}[k_t + q_{t-1}^I h_t] = 0 \quad \forall t,$$

which are associated with the Lagrange multiplier α_t in the Ramsey problem. The first order conditions of the government take the following form:

$$k_{t+1}: \quad \lambda_t = \beta\lambda_{t+1}[F_k(t+1)s_t + 1 - \delta^k] + \beta\mu_{t+1}(1-s_{t+1})G_k(t+1) + [\alpha_t - \alpha_{t+1}]\frac{u_c(t)}{1+\tau_t^c}, \tag{35}$$

$$n_t^m: \quad u_n(t) + \alpha u_{cn}(t)c_t + \lambda_t F_e(t)M_n^m(t) + [\alpha_t - \alpha_{t+1}]\frac{u_{cn}(t)}{1+\tau_t^c}[k_{t+1} + q_t^I h_{t+1}] = 0, \tag{36}$$

$$n_t^h: \quad u_n(t) + \alpha u_{cn}(t)c_t + \mu_t G_n(t) + [\alpha_t - \alpha_{t+1}]\frac{u_{cn}(t)}{1+\tau_t^c}[k_{t+1} + q_t^I h_{t+1}] = 0, \tag{37}$$

$$h_{t+1}: \quad \beta\lambda_{t+1}F_2(t+1)M_h^m(t+1) - \mu_t + \beta\mu_{t+1}[1 - \delta^k + G_h(t+1)] + [\alpha_t - \alpha_{t+1}]\frac{u_c(t)}{1+\tau_t^c}q_t^I = 0. \tag{38}$$

$$b_t: \quad [\alpha_t - \alpha_{t+1}]\frac{u_c(t)}{1+\tau_t^c} = \nu_{t-1}. \tag{39}$$

Because the first order conditions with respect to x_t^m, x_t^h and s_t do not change and the additional terms in the first order conditions for n_t^h and n_t^m are identical and disappear if (36) is subtracted from equation (37) it directly follows $\tau_t^{xm} = \tau_t^{xh} = -\tau_t^w$ and $\tau_t^{km} = \tau_t^{kh}$. From the first order conditions with respect to k_{t+1} and s_t, it follows:

$$\lambda_t = \beta\lambda_{t+1}[F_k(t+1) + 1 - \delta^k] + [\alpha_t - \alpha_{t+1}]\frac{u_c(t)}{1+\tau_t^c} \tag{40}$$

The first order condition with respect to human capital (38) can be rearranged to:

$$\lambda_t = \beta\lambda_{t+1}\frac{q_{t+1}^I}{q_t^I}[1 - \delta^h + G_h(t+1) + G_n(t+1)\frac{M_h^m(t+1)}{M_n^m(t+1)}] + [\alpha_t - \alpha_{t+1}]\frac{u_c(t)}{1+\tau_t^c}$$

Subtracting this equation from equation (40) yields:

$$1 + F_k(t+1) - \delta^k = \frac{q_{t+1}^I}{q_t^I}[1 - \delta^h + G_h(t+1) + G_n(t+1)\frac{M_h^m(t+1)}{M_n^m(t+1)}]$$

which is equivalent to equation (33) in the proof of Proposition 1. Because the first order conditions of the representative agent do not change in the presence of debt limits, the results of Proposition 1 are still optimal in the case of debt limit and in the case without government bonds (i.e. $(M_t \equiv 0 \ \forall t)$).

Proof of Proposition 3

(a) The first order conditions with respect to k_{t+1}, x_t^h, x_t^m, h_{t+1}, n_t^h, n_t^m, g_t and c_t

are:

$$c_t: \quad \frac{\lambda_t}{g_t^{Z-1}} - \Psi_1(t) + \alpha_t[\Psi_1(t) + \frac{c_t}{g_t}\Psi_{11}(t)] = [\alpha_t - \alpha_{t+1}]\frac{\Psi_{11}(t)}{1+\tau_t^c}\frac{1}{g_t}(k_{t+1} + q_t^I h_{t+1}). \tag{41}$$

$$g_t: \quad -\frac{\lambda_t}{g_t^{Z-1}} - \Psi_1(t)\frac{c_t}{g_t} + Z\Psi(t) - \alpha_t[(Z-1)\Psi_1(t) - \Psi_{11}(t)\frac{c_t}{g_t}]\frac{c_t}{g_t} = [\alpha_t - \alpha_{t+1}]\frac{(Z-1)\Psi_1(t) - \frac{c_t}{g_t}\Psi_{11}(t)}{g_t(1+\tau_t^c)}(k_{t+1} + q_t^I h_{t+1}), \tag{42}$$

$$k_{t+1}, s_t: \quad \frac{\lambda_t}{g_t^{Z-1}} = \frac{\lambda_{t+1}}{g_t^{Z-1}}\beta[F_k(t+1) + 1 - \delta] + [\alpha_t - \alpha_{t+1}]\frac{\Psi_{11}(t)}{g_t(1+\tau_t^c)}, \tag{43}$$

$$n_t^h: \quad \Psi_2(t) + \alpha_t\Psi_{12}(t)\frac{c_t}{g_t} + \frac{\mu_t}{g_t^Z}G_n(t) - [\alpha_t - \alpha_{t+1}][\frac{\Psi_{12}(t)}{g_t(1+\tau_t^c)}(k_{t+1} + q_t^I h_{t+1}) = 0, \tag{44}$$

$$n_t^m: \quad \Psi_2(t) + \alpha_t\frac{c_t}{g_t}\Psi_{12}(t) + \frac{\lambda_t}{g_t^Z}F_e(t)M_n^m(t) - [\alpha_t - \alpha_{t+1}]\frac{\Psi_{12}(t)}{g_t(1+\tau_t^c)}[k_{t+1} + q_t^I h_{t+1}] = 0, \tag{45}$$

$$x_t^h: \quad \lambda_t = \mu_t G_x(t), \tag{46}$$

$$x_t^m: \quad \lambda_t[-1 + F_e(t)M_x^m(t)] = 0, \tag{47}$$

$$h_{t+1}: \quad \lambda_t = \beta\frac{q_{t+1}^I}{q_t^I}\lambda_{t+1}[1 - \delta^h + G_h(t+1) + G_n(t+1)\frac{M_h^m(t+1)}{M_n^m(t+1)}] + [\alpha_t - \alpha_{t+1}]\frac{g_t^{Z-1}\Psi_1(t)}{(1-\tau_t^c)}. \tag{48}$$

Use the assumption to the utility function $\Psi_1(t) + \frac{c_t}{g_t}\Psi_{11}(t) = \frac{c_t}{g_t}\Psi_{12}(t)$ to replace the term $\Psi_1(t) + \frac{c_t}{g_t}\Psi_{11}(t)$ in equation (41). Now subtract equation (45) from equation (41). Some terms drop out and the result is:

$$\frac{\lambda_t}{g_t^{Z-1}}[1 + F_e(t)\frac{M_n^m(t)}{g_t}] - \Psi_1(t) + \Psi_2(t) = (\alpha_t - \alpha_{t+1})\frac{\Psi_{12} + \Psi_{11}}{g_t(1+\tau_t^c)}[k_{t+1} + \frac{h_{t+1}}{G_x(t)}]. \tag{49}$$

Multiplying (42) by $\frac{\Psi_1(t)+\frac{c_t}{g_t}\Psi_{11}(t)}{\Psi_1(t)+\frac{c_t}{g_t}\Psi_{11}(t)-Z\Psi_1(t)}\frac{g_t}{c_t}$ and subtract it from equation (41) yields:

$$\begin{aligned}&\frac{\lambda_t}{g_t^{Z-1}}[1+[1+\frac{Z\Psi_1(t)}{\Psi_1(t)+\frac{c_t}{g_t}\Psi_{11}(t)-Z\Psi_1(t)}]\frac{g_t}{c_t}]\\&-[\Psi_1(t)-[\Psi_1(t)\frac{c_t}{g_t}+Z\Psi(t)][1+\frac{Z\Psi_1(t)}{\Psi_1(t)+\frac{c_t}{g_t}\Psi_{11}(t)}]\frac{g_t}{c_t}\\=&[\alpha_t-\alpha_{t+1}]\frac{1}{g_t(1+\tau_t^c)}[k_{t+1}+q_t^I h_{t+1}][2\Psi_{11}(t)+\Psi_1(t)\frac{g_t}{c_t}]\end{aligned}$$

Use equation (49) to replace $[\alpha_t-\alpha_{t+1}]$ in the previous equation:

$$\frac{\lambda_t}{g_t^{Z-1}}=\frac{\Psi_1(t)-[\Psi_1(t)\frac{c_t}{g_t}\Psi(t)][1+\frac{Z\Psi_1(t)}{\Psi_1(t)+\frac{c_t}{g_t}\Psi_{11}(t)}]\frac{g_t}{c_t}-[\Psi_1(t)+\Psi_2(t)]\frac{2\Psi_{11}(t)+\Psi_1(t)\frac{g_t}{c_t}}{\Psi_{12}(t)+\Psi_{11}(t)}}{1+\frac{g_t}{c_t}+\frac{Z\Psi_1(t)}{\Psi_1(t)+\frac{c_t}{g_t}\Psi_{11}(t)}\frac{g_t}{c_t}\frac{2\Psi_{11}(t)+\Psi_1(t)\frac{g_t}{c_t}}{\Psi_{12}(t)+\Psi_{11}(t)}[1+\frac{F_e(t)M_n^m(t)}{g_t}]} \tag{50}$$

Suppose that there is a balanced growth path, with an endogenously given non-negative growth rate κ. By definition, time allocated to labor, n_t^h and n_t^m have to be constant over time. Along the balanced growth path with balanced budget the feasibility equation takes the following form: $c_t+g_t+(k_{t+1}-(1-\delta)k_t)+x_t^m+x_t^h=Y_t$. The proof proceeds in three steps. First, (well-known) implications of the definition of a balanced growth path for the growth rates of several variables are derived. Second, the implications of these results for optimal tax rates on a balanced growth path are derived. Third, the consequences of the tax rates for the government's problem with a balanced budget are considered.

First, it is easy to show that along a balanced growth path, c_t, k_t, g_t, x_t^m and x_t^h have to grow at the same growth rate as the economy ($\kappa=\kappa_i\ \forall i$). From the human capital technology it also directly follows that also the human capital stock has the growth rate κ due to the fact that the functions M^h and G are constant returns to scale. $\Psi(\frac{c_t}{g_t},n)$, Ψ_1, Ψ_2 and further derivatives have to be constant over time because the fraction $\frac{c_t}{g_t}$ and n are constant.

Second, we deal with the tax rates along a balanced growth path. From the homogeneity of degree one of F and G it follows that $r^m=F_k$, $w^m=F_e$, $r^h=q_t^I G_k(t)$, $w^h=q_t^I G_e$ are constant along a balanced growth path and that $M^m(t)$ and $M^h(t)$ grow at growth rate κ. The human capital accumulation technology implies that G_t has to grow at growth rate κ. From the homogeneity of degree one properties of M^h, M^m and G it follows that M_x^h, M_h^h, M_x^m and M_h^m are constant along a balanced growth path and that $M_n^m(t)$, $M_n^h(t)$, and $G_n(t)$ have to grow at growth rate κ.

Thus from equation (50) it follows that $\frac{\lambda_t}{g_t^{Z-1}}$ is constant along a balanced growth path. From (43) it follows that $\alpha_t-\alpha_{t+1}$ is constant and from (41) that α_t is

constant which yields $\alpha_t - \alpha_{t+1} = 0$. From equation (48) we get $\frac{q^I_{t+1}}{q^I_t}$ = constant. We have already shown that $\tau^w_t = -\tau^{xm}_t = -\tau^{xh}_t$, $\tau^{km}_{t+1} = \tau^{kh}_{t+1}$ and
$\tau^k_{t+1} = \tau^w_{t+1}((1 + \frac{1 - \frac{q^I_{t+1}}{q^I_t}(1-\delta^h)}{F_k(t+1)-\delta^k}), \forall t > 0.$
From the first order condition of the representative agent (16) and the assumption $\Psi_1(t) + \frac{c_t}{g_t}\Psi_{11}(t) = \frac{c_t}{g_t}\Psi_{12}(t)$ it follows along a balanced growth path:

$$1 + \tau^c_t = (1 - \tau^w_t) F_e \frac{M^m_n(t)}{g_t} \frac{\Psi_1}{(-\Psi_2)}$$

From the budget constraint of the representative agent (4) with $b_t = b_{t+1} = 0$ we get that all the tax rates in our economy have to be constant along the balanced growth path. Hence from the equations (17) compared with (43) and (20) compared with (48) it follows $\tau^{km} = 0$ and $\tau^w = 0$ due the fact that $\frac{1+\tau^c_{t+1}}{1+\tau^c_t} = 1$ along the balanced growth path. The consumption tax is now given by

$$1 + \tau^c_t = -F_e \frac{M^m_n(t)}{g_t} \frac{\Psi_1}{\Psi_2} = \frac{\Psi_1}{\Psi_2} \frac{\Psi_2 + \alpha \Psi_{12}\frac{c}{g}}{\Psi_1 + \alpha[\Psi_1 + \frac{c}{g}\Psi_{11}]} = \frac{\Psi_1 + \alpha[\Psi_{12}\frac{c}{g}]}{\Psi_1 + \alpha[\Psi_1 + \frac{c}{g}\Psi_{11}]} = 1$$

Where we used (45) and (41) to replace $-F_e\frac{M^m_n(t)}{g_t}$ and $\frac{\lambda_t}{g_t^{Z-1}}$.
We have shown that it is optimal to set all tax rates equal to zero along any balanced growth path with non-negative growth rate.
Third, we consider the government's problem along a balanced growth path with balanced budget. If the government cannot issue bonds and hence cannot transfer tax revenues to the future it is not possible to finance government expenditures along the balanced growth path. Thus it follows that $g_t = 0$ along any balanced growth path. In this case, the Ramsey problem is not well-defined anymore.

(b) The proof is an application of an argument similar as the one used to prove the Berge maximum theorem to our Ramsey problem, with the debt limits $M \equiv \{M_t\}_{t\in\mathbb{N}_0}$ as the parameters. In particular, we show that the solution of the Ramsey problem, endowed with the product topology, is an upper hemicontinuous correspondence in $\{M_t\}_{t\in\mathbb{N}_0}$, where the debt limits are also endowed with the product topology. Consider a sequence of debt limits $(M^n_0, M^n_1, ...) \in \mathbb{R}^\infty$, where $\mathbb{R}^\infty$ is endowed with the product topology, that converges to $(0, 0, ...)$. The Ramsey problem is for each n to

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t, g_t, n_t)$$

subject to $(c_t, g_t, n_t, b_t, k_t, x_t, h_t)_{t\in\mathbb{N}_0} \in \varphi(M^n)$, where $\varphi(M)$ is defined as the set of all sequences $(c_t, g_t, n_t, b_t, k_t, x_t, h_t)_{t\in\mathbb{N}_0}$ such that all constraints of the Ramsey problem (9) are satisfied in each period and such that $(c_t, g_t, k_t, x_t, h_t)$ satisfy for each t that $c_t \leq C\varsigma^t, g_t \leq G\varsigma^t$ etc. for some $\varsigma > 1$ with $\beta\varsigma^Z < 1$, which ensures that the life time utility is always finite and thus the optimization problem is well defined[11].

We first show that $\sum_{t=0}^{\infty} \beta^t u(c_t, g_t, n_t)$ is a continuous function on the graph of φ in the product topology. We have $\Psi\left(\frac{c}{g}, n\right) \leq \max\left\{\overline{\Psi}, A + B\left(\frac{c}{g}\right)^Z\right\}$ for all (c, g, n), where $\overline{\Psi}$ is a bound for the continuous function Ψ on $[0, W] \times [0, 1]$. Thus we obtain for all sequences (c_t, g_t, n_t) from the graph of φ,

$$\beta^t g_t^Z \Psi\left(\frac{c_t}{g_t}, n_t\right) \leq \beta^t g_t^Z \max\left\{\overline{\Psi}, A + B\left(\frac{c_t}{g_t}\right)^Z\right\}$$

$$\leq \beta^t G^Z \varsigma^{tZ} \overline{\Psi} + \beta^t B c_t^Z \leq \overline{L}\left(\beta\varsigma^Z\right)^t$$

Hence $\beta^t g_t^Z \Psi\left(\frac{c_t}{g_t}, n_t\right)$ is bounded by $(\beta\varsigma^Z)^t\overline{L})$ for each t (where $\overline{L}$ is some positive constant). Because by assumption $\sum_{t=0}^{\infty}\left(\beta\varsigma^Z\right)^t \overline{L} < \infty$, the Lebesgue dominated convergence theorem (applied to the counting measure on the natural numbers), yields the continuity of $\sum_{t=0}^{\infty} \beta^t u(c_t, g_t, n_t)$ on the graph of φ. To see that φ is upper hemicontinuous, note that by Theorem 16.20 in Aliprantis and Border (1999), it suffices to show that each sequence $(c^n, g^n, n^n, b^n, k^n, x^n, h^n) \in \varphi(M^n)$ has a limit point in $\varphi(M)$ if $M^n \to M$. Note that since by assumption, $(c^n, g^n, n^n, b^n, k^n, x^n, h^n)$ is coordinatewise bounded, a Cantor diagonal argument gives a pointwise convergent subsequence which converges to some limit point (c, g, n, b, k, x, h). Since the restrictions defining φ are continuous functions defined on a finite subset of coordinates, the limit point (c, g, n, b, k, x, h) satisfies all these constraints and thus $(c, g, n, b, k, x, h) \in \varphi(M)$, which proves upper hemicontinuity. By Lemma 16.30 in Aliprantis and Border (1999), the indirect utility function $v(M)$ is thus upper semicontinuous in M. Next, note that since $\varphi(0) \subseteq \varphi(M)$ for each sequence M, we have that $\liminf_{n\to\infty} v(M^n) \geq v(0)$ for each $M^n \to 0$. Thus the indirect utility function v is continuous in $M = 0$. Our goal is to show the upper hemicontinuity of correspondence of maximizers $\mu(M)$ of the Ramsey problem in $M = 0$. Using the condition for upper hemicontinuity in Theorem 16.20 in Aliprantis and Border (1999) and the Cantor diagonal argument to find a converging subsequence for some sequence $y^n \in \mu(M^n)$, we can apply the closed graph argument in the proof of Theorem 16.31 in Aliprantis and Border

[11] If we consider the usual social planer problem (without distortionary taxation), we could assume that the technology is such that all allocations that are feasible in this problem are bounded as described above. Then this bound would of course also hold for the Ramsey planer problem.

(1999) to show that the correspondence of maximizers of the Ramsey problem $\mu(M)$ is upper hemicontinuous in $M = 0$. Note that this argument only uses the upper hemicontinuity of φ and the continuity of v in $M = 0$. The definition of upper hemicontinuity and the fact that $\mu(0)$ contains by part (a) of the proposition no balanced growth path solution yield the property we wanted to show.

Proof of Proposition 4
Now consider an utility function of the form $\sum_{t=0}^{\infty} \beta^t [u(c_t + \xi(x_t^h + x_t^m), g_t, n_t)]$. with $\xi \in (0, 1)$. The new components in the utility function only affects the first order condition with respect to x_t^h and x_t^m for the optimizing problem of the representative agent:

$$1 + \tau_t^{xm} = (1 - \tau_t^w) F_e(t) M_x^m(t) + \xi(1 + \tau_t^c), \tag{51}$$
$$1 + \tau_t^{xh} = (1 - \tau_t^w) q_t^I G_e(t) M_x^h(t) + \xi(1 + \tau_t^c). \tag{52}$$

The implementability constraint is now given by $\sum_{t=0}^{\infty} \beta^t (u_c(t)(c_t + \xi(x_t^h + x_t^m))) = \hat{A}_0$ and the Euler equation for human capital does not change. The first order conditions to the Ramsey problem are therefore given by:

$$c_t: \quad \lambda_t = u_c(t) + \alpha[u_c(t) + (c_t + \xi(x_t^h + x_t^m)) u_{cc}(t)], \tag{53}$$
$$g_t: \quad \lambda_t = u_g(t) + \alpha(c_t + \xi(x_t^h + x_t^m)) u_{cg}(t), \tag{54}$$
$$k_{t+1}: \quad \lambda_t = \beta \lambda_{t+1} [F_k(t+1) s_t + 1 - \delta^k] + \beta \mu_{t+1} (1 - s_{t+1}) G_k(t+1), \tag{55}$$
$$n_t^m: \quad u_n(t) + \alpha u_{cn}(t)(c_t + \xi(x_t^h + x_t^m)) + \lambda_t F_e(t) M_n^m(t) = 0, \tag{56}$$
$$n_t^h: \quad u_n(t) + \alpha u_{cn}(t)(c_t + \xi(x_t^h + x_t^m)) + \mu_t G_n(t) = 0, \tag{57}$$
$$x_t^m: \quad \xi[u_c + \alpha[u_c + (c_t + \xi(x_t^h + x_t^m)) u_{cc}(t)] + \lambda_t[-1 + F_e(t) M_x^m(t)] = 0, \tag{58}$$
$$x_t^h: \quad \xi[u_c + \alpha[u_c + (c_t + \xi(x_t^h + x_t^m)) u_{cc}(t)] + \mu_t G_x(t) = \lambda_t, \tag{59}$$
$$s_t: \quad \lambda_t F_k(t) k_t - \mu_t G_k(t) k_t = 0, \tag{60}$$
$$h_{t+1}: \quad \beta \lambda_{t+1} F_2(t+1) M_h^m(t+1) - \mu_t + \beta \mu_{t+1} [1 - \delta^k + G_h(t+1)] = 0. \tag{61}$$

Subtracting equation (56) from equation (57) yields

$$\lambda_t F_e(t) M_n^m(t) = \mu_t G_n(t). \tag{62}$$

By using equation (53) we can reduce the first order conditions with respect to x_t^m and x_t^m ((58) and (59)) as following:

$$F_e(t) M_x^m(t) = 1 - \xi, \tag{63}$$
$$(1 - \xi) \lambda_t = \mu_t G_x(t). \tag{64}$$

With equation (62) it follows:

$$\frac{M_n^h(t)}{M_x^h(t)} = F_e(t) M_n^m(t) \frac{1}{1-\xi}. \tag{65}$$

From the first order conditions of the household we get:

$$(1-\tau_t^w) q_t^I G_e(t) M_n^h(t) = (1-\tau_t^w) F_e(t) M_n^m(t).$$

Using (52) to replace $(1-\tau_t^w) q_t^I G_e(t)$ and (65) to replace $\frac{M_n^h(t)}{M_x^h(t)}$ in the previous equation yields:

$$(1-\tau_t^w) = \frac{(1+\tau_t^{xh}) - \xi(1+\tau_t^c)}{1-\xi} \Leftrightarrow -\tau_t^{xh} = \tau_t^w - \xi\tau_t^w - \xi\tau_t^c.$$

¿From equation (51) and (63) it follows: $-\tau_t^{xm} = -\tau_t^{xh} = \tau_t^w - \xi\tau_t^w - \xi\tau_t^c$. With equation (52) we get $q_t^I = \frac{1}{G_x(t)}(1-\xi)$. With (60), (62) and (65) we have $F_k(t) = q_t^I G_k(t)$ and $\frac{\lambda_t}{\lambda_{t+1}} = \beta[\frac{G_x(t)}{G_x(t+1)}[1-\delta^h + G_e(t+1) M_h^h(t+1)] + G_x(t) F_e(t+1) M_h^m(t+1)]$ and it follows again $\tau_t^{kh} = \tau_t^{km}$. Using equation (61) together with the equations (64) and (65) and $q_t^I = \frac{1}{G_e(t) M_x^h(t)}(1-\xi)$ yields again $\frac{\lambda_t}{\lambda_{t+1}} = \beta \frac{q_{t+1}^I}{q_t^I}[1-\delta^h + G_h(t+1) + G_n(t+1)\frac{M_h^m(t+1)}{M_n^m(t+1)}]$ the results concerning the relationship between τ_t^w and τ_t^{km} do not differ from the results presented in Proposition 1.

Proof of Proposition 5
Consider the optimization problem of the representative agent. Only the first order condition with respect to h_{t+1} is affected by the change of the tax base for the labor tax rate τ_t^w and hence the new Euler equation for human capital has the following form:

$$\begin{aligned} u_c(t)\frac{q_t^I}{(1+\tau_t^c)} &= \beta u_c(t+1)[\frac{1}{(1+\tau_{t+1}^c)} - \frac{u_n(t+1)}{u_c(t+1)}\frac{1}{G_n(t+1)}[1-\delta^h - \frac{q_t^I}{q_{t+1}^I}] \\ &- \frac{u_n(t+1)}{u_c(t+1)}\left[\frac{M_h^h(t+1)}{M_n^h(t+1)} + \frac{M_h^m(t+1)}{M_n^m(t+1)}\right]] \end{aligned} \tag{66}$$

Now consider the Ramsey problem. We can again set $\gamma_t = \gamma_{t-1} = 0\ \forall t$ and therefore the first order conditions correspond to the conditions in section 2. Therefore it follows again: $F_k(t+1) + 1 - \delta^k = \frac{G_x(t)}{G_x(t+1)} \cdot [1-\delta^h + G_h(t+1) + G_n(t+1)\frac{M_h^m(t+1)}{M_n^m(t+1)}]$ which is equivalent to:

$$F_k(t+1) - \delta^k = \frac{G_x(t)}{G_x(t+1)} \cdot [1-\delta^h + G_h(t+1) + G_n(t+1)\frac{M_h^m(t+1)}{M_n^m(t+1)} - \frac{G_x(t+1)}{G_x(t)}].$$

Compare this equation to the following equation stemming from the first order conditions of the representative agent:

$$1 + (1 - \tau^{km}_{t+1})(F_k(t+1) - \delta^k) =$$
$$1 + (1 - \tau^{w}_{t+1})\frac{q^I_{t+1}}{q^I_t}[1 - \delta^h + G_h(t+1) + G_n(t+1)\frac{M^m_h(t+1)}{M^m_n(t+1)} - \frac{q^I_t}{q^I_{t+1}}].$$

With $q^I_t = \frac{1}{G_x(t)}$ it directly follows $\tau^{km}_{t+1} = \tau^w_{t+1}$.

Proof of Propositions 6 and 7 Consider the Ramsey problem. We can again set $\gamma_t = \gamma_{t-1} = 0 \ \forall t$ and therefore the first order conditions have the following form:

$$c_t: \quad \lambda_t = u_c(t) + \alpha[u_c(t) + c_t u_{cc}(t) + h_t u_{ch}(t)], \tag{67}$$
$$g_t: \quad \lambda_t = u_g(t) + \alpha(c_t u_{cg}(t) + h_t u_{gh}(t)), \tag{68}$$
$$k_{t+1}: \quad \lambda_t = \beta\lambda_{t+1}[F_k(t+1)s_t + 1 - \delta^k] + \beta\mu_{t+1}(1 - s_{t+1})G_k(t+1), \tag{69}$$
$$n^m_t: \quad u_n(t) + \alpha(u_{cn}(t)c_t + u_{hn}(t)h_t) + \lambda_t F_e(t)M^m_n(t) = 0, \tag{70}$$
$$n^h_t: \quad u_n(t) + \alpha(u_{cn}(t)c_t + u_{hn}(t)h_t) + \mu_t G_n(t) = 0, \tag{71}$$
$$x^m_t: \quad \lambda_t[-1 + F_e(t)M^m_x(t)] = 0, \tag{72}$$
$$x^h_t: \quad \mu_t G_x(t)] = \lambda_t, \tag{73}$$
$$s_t: \quad \lambda_t F_k(t)k_t - \mu_t G_k(t)k_t = 0, \tag{74}$$
$$h_{t+1}: \quad \beta\lambda_{t+1}f_2(t+1)M^m_h(t+1) - \mu_t + \beta\mu_{t+1}[1 - \delta^k + G_h(t+1)]$$
$$+\beta u_h(t+1) + \beta\alpha(u_h(t+1) + u_{hh}(t+1)h_{t+1} + u_{ch}(t+1)c_{t+1}) = 0. \tag{75}$$

From the first order conditions (69) and (74) it follows:

$$\frac{\lambda_t}{\lambda_{t+1}} = \beta[F_k(t+1) + 1 - \delta^k]. \tag{76}$$

Because $\lambda_t \neq 0$ from (72) we get:

$$F_e(t)M^m_x = 1.$$

Subtracting equation (71) from equation (70) yields $F_e(t)M^m_n(t)G_x(t) = G_n(t)$. Using equation (73) to replace μ in equation (74) yields: $F_k(t) = \frac{G_k(t)}{G_x(t)}$. Using (73) to replace μ in equation (75) and equation (72) to set $F_e(t)M^m_x = 1$ the first order condition with respect to human capital (75) can be rewritten as:

$$\frac{\lambda_t}{\lambda_{t+1}} = \beta\frac{G_x(t)}{G_x(t+1)}[1 - \delta^h + G_h(t+1) + G_n(t+1)\frac{M^m_h(t+1)}{M^m_n(t+1)}]$$
$$+\frac{G_x(t)}{\lambda_{t+1}}\left(\beta u_h(t+1) + \beta\alpha\left(u_h(t+1) + u_{hh}(t+1)h_{t+1} + u_{ch}(t+1)\,c_{t+1}\right)\right).$$

Using equation (76) to replace $\frac{\lambda_t}{\lambda_{t+1}}$ yields:

$$\begin{aligned}
&F_k(t+1)+1-\delta^k \\
&\quad = \frac{G_x(t)}{G_x(t+1)}[1-\delta^h+G_h(t+1)+G_n(t+1)\frac{M_h^m(t+1)}{M_n^m(t+1)}] \\
&\quad + \frac{G_x(t)}{\lambda_{t+1}}u_h(t+1)+\frac{G_x(t)}{\lambda_{t+1}}\alpha(u_h(t+1)+u_{hh}(t+1)h_{t+1}+u_{ch}(t+1)c_{t+1})
\end{aligned}$$

which is equivalent to

$$\begin{aligned}
&F_k(t+1)-\delta^k \\
&= \frac{G_x(t)}{G_x(t+1)}[1-\delta^h+G_h(t+1)+G_n(t+1)\frac{M_h^m(t+1)}{M_n^m(t+1)}-\frac{G_x(t+1)}{G_x(t)}+\frac{G_x(t+1)}{\lambda_{t+1}}u_h(t+1)] \\
&+ \frac{G_x(t)}{\lambda_{t+1}}\alpha x_1(t+1). \qquad (77)
\end{aligned}$$

with $x_1(t+1) := (u_h(t+1)+u_{hh}(t+1)h_{t+1}+u_{ch}(t+1)c_{t+1})$.
Now consider the optimization problem of the representative agent. By comparing the first order conditions (17) and (10) we have:

$$\begin{aligned}
&1+(1-\tau_{t+1}^{km})(F_k(t+1)-\delta^k) \\
&= 1+(1-\tau_{t+1}^w)\frac{q_{t+1}^I}{q_t^I}\left[1-\delta^h+G_h(t+1)+G_n(t+1)\frac{M_h^m(t+1)}{M_n^m(t+1)}-\frac{q_t^I}{q_{t+1}^I}\right] \\
&+ \; \frac{u_h(t+1)}{u_c(t+1)}\frac{(1+\tau_{t+1}^c)}{q_t^I}. \qquad (78)
\end{aligned}$$

From the first order condition of the representative agent (15) it follows:

$$\frac{u_h(t+1)}{u_c(t+1)}\frac{(1+\tau_{t+1}^c)}{q_t^I} = -(1-\tau_{t+1}^w)\frac{q_{t+1}^I}{q_t^I}\frac{u_h(t+1)}{u_n(t+1)}G_n(t+1)$$

Using equation (70) to replace $u_n(t+1)$ and (72) to replace $F_e(t)$ yields:

$$\begin{aligned}
&= (1-\tau_{t+1}^w)\frac{q_{t+1}^I}{q_t^I}G_x(t+1)u_h(t+1)\frac{M_n^m(t+1)}{M_m^h(t+1)} \\
&\cdot\left(\alpha(u_{cn}(t+1)c_{t+1}+u_{hn}(t+1)h_{t+1})+\lambda_{t+1}\frac{M_n^m(t+1)}{M_x^m(t+1)}\right)^{-1} \\
&= (1-\tau_{t+1}^w)\frac{q_{t+1}^I}{q_t^I}u_h(t+1)\frac{G_x(t+1)}{\lambda_{t+1}y_{t+1}}.
\end{aligned}$$

with $y_{t+1} := 1 + \frac{\alpha(u_{cn}(t+1)c_{t+1}+u_{hn}(t+1)h_{t+1})}{\lambda_{t+1}\frac{M_n^m(t+1)}{M_x^m(t+1)}}$. Replacing this term in equation (78) yields:

$$1 + (1-\tau_{t+1}^{km})(F_k(t+1)-\delta^k)$$
$$= 1 + (1-\tau_{t+1}^{w})\frac{q_{t+1}^I}{q_t^I}\left[1-\delta^h + G_h(t+1) + G_n(t+1)\frac{M_h^m(t+1)}{M_n^m(t+1)} - \frac{q_t^I}{q_{t+1}^I} + \frac{G_x(t+1)}{\lambda_{t+1}y_{t+1}}u_h(t+1)\right].$$

and with $q_t^I = \frac{1}{G_x(t)}$:

$$(1-\tau_{t+1}^{km})(F_k(t+1)-\delta^k) = (1-\tau_{t+1}^{w})$$
$$\cdot \frac{G_x(t)}{G_x(t+1)}\left[1-\delta^h + G_h(t+1) + G_n(t+1)\frac{M_h^m(t+1)}{M_n^m(t+1)} - \frac{G_x(t+1)}{G_x(t)} + \frac{G_x(t+1)}{\lambda_{t+1}}u_h(t+1)\right]$$
$$+ (1-\tau_{t+1}^{w})\frac{G_x(t)}{\lambda_{t+1}}\alpha x_2(t+1) \qquad (79)$$

with $x_2(t+1) := \frac{1}{\alpha}[\frac{1}{y_{t+1}} - 1]u_h(t+1) = \frac{1}{\alpha}\frac{-\alpha(u_{cn}(t+1)c_{t+1}+u_{hn}(t+1)h_{t+1})}{-u_n(t+1)}u_h(t+1)$ $= \frac{(u_{cn}(t+1)c_{t+1}+u_{hn}(t+1)h_{t+1})}{u_n(t+1)}u_h(t+1)$. From the Kuhn-Tucker conditions it follows that $\lambda_t > 0$. The Lagrange multiplier α for the implementability constraint has to be strictly positive because it reflects the welfare cost of the distorted margins (see Ljungqvist/Sargent p.323 and Chari/Kehoe (1999)). $G_x(t+1) > 0$ by definition. By comparing equation (77) and equation (79) we get the following result for the tax rates:

Lemma 1 *For*

$$x_1(t+1) := (u_h(t+1) + u_{hh}(t+1)h_{t+1} + u_{ch}(t+1)c_{t+1}),$$
$$x_2(t+1) := \frac{(u_{cn}(t+1)c_{t+1} + u_{hn}(t+1)h_{t+1})}{u_n(t+1)}u_h(t+1)$$

we have:

a) If $x_1(t+1) < x_2(t+1)$ it follows $\tau_{t+1}^{km} < \tau_{t+1}^{w}$.

b) If $x_1(t+1) > x_2(t+1)$ it follows $\tau_{t+1}^{km} > \tau_{t+1}^{w}$.

c) If $x_1(t+1) = x_2(t+1)$ it follows $\tau_{t+1}^{km} = \tau_{t+1}^{w}$.

Here are some examples for utility functions:

1. Consider $u(c_t, g_t, l_t, h_t) := f(c_t) + v(1-n_t) + g(h_t) + w(g_t)$ with $g(h_t) = \frac{h^{1-\sigma}}{1-\sigma}$. It follows $x_2(t) = 0$ and $x_1(t) = (1-\sigma)h_t^{-\sigma}$ and hence $\tau_t^w < \tau_t^{km} \Leftrightarrow \sigma < 1$, $\tau_t^w > \tau_t^{km} \Leftrightarrow \sigma > 1$ and $\tau_t^w = \tau_t^{km} \Leftrightarrow \sigma = 1$.

2. Consider $u(c_t, g_t, l_t, h_t) := (f(c_t) + \ln h_t)v(1 - n_t) + w(g_t)$. It follows $x_1 = 0$ and $x_2(t) = [1 + f'(c_t)c_t]\frac{\frac{1}{h_t}v(1-n_t)}{(f(c_t)+\ln h_t)} > 0$ and hence $\tau_t^w > \tau_t^{km}$.

3. Consider $u(c_t, g_t, l_t, h_t) := f(c_t) + v((1 - n_t)h_t) + w(g_t)$. It follows $x_1(t) = x_2(t) = v'((1-n_t)h_t)(1-n_t)+v''((1-n_t)h_t)(1-n_t)^2 h_t$ and hence $\tau_t^w = \tau_t^{km}$.

4. Consider $u(c_t, g_t, l_t, h_t) := c_t^\alpha(1 - n_t)^\gamma h_t^\phi + w(g_t)$. It follows $x_1(t) = x_2(t) = [\alpha + \gamma]\gamma c_t^\alpha h_t^{\gamma-1}(1 - n_t)^\phi$ and hence $\tau_t^w = \tau_t^{km}$.

5. Consider $u(c_t, g_t, l_t, h_t) := v(c_t, (1-n_t)h_t) + w(g_t)$. It follows $x_1(t) = x_2(t) = v_2(c_t, (1-n_t)h_t)(1-n_t) + v_{22}(c_t, (1-n_t)h_t)(1-n_t)^2 h_t + v_{21}(c_t, (1-n_t)h_t)(1-n_t)c_t$ and hence $\tau_t^w = \tau_t^{km}$.

6. Consider $u(c_t, g_t, l_t, h_t) = c_t^\alpha h_t^{1-\alpha} + v(1-n_t) + w(g_t)$. It follows $x_2(t) = 0$ and $x_1(t) = (1 - \alpha)c_t^\alpha h_t^{-\alpha} > 0$ and hence $\tau_t^w < \tau_t^{km}$.

7. Consider $u(c, g, n, h) = \frac{1}{1-\theta}[\theta c^\alpha + (1 - \theta)(h^\phi(1 - n)^\gamma)^\alpha]^{\frac{1-\theta}{\alpha}} + w(g)$. It follows $x_1(t) = x_2(t) = (1 - \theta)\phi[\theta c^\alpha + (1 - \theta)(h^\phi(1 - n)^\gamma)^\alpha]^{\frac{1-\theta-\alpha}{\alpha}}(1 - n)^\gamma(h^\phi(1 - n)^\gamma)^{\alpha-1}h^{\phi-1} \cdot [\alpha\phi + [\theta c^\alpha + (1 - \theta)(h^\phi(1 - n)^\gamma)^\alpha]^{-1}(1 - \theta - \alpha)[(1 - \theta)\phi h^{\alpha\phi}(1 - n)^{\gamma(\alpha-1)} + \theta c^\alpha]]$ and hence $\tau_t^w = \tau_t^{km}$.

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