Affirmative Action in Contest Games:Design Matters.[‡]

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Abstract

In this paper a contest game with heterogeneous players is analyzed in which heterogeneity could be the consequence of past discrimination. The contest designer has two normative policy options to tackle the heterogeneity of the contestants: ignore the fact that some contestants are discriminated or implement affirmative action that compensates for this heterogeneity. The consequences of these two policy options are analyzed for a simple two-person contest game and it is shown that the frequently criticized trade-off between affirmative action and aggregated effort does not exist if the affirmative action policy is designed appropriately. A generalization to the n-person case and to a case with a partially informed contest designer yields the same result only under additional restrictions.

Keywords: Asymmetric contest; affirmative action; discrimination

JEL classification: C72; D63; I38; J78

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1 Introduction

Affirmative action as an instrument to ameliorate the adverse effects of discrimination[1] is a highly debated topic in countries where those policies are in fact implemented, especially in the US. One of the reasons for this controversy seems to be the fact that its implementation goes beyond formal equal treatment considerations by addressing discriminated groups directly which is, for example, reflected by the phrase ‘positive discrimination’, or ‘preferential treatment’ as a synonym for affirmative action. However, even in contemporary societies in which formal equality is legally guaranteed and executed there exists empirical evidence of ongoing inequality with respect to specific minority groups. Hence, although open discrimination is prohibited, some minority groups may be disadvantaged out of reasons for which they cannot be held ethically responsible.[2] In such cases in which formal ‘equal treatment of equals’-legislation is ineffective because individuals are not ex-ante equal, the implementation of affirmative action instruments could be justified on ethical grounds; compare Loury (1981) and Loury (2002).

Opponents of affirmative action do not only criticize the, from their perspective, formal violation of the equal treatment principle but also they refer to possible negative effects of affirmative action. The following statement by Thomas Sowell from his book “Affirmative Action Around the World” reflects the concern of those opponents that there could exist a trade-off between affirmative action (i.e. preferential treatment) and social efficiency due to potential disincentive effects:

> Both preferred and non-preferred groups can slacken their efforts - the former because working to their fullest capacity is unnecessary and the latter because working to their fullest capacity can prove to be futile. [...] While affirmative action policies are often thought of, by advocates and critics alike, as a transfer of benefits from one group to another, there can also be net losses of benefits when both groups do less than their best. What might otherwise be a zero-sum game can thus become a negative-sum game. (Sowell (2004), p. 14)

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[1] Discrimination is interpreted here as a disadvantage of a group of individuals in social, political, or economic contexts that is based on some kind of exogenous marker, e. g. race, gender, or nationality, that is (at least initially) not related to these contexts and for which the members of these groups are personally not responsible. Alternatively, more shortly and less technical, discrimination can be described as “allowing racial identification [or gender, nationality etc.] to have a place in an individual´s life chances”; see Arrows (1998), p. 91.

[2] This persistence of discrimination could, for instance, be interpreted as the consequence of historical discrimination that affects negatively the contemporaneous generation, e.g. if investment in human capital depends on the historical segregation of work and living places along races; see Lundberg and Startz (1998).

In the next section a very stylized class of contest game is introduced to answer the question if the criticized trade-off does exist in this model, i.e. if optimizing players in these contest games would in fact reduce their respective effort levels as a consequence of affirmative action policies. The implementation of affirmative action policy is modeled as a biased contest rule where weak contestant are favored because the ethical perception interprets their weakness as the consequence of past discrimination. The alternative perception, i. e. holding the contestants ethically responsible for their heterogeneity, would then induce an unbiased contest rule. The reaction of the contestants depends on the specific design of the two policy options which could consist in, for example in the case of affirmative action, a fixed bonus specified for each discriminated minority group, or a relative bonus that also depends on the effort, bid, etc. in which the contestants compete in.
Contrary to Sowell´s prediction it is shown that in this model the optimal individual response to the implementation of appropriately designed affirmative action instruments would be to increase individual effort level in the two-player contest game (irrespectively of the fact that the individual is discriminated or not). In this sense it can be said that the design of affirmative action policies does, in fact, matter. However, under more general conditions (with more than 2 players or a partially informed contest designer) this result can only be obtained if the underlying heterogeneity is not too severe. As the model is kept sufficiently simple to facilitate tractability, it is too stylized to give any sort of direct policy implications. Nevertheless it shows that a trade-off between affirmative action and aggregated effort, as stated by Sowell, may not exist, especially if the number of competitors is rather low.

The model is formulated in general terms to reflect a variety of situations in which affirmative action is implemented in a contest-like environment. Possible real world-examples are: university admission, in which applicants compete for places in a university program by means of their high school grade point average and discriminated applicants get some kind of bonus; promotion tournaments between employees that are partly discriminated; and even sport contests, for example horse riding, in which jockeys that weight less than their competitors are forced to carry additional weight.

Related Literature

This paper contributes to the economic literature on affirmative action by introducing a different framework, i.e. a contest game to model the consequences of affirmative action. Similar models that analyze affirmative action for ex-ante heterogenous (discriminated) individuals in a local context are based on, for instance, promotion tournaments between two individuals in Schotter and Weigelt (1992); and auctions as in Corns and Schotter (1999). Both papers show theoretically and experimentally that affirmative action may lead to higher effort levels by all participants which corresponds to the results of the contest game presented below. Fu (2006) models the university admission example as an all-pay auction under complete information with two bidders and also derives similar results.

All papers mentioned so far do not specify the normative objective of affirmative action, i.e. in these papers affirmative action is considered simply as a deviation from some color-blind or equal treatment-policy. This is a crucial difference to the contest model presented below because here the normative objective of affirmative action is explicitly defined and integrated into the model which implies an additional sine-qua-non constraint on the contest design. A similar approach is used in Kranich (1994) for a two-player production economy where the jointly produced output is shared according to a 'division rule'. He introduces a normative requirement on the class of feasible division rules which is phrased equal-division-for-equal-work principle[3] and shows that there exist feasible division rules that would satisfy this principle and also pareto-efficiency. In a contest game, however, the notion of pareto-efficiency has no sense because there is no production that depends on the exerted effort of the contestants. The remaining broad class of possible contest rules that satisfy the respective normative constraint is instead reduced by solving an additional design problem, i.e. specifying the contest rule in such a way that total effort is maximized subject to the respective normative constraint. The two optimally designed and normatively determined contest rules can then be evaluated according to the total equilibrium effort that they generate.

The resulting specific form of the affirmative action policy that maximizes total effort has also an interesting normative interpretation as it is shown in a more general framework in Calsamiglia (2004). She demonstrates that the only type of affirmative action that guarantees global equality of opportunity[4] has to equalize 'reward to effort'. If

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[3] This principle is closely related to the subsequently stated two definitions of the policy alternatives in the sense that it is here adapted to the two different perceptions for the reason of the heterogeneity of the contestants. Therefore the interpretation of 'equal work' will have a different meaning depending on the ethical perception of heterogeneity.

[4] Global equality of opportunity in a contest framework would be defined as equality of welfare achieved for individuals that compete in several contests simultaneously and where the respective contest designers implement affirmative action that is based on local information (i.e. which is limited to the respective contest).

this 'equal reward to effort'-property is translated to the model presented below the type of affirmative action that maximizes total effort also equalizes reward to effort.

According to my knowledge there exists no approach that models affirmative action explicitly in a contest game framework. However, the underlying contest game is basically an asymmetric contest with heterogeneous players. Models of asymmetric contests are applied in the context of a two-stage contest; see Leininger (1993), to analyze legal presumption in trials; see Bernardo, Talley, and Welch (2000), or with the interpretation of prior probabilities; see Corchón (2000), and recently by Cornes and Hartley (2005), that are able to proof existence and uniqueness of equilibrium for the n-player asymmetric contest game. Using techniques of the last mentioned paper the subsequently presented analysis will also generalize some results of the two-player asymmetric contest in Nti (2004).

The crucial difference between the vast literature on rent-seeking contests in general, that is based on the seminal contribution of Tullock (1980)[5] and the approach that is introduced in the following sections, is the assumption on how to measure social efficiency. While exerted effort in the rent-seeking literature is usually interpreted as social loss, the situations in which affirmative action are implemented are more appropriately characterized by an interpretation that takes exerted effort as socially valuable. Therefore there also exists some relation to the literature on sport contests in which effort, i.e. the performance of the athletes, has a similar interpretation; see Szymanski (2003).

2 The Model

Affirmative action instruments are usually applied in situations of competitive social interaction. The competitive structure of these situations is captured by a contest game in which contestants compete for an indivisible prize. The contestants can increase their respective probability to win the contested prize by exerting more effort. This feature seems to be appropriate to model the basic structure of the above mentioned examples because there exists a relatively high grade of discretion on the side

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[5] Nitzan (1994) provides a literature survey, and a collection of related articles can be found in Lockard and Tullock (2001).

of the organizer of the competition. This is reflected in a contest game in which contestants face a probabilistic outcome. To guarantee analytical tractability and closed form solutions the model is formulated under complete information, i. e. the only element of uncertainty is the final winner of the contest.

2.1 The Contestants

Let $N = \{1, 2, \ldots, n\}$ denote the set of individuals that compete against each other in a contest game. Each contestant $i \in N$ exerts an effort level $e_i \in \Re_+$ and takes the effort level e_j of its rivals $j \neq i$ as given. Additionally, it is assumed that all contestants are risk-neutral and have the same positive valuation V for the contested prize. The only element of heterogeneity among the contestants is the respective 'cost function' that captures the disutility of exerting effort e_i which depends additionally on parameter β_i that (potentially) reflects the degree of historical discrimination of contestant i. It is assumed that this cost function is linear in e_i and multiplicative in β_i for all $i \in N$, with β_i normalized in such a way that for the most able contestant $\hat{\beta} = 1$ and for less able contestants $\beta \in \left(\hat{\beta}, \infty\right)$:

$$c_i(e_i) = \beta_i e_i \text{ for all } i \in N. \tag{1}$$

The contest designer sets up the rule of the contest game, or so called contest success function (CSF), that depends on the individually weighted effort levels of all contestants. The following CSF that will be applied in the model allows an asymmetric treatment of the contestants and, at the same time, preserves the probabilistic structure of the contest game:

$$p_i(x) = \frac{f_i^P(e_i)}{\sum_{j \in N} f_j^P(e_j)} \text{ for all } i \in N. \tag{2}$$

This function maps the vector of effort levels $e = (e_1, \ldots e_n)$ into win probabilities for each contestant: $p_i(x) : \Re_+^n \to \Re_+$. However, each individual effort level is weighted by a twice differentiable weighting function $f_i^P(e_i)$ with $f'(e_i) > 0$, $f''(e_i) < 0$, and $f_i^P(0) = 0$ for all contestants $i \in N$ that specifies (according to policy P that is formally introduced in the subsequent section) how the contest game is distorted with respect to each contestant. In the case that all contestants exert zero effort the prize is not awarded, i.e. $p_i(0) = 0$ for all $i \in N$.

The specification of the cost function in eq. (1) and the contest mechanism in eq. (2) are already the necessary elements to state the following expected (additive separable) utility function of contestant i:

$$u_i(e_i) = p_i(e)V - c_i(e_i) \text{ for all } i \in N. \tag{3}$$

The class of weighting function as specified in eq. (2) is rather broad which complicates the analysis of the contest game because closed form equilibria might not exist. The following result will simplify the analysis substantially because the class of CSFs under consideration can be restricted to a corresponding class of CSFs with a linear weighting function $g_i(e_i) = \alpha_i e_i + \gamma_i$ for all $i \in N$ (which implies that $p_i(e) = \frac{g_i(e_i)}{\sum_{j \in N} g_j(e_j)}$).[6] For this class of contest games with linear CSFs closed form solutions are available. The intuitive explanation for this simplification is that every equilibrium outcome of a contest game with concave weighting functions can be replicated by a contest game with appropriately designed linear weighting functions. This is shown for contest games with two players in Nti (2004) and is generalized to contest games with n players in the following proposition:

Proposition 1 *Let* $\hat{e} = (\hat{e}_1, \ldots, \hat{e}_n)$ *denote the equilibrium effort vector of a contest game with increasing, twice differentiable, and concave weighting function* $f_i(e_i)$ *where* $f_i(0) = 0$ *for all* $i \in N$.
Let $\check{e} = (\check{e}_1, \ldots, \check{e}_n)$ *denote the equilibrium effort vector of a contest game with a linear weighting function* $g_i(e_i) = \alpha_i e_i + \gamma_i$ *for all* $i \in N$.
For $\alpha_i = f_i'(\hat{e}_i) > 0$ *and* $\gamma_i = f_i(\hat{e}) - f_i'(\hat{e})\hat{e} \geq 0$ *for all* $i \in N$ *both vectors* $\hat{e}$ *and* $\check{e}$ *coincide.*

The proof is slightly more complex than in Nti (2004) and therefore relegated to the appendix. The additional difficulty is due to the fact that in the n-player contest some of the contestants could be non-active, i.e. they would prefer to exert zero effort. This implies that it is not feasible to assume that equilibrium effort is the solution of simple first order conditions, as in the two-player contest considered in Nti (2004).

Proposition 1 shows that each contest game with a concave weighting function can be replicated by a contest game with an appropriately specified linear weighting function. Therefore, the subsequent analysis is restricted to a simplified class of contest games because proposition 1 implies that no potential contest game with a concave weighting

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[6] As the following paragraphs consider any possible increasing and concave weighting function, the policy parameter P is suppressed for notational convenience.

function will be omitted by considering only contest games with the following linear weighting function:

$$g_i^P(e_i) = \alpha_i^P e_i + \gamma_i^P \text{ with } \alpha_i^P > 0, \gamma_i^P \geq 0 \text{ for all } i \in N. \tag{4}$$

The design problem of the contest designer is therefore reduced to a more tractable problem: from the choice of a concave weighting function $f_i^P(x_i)$ for all contestants to the specification of a vector $\left\{\alpha_i^P, \gamma_i^P\right\}_{i \in N}$ for the linear weighting function $g_i^P(e_i)$.

2.2 The Contest Design

The design of the weighting function is assumed to depend on two components: a normative and a strategic one. The normative component will be responsible for the implementation of a policy P that prescribes for each policy some specific constraints on the design of the weighting function. The remaining discretionary space with respect to the final design of the weighting function then allows the contest designer to strategically specify the policy parameters $\left\{\alpha_i^{P*}, \gamma_i^{P*}\right\}_{i \in N}$ of eq. (4) for the simplified contest game subject to the normative constraint given by policy P.

At first, the normative component, i.e. the reason for the implementation of a specific policy option P is discussed and summarized in formal definitions. After that, the strategic component is considered that will result in the maximization problem of eq. (8).

The choice of the policy P is assumed to be based on the ethical perception of the heterogeneity of the contestants (i.e. the different marginal cost functions)[7] which directly implies the normative objective of the respective policy alternative P. With respect to the ethical perception of the difference in cost functions there are two potential interpretations for the reason of this heterogeneity:

The first interpretation holds the contestant ethically responsible for her respective cost function in which case the probability to win the contest game (i.e. the CSF) should only depend on the vector of exerted effort. In other words, if a contestant exerts the same effort level than another contestant then both contestants should win the contest game with the same probability. This policy option would therefore treat the contestants equally with respect to their exerted effort level.

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[7] As the model is formulated under complete information, the individual marginal cost functions are common knowledge.

Definition 1 *A policy is called* ***equal treatment approach (ET)*** *if:*

$$e_i = e_j \Rightarrow p_i(e) = p_j(e) \text{ for all } i \neq j.$$

For the simplified class of contest games with linear weighting function as defined by eq. (4) equal treatment implies that:

$$\alpha_i^{ET} = \alpha^{ET}, \gamma_i^{ET} = \gamma^{ET} \text{ for all } i \in N.$$

The last line is derived by observing that for all $e_i = e_j$ it has to be the case that also $p_i(e) = p_j(e)$ for all $i \neq j$. Solving this expression for the simplified contest game with linear weighting function given in eq. (4) yields the specification for $\left\{\alpha_i^{ET}, \gamma_i^{ET}\right\}_{i \in N}$. This policy could also be interpreted as a kind of anonymity requirement in the sense that the contest success function does neither depend on the specific names nor on the exogenous characteristics of the players. However, the outcome, i.e. expected equilibrium utility, of the contest game will indirectly depend on the characteristics of the players because weaker players will exert less effort in equilibrium.

The second interpretation is based on the perception that the contestants cannot be held ethically responsible for their heterogeneity, for instance, if it is the consequence of past discrimination. As heterogeneity affects the cost function for each contestant, fairness would require that two contestants that face equal disutility induced by the chosen effort level (that could be different) should have the same probability to win the contest game.

Definition 2 *A policy is called* ***affirmative action (AA)*** *if:*

$$c_i(e_i) = c_j(e_j) \Rightarrow p_i(e) = p_j(e) \text{ for all } i \neq j. \tag{5}$$

For the simplified class of contest games with linear weighting function as defined by eq. (4) affirmative action implies that:

$$\alpha_i^{AA} = \beta_i, \gamma_i^{AA} = \gamma^{AA} \text{ for all } i \in N. \tag{6}$$

The specification in eq. (6) is derived by using the transformation $z_i = c_i(e_i)$. As $c_i(e_i)$ is linear it can be inverted: $e_i = z_i/\beta_i$. The condition in eq. (5) for the transformed model then states that if $z_i = z_j \equiv \bar{z}$ then $p_i(z/\beta) = p_j(z/\beta)$ for all $\bar{z}$ and for all $i \neq j$ where $z/\beta = (z_1/\beta_1, \ldots z_n/\beta_n)$. Solving $p_i(z/\beta) = p_j(z/\beta)$ for the simplified contest game with linear weighting function as in eq. (4) implies that: $\alpha_i^{AA}\bar{z}/\beta_i + \gamma_i^{AA} = \alpha_j^{AA}\bar{z}/\beta_j + \gamma_j^{AA}$ for all $\bar{z}$. This condition is satisfied if $\left\{\alpha_i^{ET}, \gamma_i^{ET}\right\}_{i \in N}$ is specified as in eq. (6).

The policy AA therefore specifies a bias[8] of the contest success function in favor of discriminated contestants in such a way that both contestants have the same probability to win the contest whenever they face the same disutility of effort.[9]
This definition of affirmative action is also closely related to the 'equal-division-for-equal-work' principle that is adopted in Kranich (1994). The difference is that 'equal work' in this context should be interpreted as equal disutility of effort because it is based on the "moral intuition that two people incurring equal disutility deserve equal rewards." (ibid., p. 178).

A general remark with respect to the introduced policy options should be in order at this point. Both policies, ET and AA, can be considered as principles that guarantee a notion of procedural fairness because they restrict the design of the weighting function without regard to expected equilibrium utility. An alternative approach would be a contest rule that is based explicitly on the resulting expected equilibrium utility for the contestants. In the case of AA it could therefore be alternatively argued that the outcome of the contest game should correspond to the ethical perception of the heterogeneity of the contestants in the following sense: if the contestants are perceived to be different because they are discriminated (for which they cannot be held ethically responsible) then the contest outcome should be as if they would be de facto homogenous. This directly implies that the expected utility in equilibrium should be identical for all contestants. A so called 'end-state notion' of affirmative action is captured formally in the following definition.[10]

Definition 3 *A policy is called* ***affirmative action with respect to expected equilibrium utilities (AU)*** *if the expected utility for each contestant in the contest game is identical in equilibrium:*

$$u_i(e_i^*(AU)) = u_j(e_j^*(AU) \text{ for all } i \neq j. \tag{7}$$

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[8] In fact, the relaxation of the anonymity axiom in an axiomatization of contest success functions was justified by arguing that "in many situations, however, contestants are treated differently (due to affirmative action programs for instance)" (in Clark and Riis 1998, p. 201).

[9] This specification of AA therefore coincides with the notion of equal reward to effort as in Calsamiglia (2004), as mentioned in the section on related literature.

[10] $e_i^*(P)$ denotes the equilibrium effort of contestant i under policy $P \in \{ET, AA, AU\}$.

For the simplified class of contest games as defined by eq. (4) affirmative action implies that:

$$\left\{\alpha_i^{AU}, \gamma_i^{AU}\right\}_{i \in N} \text{ such that eq. (7) holds.}$$

After the derivation of the equilibrium in the next section it can be shown that AA implies AU but not vice versa. This implies that AU is less restrictive with respect to the design of the vector of policy parameters, i. e. the class of $\left\{\alpha_i^{AU}, \gamma_i^{AU}\right\}_{i \in N}$ is broader than $\left\{\alpha_i^{AA}, \gamma_i^{AA}\right\}_{i \in N}$. An example for a lump sum affirmative action policy that satisfies AU but not AA is given in the section on two-player contests.

From definitions 1, 2, and 3 it is obvious that the contest designer still has some freedom in designing the specific vector of the policy parameters $\left\{\alpha_i^{P}, \gamma_i^{P}\right\}_{i \in N}$ for $P \in \{ET, AA, AU\}$. This final specification will influence the level of total effort in equilibrium because the contestants will react differently to different designs of the weighting function for a given policy P. Therefore, the design of the vector of policy parameters can be used as an incentive device by the contest designer. As she is assumed to be interested in total effort (in line with the real world examples mentioned before)[11], the contest designer can specify the policy parameters in such a way that aggregated equilibrium effort is maximized subject to the normative constraint given by AA, AU or ET. Formally, the finally optimal designed policy P* can be derived as the solution to the following maximization problem:

$$\left\{\alpha_i^{P*}, \gamma_i^{P*}\right\}_{i \in N} = \underbrace{argmax}_{\left\{\alpha_i^P > 0, \gamma_i^P \geq 0\right\}_{i \in N}} \quad \textstyle\sum_{i \in N} e_i^*(P) \tag{8}$$

$$\begin{array}{lll} \text{s. t.} & \alpha_i^{ET} = \alpha^{ET}, \gamma_i^{ET} = \gamma^{ET} \text{ for all } i \in N & \text{if } P = ET; \\ \text{or} & \alpha_i^{AA} = \beta_i, \gamma_i^{AA} = \gamma^{AA} \text{ for all } i \in N & \text{if } P = AA; \\ \text{or} & u_i(e_i^*(AU)) = u_j(e_j^*(AU) \text{ for all } i \neq j & \text{if } P = AU. \end{array}$$

The optimal policy that is described by the vector $\left\{\alpha_i^{P*}, \gamma_i^{P*}\right\}_{i \in N}$ will be denoted as $P^* \in \{ET^*, AA^*, AU^*\}$. In the following sections it will be shown that AU* and AA* are, in fact, identical. To avoid redundancies the following discussion is therefore restricted to the comparison of AA* versus ET*, i.e. AA* can be substituted by AU*

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[11] In the college admission example, the education authorities are interested in high effort levels, i.e. grades, by all students that are possibly affected by the policy irrespective of the fact that they are admitted or not. Also in promotion tournaments the employer is obviously interested in high effort levels by all the employees, irrespective of their final promotion (the interpretation of promotion contests as incentive devices is obvious in this case). And even in sport competitions it can be argued that spectators are sometimes interested in the overall performance of all athletes because ex-ante predictable sport competitions are usually perceived as boring.

without affecting results.
The implementation of the optimal policy P^* generates a unique level of aggregated equilibrium effort level: $E^*_P = \sum_{i \in N} e^*_i(\alpha_i^{P^*}, \gamma_i^{P^*})$ for $P^* \in \{ET^*, AA^*\}$. Based on this expression the policy comparison between AA* and ET* becomes a trivial exercise: The criticized trade-off between affirmative action and total effort would imply that total effort under AA* is lower than under ET*, i.e. that $E^*_{ET} > E^*_{AA}$. The following two sections will show that this is not true in the two-player contest and will provide conditions under which the trade-off does not exist in the n-player contest.

As the policy alternatives of the contest designer are clarified now, the timing of the complete contest game can be summarized in the following way: The heterogeneity of the contestants as manifested in their different marginal costs is observed. Based on the ethical perception of this observation a policy option $P \in \{ET, AA, AU\}$ is selected. The contest designer specifies the final policy parameters $\left\{\alpha_i^{P^*}, \gamma_i^{P^*}\right\}_{i \in N}$ for the chosen policy option $P \in \{ET, AA, AU\}$ that maximize total equilibrium effort $\sum_{i \in N} e^*_i(P)$. The contestants observe this final specification of policy parameters $\left\{\alpha_i^{P^*}, \gamma_i^{P^*}\right\}_{i \in N}$ and exert the optimal (with respect to their expected utility) equilibrium effort level $e^*_i(\alpha_i^{P^*}, \gamma_i^{P^*})$ for each $i \in N$, taking the effort levels of their rivals as given. In the last step the contest designer observes the exerted efforts and determines the winner of the contest game according to the announced policy option (which implies an eventually biased contest success function).
The two optimally designed policy options ET* and AA* (which is identical to AU*) can be compared by deriving the total equilibrium effort that they generate. This comparison then directly answers the question whether a trade-off between affirmative action and total effort does in fact exist or not.

3 The Two-Player Contest Game

Restricting the number of contestants in the two player case already yields the key result of the comparative policy analysis: in equilibrium both contestants will exert more effort under AA than under ET. Contrary to the n-player contest game this result holds without any extra assumption. The final design of ET and AA policy parameters is identical in two- and n the n-player contests but the derivation of equilibrium and the final design of the policy parameters in the two-player contest is based on simple first order conditions. Therefore, the two-player contest game is analyzed at first.

Using the simplified contest game as specified by eq. (4) the expected utility function in eq. (3) can be expressed as:

$$u_i(e_i) = \frac{\alpha_i^P e_i + \gamma_i^P}{\alpha_1^P e_1 + \gamma_1^P + \alpha_2^P e_2 + \gamma_2^P} V - \beta_i e_i \text{ for } i = 1, 2; P \in \{ET, AA, AU\}\,.$$

As the expected utility function is concave and continuous if $\left\{\alpha_i^P, \gamma_i^P\right\}_{i=1,2}$ are specified as in proposition 1, the second order conditions are satisfied. Solving first order conditions for a given policy parameter P yields the equilibrium effort level $e_i^*(P)$:[12]

$$e_i^*(P) = \frac{\alpha_i^P \alpha_j^P \beta_j}{(\alpha_1^P \beta_2 + \alpha_2^P \beta_1)^2} V - \frac{\gamma_i^P}{\alpha_i^P} \qquad \text{for } i \neq j, \tag{9}$$

with expected equilibrium utility:

$$u_i(e_i^*(P)) = \frac{(\alpha_i^P)^2 \beta_j^2}{(\alpha_1^P \beta_2 + \alpha_2^P \beta_1)^2} V + \frac{\beta_i \gamma_i^P}{\alpha_i^P} \text{ for } i \neq j.$$

Based on this equation the condition that characterizes the AU policy according to definition 3 can now be simplified:

$$\left(\frac{\beta_1}{\alpha_1^{AU}} - \frac{\beta_2}{\alpha_2^{AU}}\right) V = \left(\frac{\beta_1 \gamma_1^{AU}}{\alpha_1^{AU}} - \frac{\beta_2 \gamma_2^{AU}}{\alpha_2^{AU}}\right)\left(\frac{\beta_1}{\alpha_1^{AU}} + \frac{\beta_2}{\alpha_2^{AU}}\right). \tag{10}$$

Note that policy AA always satisfies this expression, however, not all solutions of this expression satisfy the definition of AA. Consider for instance the following AU policy that gives a lump-sum bonus to contestant 2: $\alpha_1^{AU} = \alpha_2^{AU} = 1, \gamma_1^{AU} = 0, \gamma_2^{AU} = \frac{(\beta_2 - \beta_1)V}{\beta_2(\beta_1 + \beta_2)}$. As required by definition 3 this policy parameters would equalize expected equilibrium utility but do not yield an equal probability to win the contest game for all identical effort levels $e_1 = e_2$ as required by definition 2.

The following lemma shows that this difference in interpretation and formal definition does not have any consequences for the optimal design of the policy parameters in this framework because the specification of AA* and AU* coincides.

Lemma 1 *In the two-player contest game the optimal policy vector* $\left\{\alpha_i^{P*}, \gamma_i^{P*}\right\}_{i=1,2}$ *for* $P \in \{ET, AA, AU\}$ *can be specified as:*

$$\begin{aligned} \alpha_i^{ET^*} &= \alpha^{ET^*} > 0, & \gamma_i^{ET^*} &= 0, \text{ and} \\ \alpha_i^{AA^*} &= \alpha_i^{AU^*} = \beta_i, & \gamma_i^{AA^*} &= \gamma_i^{AU^*} = 0 \text{ for } i = 1, 2. \end{aligned}$$

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[12] As e_i is defined to be non-negative, the vector $\{\alpha_i^{P^*}, \gamma_i^{P^*}\}_{i=1,2}$ has to be specified such that also $e_i^*(P)$ is non-negative.

Proof: The final design of the policy parameters is the result of the maximization problem of eq. (8) which can be stated as:

$$\left\{\alpha_i^{P^*}, \gamma_i^{P^*}\right\}_{i=1,2} = \underbrace{argmax}_{\left\{\alpha_i^P > 0, \gamma_i^P \geq 0\right\}_{i=1,2}} \frac{\alpha_1^P \alpha_2^P (\beta_1 + \beta_2)}{(\alpha_2^P \beta_1 + \alpha_1^P \beta_2)^2} - \frac{\gamma_1^P}{\alpha_1^P} - \frac{\gamma_2^P}{\alpha_2^P} \tag{11}$$

subject to the normative constraints given by $P \in \{ET, AA, AU\}$.
Maximizing the right hand side of eq. (11) implies that $\gamma_i^{P^*} = 0$ irrespective of $P \in \{ET, AA, AU\}$. The specification of α^{ET^*} becomes then irrelevant because the right hand side of eq. (11) does not depend on α^{ET^*} anymore. The specification of $\alpha_i^{AU^*}$ for $i = 1, 2$ according to eq. (10) yields the condition $\frac{\alpha_1^{AU^*}}{\alpha_2^{AU^*}} = \frac{\beta_1}{\beta_2}$ that gives the optimal designed AU* parameters $\alpha_i^{AU^*} = \beta_i$ for $i = 1, 2$ (because the CSF could be normalized appropriately) which also coincides with the specification of AA*.□

Lemma 1 shows that although policy AU is less restrictive this has no consequences with respect to the optimal design of the affirmative action policy. For contest games as specified here the procedural interpretation of affirmative action (definition 2) coincides with the end-state principle (definition 3) for the optimal design of the affirmative action policy.[13]

As AA* and AU* are identical, the following policy comparison is restricted to AA* versus ET* to ease notation. Thus, the two optimal designed policy alternatives ET* and AA* are evaluated according to the discussion in section 2, i.e. with respect to the sum of equilibrium effort $E_P^* = \sum_{i \in N} e_i^*(\alpha_i^{P^*}, \gamma_i^{P^*})$ that each policy generates.

The following proposition states the key result: Appropriately designed AA* policy, specified by vector $\left\{\alpha_i^{AA^*}, \gamma_i^{AA^*}\right\}_{i=1,2}$, will result in more individual and also aggregated effort than optimal designed ET* policy, specified by vector $\left\{\alpha_i^{ET^*}, \gamma_i^{ET^*}\right\}_{i=1,2}$, which contradicts the above mentioned critique of affirmative action. In the contest game as specified here a trade-off between affirmative action and aggregate effort does not exist.

Proposition 2 *In the two-player contest game (i) the sum of equilibrium effort, and (ii) each individual equilibrium effort level is higher in equilibrium under the optimal designed AA* policy than under optimal designed ET* policy.*

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[13] This also implies that in this framework a lump-sum bonus (as it is used, for instance, in Schotter and Weigelt 1992 to model affirmative action) is never optimal.

Proof: Using the definition of E_P^* and lemma 1 the inequality $E_{AA}^* > E_{ET}^*$ can be reduced to $\frac{\beta_1+\beta_2}{4\beta_1\beta_2}V > \frac{V}{\beta_1+\beta_2}$ which is always satisfied because it can be simplified to $(\beta_1 - \beta_2)^2 > 0$. This establishes part (i) of the proposition.
Using eq. (9) in combination with lemma 1 implies that the inequality $e_i^*(\alpha_i^{AA^*}, \gamma_i^{AA^*}) > e_i^*(\alpha_i^{ET^*}, \gamma_i^{ET^*})$ can be simplified to $(\beta_1 - \beta_2)^2 > 0$ which establishes part (ii) of the proposition.□

The reason for this at first sight surprising result lies in the fact that the implementation of the optimal designed AA* policy results in a contest game that is more balanced with respect to the characteristics of the contestants (the heterogeneity of the contestants is reduced by the distortion of the CSF). As the contestants are more similar under this AA*, the competitive pressure is higher which implies higher equilibrium effort by both contestants.[14]

In fact, the distortion required by AA* for the two-person contest game leads to a level playing field, i.e. the contestants are as similar as possible. This implies that the distortion of the CSF specified by the vector of policy parameters $\left\{\alpha_i^{P^*}, \gamma_i^{P^*}\right\}_{i=1,2}$ generates the maximal aggregated effort even for contest games that are not restricted by any normative constraint as shown by Nti (2004) for two-person contest games (where he assumes that the heterogeneity of the contestants stems from different valuations).

Proposition 3 ***(Nti2004)*** *The distortion of the CSF that is described by* $\left\{\alpha_i^{AA^*}, \gamma_i^{AA^*}\right\}_{i=1,2}$ *generates the maximal sum of equilibrium effort in the two-player contest game without any additional normative constraint.*

In the next section it will be analyzed whether these results are also valid for n-player contest games.

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[14] Similar results are known, for example, from the literature on optimal auction design: A revenue maximizing auction implies also the favoring of weak bidders (comp. McAfee and McMillan 1989).

4 The n-Player Contest Game

In this section a contest game with $n > 2$ contestants is considered. Analogously to eq. (3) the expected utility of the risk-neutral contestant i can then be expressed as:

$$u_i(e) = \frac{\alpha_i^P e_i + \gamma_i^P}{\sum_{j \in N} \alpha_j^P e_j + \gamma_j^P} V - \beta_i e_i \text{ for all } i \in N \text{ and for } P \in \{ET, AA, AU\} . \tag{12}$$

The derivation of equilibrium and the proof of existence and uniqueness for the n-player case is more involved than in the last section because, as it will be obvious later, not all contestants will always exert a strict positive effort level in equilibrium. Hence, the first-order method that was used in the two-player contest is not feasible because the equilibrium will not be interior for contestants that will exert no effort. The approach that is instead applied here is based on the notion of "share functions" as defined in Cornes and Hartley (2005) which has the advantage that the existence proof of equilibrium is reduced to a simple fixed point argument in $\Re^2$ with a single monotonic decreasing function. The equilibrium will be derived in the appendix.

The following equation provides an expression for equilibrium effort for those m contestants of set $M \subseteq N$ that are active, i.e. that exert a positive equilibrium effort:

$$e_i^*(P) = \frac{1}{\alpha_i^P} \left(1 - \frac{\beta_i}{\alpha_i^P} \frac{(m-1)}{\sum_{j \in M} \frac{\beta_j}{\alpha_j^P}} \right) \frac{(m-1)V}{\sum_{j \in M} \frac{\beta_j}{\alpha_j^P}} - \frac{\gamma_i^P}{\alpha_i^P} \text{ for all } i \in M \text{ and } P \in \{ET, AA, AU\} ; \tag{13}$$

Set M is indirectly defined by the following inequality:

$$(m-1)\frac{\beta_i}{\alpha_i^P} < \sum_{j \in M} \frac{\beta_j}{\alpha_j^P} \text{ for all } i \in M \text{ and } P \in \{ET, AA, AU\} . \tag{14}$$

Based on these two equations the optimal design of the policy parameters $\left\{\alpha_i^{P^*}, \gamma_i^{P^*}\right\}_{i \in N}$ for $P \in \{ET, AA, AU\}$ can be derived according to the maximization problem in eq. (8). The result is identical to lemma 1, however, the proof is more complex and relegated to the appendix.

Lemma 2 *In the n-player contest game the optimal policy vector* $\left\{\alpha_i^{P^*}, \gamma_i^{P^*}\right\}_{i \in N}$ *for* $P \in \{ET, AA, AU\}$ *can be specified as:*

$$\begin{aligned} \alpha_i^{ET^*} &= \alpha^{ET^*} > 0, & \gamma_i^{ET^*} &= 0, \textit{ and} \\ \alpha_i^{AA^*} &= \alpha_i^{AU^*} = \beta_i, & \gamma_i^{AA^*} &= \gamma_i^{AU^*} = 0 \textit{ for all } i \in N. \end{aligned}$$

The following corollary provides conditions for the active set for each optimally designed policy by combining lemma 2 and eq. (14).

Corollary 1 *Under the optimal designed ET* policy the active set $M \subseteq N$ of contestants is implicitly defined by the following inequality:*

$$(m-1)\beta_i < \sum_{j \in M} \beta_j \text{ for all } i \in M. \tag{15}$$

Under the optimal designed AA and AU* policy all contestants will be active.*

As before the following discussion is restricted to the optimal designed policies ET* and AA* because the specification of AA* and AU* is identical.

Lemma 2 can now be used to analyze the optimal designed policy options specified by $\left\{\alpha_i^{P^*}, \gamma_i^{P^*}\right\}_{i \in N}$ with respect to the aggregated equilibrium effort $E_P^* = \sum_{i \in N} e_i^*(\alpha_i^{P^*}, \gamma_i^{P^*})$ that they induce. However, corollary 1 already reveals that the comparison between the two policy options will not be as straight forward as in the two-player contest game because the total equilibrium effort depends on the distribution of the cost parameter that determines the active set.

The following notation will simplify the characterization of the relevant distribution for a subset $J \subseteq N$ of contestants: the arithmetic mean of the cost parameters of agents of set J will be denoted as $\bar{\beta}_J = \frac{1}{j}\sum_{i \in J} \beta_i$ (where $\bar{\beta} = \bar{\beta}_N$ to facilitate notation), and the harmonic mean respectively as: $\beta_J^H = \left[\frac{1}{j}\sum_{i \in J} \frac{1}{\beta_i}\right]^{-1}$.
The subsequent proposition states the condition under which the optimal designed AA policy generates higher aggregated effort.

Proposition 4 *In the n-player contest game the sum of equilibrium effort levels is higher for the optimal designed AA* policy than for the optimal designed ET* policy if:*

$$\frac{\bar{\beta}_M}{\beta_N^H} > \frac{\frac{m-1}{m}}{\frac{n-1}{n}}. \tag{16}$$

Proof: Calculation of the sum of equilibrium effort for the optimal designed policies ET* and AA* as defined in lemma 2 under consideration of corollary 1 yields $E_{AA}^* = \frac{n-1}{n^2} V \sum_{i \in N} \frac{1}{\beta_i}$ and $E_{ET}^* = \frac{m-1}{\sum_{i \in M} \beta_i} V$. Reformulating the inequality $E_{AA}^* > E_{ET}^*$ leads to condition (16).□

The following intuitive explanation is provided for proposition 4 which is afterwards clarified by a numerical example. As already observed in the two-player contest game, AA* induces in general higher competitive pressure because contestants are more similar than under ET*. Increasing the number of active contestants therefore yields higher total effort for both policies because this implies more intense competition. However, inducing heavily discriminated contestants to participate comes at a non-negligible cost, especially for the AA* policy because here all participants will be active. This effect is less profound for ET* because highly discriminated contestant will not participate under ET*.

Numerical Examples: Consider the following example with three contestants that have marginal costs of $(\beta_1, \beta_2, \beta_3) = (1, 2, 2)$. The underlying dispersion is measured by the coefficient of variation (defined as $CV = (1/\bar{\beta})\sqrt{var(\beta)})$ which is in this case $CV \approx 0.2828$. Implementation of AA* yields a higher total effort than ET*: $E^*_{AA} \approx 0.4444$ versus $E^*_{ET} = 0.4$. Increasing the number of contestants by adding a fourth one with $\beta_4 = 2.43$ (which has nearly the same level of dispersion $CV \approx 0.2828$) results in $E^*_{AA} \approx 0.4522$ versus $E^*_{ET} \approx 0.4038$ which is higher for each policy in comparison with the three player example. Also the difference between AA* and ET* is now even stronger: $E^*_{AA} - E^*_{ET} \approx 0.0483$. However, if the fourth contestant is highly discriminated ($\beta_4 = 10$) this would imply a decline of total effort in the case of AA*: $E^*_{AA} \approx 0.3938$. This decline is less intensive in case of ET* because here the fourth player will not participate. As only the first three contestants will be active under ET* the result is identical to the case $(\beta_1, \beta_2, \beta_3) = (1, 2, 2)$, i.e. $E^*_{ET} = 0.4$. Combining both results shows that now the result of the policy comparison switched because $E^*_{AA} < E^*_{ET}$.

This example demonstrates that the key determinants are the distribution of the discrimination parameters and the number of contestants. In general it can be stated that either a low number of contestants or a sufficiently low dispersion makes it more probable that AA* will induce more total effort than ET* because then the set of active contestants tends to be similar for both policies.[15] The exact relation between the

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[15] The fact that affirmative action distorts the participation decision of individuals which could finally dominate the effect of increased competitive pressure seems to be a more general phenomenon: In an empirical analysis of bid preferences in highway procurement auctions (Marion 2007, comp. also Krasnokutskaya and Seim 2006), it is shown that preferential treatment implied a decline in competitive pressure due to less participation of those high-cost firms that were not granted preferential treatment. This phenomenon cannot be captured by the contest model presented here because discrimination and high costs are synonyms in my approach.

distribution of discrimination parameters and the number of players is described by the inequality[16] in proposition 4 in combination with the determination of the active set in eq. (14) that also depends on this distribution.

An additional remark with respect to the relation between proposition 2 and 4 should be in order. Applying proposition 4 to a two-player contest game would yield the same result as proposition 2 because condition (16) holds irrespective of the distribution of cost parameters in the two-player case: For the optimally designed policies both contestants will exert positive equilibrium, i.e. set M and N coincide. Therefore, proposition (4) is satisfied without further restriction because condition (16) can be reduced to $\bar{\beta} > \beta_N^H$ which is always true (comp. the proof for proposition 5).

For the two-player contest game proposition 2 also contained a statement that compares individual equilibrium effort under the optimal designed policy. However, a comparable result for the n-player contest is not possible because the set of active agents depends on the underlying distribution of the discrimination parameter β. Hence, the additional assumption of full participation by all contestants under both policies shall be considered to get some further insights into the individual equilibrium behavior. This assumption would imply that the dispersion of cost parameters is sufficiently low such that also under policy ET* all contestant would be active.

The following proposition mirrors proposition 2 for this class of restricted distributions of the marginal cost parameter. Although the sum of equilibrium effort in this special case is higher under the optimal AA* policy versus the optimal ET* policy (without any further conditions) as in the two-player case, the result with respect to individual equilibrium effort is different: In the n-player contest the set of contestants that individually exert higher equilibrium effort under the optimal designed AA* policy than under the optimal designed ET* policy is restricted to contestants that are either highly able or less able than the average.

Proposition 5 *If all contestants in the n-person contest game are active under the optimally designed ET* policy, then (i) the sum of equilibrium effort levels is higher under*

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[16] Note, that the left hand side of condition (16) is lower than one for m small and larger than one for m large where m is determined according to condition (14). Inspection of the right hand side reveals that it is always lower or equal to one. This confirms the qualitative statement that condition (16) is likely to hold if the number of contestants is relatively small or the distribution is not too dispersed.

the optimal AA policy, and (ii) the individual equilibrium effort of all contestants with discrimination level* $\beta \in \left[1, \frac{1}{n-1}\bar{\beta}\right) \cup \left(\bar{\beta}, \frac{n}{n-1}\bar{\beta}\right)$ *is higher under the optimal AA* policy, while it is lower for contestants with* $\beta \in \left(\frac{1}{n-1}\bar{\beta}, \bar{\beta}\right)$*. For contestants with* $\beta \in \left\{\frac{1}{n-1}\bar{\beta}, \bar{\beta}\right\}$ *the individual equilibrium effort is the same under both policies.*

Proof: If all contestants are active, set M and N coincide, and condition (16) simplifies to $\bar{\beta} > \beta_N^H$. This inequality is always satisfied which proves the first part of the proposition.

For the second part the following inequality has to be analyzed: $e_i^*(\alpha_i^{AA^*}, \gamma_i^{AA^*}) > e_i^*(\alpha_i^{ET^*}, \gamma_i^{ET^*})$. Calculating this expression yields after some algebra the following inequality:

$$\left(\sum_{j \in N} \beta_j\right)^2 - n^2 \beta_i \left(\sum_{j \in N} \beta_j - (n-1)\beta_i\right) > 0. \tag{17}$$

This inequality is satisfied if $\beta_i \in \left[1, \frac{1}{n-1}\bar{\beta}\right)$, where the lower bound stems from the assumption that $\beta_i \geq 1$ for all $i \in N$, or if $\beta_i \in \left(\bar{\beta}, \frac{n}{n-1}\bar{\beta}\right)$, where the upper bound comes from the assumption of full participation under the optimal ET policy. The left hand side of eq. (17) is equal to zero for $\beta \in \left\{\frac{1}{n-1}\bar{\beta}, \bar{\beta}\right\}$. The continuity in β_i of the left hand side of eq. (17) implies the condition for the reversed inequality. Note also that the first interval could be empty if $n - 1 > \bar{\beta}$ which depends on the relevant underlying distribution. □

Notice that the set of contestants that individually exert more effort under the optimal AA* policy is not connected. The following intuition can be provided to explain this result: Consider first a (potentially hypothetical) level of discrimination that is identical to the mean: $\beta_k = \bar{\beta}$. Under the optimal designed AA* policy this would imply that contestant k is favored by $\alpha_k^{AA^*} = \bar{\beta}$. As the CSF is homogenous of degree zero, the vector α^{AA^*} can be normalized to the equivalent vector $\alpha' = \alpha^{AA^*}/\bar{\beta}$. For contestant k this would imply no distortion under the optimal AA policy because $\alpha_k' = \alpha_k^{AA^*}/\bar{\beta} = 1$. Additionally, he knows that under the optimal AA policy contestants that are less able than him are favored to the same amount as more able contestants are handicapped. Therefore his equilibrium effort level is not altered.

Contestants that are less able than contestant k are favored under α', i.e. their efficiency of effort in the CSF is increased ($\alpha_i' > 1$ for $i > k$) which implies that they exert higher effort level. The contrary is true for contestants that are more able than contestant k: they are handicapped ($\alpha_i' < 1$ for $i < k$) which reduces their efficiency of effort and therefore also their equilibrium effort.

However, there exist a counter effect for highly able contestants which becomes dominant for high abilities. This counter effect is due to increasing competitive pressure for highly able contestants because they are more handicapped under AA than their competitors. The cut-off ability is exactly at $\beta_c \equiv \bar{\beta}/(n-1)$. Contestants that have a lower cost parameter than contestant c will therefore exert higher equilibrium effort under AA* than under ET*.

5 An Extension: Group Contests

In the last section the implementation of the optimally designed AA* policy was based on a distortion of the contest game that was individually specified for each contestant. However, the implementation of affirmative action policies is usually not based on individual characteristics, but on group membership, e.g. minority, sex, race etc. (comp. the examples mentioned in the introduction). Reasons for this phenomenon could be incomplete information with respect to individual discrimination, or simply the fact that group members are sufficiently homogeneous to treat them identical. In the following section the latter aspect is analyzed while in the next section the informational requirements of the contest designer are relaxed.

The following model is a simplified version of the n-player contest game with the additional assumption that the n contestants belong either to group A or B that each consists out of $n_A \geq 2$ and $n_B \geq 2$ members. The members of each group are assumed to be identical, i.e. face the same discrimination parameter which is normalized for the non-discriminated group A such that $\beta_i = \beta_A = 1$ for all $i \in A$ and $\beta_i = \beta_B$ for all $i \in B$ where $\beta_B > 1$. It should be emphasized that this specification is already covered by the model of section 4 which implies that lemma 2 holds: $\alpha^{ET^*} > 0, \gamma^{ET^*} = 0$ and $\alpha_i^{AA^*} = \beta_i, \gamma_i^{AA^*} = 0$ for all $i \in N$.[17] The main objective is therefore another clarification of proposition 3 and the interplay between total effort and the active set of contestants. Additionally, the simplified model presented here can be considered as the starting point of the generalized model in the next section.

At first, the active set for the optimal designed ET* policy has to be determined (for AA* all contestants will always be active). Denote the number of active contestants

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[17] As before, the specification of AU* is identical to AA* so the following section is limited to the analysis of AA*.

of A by m_A, and m_B for group B. Starting with the less discriminated group A, it is obvious that all members of A are active because condition (29) reduces to $1 < \frac{m_A}{m_A-1}$ which is trivially satisfied for all $m_A \leq n_A$. Hence, all members of group A will be active under ET*.
Considering the members of group B, condition (14) becomes $\beta_B < \frac{n_A+m_B\beta_B}{n_A+m_B-1}$ which can be simplified to:

$$\beta_B < \frac{n_A}{n_A-1}. \tag{18}$$

Notice that the last condition does not depend on m_B anymore which implies that this condition either holds for all or for no member of B. Based on the number of group A-members and the discrimination parameter the following two cases are possible:

1. If condition (18) is satisfied both groups are active under ET*.
2. Otherwise, only members of group A are active under ET*.

Based on this two cases the aggregated equilibrium effort level under AA* and ET* can now be compared. Under case 1 all contestants are active such that proposition 5 can be used directly to conclude that AA* induces higher aggregated effort than ET*. The same proposition gives conditions for each discrimination level under which AA* induces more individually equilibrium effort than ET*. As $\beta_A = 1$ and $\beta_B > \bar{\beta}$ this implies that $\beta_A \in \left[1, \frac{1}{n-1}\bar{\beta}\right)$, $\beta_B \in \left(\bar{\beta}, \frac{n}{n-1}\bar{\beta}\right)$, and that there exists no contestant i such that $\beta_i \in \left[\frac{1}{n-1}\bar{\beta}, \bar{\beta}\right]$. However, it remains to be checked if the first interval is, in fact, non-empty, i.e. if $1 > \frac{1}{n-1}\bar{\beta}$. This inequality is satisfied if $\beta_B > \frac{n_A}{n_B}(n_A-2) + 2n_A + n_B - 1$. If this is the case, all individuals will indiviudally exert higher equilibrium effort under AA*. Otherwise, only group B members will increase their individual effort.

For the second case proposition 4 is applicable, which provides condition 16 to compare the aggregated equilibrium effort[18] under policy AA* and ET*. This condition simplifies for the contest game considered here to the following expression:

$$\beta_B < \frac{n_A(n_A+n_B-1)}{n_A(n_A+n_B-2)-n_B} \equiv \beta^*. \tag{19}$$

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[18] Proposition 4 does not mention individual equilibrium effort. For the simple contest game analyzed here, the analytical solutions for individual equilibrium effort can be compared easily to show that members of group A exert individually less effort under AA* than under ET* while members of group B trivially exert more (because they are not active under ET*).

The intuitive explanation that was given in the last section is that condition 16 is likely to hold if either the level of dispersion is sufficiently low or the number of contestants is relatively small. For the case considered here this can be verified explicitly for the simplified condition (19). In fact, it is satisfied if either β_B is low in comparison to β^* (which coincides with low dispersion), or if n_A and n_B are sufficiently low (it can be checked that β^* is decreasing in n_A and n_B).

Notice also that condition (19) is not trivial in the sense that, for instance, satisfying condition (18) automatically implies condition (19). Moreover, it can be shown that $\beta^* > \frac{n_A}{n_A-1}$. Hence, it is possible that not all contestants are active and that the sum of equilibrium effort is higher under AA* than under ET*.

5.1 A Partially Informed Contest Designer

In this section the previous contest game with groups is generalized by relaxing the assumptions on homogeneity within groups and on full information of the contest designer with respect to individual characteristics of the contestants. From now on the contestants face again individually different marginal costs that are common knowledge for the contestants. However, the contest designer is only partially informed about the heterogeneity of the contestants because, by assumption, she can only observe the group membership of each contestant and an aggregated measure of heterogeneity defined as $\bar{\beta}_A = \frac{1}{n_A}\sum_{j\in A}\beta_j$ for all contestants in group A and $\bar{\beta}_B = \frac{1}{n_B}\sum_{j\in B}\beta_j$ for all contestants in group B, respectively. Group B is supposed to be weaker in expected terms: $\bar{\beta}_B > \bar{\beta}_A$.[19]

The specification of equal treatment (definition 1) for this framework remains as before ($\alpha_i^{ET} = \alpha^{ET}$ and $\gamma_i^{ET} = \gamma^{ET}$ for all $i \in N$) because it is specified for all contestants identical (and therefore also irrespective of group membership). However, the definition of affirmative action has to be adapted to the limited informational knowledge of the contest designer because AA and AU require complete information. As the contest designer is only partially informed she has to form expectations with respect to the marginal cost of each contestant which is denoted by $E[\cdot]$, i.e. $E[\beta_i] = \bar{\beta}_i$ for all $i \in \{A, B\}$.

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[19] This is the case if, for instance, the distribution of cost parameters of group A first order stochastically dominates the distribution of group B.

Definition 2 and 3 have to be revised respectively where the argumentation remains as in section 2.

Definition 4 *A policy is called* ***affirmative action (AA')*** *in a contest game with a partially informed contest designer if:*

$$E[c_i(e_i)] = E[c_j(e_j)] \Rightarrow p_i(e) = p_j(e) \text{ for all } i \neq j. \tag{20}$$

For the simplified class of contest games with linear weighting function as defined by eq. (4) AA' implies that:

$$\alpha_i^{AA'} = \bar{\beta}_i, \gamma_i^{AA'} = \gamma^{AA'} \text{ for all } i \in \{A, B\}. \tag{21}$$

The next definition characterizes the revised affirmative action policy AU' which implies identical expected equilibrium utility for each active contestant out of the perspective of the contest designer irrespectively of group membership:[20].

Definition 5 *A policy is called* ***affirmative action with respect to expected equilibrium utilities (AU')*** *in a contest game with a partially informed contest designer if the expected equilibrium utility for each contestant is identical out of the perspective of the contest designer:*

$$E[u_i(e_i^*(AU'))] = E[u_j(e_j^*(AU')] \text{ for all } i \neq j. \tag{22}$$

For the simplified class of contest games as defined by eq. (4) AU' implies that:

$$\left\{\alpha_i^{AU'}, \gamma_i^{AU'}\right\}_{i \in N} \text{ such that eq. (22) holds.}$$

The maximization problem of eq. (8) changes slightly because definition 3 implies that the optimal designed AA'* policy can only discriminate between groups and not between individual contestant. Lemma 2 therefore changes slightly to:

Lemma 3 *In the n-player contest game the optimal policy vector* $\left\{\alpha_i^{P^*}, \gamma_i^{P^*}\right\}_{i \in N}$ *for* $P \in \{ET, AA, AU\}$ *can be specified as:*

$$\begin{aligned} \alpha_i^{ET^*} &= \alpha^{ET^*} > 0, & \gamma_i^{ET^*} &= 0, \text{ and} \\ \alpha_i^{AA'^*} &= \alpha_i^{AU'^*} = \bar{\beta}_i, & \gamma_i^{AA'^*} &= \gamma_i^{AU'^*} = 0 \text{ for all } i \in \{A, B\}. \end{aligned}$$

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[20] The expected equilibrium utility out of the perspective of each contestant could be different if his marginal cost is different from the group average.

As before the specification of AA'* and AU'* coincides so that the following analysis is restricted to AA'*.

The comparison between ET* and AA'* for this case is complex because the implementation of AA'* does not imply that all contestants are always active (corollary 1 does not hold anymore in this framework). In fact it is possible that there are different sets of active agents for each policy option. A condition that guarantees that the sum of equilibrium effort is higher under AA'* than under ET* (parallel to proposition 4) would relate the number of active agents under the two policy options with the respective expected cost parameters for each active set.[21]

To reduce the complexity of the policy comparison, the same special case as in the last section is considered, i.e. it is assumed that all contestants are active under both policy options. This implies that condition (14) is satisfied for all contestants under ET* and AA'*, i.e. that the dispersion of discrimination parameter is sufficiently low. As each contestant takes the effort level of her competitors as given, the aggregated equilibrium effort can be calculated as usually, i.e. $E_P^* = \sum_{i \in N} e_i(\alpha_i^{P^*}, \gamma_i^{P^*})$ for $P^* = \{ET^*, AA'^*\}$. The following statement about the consequences of optimal affirmative action AA'* (or AU'*) is possible:

Proposition 6 *If all contestants in a contest game with a partially informed contest designer are active under ET* and AA'*, then (i) the sum of equilibrium effort levels is higher under AA'* than under ET*, and (ii) the individual equilibrium effort is higher under AA'* than under ET* for all contestants $i \in A$ with discrimination level*

$$\beta_{i \in A} < \frac{(n_A + n_B)\bar{\beta}_A \bar{\beta}}{(n_A + n_B - 1)(\bar{\beta} + \bar{\beta}_A)}$$

and for all contestants $i \in B$ with discrimination level

$$\beta_{i \in B} < \frac{(n_A + n_B)\bar{\beta}_B \bar{\beta}}{(n_A + n_B - 1)(\bar{\beta} + \bar{\beta}_B)}.$$

Proof: For the first part the following inequality has to be analyzed: $E_{AA'}^* > E_{ET}^*$. If all contestants are active, this inequality is reduced to: $n_A/\bar{\beta}_A + n_B/\bar{\beta}_B > n^2/(\sum_{i \in N} \beta_i)$.

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[21] The condition for $E_{AA'}^* > E_{ET}^*$ is in fact:

$$\frac{(m_{AA'^*} - 1)}{m_{AA'^*}^2} \sum_{i \in M_{AA'^*}} \frac{1}{\alpha_i^{AA'^*}} > \frac{m_{ET^*}}{\sum_{i \in M_{ET^*}} \beta_i}$$

where M_{P^*} denotes the active set under policy $P^* = \{AA'^*, ET^*\}$. An interpretation of this condition does not seem to gain additional insight and is therefore omitted.

This inequality can be further reduced to $n_A n_B(\bar{\beta}_A - \bar{\beta}_B)^2 > 0$ which is always satisfied. For the second part the individual equilibrium effort has to be compared. Starting with a member of group A, the inequality $e^*_{i\in A}(\alpha_A^{AA'^*}, \gamma_A^{AA'^*}) > e^*_{i\in A}(\alpha_A^{ET^*}, \gamma_A^{ET^*})$ can be reduced to $\beta_{i\in A} < \frac{(n_A+n_B)\bar{\beta}_A\bar{\beta}}{(n_A+n_B-1)(\bar{\beta}+\bar{\beta}_A)}$ with the analogous derivation for members of group B.$\square$

Proposition 5 is intuitive because the implementation of optimal designed AA'* policy also levels the playing field with a partially informed contest designer in the following sense: Although contestants might not have identical equilibrium utilities under AA'* in comparison to ET*, the de facto-heterogeneity of the contestants is lower under AA'* because the discriminated group is favored in expected terms. This reduces the effort reducing consequences of the different cost distribution for the two groups. The assumption on full participation implies then increased competitive pressure between contestants which is translated to higher aggregated equilibrium effort.

6 Concluding Remarks

The implementation of affirmative action policies is a highly controversial topic in public policy discussion. One of the frequent critical remarks is focused on the potential disincentives generated by affirmative action policies by pointing out that affirmative action has discouraging effects on targeted and non-targeted groups that could finally result in a reduction of effort levels.

This claim is analyzed in the paper for a stylized contest game, where contestants could be heterogeneous because of past discrimination. From a normative point of view the contest rule should in this case be biased in favor of discriminated contestants to induce a level playing field. The final objective of this distortion is derived by using two alternative notions of fairness: an outcome and a procedural oriented one. Irrespective of the chosen fairness notion the bias of the contest rule can be implemented in different forms which leaves a restricted freedom in designing the final policies. Closed form equilibrium solutions for the optimal designed (i.e. effort maximizing) affirmative action versus equal treatment instruments are provided that facilitate a comparative analysis of these policies. Additionally it is shown that the two different normative interpretations of affirmative action do not matter in this framework because the optimal designed final policy parameter coincide for both cases.

The results of the policy comparison for the two-player contest proof finally that the criticized trade-off between affirmative action and aggregated effort does not only not exist but that both objectives are also closely related. The result for the n-players case and the case with a partially informed contest designer is not as straight-forward: the non-existence result of the aforementioned trade-off can only be established if the distribution of the discrimination parameter is sufficiently equal.

The underlying behavioral interpretation of affirmative action can be summarized in a the following general statement that must not be restricted to models of contest games: Discrimination increases the heterogeneity of individuals in competitive situations but affirmative action makes individuals more similar which increases competitive pressure and therefore induces higher effort by all participants. However, this argumentation only works, if discriminated individuals are in fact the weak ones[22] and if affirmative action policies are designed appropriately.

If these requirements are satisfied the critique that affirmative action instruments have adverse consequences for total effort seems to be unjustified.

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[22] In a previous version of this paper the contestants were also heterogeneous with respect to valuation for which they were not compensated for. In this case the result in proposition 2 only holds if discriminated players are sufficiently weak because otherwise the preferential treatment of discriminated players with high valuation would increase the de-facto heterogeneity.

A Appendix

A.1 Proposition 1

Nti (2004) derived proposition 1 for a two-player contest game by comparing simple first and second order conditions for the two classes of contest games defined in proposition 1. This approach is not feasible in contest games with $n > 2$ players because the simple first order approach implies that equilibrium effort is always positive which may not be the case in contest games with more than two players where some contestants prefer to remain inactive and exert zero effort. Therefore the derivation of first order conditions has to be extended by adding an explicit non-negativity constraint with respect to effort for each contestant. Additionally it has to be shown that the set of active agents will remain the same for both classes of contest games. This will be done by checking for contestant i whether and under which condition he will exert positive effort level in equilibrium, and by finally comparing the derived conditions for the two contest games.

Proof of Proposition 1:
Consider the class of contest games as specified in eq. (1) - (3) with an increasing, concave and twice differentiable weighting function $f_i(e_i)$ for all $i \in N$. The necessary Kuhn-Tucker conditions that include now the non-negativity condition on e_i can be expressed as:

$$e_i \left\{ \frac{f_i'(e_i) \sum_{j \neq i} f_j(e_j)}{\left[\sum_{j \in N} f_j(e_j)\right]^2} - \beta_i \right\} = 0 \text{ for } e_i \geq 0 \text{ for all } i \in N. \tag{23}$$

Szidarovszky and Okuguchi (1997) show that a unique equilibrium exists and can be characterized as the solution of the necessary Kuhn-Tucker conditions. Denote the vector of these solutions by $\hat{e} = (\hat{e}_1, \dots, \hat{e}_n)$.

Compare eq. (23) with the Kuhn-Tucker conditions for a contest game with the same specification as before but now with a linear weighting function $g_i(e_i) = \alpha_i e_i + \gamma_i$ for all $i \in N$:

$$e_i \left\{ \frac{\alpha_i \sum_{j \neq i} (\alpha_j e_j + \gamma_j)}{\left[\sum_{j \in N} (\alpha_j e_j + \gamma_j)\right]^2} - \beta_i \right\} = 0 \text{ for } e_i \geq 0 \text{ for all } i \in N \tag{24}$$

Denote the vector of solutions to those equations by $\check{e} = (\check{e}_1, \dots, \check{e}_n)$.

Note also, that contestant i will exert zero effort in a contest game if the expression in curly brackets is negative which implicitly determines the set of active contestants for each class of contest games. The expected utility function of the contest game with linear weighting function is concave if $\alpha_i e_i + \gamma_i \geq 0$ for all $i \in N$.

Eq. (23) and (24) coincide if $\alpha_i = f_i'(\hat{e}_i) > 0$ and $\gamma_i = f_i(\hat{e}_i) - f'(\hat{e}_i)\hat{e}_i \geq 0$ for all $i \in N$, where the inequalities are derived from the fact that $f_i(e_i)$ is increasing and concave. Therefore, the expected utility function for the case of linear weighting functions is concave. Together with the fact that it is continuous this implies that sufficiency conditions are also satisfied. Additionally, contestant i will be active under the same parameter specification for both classes of contest games because the expressions in curly brackets coincide as well. Therefore, also the set of active contestants is identical. As the equilibrium is unique both vectors $\hat{e} = (\hat{e}_1, \ldots, \hat{e}_n)$ and $\check{e} = (\check{e}_1, \ldots, \check{e}_n)$ must finally coincide.□

A.2 Equilibrium in the n-player contest

To construct the share function of contestant i, his expected utility function has to be transformed in such a way that the contest can be interpreted as an aggregative game in which the utility function of contestant i can be expressed as $\pi_i(z_i, Z)$, where $Z = \sum_{i \in N} z_i$. Consider the following transformation that yields a transformed utility function that is strategically equivalent to eq. (12): denote $z_i = z_i(e_i) = \alpha_i^P e_i + \gamma_i^P$ which can be inverted to $e_i = z_i^{-1}(z_i) = (z_i - \gamma_i^P)/\alpha_i^P$ for all $i \in N$. The resulting transformed expected utility function for contestant i, which has the aggregative game property, is denoted as:

$$\pi_i(z_i, Z) = \frac{z_i}{Z} - \delta_i^P z_i + \delta_i^P \gamma_i^P \text{ for all } i \in N \text{ and for } P \in \{ET, AA, AU\} \tag{25}$$

where $\delta_i^P = \frac{\beta_i}{\alpha_i^P V}$ and Y defined as above. This transformed contest game is now covered by the model in Cornes and Hartley (2005), p. 926, eq. 3. The share function can therefore be constructed in an analogous way by deriving the first order condition:

$$z_i \left(\frac{Z - z_i}{Z^2} - \delta_i^P \right) = 0 \text{ for } z_i \geq 0. \tag{26}$$

The best response z_i^* of player i can be expressed in terms of the aggregated equilibrium effort:[23] $z_i^*(Z) = \max\{Z - \delta_i^P Z^2, 0\}$. Finally, define player i's share function as her

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[23] It should be obvious that the best response and also the share functions depend on the policy parameter P. But as the finally implemented policy does not affect the proof of equilibrium

relative contribution

$$s_i(Z) = \frac{z_i^*(Z)}{Z} = \max\{1 - \delta_i^P Z, 0\}. \tag{27}$$

In equilibrium the aggregated effort Z^* is implicitly defined by the condition that the individual share functions sum up to one:

$$S(Z^*) = \sum_{i \in N} s_i(Z^*) = 1 \tag{28}$$

Cornes and Hartley (2005) show in their theorem 1, p. 929, that a solution to this equation exists and is unique by observing that the aggregated share function $S(Z)$ is continuous, strictly decreasing for positive Z, and has a value higher than one for Z sufficiently small and equal to zero for Z sufficiently large.

Equation (27) already indicates the possibility that for some contestant the equilibrium share will be equal to zero. To derive the set of active contestants $M \subseteq N$, i.e. the m players with strict positive share in equilibrium, it is assumed that the contestants are ordered such that $\delta_1^P \leq \delta_2^P \leq \ldots \leq \delta_n^P$. Later it will be shown that the optimally designed vector of policy parameters $\left\{\alpha_i^{P^*}, \gamma_i^{P^*}\right\}_{i \in N}$ implies the same order[24] irrespective of the chosen policy that also coincides with $\beta_1 \leq \beta_2 \leq \ldots \leq \beta_n$.

From eq. (27) it is obvious that in equlibrium $Z^* < 1/\delta_i^P$ for all $i \in M$. Combining eq. (27) and (28) yields $Z^* = \frac{m-1}{\sum_{j \in M} \delta_j^P}$. The last two expressions yield the condition that indirectly defines the set $M \subseteq N$ of active contestants:

$$(m-1)\delta_i^P < \sum_{j \in M} \delta_j^P \text{ for all } i \in M \text{ and for } P \in \{ET, AA, AU\}. \tag{29}$$

From the definition of the share function in eq. (27) the equilibrium effort level of contestant i can be calculated as $e_i^*(P) = z_i^*/\alpha_i^P - \gamma_i^P/\alpha_i^P = s_i(Z^*)Z^*/\alpha_i^P - \gamma_i^P/\alpha_i^P$. Inserting the expression for Z^* leads to eq. (13).[25]

A.3 Proof of Lemma 2

Summing up equilibrium effort $e_i^*(P)$ over all contestants yields:

$$\sum_{i \in N} e_i^*(P) = \frac{(m-1)V}{\sum_{i \in M} \frac{\beta_i}{\alpha_i^P}} \sum_{i \in M} \left[\frac{1}{\alpha_i^P} \left(1 - \frac{\beta_i}{\alpha_i^P} \frac{(m-1)}{\sum_{i \in M} \frac{\beta_i}{\alpha_i^P}} \right) \right] - \sum_{i \in M} \frac{\gamma_i^P}{\alpha_i^P}, \tag{30}$$

where the set M of active contestants is implicitly defined by eq. (14).

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existence and uniqueness, it is suppressed in this section for notational convenience.

[24] in a weak sense for $\left\{\alpha_i^{AA^*}, \gamma_i^{AA^*}\right\}_{i \in N}$ because $\delta_i^{AA^*} = \delta_j^{AA^*}$ for $i \neq j$.

[25] Stein (2002) derives a similar expression for a contest game where the weighting function is only linear and heterogeneity affects valuation instead of marginal costs.

The maximization problem of eq. (8) is solved firstly for policy ET. Definiton 1 implies that $\alpha_i^{ET} = \alpha^{ET}$ and $\gamma_i^{ET} = \gamma^{ET}$ for all $i \in N$. Eq. (30) then simplifies to $\sum_{i\in N} e_i^*(ET) = \frac{m-1}{\sum_{i\in M} \beta_i} - m\frac{\gamma^{ET}}{\alpha^{ET}}$ where set M is implicitly determined by the following inequality (from eq. 14): $(m-1)\beta_i < \sum_{j\in M} \beta_j$ for all $i \in M$. Note that the set of active agents does not depend on γ^{ET} and that $\sum_{i\in N} e_i^*(ET)$ is decreasing in γ^{ET} because γ^{ET} is non-negative. Therefore, the optimal policy parameters for policy ET are: $\alpha^{ET^*} > 0, \gamma^{ET^*} = 0$. Parameter α^{ET^*} is irrelevant again (as in the two-player contest) because $\sum_{i\in N} e_i^*(ET)$ does not depend on α^{ET^*} anymore.

Derivation of the optimal AU policy requires that expected equilibrium utility is identical for all contestants by definition 3. As before the set M is not affected by varying γ_i^{AU} (or γ_i^{AA}) and $\sum_{i\in N} e_i^*(AU)$ is decreasing in γ_i^{AU} (the same is true for AA) for all $i \in N$. Hence, setting $\gamma_i^{AU} = \gamma_i^{AA} = 0$ for all $i \in N$ is optimal. The expression for the expected equilibrium utility then simplifies to: $u_i(e_i^*(AU) = \left(1 - \frac{\beta_i}{\alpha_i^{AU}} \frac{(m-1)}{\sum_{j\in M} \frac{\beta_i}{\alpha_i^{AU}}}\right)^2 V$ for $i \in M$ and $u_i(e_i^*(AU) = 0$ for $i \in N/M$ where N/M denotes the set of non-active contestants. However, by definition 3 the equilibrium utility of active and non-active contestants must be identical. This implies that the set of non-active contestants must be empty (the set of active contestants cannot be empty because Stein (2002) shows for an equivalent contest game that at least two contestants will always exert a positive equilibrium effort level) and that $u_i(e_i^*(AU) = u_j(e_j^*(AU))$ for all $i \neq j$ and $i, j \in N$. Solving the last equation yields the condition $\frac{\alpha_i^{AU^*}}{\alpha_j^{AU^*}} = \frac{\beta_i}{\beta_j}$ for all $i \neq j$ which coincides with the AA* policy and concludes the proof.

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