Heterogeneity and Labor Demand in an Equilibrium Search Model

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Abstract: This paper combines an equilibrium search framework for heterogenous labor with a production function approach and endogenous job offer rates to analyze the effects of minimum wages on employment and wages when unemployment is both frictional and structural. This also allows to analyze wage dispersion within and between skill groups in a unified framework. We obtain analytical solutions for wage distributions within and between skill groups. For a more general class of models we simulate wage distributions often containing masspoints. We calculate the effects of minimum wages on structural unemployment and on the two types of wage dispersion. We find that labor unions minimum wages cause structural unemployment when firms labor demand reacts.

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1 Introduction

This paper combines an equilibrium search framework for heterogenous labor with a production function approach and endogenous job offer rates to analyze the effects of minimum wages on employment and wages when unemployment is both frictional and structural. This also allows to analyze wage dispersion within and between skill groups in a unified framework. We obtain analytical solutions for wage distributions within and between skill groups. For a more general class of models we simulate wage distributions often containing masspoints. We calculate the effects of minimum wages on structural unemployment and on the two types of wage dispersion. We find that labor unions minimum wages cause structural unemployment when firms labor demand reacts.

"Structural" reasons are often identified when discussing the high unemployment in European countries. This is surely correct, because the term is not very well defined and used for very different things. Structural in the sense of this paper refers to a situation where unemployment results from the market power of labor unions, setting wages too high. This type of structural reason for the high unemployment is not as easily agreed upon. In addition, it is not seen as being equally important in all European countries. However, a big part of the literature on unemployment in Germany argues that labor unions influence is an important factor in explaining the situation in Germany (see, eg, ...). We take up that point and model this explicitely.

But, economists would generally agree that even in countries where structural reasons are said to be very important, frictional unemployment is present. Frictional in the use of this (and most other) paper(s) means that individuals look for "good" jobs and they do not know where the good jobs are located. This process is time-consuming, generating waiting phases in "frictional" unemployment and potential market power for firms. We take up this idea as well and explicitely model frictional unemployment in this paper.

So far, the interplay between frictional and structural unemployment has largely been ignored in the micro-literature.[1] Both strands of the literature are largely parallel and have not inspired each other. With our approach to commonly model the two types of unemployment in an equilibrium search context, we fill this research gap. One of the few exceptions in the literature dealing with both types of unemployment in this context is Koning, Ridder, and Van den Berg (1995). They assume however, that the labor market consists of a number of separate submarkets, which do not interact. Structural unemployment in their model means that in one submarket there is no production at all, because the value of leisure exceeds the (constant) marginal productivity of the individuals. That is not the case in our model, where heterogenous agents interact.

[1] We stress the term micro, because the matching-literature does allow for the two types of unemployment. However, since we are also interested in residual wage inequality we do not take up this strand of literature.

A big part of the literature on unemployment differentials between the US and Europe focuses on the diverging experience in unemployment *and* wage dispersion. Skill-biased technological change and other factors are said to have increased the demand for high-skilled labor as compared to low-skilled labor and has therefore altered the wage structure. Numerous studies point to the fact that wage dispersion between groups increased as a reaction at least in the US (for an overview, see eg Katz and Autor (1999) or Acemoglu (2002)). Because wage regressions generally only explain little of the wage variation however (cf. Mortensen (2003)), recently the literature has become interested in the development of wage differentials within groups with comparable attributes. These literature has been stimulated by the seminal paper of Burdett and Mortensen (1998) (BM hereafter). Empirical studies show that the development of wage inequality between different skill groups and within different skill groups is far from being parallel (for Germany see, eg, Fitzenberger (1999), evtl Kohn 2006, Möller 2005).[2] Clearly, this poses the question of understanding the development of both types of wage dispersion in a unified framework.

Again, the literature on wage differentials between groups and within groups are largely parallel.[3] Our model corresponds to the challenge of understanding the development of wage dispersion within and between groups in one framework. In addition, it allows to understand the development of the unconditional wage distribution as resulting from the interplay of the development within and between groups as analyzed, eg for Germany, in Kohn (2006) or Gernandt and Pfeiffer (2006).

We extend an equilibrium search framework that naturally generates frictional unemployment and residual wage inequality to allow for wage inequality between groups and structural unemployment. Model building is complicated by the fact that in order to be able to explain different unemployment rates for different skill groups and to generate wage dispersion between groups, we allow for heterogenous labor. Under strong assumptions, we obtain analytical solutions for the wage distributions of the skill groups. These are similar in shape to the Burdett and Mortensen (1998) solution.[4] For more general model settings, we must simulate solutions. Endogenous job offer rates allow us to introduce labor demand effects in this framework. Comparatively weak assumptions are required to obtain a unique negative effect of a binding minimum wage on employment. Analytical results for effects of a binding minimum wage on the wage distribution and its moments are not easily obtained, since in equilibrium the complete distribution reacts.[5]

The plan of the paper is as follows. First, we discuss some related literature. Second, we

[2] Prasad (2004) argues based on the GSOEP that the developments are parallel for groups defined by education, experience and tenure. Lemieux (2005) argues that much if the residual wage inequality is data noise and that its importance is overstated.

[3] A notable exception is eg Acemoglu (2002).

[4] When we speak about a BM-type wage distribution, we refer to a wage density that is increasing and where the wage distribution has no masspoints.

[5] Recognize that this seems to be a stylized fact of minimum wages, see Koning, Ridder, and Van den Berg (1995).

present the model to discuss afterwards an analytical solution and extensions. Then, we give simulation results and finally introduce a labor union. The last section concludes.

2 Literature

Most of the literature on search equilibrium is based on the paper of Burdett and Mortensen (1998). It is the first paper that generates frictional unemployment and continuous residual wage dispersion in a model. Two main disadvantages arise in that model. First, individuals are homogenous with respect to all relevant characteristics, especially in their marginal productivity which is in addition assumed constant. Second, most commentators critizise the equilibrium wage distribution, that the model generates, with the argument that the increasing density is counterfactual. Part of the critique, however, seems to be a misunderstanding of the model in so far as it does not generate an unconditional wage distribution. A part of the literature seems to have the opinion that the form of the conditional density does not contradict stylized facts (see eg Gautier and Teulings (2006)).

Several attempts have been made to respond to these challenges. Heterogeneity has been incorporated in the model both on the side of the individuals and on the side of the firms. On the side of the firms heterogeneity was integrated as different productivities that are either exogenous or endogenous (see Burdett and Mortensen (1998), Mortensen (2000), Bontemps, Robin, and Van den Berg (2000)). These extensions seem to be capable of producing wage densities that have a long right tail as required for empirical wage densities (see Bontemps, Robin, and Van den Berg (2000)). On the side of the workers heterogeneity has been introduced with respect to leisure preferences by Burdett and Mortensen (1998). Van den Berg and Ridder (1998) consider individuals that are different with respect to their marginal productivities, but assume that the labor market is completely segmented. Ridder and Van den Berg (1997b), building on Manning (1992) and Mortensen and Vishwanath (1994), consider the case where marginal productivity of the individuals varies with the amount of labor employed in a particular firm, ie a production function with one production factor. Postel-Vinay and Robin (2002) model jointly productive heterogeneity on the side of the employers and on the side of the employees. Their assumptions somewhat differ from the standard BM-case. Still, they maintain the assumption that marginal productivity of the individual does not depend on the employment.[6]

The first attempt of modelling several skill groups jointly and to link their employment in firms via a production function in the equilibrium search context is Holzner and Launov (2006). In addition, they allow for heterogenous firm productivities. To solve their model they have to assume supermodularity and their model does not allow for

[6]For surveys on new developments in the search literature, compare Garloff (2007), Rogerson, Shimer, and Wright (2005) and the book of Mortensen (2003).

labor demand effects via an endogenous job offer rate. Since their model entails several skill groups it generates wage inequality within and between groups. In addition, it is able to generate a long right tail for the wage density. Our model extends their results by allowing for labor demand effects, by employing different assumptions for obtaining an analytical solution and by giving simulation results.

3 An equilibrium search model with heterogenous labor

We consider a BM-type labor market, where there is a huge amount of workers of two skill groups ($i = 1, 2$ of masses N_i and firms (of mass 1). Individuals are maximizing their expected lifetime income, while firms maximize expected profits. The labor market is frictional in the sense that finding a job requires time, and in the sense that jobs are destroyed from time to time. This is reflected in the job offer rates for unemployed individuals (λ_i) and for employed individuals ($\lambda_{i,L}$) on the one hand and by the job destruction rate (δ_i) on the other hand.

3.1 Individuals behavior

We now derive the optimal behavior of the individuals. As usual the optimal behavior of the individuals is characterized by an optimal stopping property, ie a reservation wage.

Proposition 1: Individual behavior and therefore the reservation wage depends only on the marginal wage distributions.

Proof: see Appendix.

The reservation wage is given by the wage w_i for individuals employed at wage w_i (see Mortensen and Neumann (1988)). It is determined by the value equations for unemployed individuals (skill group $i = 1, 2$ and is given by:[7]

$$(1) \quad w_{i,R} = z_i + (\lambda_i - \lambda_{i,L}) \int_{w_{i,R}}^{\hat{w}_i} \frac{1 - H_i(w_i)}{r + \delta_i + \lambda_{i,L}(1 - H_i(w_i))} dw_i$$

$w_{i,R}$ is the reservation wage for an unemployed individual of skill group i, z_i is the alternative income of the unemployed, ie, unemployment benefits net of search cost, and $H_i(w_i)$ is the (marginal) wage offer distribution for skill group i.[8]

[7]Given that individuals care in their decision only about the marginal distributions, the derivation of the reservation wage is standard and can be found in Garloff (2007).

[8]This is the distribution from which an individuals draws, when it receives a job offer according to the respective job offer rate. The probability of drawing a job offer paying a wage above w_i is $(1 - H_i(w_i))$ irrespectively of whether or not an individual is employed and where it is employed.

The reservation wage completely characterizes the decision of an individual. The unemployed (employed) individual accepts all job offers that pay wages above $w_{i,R}$ (w_i) and rejects otherwise.

Given this optimal behavior, we can describe the employment dynamics for firms. Consider thy entity of firms that pay wages above a wage w_i. Let $K(w_i) \equiv N_i - U_i - L_i(w_i)$ be the employment in these firms, where U_i is the number of unemployed and $L_i(w_i)$ is the number of employed in firms paying wages below w_i. Their employment develops according to:

$$(2) \quad \dot{K}_i(w_i) = (\lambda_i U_i + \lambda_{L,i} L_i(w_i))(1 - H_i(w_i)) - \delta_i K_i(w_i)$$

Under stationarity, we have $\dot{K}_i(w_i) = 0$ and from this we can calculate the connection between the cross-sectional wage distribution $G_i(w_i) = \frac{L_i(w_i)}{(N_i - U_i)}$ [9] and the wage offer distribution $H_i(w_i)$ [10]. It is given by:

$$(3) \quad G_i(w_i) = \frac{\delta_i H_i(w_i)}{\lambda_{i,L}(1 - H_i(w_i)) + \delta_i}$$

Imposing stationarity $\dot{K}_i(w_i) = 0$ and twice differentiating the resulting condition with respect to w_i gives us:

$$(4) \quad \frac{l_1'(w_1)}{l_1(w_1)} = \frac{2\lambda_{1,L} h_1(w_1)}{(\lambda_{1,L}(1 - H_1(w_1)) + \delta_1)}$$

This is a central equation characterizing the dynamics of employment in a firm that offers a wage w_i depending on the entity of wage strategies of other firms and the frictions. This dynamics follows from the optimal behavior of individuals.

We can calculate $l_1(w_1)$ from $L_1'(w_1) = l_1(w_1) h_1(w_1)$ as

$$(5) \quad l_1(w_1) = \frac{(N_1 - U_1)\delta_1 (\lambda_{1,L} + \delta_1)}{(\lambda_{1,L}(1 - H_1(w_1)) + \delta_1)^2}$$

and $l_1'(w_1) > 0$.

Next, we consider optimal behaviour of firms.

[9] The distribution of wages when randomly sampling an individual.

[10] The wage distribution when randomly sampling a firm. These are different because firms differ in size.

3.2 Firms behavior

Firms maximize the expected profits, which are given as

$$\Pi(w_1, w_2) = y(l_1(w_1), l_2(w_2)) - w_1 l_1(w_1) - w_2 l_2(w_2) \tag{6}$$

by choosing an optimal wage-vector w. Recognize that this is similar to saying that firms choose an optimal employment level, because the wage uniquely determines the employment level (see equation (5)). Recognize that Proposition 1 implies together with the above derivations that the employment for skill group i only depends on the wage for skill group i and not on w_j and therefore the above profit function is correct.

Firms optimal choice of w_1 implies

$$\begin{aligned} \frac{\partial \Pi(w_1, w_2)}{\partial w_1} &= \frac{\partial y()}{\partial l_1}\frac{\partial l_1}{\partial w_1} - l_1(w_1) - w_1 \frac{\partial l_1()}{\partial w_1} = 0 \\ \frac{l_1'(w_1)}{l_1(w_1)} &= \frac{1}{\left(\frac{\partial y()}{\partial l_1} - w_1\right)} = \frac{1}{y'(l_1) - w_1} \end{aligned} \tag{7}$$

a condition describing a connection between the relative change of the employment density with w_i and the profit per worker.

3.3 Equilibrium

To solve the model, we now equate the conditions 7 and 4 yielding the following condition of optimality.

$$2h_1(w_1)\left(\frac{\partial y()}{\partial l_1}(H_1(w_1), H_2(w_2)) - w_1\right) + H_1(w_1) - \frac{\lambda_{1,L} + \delta_1}{\lambda_{1,L}} = 0 \tag{8}$$

In this equation we have substituted $H_i(w_i)$ for $l_1(w_1)$ as arguments in the production function. In the appendix, we show that this differential equation has in general a solution and that this solution is unique. However, we must specify the production function in order to be able to say something about the form of the solution of this differential equation. Moreover, in this form the equation looks like a partial differential equation. We will argue below that it is not a partial differential equation but only an ordinary. Assume for the moment a Cobb-Douglas form $y = Al_1^{\alpha_1} l_2^{\alpha_2}$. We can rewrite the central differential equation as

$$(9)\qquad h_1(w_1)=\frac{-H_1(w_1)+\frac{\lambda_{1,L}+\delta_1}{\lambda_{1,L}}}{2\left(\alpha_1 A\left(\frac{(N_1-U_1)\delta_1\left(\lambda_{1,L}+\delta_1\right)}{\left(\lambda_{1,L}(1-H_1(w_1))+\delta_1\right)^2}\right)^{\alpha_1-1}\left(\frac{(N_2-U_2)\delta_2\left(\lambda_{2,L}+\delta_2\right)}{\left(\lambda_{2,L}(1-H_2(w_2))+\delta_2\right)^2}\right)^{\alpha_2}-w_1\right)}$$

This equation does not look very nice either. However, we can use it to prove our proposition 2.

Proposition 2: The optimal choice of w_1 uniquely determines the optimal choice of w_2, ie $w_1 = z(w_2)$. (Deterministic wage strategy) In addition, $z' > 0$.

Proof. see Appendix.

Using this proposition considerably simplifies the problem, since it makes clear that we only have to solve an ordinary differential equation. We can replace $H_1(w_1)$ for $H_2(w_2)$ in the above equation by the transformation theorem for densities(see, eg, Mood, Graybill, and Boes (1974)). Still, the above equation looks not very nice. We do not see any possibility to find a general solution for this non-linear, non-autonomous ordinary differential equation.

4 Solving the model

4.1 An analytical solution

For an analytical solution to this problem, we now constrain ourselves to a special case with parameter restrictions. We discuss some other solution strategies in the appendix. Let us first assume that our production technology has constant returns to scale, ie $\alpha_1 + \alpha_2 = 1$. Second, we assume that the ratio of job destruction rate and job offer rate on the job is identical for both skill groups, ie

$$(10)\quad \frac{\delta_1}{\lambda_{1,L}} = \frac{\delta_2}{\lambda_{2,L}}$$

This assumption can be justified on empirical reasons (see, Van den Berg and Ridder (1998)). The empirical findings of Van den Berg and Ridder (1998) even allow for a stronger assumption, since they note that it is "... important to allow for unobserved heterogeneity in λ but not necessarily in λ_L or in δ." (p.1213)

We obtain:

$$(11)\quad h_1(w_1) = \frac{-H_1(w_1) + \frac{\lambda_{1,L}+\delta_1}{\lambda_{1,L}}}{2\left(\alpha_1 A\left(\frac{(N_1-U_1)\delta_1(\lambda_{1,L}+\delta_1)}{(\lambda_{1,L}(1-H_1(w_1))+\delta_1)^2}\right)^{\alpha_1-1}\left(\frac{(N_2-U_2)\delta_2(\lambda_{2,L}+\delta_2)}{(\lambda_{2,L}(1-H_1(w_1))+\delta_2)^2}\right)^{\alpha_2} - w_1\right)}$$

This differential equation is similar to the BM-case and can be solved using separation of variables (see Appendix).

We get a closed form solution for the wage distribution of skill group 1 and by symmetry for skill group 2:

$$H_i(w_i) = \frac{\delta+\lambda_L}{\lambda_L}\left(1 - \sqrt[2]{\frac{(\alpha_i A(\lambda_{i,L}+\delta_i)^{\alpha_i-1}(N_i-U_i)^{\alpha_i-1}(\lambda_{j,L}+\delta_j)^{1-\alpha_i}(N_j-U_j)^{1-\alpha_i}-w_i)}{(\alpha_i A(\lambda_{i,L}+\delta_i)^{\alpha_i-1}(N_i-U_i)^{\alpha_i-1}(\lambda_{j,L}+\delta_j)^{1-\alpha_i}(N_j-U_j)^{1-\alpha_i}-w_i^R)}}\right)$$

A steady state labor market equilibrium solution to this model is given by H_i, H_j, w_i^R, w_j^R, where individuals maximize expected lifetime income, firms maximize expected profits. This is the solution to an equilibrium search model with two skill groups which are linked via a Cobb-Douglas production function, where both firms and individuals behave rationally with respect to their preferences.[11]

Of course, the resulting mixture wage distribution of the two skill groups is of special interest, since it allows to decompose the variance of the wage distribution in a component that is due to the variance within a skill group (ie, because of frictions), and to a component that is due to the variance between skill groups (ie, because of human capital differences.

Using that by proposition 2 $H_2(w_2) = H_1(z^{-1}(w_2))$ the function z is determined by the above two equations. We obtain

(12)

$$z^{-1}(w_2) = C + \frac{((\alpha_1 A(\lambda_{1,L}+\delta_1)^{\alpha_1-1}(N_1-U_1)^{\alpha_1-1}(\lambda_{2,L}+\delta_2)^{1-\alpha_1}(N_2-U_2)^{1-\alpha_1}-w_1^R)}{((1-\alpha_1)A(\lambda_{2,L}+\delta_2)^{-\alpha_1}(N_2-U_2)^{-\alpha_1}(\lambda_{1,L}+\delta_1)^{\alpha_1}(N_1-U_1)^{\alpha_1}-w_2^R)} w_2$$

where $C = \alpha_1 A(\lambda_{1,L}+\delta_1)^{\alpha_1-1}(N_1-U_1)^{\alpha_1-1}(\lambda_{2,L}+\delta_2)^{1-\alpha_1}(N_2-U_2)^{1-\alpha_1}$.

and

[11] Recognize that differences in marginal productivity of these two skill groups stem from differences in the equilibrium unemployment rate, from the friction parameters and from differences in the α attached to them.

$$\frac{dz^{-1}(w_2)}{dw_2} > 0$$

.

We checked that the solutions satisfy the original differential equations (ie, equation 11 is satisfied).

4.2 properties of the solution

4.2.1 Increasing wage densities

The wage offer density from the model is given as:

$$h_1(w_1)=\frac{\delta+\lambda_L}{2\lambda_L}\sqrt[2]{\frac{1}{(\alpha_1 A(N_1-U_1)^{\alpha_1-1}(N_2-U_2)^{1-\alpha_1}-w_1)(\alpha_1 A(N_1-U_1)^{\alpha_1-1}(N_2-U_2)^{1-\alpha_1}-w_1^R)}}$$

Taking derivatives yields the following

$$\frac{\partial h_1(w_1)}{\partial w_1}=\frac{\delta+\lambda_L}{4\lambda_L}(\alpha_1 A(N_1-U_1)^{\alpha_1-1}(N_2-U_2)^{1-\alpha_1}-w_1^R)^{-1/2}(\alpha_1 A(N_1-U_1)^{\alpha_1-1}(N_2-U_2)^{1-\alpha_1}-w_1)^{-3/2}>0$$

which is positive. If the density of the wage offers is increasing the density of the cross-sectional wages increases as well $\frac{\partial g_1(w_1)}{\partial w_1} > 0$.[12] Recognize that the equilibrium connection between H_1 and G_1 is given by equation (3). So, introducing a production function the way we do it does not lead to a wage density with a long right tail. The mixture of the two might come a bit closer to the desired form, but two skill groups clearly do not suffice to generate the required form (see Holzner and Launov (2006)).

4.2.2 Constant marginal productivity over the wage distribution

One of the reasons, why we get a closed form solution for our special parameter constellation is that we have a constant marginal productivity across the wage distribution. This brings us more or less back in the BM-world for each skill group, given it is optimal for the firms to cover the same position in both wage distributions.

To see why this is the case, observe that the wage uniquely determines the size of the work force of each firm. For our parameter constellation the effect of a decreasing marginal productivity when increasing the wage and therefore employment of one skill group is exactly offset by the corresponding effect of the increasing marginal productivity because the employment of the other skill group increases as well. This increase of the other skill group arises because

[12]When the wage offer density increases, that means that there are more firms offering a higher wage. In addition, firms paying a higher wage are bigger ($l_1'(w_1) > 0$, see equation (5)). This implies that the wage density is steeper than the wage offer density.

firms cover the same position in both wage distributions. In the formula this can be seen, because the part which contains the wage distribution cancels out.

$$\frac{\partial y}{\partial l_1} = \alpha_1 A \left(\frac{(N_1 - U_1)\delta_1 (\lambda_{1,L} + \delta_1)}{(\lambda_{1,L}(1 - H_2(w_2)) + \delta_1)^2} \right)^{\alpha_1 - 1} \left(\frac{(N_2 - U_2)\delta_2 (\lambda_{2,L} + \delta_2)}{(\lambda_{2,L}(1 - H_2(w_2)) + \delta_2)^2} \right)^{\alpha_2} \tag{13}$$

$$= \alpha_1 A (N_1 - U_1)^{\alpha 1 - 1} (\lambda_{1,L} + \delta_1)^{\alpha_1 - 1} (N_2 - U_2)^{1-\alpha_1} (\lambda_{2,L} + \delta_2)^{1-\alpha_1} \tag{14}$$

4.2.3 Reservation wage and upper bound and moments of the wage distribution

Having at hand the wage distribution and the marginal productivity of the individuals we can calculate the reservation wage and the upper bound of the wage distribution(s) according to the following two formulas:

$$w_1^R = z_1 + (\lambda_1 - \lambda_{1,L}) \int_{w_1^R}^{\hat{w}} \frac{1 - H_1(w_1)}{r + \delta_1 + \lambda_{1,L}(1 - H_1(w_1))} dw_1 \tag{15}$$

and from setting $H_1(w_1) = 1$:

$$w_1^o = \frac{\partial y}{\partial l_1} - (\frac{\partial y}{\partial l_1} - w_1^R) \left(\frac{\delta_1}{\delta_1 + \lambda_{1,L}} \right)^2 \tag{16}$$

which for reasonable parameter choices is close to the marginal productivity.

Constraining ourselves to a case, where the lower bound of the wage distribution is not given by the reservation wage, but by an exogenous restriction, as say a minimum wage, in addition, we can calculate the moments of the wage distribution according to the following formulas (see Ridder and Van den Berg (1997a)):

$$Exp_{G_1}(w_1) = \left(y_1' - mw_1\right) \left(1 - \frac{\delta_1}{\delta_1 + \lambda_{1,L}}\right) + mw_1$$

, where $y_1' = \frac{\partial y}{\partial l_1}$ is the marginal productivity and mw_1 is the binding minimum wage. Using the same notation as above, the variance is given by the following formula (ibid.):

$$var_{G_1}(w_1) = \frac{1}{3} \left(y_1' - mw_1\right)^2 \frac{\delta_1}{\delta_1 + \lambda_{1,L}} \left(1 - \frac{\delta_1}{\delta_1 + \lambda_{1,L}}\right)^2 = \frac{1}{3} \left(y_1' - mw_1\right)^2 \frac{\lambda_{1,L}^2 \delta_1}{(\delta_1 + \lambda_{1,L})^3}$$

.

By symmetry these formulas apply for skill group 2 as well.

A nice feature of the model is that we do not have to make the strong assumption that $\lambda = \lambda_L$ which is often made in advanced search models, see eg Ridder and Van den Berg (1997a). Van den Berg and Ridder (1998) note that it is important to allow for unobserved heterogeneity in λ but not in λ_L, which we do here. That is also the reason why the reservation wage is (considerably) different from the alternative income of the unemployed z_1.

4.3 Simulation results

The general problem which we want to solve is equation (11). The problem with this differential equation is that there is no easy way to solve a non-autonomous, non-linear ordinary differential equation. Above, we invoke some assumptions which allow for a analytical solution. Deviating from these assumptions, we assess the solution by trying numerical methods. For all simulations, though, we maintain the assumption that the choice of the wage for one skill-group determines the wage of the other skill group. That is we avoid the problem of solving a system of partial differential equations.

We start by assessing whether the numerical solution to the variant which we can solve analytically reproduces the analytical results, which - fortunately - it does.[13] As a starting point, we consider two parameter constellations: a labor market with a high degree of frictions and a labor market with a low degree of frictions (see, table 2 in the Appendix for the chosen values). Table 1 gives the results for the two parameter constellations.

Table 1: Simulation results: CRS

Parameter	low frictions	high frictions
u_1	0.091	0.286
u_2	0.063	0.118
$y_1^{'}$	0.407	0.161
$y_2^{'}$	0.593	0.913
w_1^r	0.137	0.035
w_2^r	0.277	0.467
w_1^o	0.404	0.146
w_2^o	0.588	0.859
$E_{G_1}(w_1)$	0.371	0.116
$E_{G_2}(w_2)$	0.535	0.665
$Var_{G_1}(w_1)$	0.003	0.001
$Var_{G_2}(w_2)$	0.007	0.025
Π	0.064	0.231

Clearly, the market with the high degree of frictions has the higher unemployment rates. The marginal productivity for the low-skilled is higher in the market with a low degree of frictions. This is the case because of the higher α attached to them and because own employment is much lower (whereas the employment of the other skill group is not that much lower). The contrary is true for the marginal productivity of the other skill group, by the same arguments. The reservation wage is a premium of about 35% (17%) for skill group 1 and 175% (125%) for skill group 2 in the case of low frictions (high frictions). The lower premium in the frictional case stems from the fact that market prospects are worse. By the same, the premium for skill group 2 is higher. The profit of the firms is higher in the frictional case, since a higher amount of frictions is associated with a higher degree of monopsony power for the firms.

Now we analyze the impact of small deviations from the parameter restrictions on the equilibrium. We find that altering one of the parameters ($\lambda_{1,L}, \lambda_{2,L}, \delta_1, delta_2$) by a small amount

[13]For all simulations, we used Mathematica's routine NDSolve. This routine chooses between different methods to find numerical solutions, most of them being a variant of the Runge-Kutta method. For a description of Runge-Kutta methods, see Bronstein, Semendjajew, Musiol, and Mühlig (2000). For a description of Mathematica's NDSolve routine, see http://documents.wolfram.com/mathematica/.

immediately destroys the double BM property in the case where frictions are low. We get a mass point at the top of the distribution. This happens already if we increase δ_1 by 2.5%. We get an inverse-u-shaped form for the wage density of skill group 1. The reason for the mass point at the top of the distribution is that the marginal productivity is no more constant across the wage distribution and that the wage attains the marginal productivity at a value where $H_w < 1$. Thus further increasing the employment is no more valuable. In the case of a high degree of frictions the double BM property is more stable. So, increasing δ_1 by 25% does not change the double BM-type-solution. We get the masspoint in this case as well, but it appears on the right side of the support of w.[14]

Considering now decreasing returns to scale, keeping the parameter restrictions for δ, λ_L, yields results with double BM-type solution but this is not an equilibrium since firms at the upper bound of the support make lower profits than firms at the lower bound of the support. Relaxing in addition the parameter restrictions as for the analytical case, mostly yields a masspoint in the distribution of wages.

The case of increasing returns to scale is more interesting. Clearly, we maintain the double BM-property when for the case which we are able to solve analytically we increase the α's a bit. However, firms at the top of the distribution make higher profits than firms at the bottom of the wage distribution. In addition, sometimes there seems to be a problem with the uniqueness of the reservation wage. Increasing the α's stronger increases the difference in profits at the lower and the upper bound. Increasing returns to scale strengthen the original effect of an increasing density, which gets very steep when the α's get big.

5 A labor union in an equilibrium search model

So far we have said nothing about unemployment. As with most search models our model has the characteristic that minimum wages do not change employment as long as they remain under the marginal productivity of the individuals. To see this, recognize that as long as the inflow in and outflow rates out of unemployment are unchanged, equilibrium unemployment is unchanged in a stationary equilibrium. However, the inflow rate is considered exogenous and equal to δ, while the outflow rate is $\lambda(1-H_1(mw_1))$. However, since offering wages below the minimum wage mw_1 is illegal and since we assume the minimum wage to be binding, there are no wages below the minimum wage and thus:[15]

$$(17) \quad u_1 = \frac{U_1}{N_1} = \frac{\delta_1}{(\delta_1 + \lambda_1)}$$

The fact that minimum wages affect unemployment only if it is above marginal productivity is not different from a competitive model, but, in a competitive model all wages correspond to the marginal productivities of an individual while in our model *all* paid wages lie below the marginal productivity. The reason for this is that search frictions and the assumed wage setting mechanism imply that firms have monopsony power.[16] Therefore, a labor union (or

[14]Recognize that in the analytical case, we get a masspoint at the marginal productivity, which lies to the right of the distribution function.

[15]Of course, the same reasoning holds for a non-binding minimum wage. Then, the (common) reservation wage is the lower bound of the wage distribution.

[16]For an extensive treatment of the effects of monopsony power on labor market outcomes, see Manning (2003).

the state) that increases a minimum wage compresses the wage distribution and redistributes rents but does not alter employment.[17]

Now, imagine that the minimum wage increases above the marginal productivity of the individuals of say skill group 1. In the BM-case, employment would be zero because individuals have a constant marginal productivity. Here, however it is still optimal to employ some members of skill group 1, since when employment is lower, marginal productivity increases. To see this, just raise the minimum wage slightly above the level of the marginal productivity of the workers. Now, half the number of individuals in skill group 1, ie $N_1^* = \frac{N_1}{2}$. Under these circumstances we get a new solution with two BM-type wage distributions, where firms make positive profits.

We take up this idea in the following and introduce rationing of jobs by endogenizing λ. Assume that λ is determined by the aggregate search effort of firms. Assume further that hiring is costly to firms and that firms cannot distinguish prior to the search effort whether an individual is employed or unemployed, implying that $\lambda_i = \lambda i, L$ (see Mortensen (2003)). [18] Assume for simplicity in addition that only hiring in skill group 1 is costly and that hiring cost per worker contact are linear and given by c.

Consider a firm that pays a wage w_1 for skill group 1, where its stationary employment is given by $l_1(w_1)$ as given by equation 5.[19] The firm has to contact enough workers to exactly offset the outflows, taking into account the acceptance probability $\gamma(w_1)$ of an individuals contacted. The acceptance probability is given by:

$$\gamma(w_1) = u_1 + (1 - u_1)G_1(w_1)$$

where $G_1(w_1)$ is given by equation 3. For the total amount of worker contacts necessary to maintain firm size, we obtain:

$$(18)\quad \frac{(\lambda_1(1 - H_1(w_1)) + \delta_1)l_1(w_1)}{\gamma(w_1)} c_1 = \lambda_1 N_1 c_1$$

.

Using the number of contacts necessary for stationarity in the profit equation, we get:

$$(19)\quad \Pi(w_1, w_2) = y(l_1(w_1), l_2(w_2)) - w_1 l_1(w_1) - w_2 l_2(w_2) - \lambda_1 N_1 c_1$$

This implies that the first order conditions in optimally choosing the wage vector w are unchanged and thus for any parameter constellation that yields a solution of the BM-type without considering contact costs and which yield positive (zero) profits considering contact costs, the equilibrium is unchanged.

[17]We consider labor unions here, because in the German context there is - with some exceptions - no minimum wage by the state. But labor unions contracts in connection with the ,,to-the-workers-advantage principle" (,,Günstigkeitsprinzip") closely resembles minimum wages.

[18]See, eg, Manning (1992), for one of the few attempts to introduce rationing of jobs in an equilibrium search framework.

[19]Recognize that this requires that there is no masspoint at w_1.

Now, consider, that the contact rate is endogenous. Consider the decision of the firms of contacting an additional worker. As long as the (expected) profits (per worker contacted) are positive the firms will enhance worker contacts (see Mortensen (2003)), ie:

$$(20)\quad \frac{\Pi(w_1,w_2)}{\lambda_1 N_1} = \frac{1}{\lambda_1 N_1}\left(y(l_1(w_1),l_2(w_2)) - w_1 l_1(w_1) - w_2 l_2(w_2)\right) - c_1 = 0$$

Maintaining the assumptions required for Proposition 2, and using the fact that we have no masspoint at (w_1,w_2) yields:

$$(21)\quad \frac{1}{\lambda_1 N_1} A\left(\frac{\delta_1\lambda_1 N_1}{(\delta_1+\lambda_1(1-H_1(w_1)))^2}\right)^{\alpha}\left(\frac{\delta_2\lambda_2 N_2\frac{\lambda_{2,L}+\delta_2}{\lambda_2+\delta_2}}{(\lambda_{2,L}(1-H_1(w_1))+\delta_2)^2}\right)^{1-\alpha}$$
$$-w_1\frac{\delta_1}{(\delta_1+\lambda_1(1-H_1(w_1)))^2} - w_2\frac{\delta_2\lambda_2 N_2\frac{\lambda_{2,L}+\delta_2}{\lambda_2+\delta_2}\frac{1}{\lambda_1 N_1}}{(\lambda_{2,L}(1-H_1(w_1))+\delta_2)^2} = c_1$$

Following Mortensen (2003), this must hold for every wage that is paid in equilibrium, and we can write (setting $H_1(w_1)$ to zero):

(22)

$$N_1^{\alpha-1}A\left(\frac{\delta_1\lambda_1}{(\delta_1+\lambda_1)^2}\right)^{\alpha}\left(\frac{\delta_2\lambda_2 N_2\frac{\lambda_{2,L}+\delta_2}{\lambda_2+\delta_2}}{(\lambda_{2,L}+\delta_2)^2}\right)^{1-\alpha} - z_1\frac{\delta_1\lambda_1}{(\delta_1+\lambda_1)^2} - w_2^R\frac{\delta_2\lambda_2 N_2\frac{\lambda_{2,L}+\delta_2}{\lambda_2+\delta_2}\frac{1}{N_1}}{(\lambda_{2,L}+\delta_2)^2} = c_1\lambda_1$$

Let us have a look how a λ that solves the above equation varies with the lower bound of the wage distribution, be it the reservation wage or the binding minimum wage mw_1.

Define:

$$G(\lambda_1) = N_1^{\alpha-1}A\left(\frac{\delta_1\lambda_1}{(\delta_1+\lambda_1)^2}\right)^{\alpha}\left(\frac{\delta_2\lambda_2 N_2\frac{\lambda_{2,L}+\delta_2}{\lambda_2+\delta_2}}{(\lambda_{2,L}+\delta_2)^2}\right)^{1-\alpha} - z_1\frac{\delta_1\lambda_1}{(\delta_1+\lambda_1)^2} - w_2^R\frac{\delta_2\lambda_2 N_2\frac{\lambda_{2,L}+\delta_2}{\lambda_2+\delta_2}\frac{1}{N_1}}{(\lambda_{2,L}+\delta_2)^2} - c_1\lambda_1 = 0$$

Implicitely differentiating yields:

$$\frac{d\lambda_1}{dz_1} = \frac{\frac{\delta_1\lambda_1}{(\delta_1+\lambda_1)^2}}{\frac{\delta_1^2-\delta_1\lambda_1}{(\delta_1+\lambda_1)^3}\left(\alpha A\left(\frac{\delta_1\lambda_1 N_1}{(\delta_1+\lambda_1)^2}\right)^{\alpha-1}\left(\frac{\delta_2\lambda_2 N_2\frac{\lambda_{2,L}+\delta_2}{\lambda_2+\delta_2}}{(\lambda_{2,L}+\delta_2)^2}\right)^{1-\alpha} - z_1\right) - c_1}$$

The denominator is negative if, a) $\lambda_1 > \delta_1$ and b) $\frac{\partial y}{\partial l_1}(\lambda_1) > mw_1$. Both requirements are highly likely to hold, since the first assumption is simply assuming that the unemployment rate is smaller than 50% (see equation 17), while the second says that the marginal productivity must exceed the minimum wage, which it always does if there is production. Recognize that for these arguments to hold we have assumed that at the lower bound of the wage distribution there is no mass point. Since optimality implies that profits are identical on the support of the wage distribution, this result holds also with mass points in the wage distribution as long as there is a continuous part. When the wage distribution shrinks to a single point the above arguments do not apply. In deed, the theoretical results of Ridder and Van den Berg (1997a) suggest that there are many cases where there is a wage distribution with a continuous part. Our simulation results for all parameter constellations resulted in wage distributions that are continuous at the lower bound of the distribution.

5.1 Frictional and structural unemployment

We can now decompose the total unemployment rate in a frictional component, caused by labor market frictions, and a structural component, caused by a labor unions minimum wage. We define frictional unemployment as the unemployment rate in the absence of a minimum wage. Let z_1 be the reservation wage of the individuals and let $mw_1 > z_1$ be the binding minimum wage, then frictional unemployment is given as:

$$(23) \quad u_1^f = \frac{\delta_1}{(\delta_1 + \lambda_1(z_1))}$$

while structural unemployment is the difference of total unemployment and frictional unemployment.

$$(24) \quad u_1^s = u_1 - u_1^f = \frac{\delta_1}{(\delta_1 + \lambda_1(mw_1))} - \frac{\delta_1}{(\delta_1 + \lambda_1(z_1))}$$

Because λ_1 decreases with the minimum wage, structural unemployment increases with the minimum wage.

5.2 Wage dispersion

Next, we consider the wage distribution in the case where the labor union sets a minimum wage. For the above derivations, we had only to assume that the wage distribution has a continuous part, which we argued is likely to be the case. If we impose parameters that guarantee a BM-type solution, we can calculate the moments of the wage distributions and calculate comparative static results for changing the minimum wage for these moments. In order to derive comparative static results, consider now the special case, where the job offer rate is identical across states and skill groups, ie, $\lambda_1 = \lambda_2$ and $\lambda_{i,L} = \lambda_i$, where as above λ is determined by the hiring process for skill group 1.[20] In addition, we maintain the assumption we imposed for the analytical solution, which implies that the destruction rates are identical across groups, ie $\delta_1 = \delta_2$.

For this case, we obtain a closed form solution for λ

[20] Admittedly, from an economic point of view, the assumption that λ is determined by the hiring process for skill group 1, but also holds for skill group 2 is hard to justify.

$$\lambda = -\delta + \sqrt[2]{\frac{\delta}{c_1}\left(N_1^{\alpha-1}N_2^{1-\alpha}A - z_1 - w_2^R\frac{N_2}{N_1}\right)}$$

which solves the zero profit condition. We can show easily that the effect of z_1 on λ is uniquely negative, and we do require neither $\lambda_1 > \delta_1$ nor $\frac{\partial y}{\partial l_1}(\lambda_1) > z_1$ to hold.

$$\frac{\partial\lambda}{\partial z_1} = -\frac{1}{2}\left(\frac{\delta}{c_1}\left(N_1^{\alpha-1}N_2^{1-\alpha}A - z_1 - w_2^R\frac{N_2}{N_1}\right)\right)^{-0.5}$$

Since the introduction of hiring costs does not change the first order conditions (s.a.), the parameter constellation chosen guarantees the existence of a BM-type solution as long as profits are positive. The wage offer distribution is given by:

$$H_1(w_1) = \frac{\delta+\lambda}{\lambda}\left(1 - \sqrt[2]{\frac{(\alpha_1 A(N_1)^{\alpha_1-1}(N_2)^{1-\alpha_1} - w_1)}{(\alpha_1 A(N_1)^{\alpha_1-1}(N_2)^{1-\alpha_1} - z_1)}}\right)$$

To get a feeling on the effect of an increasing minimum wage on the wage distribution, start by calculating the upper bound of the wage distribution(s).[21]

From

$$(25)\quad 1 = \frac{\delta+\lambda}{\lambda}\left(1 - \sqrt[2]{\frac{(y_1' - w_1^o)}{(y_1' - w_1^R)}}\right)$$

obtain

$$(26)\quad w_1^o(z_1) = y_1' - \left(1 - \frac{\lambda}{\delta+\lambda}\right)^2 (y_1' - z_1)$$

Since there is no effect of λ on the marginal productivity, we obtain:

$$(27)\quad \frac{\partial w_1^o}{\partial z_1} = 2\left(\frac{\delta^2}{(\delta+\lambda)^3}\right)\frac{\partial\lambda}{\partial z_1}(y_1' - z_1) + \left(1 - \frac{\lambda}{\delta+\lambda}\right)^2$$

The effect of increasing the minimum wage on the upper bound of the wage distribution(s) is not unique. The reason is that there are two counteracting effects. On the one hand, an increasing minimum wage shifts the whole distribution, including the upper bound to the right if λ is constant. On the other hand, λ decreases as a reaction to the increasing minimum wage and therefore the market gets less competitive, thereby decreasing the upper bound.

Next, we analyse the effect of an increasing minimum wage on the expectation of the wage distribution. The expectation is given as (see Van den Berg and Ridder (1993)):

[21] Recognize, that the upper bound of the distribution of wage offers is also the upper bound of the distribution of paid wages.

$$Exp_{G_1}(w_1 - z_1) = (y_1' - z_1)\left(1 - \frac{\delta}{\delta + \lambda}\right)$$

, where $y_1' = \frac{\partial y}{\partial l_1}$.

We obtain

$$\frac{\partial}{\partial z_1} Exp_{G_1}(w_1 - z_1) = -1\left(1 - \frac{\delta}{\delta + \lambda}\right) + (y_1' - z_1)\frac{\delta}{(\delta + \lambda)^2}\frac{\partial \lambda}{\partial z_1} < 0$$

which is uniquely negative. That is, the expectation of w_1 increases less than z_1. Looking however at $\frac{\partial}{\partial z_1} Exp_{G_1}(w_1)$ causes a non-unique sign.

$$\frac{\partial}{\partial z_1} Exp_{G_1}(w_1) = \frac{\delta}{\delta + \lambda}\left(1 + \frac{(y_1' - z_1)}{\delta + \lambda}\frac{\partial \lambda}{\partial z_1}\right)$$

Again the non-uniqueness stems from the fact that the direct effect from increasing z_1 on the expectation is positive. However, decreasing λ counteracts by making the market less competitive. If λ does not react again the effect is uniquely positive $\frac{\partial}{\partial z_1} Exp_{G_1}(w_1) = \frac{\delta}{\delta+\lambda}$.

For the variance we have:

$$var_{G_1}(w) = \frac{1}{3}(y_1' - z_1)^2 \frac{\delta}{\delta + \lambda}\left(1 - \frac{\delta}{\delta + \lambda}\right)^2 = \frac{1}{3}(y_1' - z_1)^2 \frac{\lambda^2 \delta}{(\delta + \lambda)^3}$$

.

The partial derivative is calculated as:

$$\frac{\partial var_{G_1}(w)}{\partial z_1} = \frac{1}{3}(y_1' - z_1)\frac{\lambda \delta}{(\delta + \lambda)^3}\left(\frac{\partial \lambda}{\partial z_1}\frac{(2\delta - \lambda)}{\delta + \lambda} - 2\lambda\right)$$

which is uniquely negative if $\lambda < 2\delta$ and thus if the unemployment rate is above 33% and depends on the parameters otherwise.

The effects on the wage dispersion discussed so far, all concern the wage distribution of skill group 1. Recognize that when the labor unions increase the minimum wage of skill group 1 it also alters the moments of the wage distribution of skill group 2. This is the case because as labor demand reacts the marginal productivity for skill group 2 decreases, and therefore the expectation and the variance change. A second reason for this is that we assume here that the λ's are identical and thus also labor demand for skill group 2 reacts. In the absence of the second factor, ie holding $\lambda_2, \lambda_{2,L}$ constant, the effect of an increasing minimum wage for skill group 1 on the expectation and on the variance of the wage distribution of skill group 2 is uniquely negative. This means that the dispersion between groups changes as well.

6 Labor Unions objectives

So far, we have assumed that the labor unions have the possibility to impose a minimum wage that is binding. This can be justified for the German labor market by acknowledging that negotiated wages have indeed a character of minimum wages (see, eg, Franz (2003) and Pfeiffer (2003) for empirical results). Now, we ask the question which minimum wage a labor union chooses optimally. The trade-off in the above model is as follows: increasing the minimum wage always decreases employment and can increase the expectation and decrease the variance of the wage distribution.

Let us start by assuming that the labor union maximizes the wage sum. In order to derive an analytical form for the objective function, we refer to the above special case, where $\lambda_1 = \lambda_2$, $\lambda_L = \lambda$ and $\delta_1 = \delta_2$.

$$WS=(N_1-U_1)E_G(w_1)=N_1\frac{\lambda}{\delta+\lambda}\left[\left(y_1'-mw_1\right)\left(1-\frac{\delta}{\delta+\lambda}\right)+z_1\right]=N_1\left(y_1'-mw_1\right)\left(\frac{\lambda}{\delta+\lambda}\right)^2+N_1\frac{\lambda}{\delta+\lambda}z_1$$

The labor union will choose a minimum wage mw_1 that maximizes this sum.

$$\frac{\partial WS}{\partial mw_1}=N_1\left(\frac{\partial}{\partial mw_1}\left(y_1'-mw_1\right)\left(\frac{\lambda}{\delta+\lambda}\right)^2+\left(y_1'-mw_1\right)\frac{\partial}{\partial mw_1}\left(\frac{\lambda}{\delta+\lambda}\right)^2\right)+\frac{\partial}{\partial mw_1}N_1\frac{\lambda}{\delta+\lambda}mw_1$$

We obtain the optimal chosen minimum wage mw_1 of the labor union as

$$(28)\quad mw_1 = \frac{-\lambda\left((\delta+\lambda)+2y_1'\frac{\partial\lambda}{\partial mw_1}\right)}{\frac{\partial\lambda}{\partial mw_1}\left((\delta+\lambda)^2-2\lambda\right)}$$

.

Now, consider a labor union that also cares about the income of the unemployed. Its maximization problem is:

$$WU = (N_1-U_1)E_G(w_1)+z_1U_1 = N_1\left(y_1'-mw_1\right)\left(\frac{\lambda}{\delta+\lambda}\right)^2+N_1\frac{\lambda}{\delta+\lambda}mw_1+z_1N_1\frac{\delta}{\delta+\lambda}$$

We obtain

$$(29)\quad mw_1 = \frac{\lambda\left(\delta+\lambda\right)+y_1'2\lambda\frac{\partial\lambda}{\partial mw_1}-z_1\left(\delta+\lambda\right)}{\frac{\partial\lambda}{\partial mw_1}\left(2\lambda-(\delta+\lambda)^2\right)}$$

,

which is higher as the corresponding value of the labor union that does not care about the unemployed if we require that $\lambda > \delta$ and that both λ and δ are positive. The reason for this is that in the case where the labor union cares about the unemployed they have still a positive contribution to the objective function in the case where they are unemployed while in the case where the labor union does not care about the unemployed, they get a value of zero in the objective function.

7 Conclusion

We have constructed an equilibrium search model where two types of labor are linked over a production function. We introduce labor demand effects in this framework and consider labor unions behavior and its influence on employment and wage dispersion via a binding minimum wage. We show that optimal strategies imply that firms cover the same position in the wage distribution for both skill groups. We solve the model for particular parameter constellations and simulate solutions otherwise. Labor unions minimum wages are higher when the union cares about the unemployed; they generate structural unemployment and alter the wage distribution. In general, the effect on the wage distribution is not unique as far as its moments are concerned, the reason being that the labor demand effect counteracts the increase in the minimum wage.

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A Appendix

A.1 Proof of Proposition 1

Sampling one firm at random is a random draw (w_1, w_2) from the wage offer distribution H, and $H(w_1, w_2)$ is the probability that the wages w_1 and w_2 are below these specific values (More precisely that the random variables $W_1 <= w_1$ and $W_2 <= w_2$). Sampling one worker from each skill group at random, instead implies two random draws from the two marginal distributions of paid wages $G_1(w_1)$ and $G_2(w_2)$. If an individual samples a firm and decides whether to accept the wage offer (w_1, w_2) it only cares about one of these wages, implying that the decision is based upon the Marginal.

The marginal distributions $H_1(w_1), L_1(w_1)$ are given by integrating out the other wage variable:

$$H_1(w_1) = \int_0^{w_1} h_1(w_1)dw_1 = \int_0^{w_1} \left[\int_0^{\infty} h(w_1, w_2)dw_2\right] dw_1 = \int_0^{\infty} h_2(w_2)H_{w_1|w_2}(w_1|w_2)dw_2$$

$$\frac{L_1(w_1)}{(N_1 - U_1)} = G_1(w_1) = \int_0^{w_1} \int_0^{\infty} g(w_1, w_2)dw_2dw_1$$

The interpretation of the joint wage density $g(w_1, w_2)$ is simply the probability of drawing one worker of each skill group at random, ie the product of the two marginals $g(w_1, w_2) = g_1(w_1)g_2(w_2)$. The marginal distribution of paid wages $G_1(w_1), G_2(w_2)$ themselves follow from $H_1(w_1), H_2(w_2)$ and from the dynamics of employment following from the individual efforts to look for a higher paying jobs (from the marginal wage offer distribution) and from job destruction.

Concerning the strategies of the firms, there are three possibilities:

- Deterministic strategy: Fixing a wage for skill group i determines the optimal wage for skill group $j \neq i$. Then, sampling a firm at random yields a random draw (w_1, w_2) and $H(w_1, w_2) = H_1(w_1) = H_2(w_2)$. (analogous to Holzner and Launov (2006))
- Independent strategy: Fixing a wage for skill group i does not influence the optimal wage for skill group $j \neq i$. Then, sampling a firm at random yields a random draw (w_1, w_2) and $H(w_1, w_2) = H_1(w_1)H_2(w_2)$.

- Mixed strategy: Conditional on paying a wage w_1 for skill group 1 there exists a wage distribution for skill group 2. This distribution however is different from the marginals. Then, sampling a firm at random yields a random draw (w_1, w_2) and $H(w_1, w_2) \neq H_1(w_1) * H_2(w_2)$.

In proposition 2, we prove that under some restrictions we obtain a deterministic strategy.

A.2 Solution Strategies

One solution possibility is to assume that the wage for one skill group is fixed exogenously. In this case, individuals of this skill group evenly distribute across firms, as long as firms make positive profits per worker of this skill group (otherwise there is rationing of jobs). For the other skill group, we can then apply the results of Ridder and Van den Berg (1997a). That is, for some parameter constellations we obtain BM-type solution, for some we obtain a wage distribution with mass point and for some we obtain only a mass point.

Another solution possibility is to assume that the labor market for one of the skill groups (for example of the high-skilled, ie $i = 2$) is competitively organized, meaning marginal productivity pay. Assume that $\alpha_1 < 1$ and $\alpha_2 < 1$. Now, assume that we start from a BM-type solution for the non-competitive skill group ($i = 1$). This can only be an equilibrium, when labor of skill group 2 distributes over firms in a way that exactly offsets the differences in marginal productivity resulting from the different sizes in skill group 1. More formally it must hold that:

$$(30) \quad \frac{\partial y}{\partial l_2} = \alpha_2 A l_1^{\alpha_1} l_2^{\alpha_2 - 1} = \alpha_2 A \left(\frac{(N_1 - U_1)\delta_1(\lambda_{1,L} + \delta_1)}{(\delta_1 + \lambda_{1,L}(1 - H_1(w_1)))^2} \right)^{\alpha_1} l_2^{\alpha_2 - 1} = w_2$$

is identical for every paid wage w_1 and the according employment of skill group 2. The fact that firms make identical profits implies an equilibrium.[22]

A.3 Solving the differential equation

Taking the homogenous part of the differential equation (equation 11), we write the equation as:

$$(31) \quad \frac{h_1(w_1)}{H_1(w_1)} = -\frac{1}{2\left(\alpha_1 A(\lambda_{1,L} + \delta_1)^{\alpha_1 - 1}(N_1 - U_1)^{\alpha_1 - 1}(\lambda_{2,L} + \delta_2)^{1-\alpha_1}(N_2 - U_2)^{1-\alpha_1} - w_1\right)}$$

Then integrating both sides and exponentiating yields

$$(32) \quad H_1(w_1) = A_2 \sqrt[2]{(\alpha_1 A(\lambda_{1,L} + \delta_1)^{\alpha_1 - 1}(N_1 - U_1)^{\alpha_1 - 1}(\lambda_{2,L} + \delta_2)^{1-\alpha_1}(N_2 - U_2)^{1-\alpha_1} - w_1)}$$

[22]It is not clear how this equilibrium would arise, because all firms pay identical wages and therefore the distribution of employment over firms is arbitrary. But there is no reason to deviate once this equilibrium is achieved. The above equation follows from the FOC of the profit function with respect to l_2.

where A_2 is an integration constant to be determined by an initial condition.

A particular solution is given, when setting $h_1(w_1) = 0$ in 11, $H(w) = \frac{\delta+\lambda_L}{\lambda_L}$ yielding the following:

$$H_1(w_1) = A_2 \sqrt[2]{(\alpha_1 A(\lambda_{1,L} + \delta_1)^{\alpha_1 - 1}(N_1 - U_1)^{\alpha_1 - 1}(\lambda_{2,L} + \delta_2)^{1-\alpha_1}(N_2 - U_2)^{1-\alpha_1} - w_1)} + \frac{\delta + \lambda_L}{\lambda_L}$$

Using that $H_1(w_1^R) = 0$ finally helps determining the integration constant:

(33)

$$A_2 = -\frac{\delta + \lambda_L}{\lambda_L} \frac{1}{\sqrt[2]{(\alpha_1 A(\lambda_{1,L} + \delta_1)^{\alpha_1 - 1}(N_1 - U_1)^{\alpha_1 - 1}(\lambda_{2,L} + \delta_2)^{1-\alpha_1}(N_2 - U_2)^{1-\alpha_1} - w_1^R)}}$$

Thus, the solution for the wage distribution of skill group 1 becomes:

$$H_1(w_1) = \frac{\delta + \lambda_L}{\lambda_L}\left(1 - \sqrt[2]{\frac{(\alpha_1 A(\lambda_{1,L} + \delta_1)^{\alpha_1 - 1}(N_1 - U_1)^{\alpha_1 - 1}(\lambda_{2,L} + \delta_2)^{1-\alpha_1}(N_2 - U_2)^{1-\alpha_1} - w_1)}{(\alpha_1 A(\lambda_{1,L} + \delta_1)^{\alpha_1 - 1}(N_1 - U_1)^{\alpha_1 - 1}(\lambda_{2,L} + \delta_2)^{1-\alpha_1}(N_2 - U_2)^{1-\alpha_1} - w_1^R)}}\right)$$

A.4 Simulation parameters

In magnitude the friction parameter (δ, λ) we choose are comparable with the estimation results of equilibrium search models for the Dutch and for the German labor market (see Van den Berg and Ridder (1998), Launov and Wolf (2005)).

Table 2: Parameters chosen for simulation

Parameter	low frictions	high frictions
δ_1	0.004	0.008
λ_1	0.04	0.02
$\lambda_{1,L}$	0.03	0.015
N_1	1	3
α_1	0.6	0.3
δ_2	0.008	0.016
λ_2	0.12	0.08
$\lambda_{2,L}$	0.06	0.03
N_2	1	1
α_2	0.4	0.7
A	1	1
r	0.02	0.04
z_1	0.1	0.03
z_2	0.1	0.2