Nominal Interest Rates and Optimal Disinflation in New Keynesian Models*

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Abstract

Central bankers' conventional wisdom suggests that nominal interest rates should be raised to implement a lower inflation target. In contrast, I show that the standard New Keynesian monetary model predicts that nominal interest rates should be decreased to attain this goal. This result holds both in a basic New Keynesian model and in recent vintages of New Keynesian models with sticky wages, price and wage indexation and habit formation in consumption.

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1 Introduction

The standard strategy to assess the quantitative performance of monetary business cycle models is to investigate impulse responses to (monetary policy) shocks. Whereas New Keynesian models perform very well in these experiments (Woodford (2003), Christiano et al. (2005)), I show that there is an inconsistency between one of the model's main predictions and observed monetary policy. Suppose the central bank wants to implement a lower inflation target. The most prominent example for such a regime change are presumably the 1970s, a period of high inflation, followed by the Volcker disinflation.[1] Once a lower inflation regime is considered to be optimal, central bankers' conventional wisdom suggests that nominal interest rates should be *increased*.[2] But this is not what standard New Keynesian models predict. In these models the optimal policy response is to implement a *lower* nominal interest rate right away.

The reason for this inconsistency is clear if prices are flexible. In the absence of pricing frictions it is optimal to immediately adjust inflation to its new target level. The Fisher equation – the nominal interest rate equals the real interest rate plus the inflation rate – then implies that the nominal interest should be lowered immediately. This mechanism is related to what is typically referred to as the 'expectations channel'. The central bank sets nominal interest such that they are consistent with the private sector's expectations of lower inflation rates in the future.

With sticky prices this expectations channel is also available but there is an additional

[1]Primiceri (2005) and Sargent et al. (2005) support the view that this was indeed a target change. They both explain the high inflation and the subsequent disinflation as the optimal policy outcome of a rational policy maker who has to learn the "true" data generating mechanism. In both papers the government's perception was that disinflation was too costly during the 1970s. The perceived inflation-unemployment trade-off became favorable, relative to the level of inflation, only in the late 1970s, what then led to a disinflation. Ireland (2005) and Milani (2006) estimate the Fed's inflation target and find a sharp drop in its level in the late 1970s.

[2]This conventional is very well conveyed in the excellent historical review of the Volcker disinflation by Lindsey et al. (2005). Erceg and Levin (2003) provide further references and state that the federal funds rate remained the main instrument of monetary policy, although the Federal Reserve's stated operational target involved the stock of nonborrowed reserves from 1979:4 to 1982:3.

‘aggregate demand’ channel, which links lower aggregate demand to lower inflation. Increasing nominal interest then, because prices are sticky, lowers aggregate demand what then leads to lower inflation rates. Using this channel is however quite costly since it requires an output contraction, which can be avoided when the expectations channel is used. Even with sticky prices it is then optimal to only use the expectations channel with the consequence that nominal interest rates are uniformly lowered to implement a lower inflation target. An immediate adjustment of inflation to its target level however is not necessarily optimal in the presence of pricing frictions. Instead, inflation and nominal interest rates are only gradually adjusted.

The qualitative properties of optimal policy do not change if several features, that are part of recent vintages of New Keynesian models, such as habit formation in consumption, sticky wages and wage and price indexation are allowed for. Nominal interest rates need to be lowered uniformly in order to implement a lower inflation target.

This result may appear counterintuitive since model-generated impulse response functions fit the data well. In particular, the inflation rate drops in response to a short-lived increase in nominal interest rates. The two experiments - implementing a lower inflation target on the one hand and monetary policy shocks on the other hand - thus lead to different conclusions. How can this apparent contradiction be reconciled?

There are two reasons. First, a short-lived increase in nominal interest rates does not create expectations of a lower inflation rate in the long-run. As a result the role of the expectations channel is diminished in the second experiment. Second, a positive shock to the nominal interest rate leads to a contraction in output and to lower inflation rates. Whereas it is optimal not to use this channel in the first experiment, an output contraction is an avoidable consequence of a positive shock to nominal interest rates in the second experiment.

Although the expectations channel is the key mechanism, the results of this paper do not depend on expectations being rational, the standard assumption in the New Keynesian literature. I show that, if inflation expectations are linked to current inflation, the inconsistency between the model and monetary policy remains. Even with non-rational expectations, it is optimal to uniformly lower inflation rates, what leads to uniformly lower inflation expectations and thus to uniformly lower nominal interest rates.

The results in this paper characterize optimal policy and need not hold if policy is not optimal. Erceg and Levin (2003) for example consider a New Keynesian model where the private sector has to learn the central bank's inflation target.[3] They find that nominal interest rates are increased in response to a persistent drop in the inflation target. However, this finding depends on their specification of the monetary policy rule, which does not describe the optimal policy.[4]

Concerning the implications of a disinflation for output most macroeconomists' view is that a disinflation is associated with a recession. In the basic New Keynesian model however the opposite result holds: a disinflation causes an output boom (Ball (1994) and Ball et al. (2005)). The reason is that a lower inflation rate in the future leads to larger price decreases by firms, who can adjust the price in the current period, since they may not be able to adjust the price in the future (because of Calvo (1983)-pricing). These preemptive price cuts stimulate demand and lead to an expansion of output in the current period. With sufficiently strong indexation of prices as for example in Giannoni and Woodford (2004), the incentives for preemptive price cuts disappear since prices are automatically lowered when other firms lower their prices in the future. Therefore a disinflation does not necessarily lead to an immediate expansion. The New Keynesian model, amended with full price indexation, is thus inconsistent with conventional wisdom about nominal interest rates but consistent with conventional wisdom about output.

In the next section I consider a simple, analytically tractable sticky price model that aims at providing the intuition for the main results. Section 3 describes the model of Gi-

[3] Ball (1995a) also considers a disinflation in a simple New Keynesian model with imperfect credibility and Ireland (1995, 1997) computes the optimal disinflation path. The focus of these papers is on the welfare and output effects of a disinflation and not on nominal interest rates.

[4] Specifically, they use $i_t = 1.296\frac{\pi_t+\pi_{t-1}+\pi_{t-2}+\pi_{t-3}}{4} - 0.506\pi^* + \ldots$, where i is the nominal interest rate, π is the inflation rate and π^* is the inflation target. A drop in π^* then mechanically leads to an increase in i_t if π does not fall fast enough (because of learning). But this mechanical increase is not optimal. However, deriving the optimal policy in their framework is very hard, since it requires a characterization of optimal policy in a dynamic signaling game.

annoni and Woodford (2004), which features habit formation in consumption, sticky wages and sticky prices and indexation of prices and wages. The parameter estimates of Giannoni and Woodford (2004) and the results for the optimal paths of inflation and nominal interest rates are presented in Section 4. Section 5 provides a sensitivity analysis and Section 6 concludes. All proofs are delegated to the appendix.

2 A Simple Model

I now present a basic New Keynesian model which includes, following Clarida et al. (1999)(CGG), both cost-push shocks and shocks to the natural rate of interest. This model allows for theoretical results since it abstracts from several features such as habit formation in consumption, sticky wages and wage and price indexation. All of these elements will be present in the general model below. The purpose of this simple model is to understand which properties of the model are crucial for the results.

The economy is described by two equations. I follow Woodford (2003) and consider, for tractability, a log-linearized version.[5] The first equation, the Phillips curve, summarizes the optimal price setting behavior of monopolistically competitive firms under a Calvo (1983)-style price adjustment mechanism:

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u_t, \tag{1}$$

where π_t is the inflation rate, x_t is the output gap - the difference between log output with sticky prices and log output when prices are flexible - in period t and u_t is, in the terminology of CGG, a cost-push shock. The discount factor of the representative household is denoted $\beta \in (0,1)$ and $\kappa > 0$ is the "slope" of the Phillips curve, which depends on features such as the frequency of price changes and the sensitivity of prices to changes in marginal cost.

The second equation, the IS equation, is derived from the standard consumption Euler

[5] Benigno and Woodford (2006) show that any optimal policy problem can be approximated through a problem with (L)inear constraints and a (Q)uadratic objective function. See Benigno and Woodford (2006) for a discussion of the advantages of the LQ approach.

equation of the representative household:

$$x_t = E_t x_{t+1} - \sigma E_t(i_t - \pi_{t+1} - r_t^n), \tag{2}$$

where i_t is the nominal interest rate in period t and r_t^n is the real interest rate in period t if prices were flexible.[6]

The policy experiment is as follows. At time $t = 0$ the central bank is told to implement an inflation target $\hat{\pi}^*$ that is lower than the current inflation target $\overline{\pi}^*$. The goal is then to compute the sequence of nominal interest rates that implement the regime change.

An optimal policy is a sequence π_t and x_t which minimizes the loss function

$$\sum_{t=0}^{\infty} \beta^t [(\pi_t - \pi^*)^2 + \lambda_x (x_t - x^*)^2], \tag{3}$$

subject to constraints (1) and (2).

Here π^* is the inflation target and equals $\overline{\pi}^*$ without a regime change and equals $\hat{\pi}^* < \overline{\pi}^*$ with a regime change. All results in this section hold for all values of $\hat{\pi}^* < \overline{\pi}^*$, but for the linearization to be appropriate, one should think of inflation targets sufficiently close to zero.

The output target is denoted x^* and λ_x is the weight that is assigned to output stabilization. Two cases are considered for how the choice of x^* is related to the inflation target π^*. Either the output target x^* is the same for both inflation targets or it is chosen to be consistent with the inflation target and the Phillips curve (1), that is $x^* = (1 - \beta)\pi^*/\kappa$.

I now characterize $i_t(\overline{\pi}^*)$ and $i_t(\hat{\pi}^*)$, the paths for nominal interest rates under the two different regimes. The same notation is used for π and x to denote the dependence on the inflation target ($\pi_t(\overline{\pi}^*)$, $\pi_t(\hat{\pi}^*)$, $x_t(\overline{\pi}^*)$ and $x_t(\hat{\pi}^*)$).

In two special cases - if prices are assumed to be flexible or the weight assigned to output stabilization λ_x is zero - the characterization of optimal policy is simple. The inflation rate is always set equal to its target level since either the output gap is zero (if prices are flexible) or not a concern (if $\lambda_x = 0$). The nominal interest then equals $r_t^n + \overline{\pi}^*$ without a target change and $r_t^n + \hat{\pi}^*$ with the new target. Thus the central bank immediately reduces the nominal interest by $\overline{\pi}^* - \hat{\pi}^* > 0$ to implement the lower inflation rate.

[6] All variables, except for inflation, are log deviations from the their trend values.

Proposition 1 (Two special cases) *If either prices are flexible or* $\lambda_x = 0$*, the nominal interest rate is uniformly lower in the new regime:* $i_t(\hat{\pi}^*) - i_t(\overline{\pi}^*) = \hat{\pi}^* - \overline{\pi}^* < 0$*.*

An immediate adjustment of inflation, nominal interest rates and output to their new target levels is also optimal in a model with sticky prices and $\lambda_x > 0$ if there are no cost-push shocks ($u_t \equiv 0$) and the output target is consistent with the inflation target ($x^* = (1-\beta)\pi^*/\kappa$). If one of these two assumptions is relaxed - there are cost-push shocks or $x^* \neq (1-\beta)\pi^*/\kappa$ - an inflation-output trade-off exists. The optimal adjustment of inflation to its new target level is then only gradually.

But whether the adjustment of inflation is immediate or not, the implications for the path of nominal interest rates always has one property: if the new inflation target is lower ($\hat{\pi}^* < \overline{\pi}^*$), then the nominal interest rate is uniformly lower $i_t(\hat{\pi}^*) - i_t(\overline{\pi}^*) < 0$ for all t.

The reason is that it is optimal to uniformly and right away lower the inflation rate when the inflation target is decreased. The Fisher equation - the nominal interest rate i equals inflation π plus the real interest rate r - then implies that nominal interest rates track the inflation rate. As a consequence, nominal interest rates are lowered uniformly and right away. This optimal policy avoids the costly aggregate demand channel, which prescribes that real interest rates should be increased to contract output and thus lower inflation. Indeed, the real interest rate is (weakly) lower for a lower inflation target ($r_t(\hat{\pi}^*) - r_t(\overline{\pi}^*) \leq 0$ for all t). By the Fisher equation ($i = r + \pi$) a lower r leads to lower nominal interest rates by itself. However, it turns out in the quantitative exploration of the general model in the subsequent sections, that the real interest rate only moves within narrow bands around its steady state level. The quantitatively important reason for lower nominal interest rates are thus lower inflation rates and not lower real interest rates.

The result that nominal interest rates are lowered holds for any size of pricing frictions, parameterized through κ. But the optimal policy changes if prices become less (κ increases) or more sticky (κ decreases). For example, a smaller κ decreases $|\pi_t(\hat{\pi}^*) - \pi_t(\overline{\pi}^*)|$, this means that it is optimal to slow down the speed of convergence to the new inflation target. The same arguments apply to an increase in λ_x, the weight on output in the loss function. A higher λ_x slows down adjustment, that is it lowers $|\pi_t(\hat{\pi}^*) - \pi_t(\overline{\pi}^*)|$. This result is consistent with proposition 1, which considers the extreme case $\lambda_x = 0$: If the weight on output is zero, immediate adjustment is optimal. Another interpretation of this result is that both a weak (a high λ_x) and a tough (a low λ_x) central banker decrease nominal interest rates and only the speed of the disinflation process differs.[7]

To get an analytical characterization of optimal policy, I assume that the zero bound on nominal interest rates is not binding. I can then derive all results for arbitrary sequences of shocks with a simple outcome. Additivity of shocks and the linear-quadratic nature of the problem imply that the differences $\pi_t(\hat{\pi}^*) - \pi_t(\overline{\pi}^*)$, $x_t(\hat{\pi}^*) - x_t(\overline{\pi}^*)$ and $i_t(\hat{\pi}^*) - i_t(\overline{\pi}^*)$ are unaffected by shocks. But the assumption that the zero bound on nominal interest rates is not binding is needed since the optimal sequences π_t, x_t and i_t are affected by shocks.

Proposition 2 (No cost-push shocks) *Assume that the zero bound on nominal interest rates is never binding.*
Without cost-push shocks ($u_t \equiv 0$), the nominal interest is uniformly lower in the new regime: $i_t(\hat{\pi}^*) - i_t(\overline{\pi}^*) < 0$ for all $t \geq 0$.
If in addition $x^* = (1-\beta)\pi^*$, both the inflation rate and the nominal interest rate are adjusted immediately to their new target levels, $\pi_t = \hat{\pi}^*$ and $i_t(\hat{\pi}^*) - i_t(\overline{\pi}^*) = \hat{\pi}^* - \overline{\pi}^* < 0$ for all $t \geq 0$.

Proposition 3 (Cost-push shocks) *Assume that the zero bound on nominal interest rates is never binding. With cost-push shocks the nominal interest is uniformly lower in the new regime: $i_t(\overline{\pi}^*) - i_t(\hat{\pi}^*) < 0$ for all $t \geq 0$.*

I so far made the standard assumption in the literature that expectations are rational. McCallum (2005) however argues that inflation expectations adjust slowly to a regime change. If this view of the economy is also what central bankers have in mind then central bankers' conventional wisdom could rely on some form of adaptive expectations. The next section shows that this is not the case. If inflation expectations are linked to current inflation, optimal policy in the New Keynesian model is still not consistent with conventional wisdom.

[7] See for example Backus and Driffill (1985), Barro (1986) and Ball (1995b) for models where policy makers can be either weak or tough.

2.1 Adaptive Expectations

The model is the same as in the previous section except for one difference. Expected inflation $E_t\pi_{t+1}$ is linked to current inflation here:

$$E_t\pi_{t+1} = (1-\gamma)\pi_{t+1} + \gamma\pi_t, \tag{4}$$

for some $\gamma \in [0,1]$, whereas expectations are rational in the previous section, $E_t\pi_{t+1} = \pi_{t+1}$. The formulation in this section includes both the case of purely adaptive expectations if $\gamma = 1$ and the case of rational expectations if $\gamma = 0$. However, an intermediate value of $\gamma \in (0,1)$ presumably describes the data best, since agents, as Erceg and Levin (2003) document for the Volcker disinflation, adapted their inflation expectations to the shift in monetary policy and did not base their expectations on current inflation rates only.[8]

Two equations then describe an equilibrium:

$$\pi_t = \kappa x_t + \beta((1-\gamma)\pi_{t+1} + \gamma\pi_t) \tag{5}$$

$$x_t = x_{t+1} - \sigma(i_t - ((1-\gamma)\pi_{t+1} + \gamma\pi_t) - r^n). \tag{6}$$

Note that I set all shocks equal to zero (the same arguments as in the previous section would establish that results would be unchanged if shocks were added).

Again, the policy experiment is to implement an inflation target $\hat{\pi}^*$ that is lower than the current inflation target $\bar{\pi}^*$. An optimal policy is then a sequence π_t and x_t which minimizes the loss function

$$\sum_{t=0}^{\infty} \beta^t[(\pi_t - \pi^*)^2 + \lambda_x(x_t - x^*)^2], \tag{7}$$

subject to the two constraints (5) and (6).

The next proposition states that allowing for adaptive expectations does not change the main conclusions of this section. Nominal interest rates are lowered to implement a lower inflation target.

[8] In numerical examples (analytical results are not available) I also allowed for a learning component $\rho(\pi_t - \pi_{t-1})$ so that $E_t\pi_{t+1} = (1-\gamma)\pi_{t+1} + \gamma(\pi_t + \rho(\pi_t - \pi_{t-1}))$. The conclusions of the paper remain unchanged.

Proposition 4 *Assume that the zero bound on nominal interest rates is never binding. Then nominal interest rates are uniformly lower in the new regime:* $i_t(\hat{\pi}^*) - i_t(\overline{\pi}^*) < 0$ *for all* $t \geq 0$.

The reason for this result is the same as in the case with rational expectations. It is always optimal to uniformly lower inflation in response to a drop in the inflation target. The Fisher equation then implies that nominal interest rates have to be uniformly lowered as well.

This reasoning invalidates the intuition that nominal interest should be increased to signal that the central bank is tough on inflation. Instead, a central bank who wants to be tough on inflation - bring down inflation fast and put a small weight on output - should decrease nominal interest rates fast. An increase in nominal interest rates on the other hand would only signal higher future inflation rates.

Another assumption that I make throughout the paper is that of full commitment to future policies. Although this is the standard assumption in New Keynesian models, a literature, initiated by Kydland and Prescott (1977) and Barro and Gordon (1983), assumes that the government does not have the ability to commit to future choices, but can re-optimize every period. In the next subsection I show that adopting this assumption does not change the conclusions of this paper.

2.2 Discretionary Monetary Policy

When the policymaker re-optimizes every period in the basic New Keynesian model described above, the first-order condition in period t is:

$$(\pi_t - \pi^*) + \frac{\lambda_x}{\kappa}(\frac{\pi_t - \beta\pi_{t+1}}{\kappa} - x^*), \tag{8}$$

where I already incorporated that inflation expectations are rational. Since the choice problem is the same in every period, the optimal level of inflation is the same for all t. The discretionary inflation π^{DMP} then equals

$$\pi^{DMP} = \frac{\pi^* + x^* \frac{\lambda_x}{\kappa}}{1 + (1-\beta)\frac{\lambda_x}{\kappa^2}}. \tag{9}$$

The result that the inflation rate is constant implies that both the output level and the nominal interest are constant as well, and leads to the following proposition.

Proposition 5 *The nominal interest rate is immediately adjusted to its new level and is uniformly lower in the new regime:* $i_t(\hat{\pi}^*) - i_t(\overline{\pi}^*) = \frac{\hat{\pi}^* - \overline{\pi}^*}{1+(1-\beta)\frac{\lambda_x}{\kappa^2}} < 0.$

For optimal policy in a New Keynesian model to be consistent with conventional wisdom it is necessary that one of the two following conditions hold. It is either optimal to increase inflation or at least inflation expectations have to increase if the inflation target is lowered. The model with rational expectations, the model with adaptive expectations and the model with time-inconsistent policy all do not satisfy these conditions.

3 The General Model

Giannoni and Woodford (2004)(GW) extend the Rotemberg and Woodford (1997) sticky price model to allow for sticky wages, indexation of wages and prices to the lagged price index and habit persistence in private consumption expenditures. I use their linearized model except for one feature. GW assume that expenditures decisions are predetermined two quarters in advance and prices and wages are predetermined one quarter in advance. To simplify notation I omit this complication and assume that there are no decision lags. Section 5 shows that this assumption is inessential for the results.

3.1 Optimal Consumption Decisions

Optimal consumption decisions imply that the intertemporal consumption Euler equation holds. With habit persistence (that is current utility depends on $x_t - \eta x_{t-1}$ and not on the output gap x_t only[9]), the linearized version of the Euler equation is a generalization of the IS-equation (2) and has the form

$$\tilde{x}_t \quad = \quad E_t \tilde{x}_{t+1} - \varphi^{-1} E_t (i_t - \pi_{t+1} - r_t^n), \tag{10}$$

[9]GW assume that current utility depends on the household's own past consumption level, and not on that of other households, this means they have an internal rather than an external habit.

where $\tilde{x}_t = (x_t - \eta x_{t-1}) - \beta\eta E_t(x_{t+1} - \eta x_t)$, i_t is the nominal interest rate at t, π_t is the inflation rate at t and r_t^n is the real interest rate that would prevail if prices and wages are flexible. In a steady state $r_t^n = 1/\beta - 1$.[10] The coefficient $0 \leq \eta \leq 1$ is the degree of habit persistence and φ^{-1} is the intertemporal elasticity of substitution, adjusted for habit persistence.

Without habit persistence ($\eta = 0$) equation (10) reduces to the standard Euler/IS equation (2). With habit persistence ($\eta > 0$), an increase in the output gap x_t decreases marginal utility in period t (which also depends on x_{t-1}) and decreases marginal utility in period t+1 (which also depends on x_{t+1}). This is why $\tilde{x}_t$ and not only x_t is the relevant variable for the Euler equation.

3.2 Optimal Wage and Price Setting

A discrete version of the optimizing model of staggered pricing setting following Calvo (1983), modified to allow for indexation of the price index during periods of no re-optimization, leads to the following log-linearized aggregate-supply relation:

$$\pi_t - \gamma_p \pi_{t-1} = \xi_p \omega_p x_t + \xi_p (w_t - w_t^n) + \beta E_t(\pi_{t+1} - \gamma_p \pi_t), \quad (11)$$

where $0 \leq \gamma_p \leq 1$ is the degree of automatic indexation to the (lagged) aggregate price index. The parameters ξ_p and ξ_w measure the degree to which prices and wages are sticky respectively. Specifically, ξ_p indicates the responsiveness of price-inflation to the gap between marginal cost and current prices and ξ_w indicates the responsiveness of wage-inflation to the gap between households' marginal rate of substitution (the wage on agents' supply curve) and current wages. The coefficient ω_p is the quantity-elasticity of marginal cost and ω_w is the quantity-elasticity of households' marginal rate of substitution.[11] The real wage is denoted w_t and w_t^n is the "natural real wage", the equilibrium real wage when both wages and prices are flexible. Sticky wages thus induce real disturbances $w_t - w_t^n$, that

[10] Note, that I do not subtract the steady state values from i and r. All other variables, except for inflation, are still log-deviations from their steady state value.

[11] For more details on these coefficients, in particular how they are related to features such as the frequency of price and wage adjustment, see GW and Woodford (2003).

have similar consequences as the cost-push shocks in section 2.

To model sticky wages, GW follow Erceg et al. (2000) and assume staggered wage setting analogous to the staggered price setting in Calvo (1983). This gives the second equation of the supply side:

$$\pi_t^w - \gamma_w \pi_{t-1} = \xi_w(\omega_w x_t + \varphi \tilde{x}_t) + \xi_w(w_t^n - w_t) + \beta E_t(\pi_{t+1}^w - \gamma_w \pi_t), \tag{12}$$

where π^w is nominal wage inflation that satisfies the identity

$$w_t = w_{t-1} + \pi_t^w - \pi_t. \tag{13}$$

Equation (12) can equivalently be rewritten as

$$\pi_t^w - \gamma_w \pi_{t-1} = \kappa_w[(x_t - \delta x_{t-1}) - \beta\delta E_t(x_{t+1} - \delta x_t)] + \xi_w(w_t^n - w_t) + \beta E_t(\pi_{t+1}^w - \gamma_w \pi_t), \tag{14}$$

where $0 \leq \delta \leq \eta$ is the smaller root of $\eta\varphi(1+\beta\delta^2) = [\omega_w + \varphi(1+\beta\eta^2)]\delta$ and $\kappa_w = \xi_w \eta\varphi/\delta$.

3.3 Loss function and constraints

To compute the optimal deflation policy, I have to specify a loss function and I simplify the constraints (10), (11), (13) and (14), which together characterize an equilibrium for a given policy.

To isolate the effects of a lower inflation target, I abstract from any real shocks.[12] I set w_t^n and r_t^n to their steady state values, $w_t^n = 0$ and $r_t^n = 1/\beta - 1$.

Next, I solve equation (13) for $\pi_t^w = w_t - w_{t-1} + \pi_t$ and substitute it into the wage setting equation (14). Perfect-foresight equilibrium paths for inflation, output, wages and nominal interest rate are therefore characterized through two aggregate supply equations

$$\pi_t - \gamma_p \pi_{t-1} = \xi_p \omega_p x_t + \xi_p w_t + \beta(\pi_{t+1} - \gamma_p \pi_t), \tag{15}$$

$$w_t - w_{t-1} + \pi_t - \gamma_w \pi_{t-1} = \kappa_w[(x_t - \delta x_{t-1}) - \beta\delta(x_{t+1} - \delta x_t)] - \xi_w w_t + \beta(w_{t+1} - w_t + \pi_{t+1} - \gamma_w \pi_t), \tag{16}$$

[12] I showed in section 2, that shocks do not affect $i_t(\hat{\pi}^*) - i_t(\overline{\pi}^*)$, the difference between nominal interest rates with and without a change in the inflation target.

and through the Euler/IS equation

$$i_t - \pi_{t+1} - (1/\beta - 1) \;\; = \;\; \varphi(\tilde{x}_{t+1} - \tilde{x}_t), \tag{17}$$

The objective of monetary policy is assumed to minimize deviations of price-inflation, output, wage-inflation and nominal interest rates from its target values. The discounted loss function then equals

$$\sum_{t=0}^{\infty} \beta^t [(\pi_t - \pi^*)^2 + \lambda_x (x_t - x^*)^2 + \lambda_w (\pi_t + w_t - w_{t-1} - \pi_w^*)^2 + \lambda_i (i_t)^2], \tag{18}$$

where π^*, x^* and π_w^* are the target values for price-inflation, output and wage-inflation respectively and where I used the identity $\pi_t^w = w_t - w_{t-1} + \pi_t$. Note, that the objective function (18) depends on the levels of π and x whereas in Woodford (2003), quasi-differences $x_t - \eta x_{t-1}$, $\pi_t - \gamma_p \pi_{t-1}$, $\pi_t^w - \gamma_w \pi_{t-1}$ enter the objective function. While I abstract from this complication here, I will discuss in section 5 that this simplification is inessential for the results. Following Woodford (2003), I also allow for monetary frictions here (reflected by the term $\lambda_i (i_t)^2$ in the loss function), but I will also consider $\lambda_i = 0$ in the sensitivity analysis.[13]

4 Optimal Disinflation

The policy experiment is the same as in section 2. At date $t = 0$ the inflation target π^* is lowered from $\bar{\pi}^*$ to $\hat{\pi}^*$. The monetary authority chooses sequences for the inflation rate $\{\pi_t\}_{t=0}^{\infty}$, the output gap $\{x_t\}_{t=0}^{\infty}$, wages $\{w_t\}_{t=0}^{\infty}$ and nominal interest rates $\{i_t\}_{t=0}^{\infty}$ to minimize the loss function (18) such that the constraints for optimal price setting (15), optimal wage setting (16) and optimal consumption decisions (17) are fulfilled.

[13] The fact that $\lambda_i > 0$ allows Woodford (2003) to derive an optimal interest rate rule, which is equivalent to a first-order condition is his optimization problem. He finds that lower(higher) inflation rates require lower(higher) nominal interest rates. This findings is consistent with the results of my paper, once it is recognized that it is optimal to lower inflation rates during a disinflation. Another interpretation of a positive λ_i is that central banks apparently care about reducing the volatility of nominal interest rates (Goodfriend (1991)).

The main difference between the models in sections 2 and 3 is that past values, for example lagged inflation rates, affect current allocations in the general model but not in the simple model. This makes it necessary to specify initial conditions for these variables. I assume that the economy is in a steady state with $\pi = \bar{\pi}^*$ before the policy change. The steady state values of the three other endogenous variables - output, wages and nominal interest rates - have to fulfill the steady state versions of equations (15), (16) and (17). This choice seems reasonable since the paper wants to capture a regime change, where a low inflation rate is the new target after a period of high inflation.

In the appendix I compute the first-order conditions and show that they, together with the three constraints (15), (16) and (17), can equivalently be expressed as a difference equation of the form

$$z_{t+1} = Az_t, \tag{19}$$

where $z_t = (\pi_{t-1}, \pi_t, x_{t-2}, x_{t-1}, x_t, w_{t-1}, w_t, \mu_{t-1}, \mu_t, \chi_{t-1}, \chi_t, i_{t-1}, i_t)$ and some matrix A. The next step makes it necessary to compute the eigenvalues and eigenvectors of the matrix A, which is possible only once numerical values for all parameters are specified. The details are again laid out in the appendix. I now describe how I choose the parameters.

4.1 Parameter Values

The parameters are exactly those found in the quarterly model of Giannoni and Woodford (2004). They follow Rotemberg and Woodford (1997) and choose the parameters to minimize the distance between the theoretical model impulse response function and the estimated VAR impulse response functions. Table 1 shows their results. Two parameters

Table 1: Estimated Parameter Values from Giannoni and Woodford (2004).

η	γ_p	γ_w	ξ_p	ξ_w	φ	ω_w
1	1	1	0.002	0.0042	0.7483	19.551

are calibrated directly in GW. β is set equal to 0.99 to match an steady state real interest rate of one percent. They set $\omega_p = 1/3$ to match the output elasticity with respect to hours. Two parameters, κ_w and δ, are functions of other parameters as described in the last section. The values are shown in table 2.

What remains to be determined are the welfare weights for output, wages and nominal interest rates (the welfare weight for inflation is normalized to 1). There are two natural possibilities. The first one is to choose the welfare weights such that the loss function is a second-order approximation to the utility function of the representative agent. The second possibility is to pick the welfare weights to reflect conventional wisdom about the central bank's objective - stabilization of inflation and output. Since I want to compare the optimal policy in the theoretical model to conventional wisdom about policy I follow the latter possibility and consider the first possibility, utility-based welfare weights, in Section 5. I thus assume high weights for both inflation and output stabilization and I set $\lambda_x = 1$. For λ_w, GW find that wage inflation stabilization on top of price inflation stabilization is of minor importance for the central bank. I thus choose $\lambda_w = 0.004$ as in GW. Finally, Woodford (2003) finds $\lambda_i = 0.077$, but he considers this to be an upper bound. Since a higher value for λ_i implies that lowering nominal interest rates becomes more important, I choose $\lambda_i = 0.02$, the lower bound in Woodford (2003). In addition, to isolate the effect of the change in the inflation target, I report results for the difference in nominal interest rates $i_t(\hat{\pi}^*) - i_t(\overline{\pi}^*)$ as in section 2 (there I considered the difference to isolate the effect of target changes from the effects of shocks.).

Table 2: Additional Parameter Values.

β	λ_x	λ_w	λ_i	ω_p	κ_w	δ
0.99	1	0.004	0.02	1/3	0.0883	0.0356

4.2 Results

Now that I have specified the model and its parameters, I can compute the optimal policy response to a change in the inflation target for this model. The details of the procedure are described in the appendix. Figure 1 shows the optimal sequence of nominal interest rates i_t

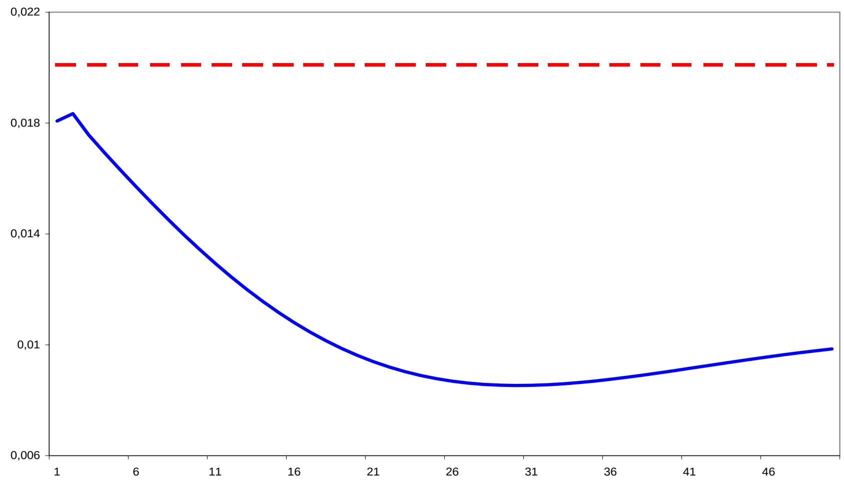

Figure 1: Optimal nominal interest rates to implement a drop in the inflation target. The dashed line is the steady state nominal interest rate without a target change. Parameter values are given in tables 1 and 2.

to implement the inflation target $\hat{\pi}^* = 0$. The dashed line at $\overline{\pi}^* + 1/\beta - 1 = 0.01 + 1/\beta - 1 = 0.02$ is the nominal interest rate in the steady state before the target change. The nominal interest rate after the target change takes lower values lower than $\overline{\pi}^* + 1/\beta - 1 = 0.02$ in all periods $t \geq 0$. This says that nominal interest rates should be uniformly lowered to implement a lower inflation target.

As discussed above, to isolate the effect of the change in the inflation target (for example from the need to reduce monetary frictions), figure 2 shows the difference $i_t(\hat{\pi}^*) - i_t(\overline{\pi}^*)$ between nominal interest rates with and without a target change. Again the nominal interest rates are uniformly lower if the inflation target is smaller. This conclusion does not change for very high welfare weights on output, such as $\lambda_x = 10$ or for very low values of $\lambda_x = 0.002$.

The central bank can use two channels to lower the inflation rate, the aggregate demand channel and the expectations channel. Since the aggregate demand channel involves higher real interest rates and thus unnecessary output contractions, it is optimal to use the expectations channel only. The Fisher equation then implies that the nominal interest rate

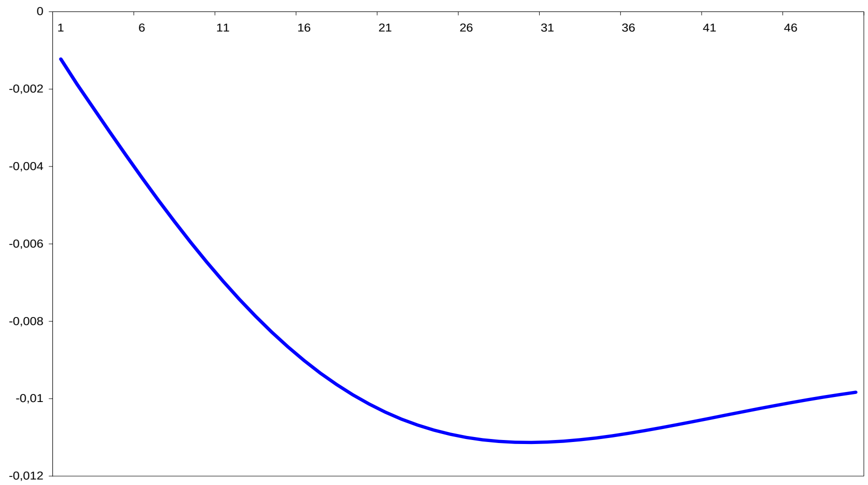

Figure 2: Difference $i_t(\hat{\pi}^*) - i_t(\overline{\pi}^*)$ in optimal nominal interest rates for inflation targets $\hat{\pi}^*$ and $\overline{\pi}^*$, where $\hat{\pi}^* < \overline{\pi}^*$. Parameter values are given in tables 1 and 2.

tracks the inflation rate. If the inflation target is decreased, it is optimal to uniformly lower the inflation rate (Figure 3 shows the optimal inflation path) and therefore to uniformly lower nominal interest rates.

Although the aggregate demand channel is not used, the Phillips curve implies that output cannot be fully stabilized. The fluctuations in output are however quite small, as Figure 4 shows. Output never falls below -0.3% and is never higher than 0.1% (relative to its steady-state level). Consistent with conventional wisdom, after a drop in the inflation target, output is lower, at least for the first 8 years (= 32 quarters), than output without such a drop.

At the same time, the real interest rate hardly moves. Figure 5 shows that the real interest rate stays within a 0.1 percentage point band around its steady-state value $\frac{1}{\beta} - 1$, and has almost converged to it after a year. A comparison of the real interest rate with and without a drop in the inflation target strengthens this observation. The difference of the real interest rate between these two regimes is about 0.001%, i.e. virtually zero. In other words, variations in real interest rates are kept to a minimum and the aggregate demand channel is inactive.

The quantitative results in this section show that the theoretical conclusions drawn

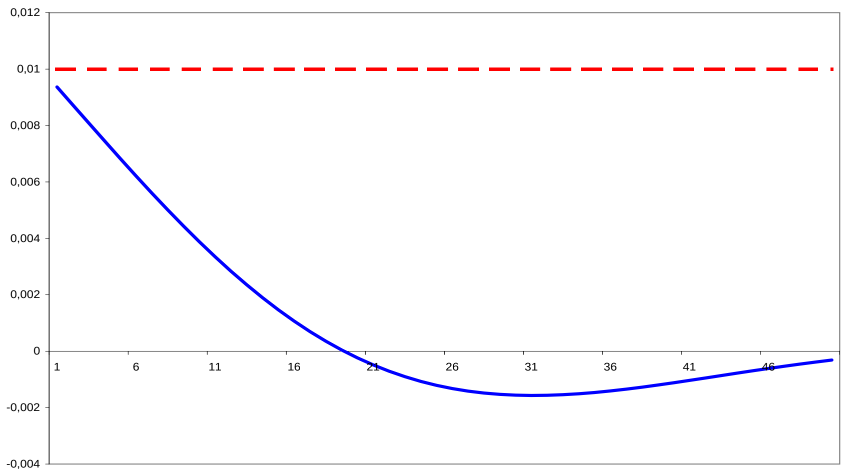

Figure 3: The solid line is the optimal inflation rate path after a drop in the inflation target. The dashed line is the inflation rate $\overline{\pi}^*$ before the target change. Parameter values are given in tables 1 and 2.

from the restricted model in section 2 do not change once features such as habit persistence, indexation and sticky wages are added. Nominal interest are uniformly lowered to implement a lower inflation target.

5 Sensitivity Analysis

In this section I investigate the robustness of the results for different parameter values, for different welfare criteria and if decision lags in consumption, prices and wages are allowed for.

For each robustness check I only show the results for the path of nominal interest rates but the conclusions drawn from these experiments remain unchanged if I consider the difference $i_t(\hat{\pi}^*) - i_t(\overline{\pi}^*)$ (as in figure 2).

5.1 Parameter Values

GW choose the parameters to minimize the distance between the model and empirical impulse response functions. Two robustness checks seem necessary. First, the parameters

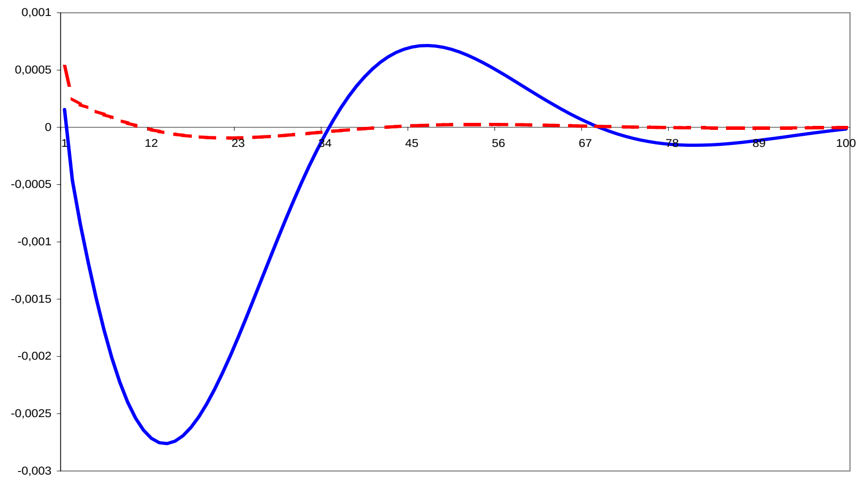

Figure 4: The solid line is the optimal output after a drop in the inflation target. The dashed line is optimal output path withou a target change.

ξ_p, ξ_w, φ and ω_w are imprecisely estimated. Second, to estimate parameters from impulse response functions, it is necessary to specify the length of the horizon following the shock. The results in table 1 are based on a horizon of 12 quarters, but this choice is somewhat arbitrary. I now compute how different parameter values and how different time horizons change the results.

Estimated Parameters for Alternative Horizons

GW provide estimates for different horizons namely 6, 8, 12, 16 and 20 quarters. Table 3 in the appendix shows their estimated values and figure 6 shows the results for all 5 possible horizons including the benchmark, 12 quarters. It is quite evident that the choice of the horizon has a negligible impact on the results.

Different Parameter Values

Whereas the upper bound of 1 is binding for the parameters η (degree of habit persistence), γ_p (degree of price indexation) and γ_w (degree of wage indexation), the other parameters, ξ_p, ξ_w, φ and ω_w, are imprecisely estimated. I therefore vary these four parameters to check the robustness of the results. Note that η, γ_p and γ_w are not only precisely estimated but that, as already demonstrated in section 2, eliminating persistence (setting $\eta = \gamma_p = \gamma_w = 0$) would not change the conclusions.

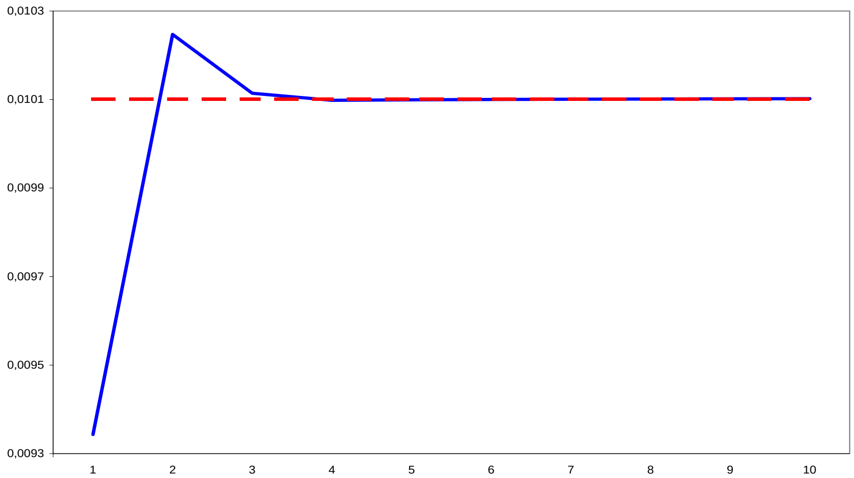

Figure 5: The solid line is the optimal path of the real interest rate a drop in the inflation target. The dashed line is the steady state real interest rate $\frac{1}{\beta} - 1$.

Figure 7 to 10 show the results, separately for each parameter, when ξ_p takes values $0.001, 0.002$ and 0.1 ξ_w takes values $0.001, 0.0042$ and 0.1, φ takes values $0.1, 0.7483$ and 10 and ω_w takes values $5, 19.551$ and 35. Note that the second number is the benchmark value (table 1). In all cases the conclusions do not change although the parameter variations are quite big and would lead to a deterioration of the model's empirical performance, when assessed through comparing impulse responses in the model and in the data. If for example $\varphi = 10$, changes in nominal interest rates would have very small output effects in contrast to the hump-shaped response in the data. The high value for φ also affects the path of nominal interest rates, which are immediately sharply decreased. The small effect of nominal interest rates on output implies that output is not a concern for monetary policy and monetary frictions become more important ($\lambda_i = 0.02$). This leads to a drop in nominal interest rates in figure 9 if $\varphi = 10$, similar to the results in figure 1.

5.2 Predetermined Decisions

There is a small difference between the model used in section 4 and the model estimated in GW. GW assume that consumption decisions are predetermined two periods in advance and prices and wages are set one period in advance, whereas there are no decision lags in consumption, prices or wages in section 4. In this section I check whether different assumptions about the predeterminedness of agents' decisions change the results. Specifically I compute the optimal policy when

a) consumption is predetermined two periods in advance and prices and wages are predetermined one period in advance (as in GW)

b) consumption, prices and wages are predetermined one period in advance

c) consumption is predetermined one period in advance and prices and wages are predetermined two periods in advance

and compare it to the benchmark in section 4. Figure 11 shows that the conclusion is robust for all four assumptions about predeterminedness: nominal interest rates are uniformly lowered. Note, that the figures show nominal interest rates only for periods where households make a consumption/saving decision (for example in case a), the interest rates in the first two periods are not determined).

If all variables are predetermined one period in advance (case b)), then the path for optimal nominal interest is shifted one period to the right. This is a general property of decision lags. More periods of predeterminedness just shift the paths of all variables to the right. This is why decision lags improve the impulse response of the model since in the data some variables respond with a lag only. For the experiment in this paper - lowering the inflation target - the length of the decision lags does not affect the conclusions.

5.3 Different Loss Criteria

The loss/welfare function used in section 4 is based on two assumptions. First, only levels (e.g. π_t) and not quasi-differences ($\pi_t - \gamma_p \pi_{t-1}$) matter. Second, the welfare weights λ_x, λ_w and λ_i used in section 2 may still not coincide with a 'classic' central banks objective function. I address these possible concerns in this section.

Different Loss Function

If minimizing the loss function is equivalent to maximizing the utility of a representative household (or a quadratic approximation thereof as in GW), then quasi-differences and not levels enter the objective function. Specifically, the objective is to minimize the deviations of $\pi_t - \gamma_p \pi_{t-1}$, $\pi_t^w - \gamma_p \pi_{t-1}$ and $x_t - \delta x_{t-1}$ from their target levels instead of minimizing the deviations of π_t, π_t^w and x_t from their target levels. I show now that this is inessential for the results. For output x this is not very surprising since δ is quite small (equal to 0.0356). For price and wage inflation, the coefficients are at their maximum level $\gamma_p = \gamma_w = 1$. A high value of γ_p leads to two problems. First, if $\gamma_p = 1$, the steady state level of inflation is irrelevant for welfare since only changes in inflation, $\pi_t - \pi_{t-1}$, matter. The experiment 'lowering the inflation target' would be meaningless. Second, since inflation is of minor importance, monetary frictions dominate optimal policy, what can render the zero-bound on nominal interest rates binding in some periods. I therefore assume that $\gamma_p = 0.9$. Figure 12 shows both the results when $x_t - \delta x_{t-1}$ matters, when $\pi_t - \gamma_p \pi_{t-1}$ and $\pi_t^w - \gamma_p \pi_{t-1}$ matter and compare them to the benchmark. As expected, the path of nominal interest rates does not change much if habit persistence enters the welfare function. In contrast adding indexation to the welfare function changes the path substantially. As explained above, indexation reduces the importance of reducing inflation relative to reducing monetary frictions. The long-run optimal nominal interest rate then equals 0.356% and the optimal path of nominal interest rates is shifted downwards. The size of this shift would be smaller for higher values of λ_x or lower values of λ_i but wouldn't change the conclusion. Nominal interest rates are always uniformly lowered.

I next compute the optimal policy for the full GW specification of the loss function. Quasi-differences of price-inflation, wage-inflation and output enter the objective function with the weights as in table 2, except for $16\lambda_x = 0.0026$ and λ_i. Monetary frictions are no concern here, what is equivalent to $\lambda_i = 0$. I set $\gamma_p = 0.99 < 1$ to make the experiment 'lowering the inflation target' meaningful. Figure 14 shows the result for two different assumptions about predeterminedness. First, as in GW, when consumption is predetermined two periods and price and wages are determined one period in advance and second when there is no predeterminedness. Note, that in the first case interest rates are shown from period 2 on only, since they are not determined before. Since $\gamma_p = 0.99$ the adjustment of inflation to its new target is very slow and so is the adjustment of nominal interest rates. But the conclusion remains unchanged.

Different Welfare Weights

GW choose the welfare weights such that minimizing the loss function is equivalent to maximizing a quadratic approximation of the expected utility of a representative household. The resulting loss function differs from the 'classic' objective function of a central bank, which stabilizes output and inflation only (see for example Clarida et al. (1999)). I therefore consider a 'classic' loss function $(\pi_t - \pi^*)^2 + (x_t - x^*)^2$. Figure 13 shows the result. Again the Fisher effect dominates optimal policy and nominal interest rates are lowered to implement a lower target.

6 Conclusion

The results in this paper imply that there is an inconsistency between central bankers' conventional wisdom and one implication of New Keynesian models. Conventional wisdom suggests that nominal interest rates should be increased to implement a lower inflation target. In contrast, the optimal policy in a New Keynesian model is to uniformly lower nominal interest rates. This result holds both in a basic New Keynesian model with sticky prices and in extensions of this model, such as Giannoni and Woodford (2004), which allow for sticky wages, price and wage indexation and habit formation in consumption.

The reason is that the aggregate demand channel, which raises real interest rates to contract aggregate demand what then leads to lower inflation rates is too costly relative to the expectations channel. The expectations channel sets nominal interest rates consistent with the private sector's expectations of lower inflation rates in the future. Real interest rates are basically constant and thus a costly output contraction is avoided.

Assuming adaptive instead of rational expectations does not change these conclusions. Inflation again falls uniformly and so do inflation expectations (with a lag) and also nominal interest rates. These experiments suggest that in models where the low inflation target is not perfectly credible, as for example in Ball (1995a), nominal interest rates should still be decreased. The reason is that, as in the model with adaptive expectations, both inflation and inflation expectations come down uniformly. Indeed, in these kind of models, imperfect credibility leads to slower decreases of expected inflation and thus to slower decreases of nominal interest rates, because of uncertainty about a potential reversal to a higher inflation regime. Thus, whereas imperfect credibility changes the output implications of a disinflation, it cannot change the conclusions of this paper.[14]

Recent work by Christiano et al. (2005) adds two more feature to the model of Giannoni and Woodford (2004): capital formation (with adjustment costs and variable utilization rates) and firms must borrow working capital to finance their wage bill.

Adding capital puts an additional constraint on the real interest rate - it has to equal the marginal productivity of capital - and thus makes the aggregate demand channel less effective. The expectations channel is not affected since real interest rates are basically kept constant anyway when nominal interest rates track the inflation rate. These arguments are consistent with the experiments in Christiano et al. (2005). When, for example, they drop the assumption of variable capital utilization, they find larger increases of inflation in response to a decrease in nominal interest rates (see row 1 of figure 6 in Christiano et al. (2005)).

The assumption that firms finance their wage bill through borrowing capital would further strengthen my results. An increase in nominal interest rates increases firms' marginal costs and thus leads to price increases. Adding this feature to the model would make increasing nominal interest rates an even worse choice to implement a lower inflation target. Again Christiano et al. (2005) conduct experiments that support these arguments. When they drop the assumption that firms have to borrow their wage bill, a decrease in nominal interest rates leads to larger increases in inflation rates (see row 5 of figure 6 in Christiano et al. (2005)).

[14] There is large literature that assumes that agents have imperfect knowledge of the economy. The key result in this literature is that the persistence of inflation (expectations) is raised and that the trade-off between inflation and output stabilization is distorted. This result for example helps to account for inflation scars (Orphanides and Williams (2005b), leads to different conclusion about optimal monetary policy (Orphanides and Williams (2004, 2005a, 2006), Gaspar et al. (2006)), and improves the fit of DSGE models (Milani (2005)).

The reasoning in this paper suggests that two deviations from the New Keynesian model seem promising to reconcile the conventional wisdom with the predictions of an economic model. First, changes in nominal interest rates should have strong effects on real interest rates. A one percent increase in nominal interest rates would then not lead to a one percent increase in inflation rates. Second, changes in nominal interest rates should, for an unchanged real interest rate, have output effects. A decrease in nominal interest rates would then necessarily lead to an output expansion and thus to some upward pressure on prices.

New Keynesian models, as shown in this paper, are not a promising candidate to overcome these problems due to the absence of a prominent role for liquidity (effects). Motivated by these arguments I develop a quantitative model in Hagedorn (2006), which indeed has a strong liquidity effect. The new monetary transmission mechanism in this paper is thus a candidate to reconcile central bankers' conventional wisdom with economic theory.

Appendix

Proof of Results in Section 2

An optimal perfect-foresight policy $\{\pi_t, x_t\}_{t \geq o}$ minimizes

$$\sum_{t=0}^{\infty} \beta^t [(\pi_t - \pi^*)^2 + \lambda_x (x_t - x^*)^2], \tag{20}$$

such that

$$\pi_t \quad = \quad \kappa x_t + \beta \pi_{t+1} + u_t. \tag{21}$$

The IS-equation is not included since minimizing monetary frictions does not enter the objective function in section 2. It just determines i once the optimal π and x are known. I solve the aggregate-supply relation for x_t

$$x_t = (\pi_t - \beta \pi_{t+1} - u_t)/\kappa \tag{22}$$

and plug it into the objective function

$$\min_{\pi_t, t\geq 0} \sum_{t=0}^{\infty} \beta^t [(\pi_t - \pi^*)^2 + \lambda_x((\pi_t - \beta\pi_{t+1} - u_t)/\kappa - x^*)^2], \tag{23}$$

where $\pi^* = \overline{\pi}^*$ without a regime change and $\pi^* = \hat{\pi}^*$ with a regime change. The first order necessary and sufficient conditions are:

$$\begin{aligned} \pi_t - \pi^* + \lambda_x/\kappa(x_t - x_{t-1}) &= 0 \qquad \text{for } t \geq 1 \qquad (24) \\ (\pi_0 - \pi^*) + \lambda_x/\kappa(x_0 - x^*) &= 0 \qquad \text{for } t = 0. \qquad (25) \end{aligned}$$

The first order condition (24) yields a difference equation for π:

$$\pi_{t+1} = \frac{\kappa^2}{\lambda_x \beta}(\pi_t - \pi^*) + \frac{1+\beta}{\beta}\pi_t - \frac{1}{\beta}\pi_{t-1} - \frac{1}{\beta}(u_t - u_{t-1}) \qquad \text{for } t \geq 1. \tag{26}$$

The characteristic polynomial, $z^2 - z(\frac{\kappa^2}{\lambda_x \beta} + \frac{1+\beta}{\beta}) + \frac{1}{\beta}$ has one root $\delta = b/2 - \frac{\sqrt{b^2 - 4/\beta}}{2} \in (0,1)$, where $b = \frac{\kappa^2}{\beta \lambda_x} + \frac{1+\beta}{\beta}$.

A solution to (26) is (plugging in verifies the claim)

$$c\delta^t + \delta^t \sum_{k=1}^{t} \eta_k \delta^{-k} \frac{1 - (\delta^2\beta)^{-(t-k+1)}}{1 - (\delta^2\beta)^{-1}} + \pi^* \tag{27}$$

for some c, where $\eta_t = -\frac{1}{\beta}(u_{t-1} - u_{t-2})$ for $t \geq 2$, $\eta_1 = \eta_0 = 0$.

c is chosen to satisfy the initial condition, the first order condition with respect to π_0 (equation (25)),

$$\pi_1 = \frac{\kappa^2}{\lambda_x \beta}(\pi_0 - \pi^*) + \frac{1}{\beta}(\pi_0 - u_0 - \kappa x^*) \tag{28}$$

This gives

$$c = \frac{(\beta - 1)\pi^* + u_0 + \kappa x^*}{1 + \kappa^2/\lambda_x - \beta\delta} \tag{29}$$

Without cost push shocks and $x^* = (1-\beta)\pi^*/\kappa$, $u_0 = 0$, c equals 0 and thus $\pi_t = \pi^*$ and $x_t = x^*$ for all t. It follows that $i_t^* = r_t^n + \pi^*$ and $i_t(\hat{\pi}^*) - i_t(\overline{\pi}^*) = \hat{\pi}^* - \overline{\pi}^* < 0$.

With cost-push shocks and $x^* = (1-\beta)\pi^*/\kappa$, c equals $\frac{u_0}{\beta\delta - \kappa^2/\lambda_x + 1}$. Since c is independent from π^*, $\pi_t(\hat{\pi}^*) - \pi_t(\overline{\pi}^*) = \hat{\pi}^* - \overline{\pi}^*$ and $x_t(\hat{\pi}^*) - x_t(\overline{\pi}^*) = 0$. It follows that $i_t(\hat{\pi}^*) - i_t(\overline{\pi}^*) =$

$\hat{\pi}^* - \overline{\pi}^* < 0.$

If x^* is independent from π^*, the difference in c equals

$$c(\hat{\pi}^*) - c(\overline{\pi}^*) = \frac{(1-\beta)(\hat{\pi}^* - \overline{\pi}^*)}{\beta\delta - \kappa^2/\lambda_x - 1} > 0 \tag{30}$$

Since $1 - \frac{(1-\beta)}{1+\kappa^2/\lambda_x - \beta\delta} > 0$,

$$\pi_t(\hat{\pi}^*) - \pi_t(\overline{\pi}^*) = (\hat{\pi}^* - \overline{\pi}^*)(1 - \delta^t \frac{(1-\beta)}{1+\kappa^2/\lambda_x - \beta\delta}) < 0 \tag{31}$$

From equation (21) output equals

$$x_t(\pi^*) = \frac{\pi_t(\pi^*) - \beta\pi_{t+1}(\pi^*) - u_t}{\kappa} \tag{32}$$

and output growth equals

$$x_{t+1}(\pi^*) - x_t(\pi^*) = \frac{\pi_{t+1}(\pi^*) - \beta\pi_{t+2}(\pi^*) - \pi_t(\pi^*) + \beta\pi_{t+1}(\pi^*) - u_{t+1} + u_t}{\kappa} \tag{33}$$

Plugging the solution for π_t from (31) into (33) and simplifying yields:

$$\begin{aligned}(x_{t+1}(\hat{\pi}^*) - x_t(\hat{\pi}^*)) - (x_{t+1}(\overline{\pi}^*) - x_t(\overline{\pi}^*)) &= \frac{\hat{\pi}^* - \overline{\pi}^*}{\kappa}\alpha\delta^t(1-\delta)(1-\beta\delta) \\ &< 0,\end{aligned} \tag{34}$$

where $\alpha = \frac{(1-\beta)}{1+\kappa^2/\lambda_x - \beta\delta}$.

Plugging the solution for inflation and output growth into the IS-equation

$x_t = x_{t+1} - \sigma(i_t - \pi_{t+1} - r_t^n)$ yields:

$$\begin{aligned}i_t(\hat{\pi}^*) - i_t(\overline{\pi}^*) &= \frac{(x_{t+1}(\hat{\pi}^*) - x_t(\hat{\pi}^*)) - (x_{t+1}(\overline{\pi}^*) - x_t(\overline{\pi}^*))}{\sigma} + \pi_{t+1}(\hat{\pi}^*) - \pi_{t+1}(\overline{\pi}^*) \\ &< 0\end{aligned}$$

Effects of changes in κ on $\mid \pi_t(\hat{\pi}^*) - \pi_t(\overline{\pi}^*) \mid$:

$$\frac{\partial \mid \pi_t(\hat{\pi}^*) - \pi_t(\overline{\pi}^*) \mid}{\partial\kappa} = (\hat{\pi}^* - \overline{\pi}^*)\delta^{t-1}\alpha\{\frac{\partial\delta}{\partial\kappa} - \frac{\delta}{1+\kappa^2/\lambda_x - \beta\delta}(2\kappa/\lambda_x - \beta\frac{\partial\delta}{\partial\kappa}\}$$

Since

$$\frac{\partial\delta}{\partial\kappa} = \frac{\kappa}{\beta\lambda_x}(1 - \frac{b}{\sqrt{b^2 - 4/\beta}}) < 0$$

it follows that

$$\frac{\partial \mid \pi_t(\hat{\pi}^*) - \pi_t(\overline{\pi}^*) \mid}{\partial \kappa} > 0$$

The same computations show that

$$\frac{\partial \mid \pi_t(\hat{\pi}^*) - \pi_t(\overline{\pi}^*) \mid}{\partial \lambda_x} < 0$$

Proof of Results in Section 2.1

The same arguments used for rational expectations prove the results in the model with adaptive expectations.

Solving the aggregate-supply relation (5) for x_t and plugging it into the objective function gives

$$\min_{\pi_t, t\geq 0} \sum_{t=0}^{\infty} \beta^t[(\pi_t - \pi^*)^2 + \lambda_x((\pi_t - \beta((1-\gamma)\pi_{t+1} + \gamma\pi_t))/\kappa - x^*)^2], \tag{35}$$

where (6) again just determines i once the optimal π and x are found.

For $\gamma < 1$ ($\gamma = 1$ will be treated below), the first order conditions yield a difference equation for π

$$\pi_{t+1} = -\tilde{b}\pi_t - \frac{1}{\beta}\pi_{t-1} - k(\pi^*) \qquad \text{for } t \geq 1, \tag{36}$$

where $\tilde{b} = \frac{1-\beta\gamma}{\beta(1-\gamma)} + \frac{1-\gamma}{\beta(1-\beta\gamma)} + \frac{\kappa^2}{\lambda_x(1-\gamma)(1-\beta\gamma)}$, $k(\pi^*) = \pi^*\frac{\kappa^2}{\beta\lambda_x(1-\beta\gamma)(1-\gamma)} + x^*\frac{\kappa\gamma(1-\beta)}{\beta(1-\gamma)(1-\beta\gamma)}$ and an initial condition (the first-order condition for π_0):

$$(\pi_0 - \pi^*) + \frac{\lambda_x(\pi_0 - \beta((1-\gamma)\pi_1 + \gamma\pi_0) - x^*)}{\kappa} \quad = \quad 0. \tag{37}$$

Computing the roots of the associated characteristic polynomial gives $\tilde{\delta} = \tilde{b}/2 - \frac{\sqrt{\tilde{b}^2 - 4/\beta}}{2}$ as the only root in $(0,1)$. The long-run value of inflation π^{ss} is the solution to the steady-state version of the first-order condition of π_t:

$$\pi^{ss} = \frac{\pi^*\kappa^2 - x^*\lambda_x\beta\gamma(1-\beta)}{\kappa^2 + (1-\beta)^2\lambda_x\gamma} \tag{38}$$

The solution for π_t then equals

$$\tilde{c}\tilde{\delta}^t + \pi^{ss}, \tag{39}$$

where $\tilde{c}$ is chosen to satisfy the initial condition (37). This gives[15]

$$\tilde{c}(\pi^*) = \frac{\pi^{ss}(1-\beta) - \kappa x^*}{\beta(1-\gamma) + \frac{\beta\gamma - 1}{\tilde{\delta}}}. \tag{40}$$

Since $\tilde{c}(\hat{\pi}^*) - \tilde{c}(\overline{\pi}^*) > 0$, $\pi^{ss}(\hat{\pi}^*) - \pi^{ss}(\overline{\pi}^*) < 0$ and $1 - \frac{1-\beta}{\beta(1-\gamma)+\frac{\beta\gamma-1}{\tilde{\delta}}} > 0$ it follows that

$$\begin{aligned}
\pi_t(\hat{\pi}^*) - \pi_t(\overline{\pi}^*) &= (\pi^{ss}(\hat{\pi}^*) - \pi^{ss}(\overline{\pi}^*)) + \tilde{\delta}^t(\tilde{c}(\hat{\pi}^*) - \tilde{c}(\overline{\pi}^*)) && (41)\\
&< (\pi^{ss}(\hat{\pi}^*) - \pi^{ss}(\overline{\pi}^*)) + (\tilde{c}(\hat{\pi}^*) - \tilde{c}(\overline{\pi}^*)) && (42)\\
&= (\pi^{ss}(\hat{\pi}^*) - \pi^{ss}(\overline{\pi}^*))(1 - \frac{1-\beta}{\beta(1-\gamma) + \frac{\beta\gamma-1}{\tilde{\delta}}}) && (43)\\
&< 0 && (44)
\end{aligned}$$

I now use the solution for π and (5) to derive an expression for output growth.

$$\begin{aligned}
x_{t+1}(\pi^*) - x_t(\pi^*) &= \frac{c(\pi^*)}{\kappa}\{(1-\beta\gamma)\tilde{\delta}^{t+1} - \beta(1-\gamma)\tilde{\delta}^{t+2} - (1-\beta\gamma)\tilde{\delta}^t + \beta(1-\gamma)\tilde{\delta}^{t+1}\}\\
&= \frac{c(\pi^*)}{\kappa}\tilde{\delta}^t(1-\tilde{\delta})\{\beta\gamma - 1 + \tilde{\delta}\beta(1-\gamma)\}
\end{aligned}$$

Thus the difference in output growth equals

$$\begin{aligned}
(x_{t+1}(\hat{\pi}^*) - x_t(\hat{\pi}^*)) - (x_{t+1}(\overline{\pi}^*) - x_t(\overline{\pi}^*)) &= \frac{\tilde{c}(\hat{\pi}^*) - \tilde{c}(\overline{\pi}^*)}{\kappa}\tilde{\delta}^t(1-\tilde{\delta})\{\beta\gamma - 1 + \tilde{\delta}\beta(1-\gamma)\}\\
&< 0
\end{aligned}$$

Plugging the solution for inflation and output growth into the IS-equation (6) again yields the result that nominal interest rates are uniformly lower if the inflation target is lowered:

$$i_t(\hat{\pi}^*) - i_t(\overline{\pi}^*) < 0 \tag{45}$$

If $\gamma = 1$ then $x_t = \frac{1-\beta}{\kappa}\pi_t$ and thus π_t is chosen to minimize $(\pi_t - \pi^*)^2 + \lambda_x(\frac{1-\beta}{\kappa}\pi_t - x^*)^2$. Thus an immediate adjustment of inflation to the steady state value $\frac{\kappa^2\pi^* + \kappa\lambda_x(1-\beta)x^*}{\kappa^2 + \lambda_x(1-\beta)^2}$ is optimal. As a consequence output and nominal interest rates also immediately adjust to their steady state levels. In particular, nominal interest rates are lowered when a lower inflation target is implemented, $i_t(\hat{\pi}^*) - i_t(\overline{\pi}^*) < 0$ for all $t \geq 0$.

[15] Note that for $\gamma = 0$, $\tilde{c} = c$ since $\beta\delta = b\beta + \frac{1}{\delta}$.

Proof of Proposition 5 in Section 2.2

The result, established in the main text, that π is constant at π^{DMP} implies that the output level is constant at

$$x^{DMP} = \frac{\pi^{DMP}(1-\beta)}{\kappa}, \tag{46}$$

and the nominal interest rate is constant at

$$\pi^{DMP} + r^n. \tag{47}$$

The difference between nominal interest with and without a drop in the inflation target then equals

$$i_t(\hat{\pi}^*) - i_t(\overline{\pi}^*) = \pi^{DMP}(\hat{\pi}^*) - \pi^{DMP}(\overline{\pi}^*) = \frac{\hat{\pi}^* - \overline{\pi}^*}{1 + (1-\beta)\frac{\lambda_x}{\kappa^2}} < 0. \tag{48}$$

Proof of Results in Section 4

Minimizing the loss function (18) such that the constraints (15), (16) and (17) are fulfilled results in the following first-order conditions:

$$\begin{aligned} &2(\pi_t - \pi^*) + 2\lambda_w(\pi_t + w_t - w_{t-1} - \pi_w^*) \\ &+\mu_t(1+\beta\gamma_p) - \mu_{t-1} - \gamma_p\beta\mu_{t+1} + \chi_t(1+\beta\gamma_w) \\ &-\chi_{t-1} - \gamma_w\beta\chi_{t+1} + \psi_{t-1}/\beta = 0 \end{aligned} \tag{49}$$

$$\begin{aligned} &2\lambda_w(\pi_t + w_t - w_{t-1} - \pi_w^*) - \beta 2\lambda_w(\pi_{t+1} + w_{t+1} - w_t - \pi_w^*) \\ &-\xi_p\mu_t + \chi_t(1+\beta+\xi_w) - \beta\chi_{t+1} - \chi_{t-1} = 0 \end{aligned} \tag{50}$$

$$\begin{aligned} &2\lambda_x(x_t - x^*) - \mu_t\xi_p\omega_p - \chi_t(1+\beta\delta^2)\kappa_w + \chi_{t+1}\beta\delta\kappa_w + \chi_{t-1}\delta\kappa_w \\ &+\eta\phi\beta\psi_{t+1} - \phi(\eta+1+\beta\eta^2)\psi_t + \psi_{t-1}/\beta\phi(1+\beta\eta^2-\beta\eta) = 0 \end{aligned} \tag{51}$$

$$\psi_t - 2\lambda_i i_t = 0, \tag{52}$$

where $\beta^t\mu_t$, $\beta^t\chi_t$ and $\beta^t\psi_t$ are the Lagrange multipliers for constraints (15), (16) and (17) respectively.

The next step is to rewrite the dynamics of this system as

$$z_{t+1} = Az_t, \tag{53}$$

for $z_t = (\pi_{t-1}, \pi_t, x_{t-2}, x_{t-1}, x_t, w_{t-1}, w_t, \mu_{t-1}, \mu_t, \chi_{t-1}, \chi_t, i_{t-1}, i_t)$ and a matrix A.

7 equations, four first-order conditions and three constraints, describe the system. As a first step I use condition (52) to solve for $\psi_t = -2\lambda_i i_t$ and substitute ψ_t into the remaining 6 equations. Next I solve 6 equations – the remaining first order conditions for x_t, π_t and w_t and the constraints (17) for period $t-1$ and (15) and (16) for period t – for x_{t+1}, π_{t+1}, w_{t+1}, n_{t+1}, μ_{t+1} and χ_{t+1} (Note that I, for pure mathematical convenience, shift the IS-equation by one period).

This expresses x_{t+1}, π_{t+1}, w_{t+1}, n_{t+1}, μ_{t+1} and χ_{t+1} as functions of x_t, x_{t-1} π_t, w_t, n_t, μ_t and χ_t. Rewriting these expressions in matrix form and adding the identities $\pi_t = \pi_t, \ldots$ results in a matrix equation $z_{t+1} = Az_t$. For example, the first row of A has a 1 in the second column and zeros elsewhere. The second row is then the expression for $\pi_{t+1}, \ldots$.

Now that the dynamics are rewritten in matrix form, I can compute the eigenvalues and eigenvectors. After plugging in parameter values, I find 6 non-explosive (different) eigenvalues $\nu_1, \ldots \nu_6$ and corresponding (column) eigenvectors $v_1, \ldots v_6$. This is true for the benchmark (see table 1) and all the robustness checks in section 5.

Let $\bar{x}$, $\bar{\pi}$, $\bar{w}$, $\bar{i}$, $\bar{\mu}$ and $\bar{\chi}$ be the steady state solution to the 6 equations (this means all time indices are dropped). Set $\mathcal{L}$ equal to the column vector $(\bar{\pi}, \bar{\pi}, \bar{x}, \bar{x}, \bar{x}, \bar{w}, \bar{w}, \bar{\mu}, \bar{\mu}, \bar{\chi}, \bar{\chi}, \bar{i}, \bar{i})$.

The theory of difference equation implies that there exist numbers $c_1, \ldots c_6$ such that every solution to $z_{t+1} = Az_t$ has the form:

$$z_t = \sum_{k=1}^{6} c_k \nu_k^t v_k + \mathcal{L}. \tag{54}$$

Since π_t is the second entry of z, the solution for π_t is the second entry of the right hand side.

What remains to be determined are the six numbers $c_1, \ldots c_6$. Plugging in the solutions for x_t, π_t, w_t, n_t, μ_t and χ_t from (54) into the first order conditions for x_0, π_0 and w_0 and the constraint (15), (16) and (17) for period 0, results in 6 equations in the 6 unknowns $c_1, \ldots, c_6$.

This gives six values $c_1^*, \ldots c_6^*$. Note, that the period 0 constraints and thus the period 0 first order conditions are different from the period t constraints, a well known fact from optimal taxation (see Chari & Kehoe (1999)).

The unique solution to the optimal deflation problem then equals

$$z_t = \sum_{k=1}^{6} c_k^* \nu_k^t v_k + \mathcal{L}, \tag{55}$$

where as before the solution for π_t can be read off in row 2, the solution for x_t in row 5, the solution for w_t in row 7 and the solution for i_t in row 13.

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Estimated Parameters for Alternative Horizons

Table 3: Parameter Values from Giannoni and Woodford (2004) estimated for different horizons following the shock.

	Horizon				
	6	8	12	16	20
Parameters					
η	1	1	1	1	1
γ_p	0.937	1	1	1	1
γ_w	1	1	1	1	1
ξ_p	0.0065	0.0036	0.0020	0.0017	0.0013
ξ_w	0.0073	0.0056	0.0042	0.0081	0.0203
φ	0.5739	0.6635	0.7483	0.7769	0.7502
ω_w	19.559	19.545	19.551	9.4925	4.2794
κ_w	0.1510	0.1167	0.0883	0.0890	0.1152
δ	0.0277	0.0318	0.0356	0.0707	0.1322

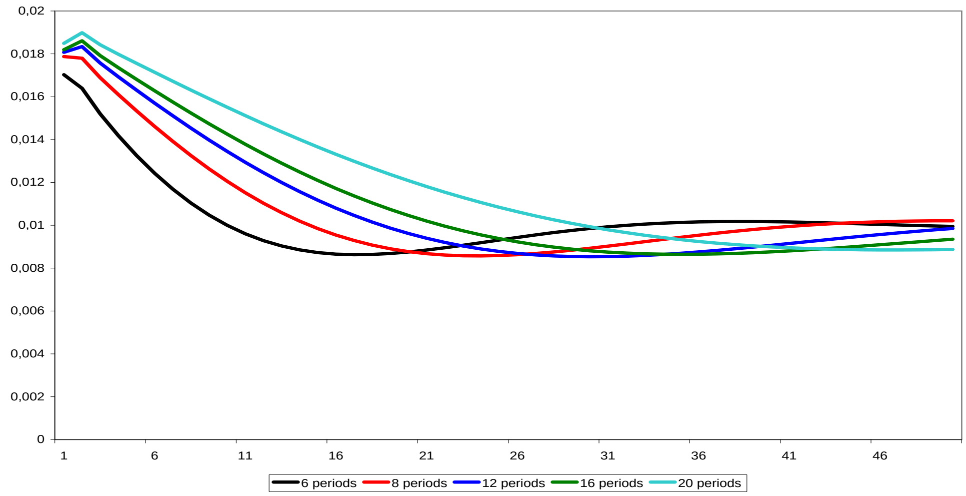

Figure 6: Optimal nominal interest rates for a welfare weight $\lambda_x = 1$ and estimates based on impulse response functions with horizons of 6,8,12,16,20 quarters following the shock.

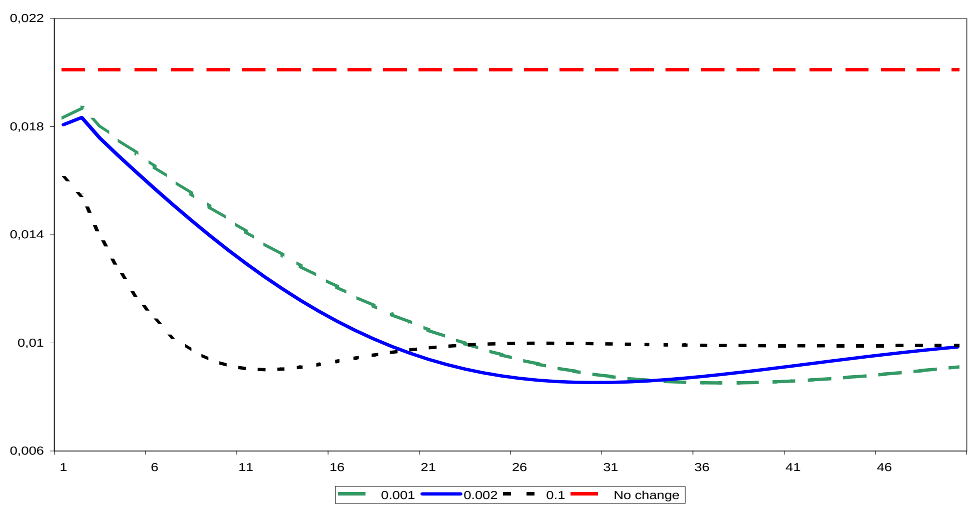

Figure 7: Optimal nominal interest rates for a welfare weight $\lambda_x = 1$ and $\xi_p = 0.001, 0.002, 0.1$. The horizontal line at 0.02 is the steady state nominal interest rate without a target change.

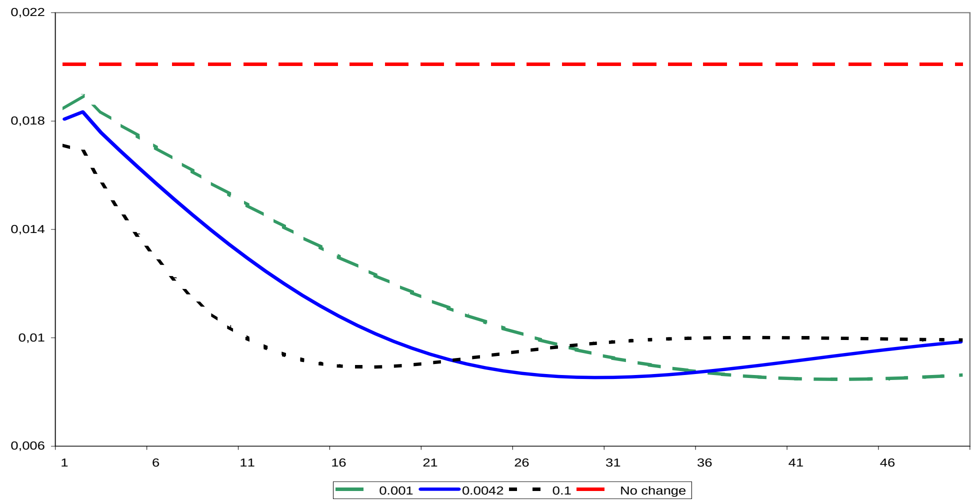

Figure 8: Optimal nominal interest rates for a welfare weight $\lambda_x = 1$ and $\xi_w = 0.001, 0.0042, 0.1$. The horizontal line at 0.02 is the steady state nominal interest rate without a target change.

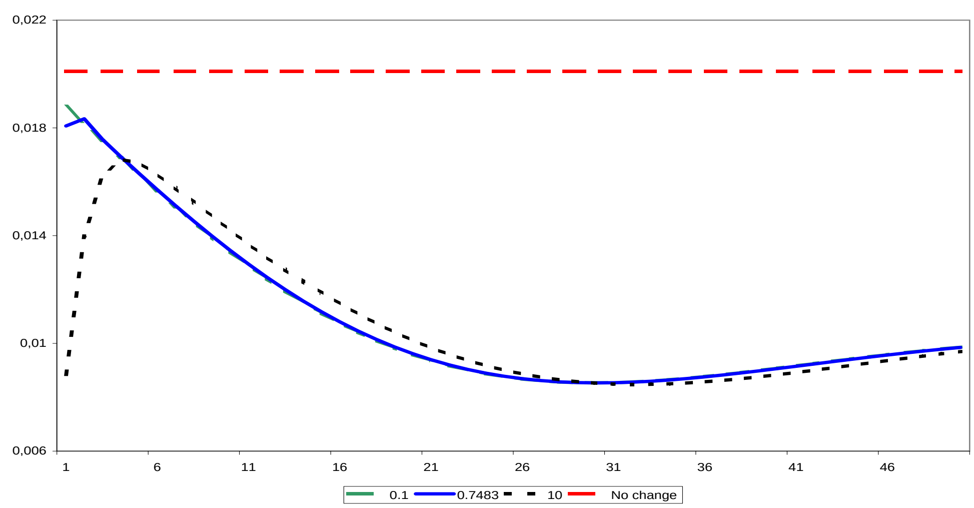

Figure 9: Optimal nominal interest rates for a welfare weight $\lambda_x = 1$ and $\varphi = 0.1, 0.7483, 10$. The horizontal line at 0.02 is the steady state nominal interest rate without a target change.

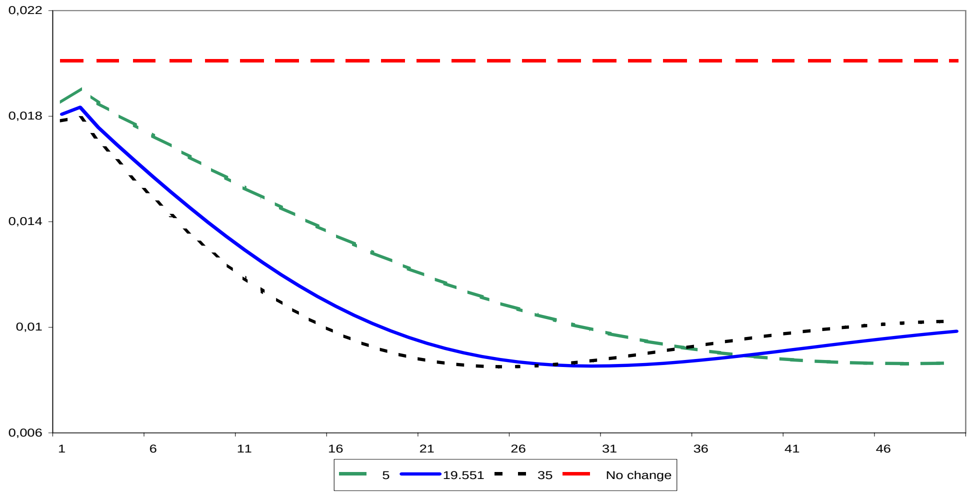

Figure 10: Optimal nominal interest rates for a welfare weight $\lambda_x = 1$ and $\omega_w = 5, 19.551, 35$. The horizontal line at 0.02 is the steady state nominal interest rate without a target change.

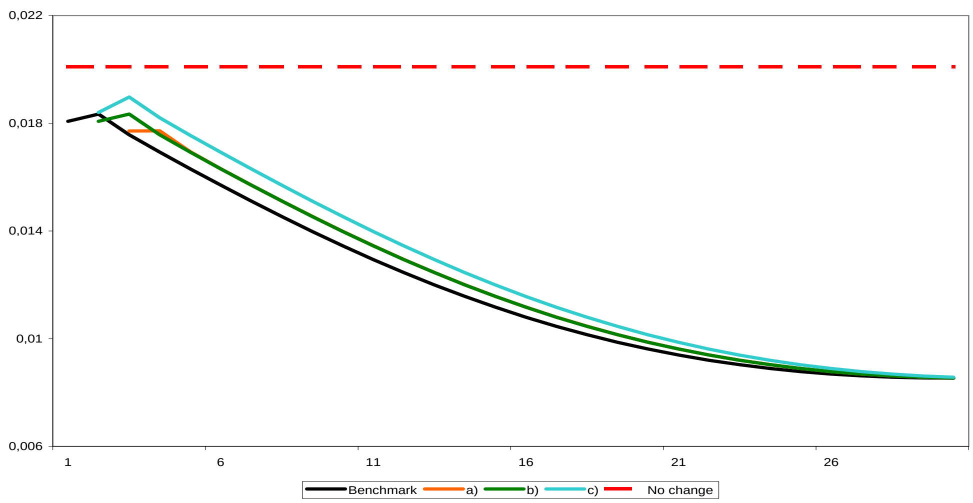

Figure 11: Optimal nominal interest rates for various degrees of predeterminedness. No predeterminedness (benchmark), a) consumption 2 periods, wages and prices 1 period, b) consumption, prices and wages 1 period, c) consumption 1 period, wages and prices 2 periods. The horizontal line at 0.02 is the steady state nominal interest rate without a target change.

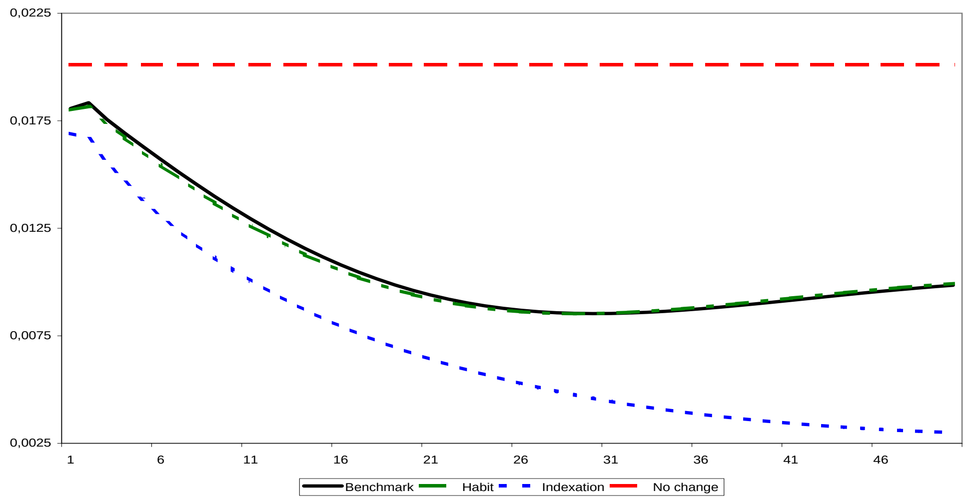

Figure 12: Optimal nominal interest rates when quasi-differences $x_t - \delta x_{t-1}$ (habit) and $\pi_t - \gamma_p \pi_{t-1}$ and $\pi_t^w - \gamma_p \pi_{t-1}$ (indexation) enter the welfare function. The horizontal line at 0.02 is the steady state nominal interest rate without a target change. The benchmark is for $\lambda_x = 1$.

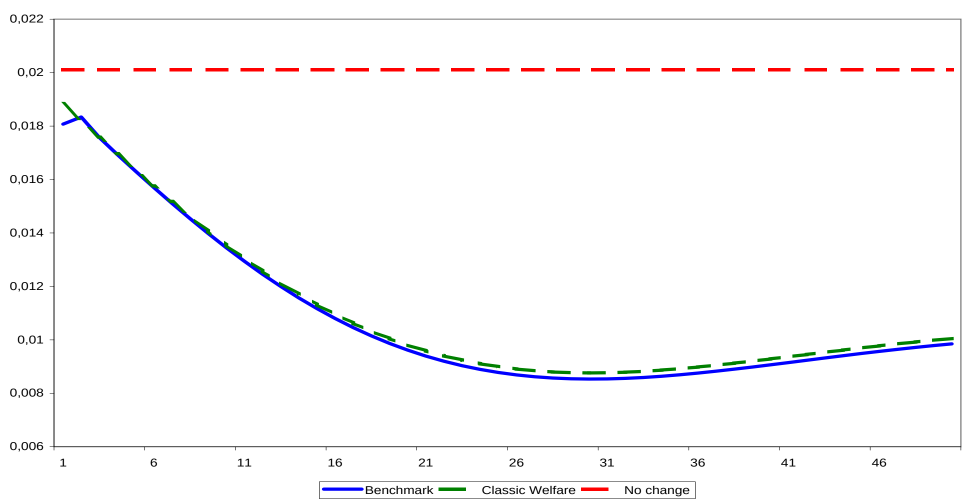

Figure 13: Optimal nominal interest rates for a 'classic' loss (period) function $(\pi_t - \pi^*)^2 + (x_t - x^*)^2$. The horizontal line at 0.02 is the steady state nominal interest rate without a target change. The benchmark is for $\lambda_x = 1$.

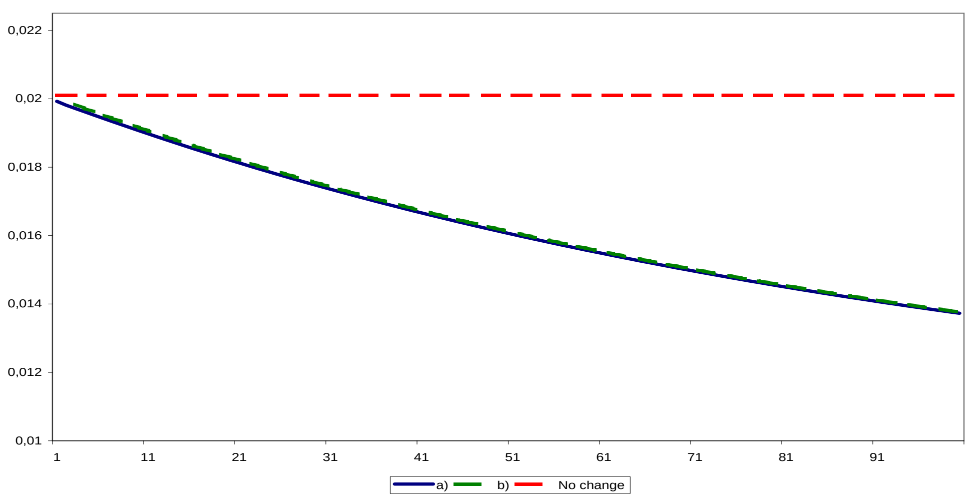

Figure 14: Optimal nominal interest rates for the GW-welfare specification when a) there is no predeterminedness and b) consumption is predetermined 2 periods and wages and prices are predetermined 1 period in advance. The horizontal line at 0.02 is the steady state nominal interest rate without a target change.