The Role of the Capital Market in a Dynamic Macroeconomic Model with Stochastic Rationing

Annette Förster*
Gerd Weinrich**

Abstract: This paper concerns a dynamic macroeconomic model with a capital market and stochastic rationing. Transactions take place in each period, even when prices are not at their market clearing values. At the end of a period, price, wage, and interest rate are adjusted according to the strength of rationing which can be measured in a satisfactory manner due to stochastic rationing. A special case of the model in which the capital market is excluded and a model variant with permanent capital market clearing are considered as well. The dynamic adjustment equations are derived as prerequisites for numerical simulations. Our numerical results provide evidence of stationary states with credit rationing. This contradicts the commonly accepted assumption that slowly adjusting interest rates will only lead to temporary credit rationing. Moreover, credit rationing seems to generate an increasing complex dynamic behaviour.

Keywords: Dynamics, Stochastic Rationing, Credit Rationing

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*Hochschule Heilbronn, Germany
**Università Cattolica del Sacro Cuore, Milano, Italy

1 Introduction

The phenomenon of credit rationing, its causes and consequences are of recurring interest. A recent example are the discussions about the existence of credit rationing in Germany in the recession years 2001-2003. Fact is, that the decline in GDP was accompanied by a weak credit growth. However, it is still an open question, whether the credit supply or the credit demand side is responsible for the modest credit development. According to the German Council of Economic Advisors (2004) and the Deutsche Bundesbank (2005) credit weakness in these years was mainly demand-sided driven. The economists base their thesis on econometric estimates (Deutsche Bundesbank (2005)). Furthermore, they argue that German enterprises reduced their investment activity and therefore their credit demand in view of the bad economic outlook. Nevertheless, they admit that “there are signs of a certain lending restraints among banks” (Deutsche Bundesbank (2002)). On the contrary, the ifo institute in Munich (Westermann (2003)) and the RWI institute (Nehls and Schmidt (2003)) in Essen find clear indications for a credit rationing, e.g. the drop in earnings in the banking sector. In any case, economists feared that a credit rationing in this recession would hinder entrepreneurs from investing and thus would accelerate and reinforce the econcomic downward trend.

What are the theoretical contributions to this subject?
Microeconomic theory provides three major concepts why interest rates are rigid and do not react to an imbalance of capital supply and demand: First, the Government intervenes in the market process, determining an upper limit for the interest rate. Second, interest rates do not adjust instantaneously to imbalances, but with a certain time lag. Third, banks derive from their rational calculation an optimal interest rate, which lies below the marketclearing one and which the banks do not want to raise. Among a large body of literature we want to mention the classical papers of Jaffee and Modigliani (1969) dealing with legal restrictions on interest rates, and, with respect to the third topic, the asymmetric information approach by Stiglitz and Weiss (1981) and Devinney (1983). For a general thorough survey the reader is referred to Baltensperger and Devinney (1985).
Macroeconomic investigations concentrate on two questions: (i) how effective will be monetary policy if credit demand is rationed? (ii) do restrictions on the credit market aggravate the downward trend in GDP and employment? Corresponding studies assume hereby in general an optimal bank interest rate (e.g. Kyiotaki and Moore (1997), Mendicino (2006)), often in the Stiglitz and Weiss (1981) framework. The macroeconomic effects of credit rationing are controversily discussed in literature. For example, Eickmeier et al. (2006) found no evidence “that loans amplify the transmission of macroeconomic fluctuations”,

while Mendocino (2006) obtained the result that there is a correlation between the size of the credit market and output volatility.

In order to get a more profound understanding of macroeconomic effects of credit rationing, we construct a dynamic macroeconomic model with a capital market, following a Keynesian-type model (called BCW-model) presented in Bignami, Colombo and Weinrich (2000).[1] Its distinctive feature is that prices react more slowly than quantities. This aspect constitutes an important conceptual improvement with respect to existing literature, where the possibility of slowly adjusting interest rates has been ignored, because the resulting credit rationing has only been regarded as a temporary phenomenon. In our model, time is devided into periods and transactions take place in every period even if prices are not at their Walrasian values. Such an approach raises the question of the existence and description of a consistent allocation at non market clearing values. In this model this problem has been solved by considering rationing and quantity signals. In the literature this solution is known as the concept of a temporary equilibrium with quantity rationing which mainly originates in Benassy (1975) and Drèze (1975). A key-characteristic of the present model is the assumption of a stochastic rationing mechanism. As has been pointed out by Weinrich (1984), this approach allows to develop a reliable measure for the size of disequilibrium, which is decisive for the price adjustment. Modelling dynamical adjustment processes within the framework of a temporary equilibrium model enables us to investigate if temporary credit rationing generates complex behaviour of the economy. Moreover, steady states with fixed prices involving permanent unemployment would emerge as fixed points of the dynamical system (Böhm, Lohmann and Lorenz (1994)).

Models of this type are characterized by highly complicated, nonlinear dynamical systems. Due to this fact an analytical determination of the trajectories is impossible. Nevertheless, we were able to develop two algorithms, one for the dynamic evolution of this model and one for a variant of it. These are the basic requirements for corresponding simulation programs. The variation consists in the assumption of permanent capital market clearing. Since the BCW-model is incorporated in our model we can simulate the dynamics of an economy without a capital market, with credit rationing and with capital market clearing and we can compare them to each other.

[1]The BCW-model provides only for a labour and a commodity market.

Our paper is organized as follows: In section 2 at first we explain the time structure of our model. Next we derive detailed behavioural functions of producers, consumers and the government under stochastic rationing. The subsequent section contains the definition of a temporary equilibrium and examplary analytical as well as graphical illustrations of two temporary equilibrium systems. Afterwards we prove that the BCW-model is a special case of our model. Then we partition the price-wage-interest rate-space into different equilibrium systems for a numerical example. The next subsection is concerned with theoretical considerations about dynamics and ends with a portrayal of the model in form of a flow diagram. The modification of our model and its consequences are explained thereafter. In section 3 we present our simulation results. Finally we give a summary of our work and conclude with some suggestions for further research. Proofs of some technical results and the system-specific equations including the algorithms are added as appendix.

2 The Model

2.1 Behavioural assumptions

Our model consists of a labour market, a capital market, and a goods market. We distinguish between producers, the government, and consumers for which we assume an overlapping generations structure. Each consumer lives two periods and in each period there exist a young and an old generation. The consumers and the government together form the consumption sector. Time is divided into periods of equal length. The prevailing nominal wage w_t, the nominal interest rate r_t and the price of goods p_t at the beginning of a period t remain fixed for the length of the period and transactions take place at these prices. If the supply does not coincide with the demand in a market, the respective price will be adjusted at the period's end. We assume a temporal sequential structure. The labour market is visited first and the capital market next. After the decisions on the factor markets are taken, agents express their transaction offers for the goods market. In the following we want to explain the time structure of the model illustrated in figure 1.

At the beginning of a period the producers (P) demand labour and pay for the current employment L_t a salary of w_tL_t to the young consumers (C^Y) who supply the labour. Since at that moment the producers lack money, we suppose that they receive a bank loan to finance the work L_t. At the end of the same period, producers repay the loans with the proceeds of the sales of their goods. For simplicity, we assume that the bank charges no interest and is endowed with a stock of money with which the demands for credit can always be satisfied. Hence the financing of the labour employment has no effect on the model.

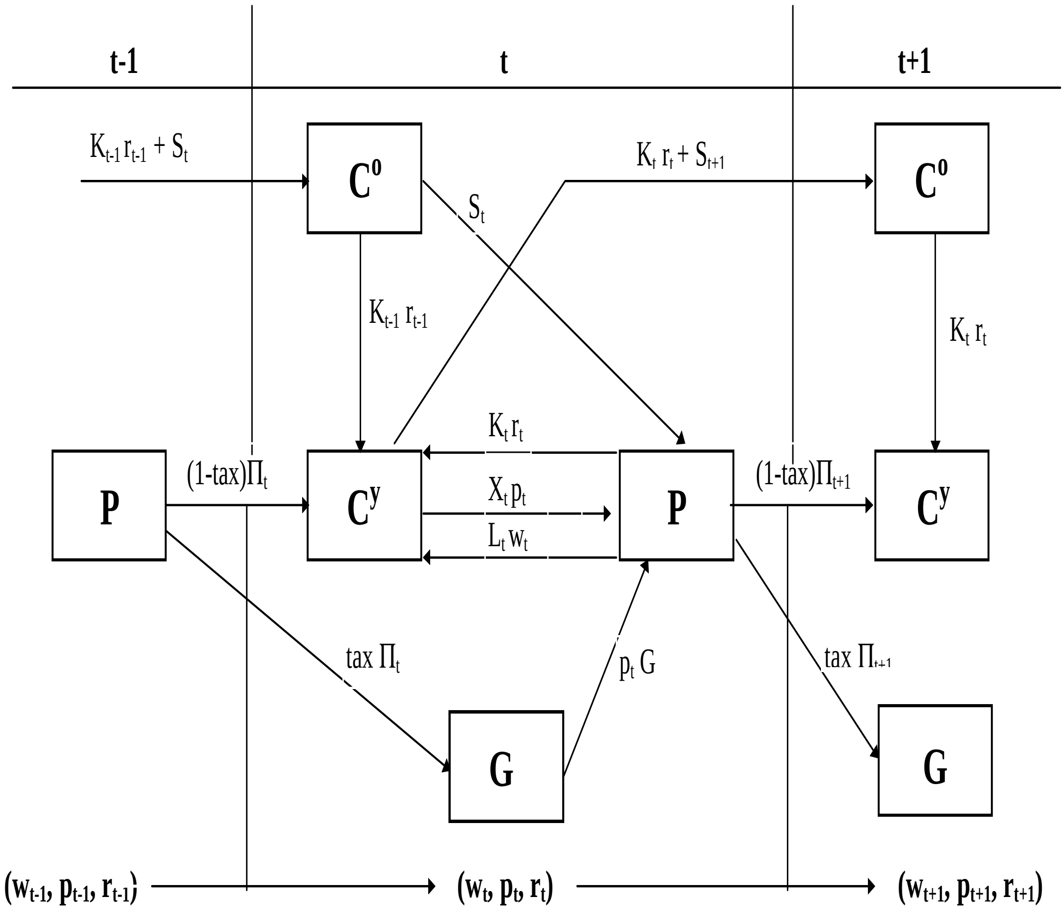


figure 1

The young consumers offer to the producers for the length of one period the amount[2] $K_t p_t$ to finance the capital goods and receive in return a nominal interest of $K_t r_t$ from them. The producers purchase the capital stock abroad, use it once, and sell it back without depreciation. Rationings in the foreign capital goods markets are excluded. This simple model enables us to study explicitly the consequences of rationing in the financial sphere. The goods sales to the government (G) $p_t G_t$, to the old consumers (C^o) S_t, and to the young consumers $p_t X_t$ minus the costs for the employment of the production factors $w_t L_t + K_t r_t$ yield a profit Π_{t+1}. The government receives a part of this profit in form of taxes $(\text{tax}\,\Pi_{t+1})$, the rest of the profit is distributed among the young consumers in period $t+1$.

On the other hand the young consumers receive, in addition to their labour income $w_t L_t$ and the net profit of the previous period $(1-\text{tax})\Pi_t$, from the older generation the interest payment $K_{t-1} r_{t-1}$ (earned the period before).

[2]This financial flow is not shown in figure 1.

They spend part of the total income for the consumption of the current period $p_t X_t$ and save the rest S_{t+1} for the second period of their life. It is important to bear in mind that S_{t+1} consists of the desired savings which correspond to the capital supply $K_t p_t$ and a stock of money. This stock arises if the demand for goods is rationed. The transfer of the interest and the profit of period t to the young consumer generation of the subsequent period $t+1$ ensures that even in the case of unemployment they hold a minimum stock of money. Moreover the assumption regarding the interest payment has the technical advantage that expected rationings of the capital supply are of no importance for the capital giver and are therefore not considered in their optimization calcul.

The old generation can do nothing but spend its savings S_t for consumption purposes. The quantity of goods G demanded by the government is fixed[3] and public expenditures $p_t G$ are financed from tax revenue and, if necessary, by means of money creation.

The following identity for savings arises from the structure of the model:

$$S_{t+1} = w_t L_t + (1 - tax)\Pi_t + K_{t-1} r_{t-1} - p_t X_t.$$

Taking into account the aggregate consumption Y_t which is defined via the relation

$$Y_t = X_t + G + \frac{S_t}{p_t}$$

leads to

$$S_{t+1} = w_t L_t + (1 - tax)\Pi_t + K_{t-1} r_{t-1} - p_t Y_t + p_t G + S_t.$$

After simple transformations we obtain the final result:

$$S_{t+1} - S_t + \Pi_{t+1} - \Pi_t + K_t r_t - K_{t-1} r_{t-1} = p_t G - tax\Pi_t.$$

The financing deficit of the government corresponds to the variation in the money stock held by the private sector, hence the model is closed. We suppose that budget surplusses are taken out of circulation, whereas budget deficits are compensated with money creation. Consequently the quantity of money is variable.

The Consumption Sector

The consumption sector consists of $2n$ agents, n of them are identical young consumers and n identical old ones. Each young consumer is informed of the prices p_t, w_t, r_t. For his decision process, the prices as well as his expected

[3]Henceforth we omit the subscript t.

transactions are crucial. The agent receives quantity signals for all three markets in form of quotients of aggregate supply and aggregate demand, assuming that they cannot be influenced by his own actions. The signal for the labour market is $\lambda_t^s = \min\{L_t^d/L^s, 1\}$, with L_t^d and L^s denoting aggregate labour demand and aggregate labour supply, respectively. Hence a stochastic rationing mechanism of type all-or-nothing in the labour market, that is either full employment or unemployment, looks like the following:

$$l_t = \begin{cases} l^s & \text{with probability} \quad \lambda_t^s \\ 0 & \text{with probability} \quad 1-\lambda_t^s. \end{cases}$$

The variable l_t denotes the actual labour employment, whereas l^s indicates the individual labour supply. We suppose that a young consumer derives utility only from the consumption of goods and not from leisure. Consequently it is rational for him to supply his total labour endowment. Then the labour supply l^s is invariant with respect to the wage and the rationing probability, and there is no need for a time index. The aggregate supply of labour is, therefore:

$$L^s = nl^s.$$

After the transactions on the labour market have taken place, the young consumer knows his real income. In the case of unemployment it amounts to[4]:

$$\omega_t^0 = \frac{(1-tax)\Pi_t}{np_t} + \frac{K_{t-1}r_{t-1}}{np_t},$$

and in the case of employment

$$\omega_t^1 = \frac{(1-tax)\Pi_t}{np_t} + \frac{K_{t-1}r_{t-1}}{np_t} + \frac{w_t}{p_t}l^s.$$

The young consumer uses his income for consumption in both periods, x_t and x_{t+1}. In taking any decision he has to consider the constraints

$$\begin{aligned} &x_t \geq 0, \qquad x_{t+1} \geq 0, \\ &x_t \leq \omega_t^i, \qquad x_{t+1} \leq (\omega_t^i - x_t)\frac{p_t}{p_{t+1}}, \qquad i = 0, 1. \end{aligned}$$

Moreover on the commoditiy market he is confronted with the following rationing mechanism:

$$x_t = \begin{cases} x_t^d & \text{with probability} \quad \rho\,\gamma_t^d \\ c_t x_t^d & \text{with probability} \quad 1-\rho\,\gamma_t^d. \end{cases}$$

Here the variable x_t^d stands for an agent's effective demand. Therefore his demand is completely satisfied with probability $\rho\gamma_t^d$ and with probability $1-\rho\gamma_t^d$ he gets only the part c_t of his desired quantity. By assumption it is

[4]We assume that interest and profit are evenly distributed.

determined by means of the constant parameter $\rho \in [0,1]$ and the endogenous parameter[5] $\gamma_t^d \in [0,1]$ as follows:

$$c_t = \frac{\gamma_t^d - \rho\gamma_t^d}{1 - \rho\gamma_t^d}.$$

This implies for the expected transaction of the consumer:

$$Ex_t = \rho\gamma_t^d x_t^d + (1 - \rho\gamma_t^d)c_t x_t^d = \gamma_t^d x_t^d.$$

The rationing mechanism for the capital market is supposed to work in the same way. This yields an expected transaction $\mu_t^s k_t^s$ for the young consumer, where k_t^s denotes the individual capital supply. The consequence of rationing in the capital market for the young consumer is that, on a portion of his savings, no interest is paid. As already mentioned before, due to the interest transfer to the next young generation the rationing probability in the capital market is of no relevance for his optimization problem. The effective goods demand results now from the maximization of the expected utility:

$$x_t^{di} = \operatorname*{argmax}_{x_t \le \omega_t^i} \rho\gamma_t^d u\left(x_t, \frac{\omega_t^i - x_t}{\theta_t^e}\right) + (1 - \rho\gamma_t^d)u\left(c_t x_t, \frac{\omega_t^i - c_t x_t}{\theta_t^e}\right)$$

$$\text{with} \quad \theta_t^e = \frac{p_{t+1}^e}{p_t}$$

where we denote with p_{t+1}^e the expected price for the period $t+1$. For a generic utility function it is not possible to determine the optimal x_t^d but, as has been shown by Bignami, Colombo and Weinrich (2000), with the assumptions of a Cobb-Douglas-utility function $u(x_t, x_{t+1}) = x_t^h x_{t+1}^{1-h}$ and $\rho = 1$ we obtain the following explicit solution:

$$x_t^{di} = h\omega_t^i.$$

Note that planned savings, which equal the optimal capital supply k_t^{si}, corrected by the relative price of consumption in periods t and $t+1$, lead to the planned real demand of period $t+1$:

$$k_t^{si} = (1-h)\omega_t^i, \qquad x_{t+1}^{di} = (1-h)\omega_t^i \frac{p_t}{p_{t+1}}. \qquad i = 0, 1.$$

It is remarkable that the demand for goods does not depend on the signal γ_t^d. The cause of this result rests in the assumption of $\rho = 1$, which implies a 0/1-rationing mechanism. In this particular case the consumer would only worsen his situation, if he deviated from his initial plan due to possible rationing.

[5]The definition of equilibrium will reveal why the parameter is endogenous.

The individual transaction offers lead to the aggregate supply of capital

$$K_t^s = \lambda_t^s n k_t^{s1} + (1-\lambda_t^s) n k_t^{s0} = \lambda_t^s K^{s1}(\omega_t^1) + (1-\lambda_t^s) K^{s0}(\omega_t^0) \tag{1}$$
$$= (1-h)\left[\lambda_t^s \frac{w_t}{p_t} L^s + \frac{(1-tax)\Pi_t}{p_t} + \frac{K_{t-1} r_{t-1}}{p_t}\right] = K^s(\lambda_t^s, w_t, p_t, \Pi_t, Z_t),$$

and the aggregate demand of commodities becomes

$$X_t^d = \lambda_t^s n x_t^{d1} + (1-\lambda_t^s) n x_t^{d0} = \lambda_t^s X^{d1}(\omega_t^1) + (1-\lambda_t^s) X^{d0}(\omega_t^0) \tag{2}$$
$$= h\left[\lambda_t^s \frac{w_t}{p_t} L^s + \frac{(1-tax)\Pi_t}{p_t} + \frac{K_{t-1} r_{t-1}}{p_t}\right] = X^d(\lambda_t^s, w_t, p_t, \Pi_t, Z_t).$$ [1]

To study the temporary equilibrium regimes it will be useful to illustrate the aggregate transaction offers and the expected aggregate transactions for varying values of the quantity signals. The locus of the aggregate transaction offers is given by

$$H := \{(L^s, K^s(\lambda^s), X^d(\lambda^s)) \mid \lambda^s \in [0,1]\}.$$

Analogously to the individual transactions $\lambda_t^s l^s, \mu_t^s k_t^s$ and $\gamma_t^d x_t^d$ expected by the young consumers we can formulate the expected aggregate transactions, the locus of which is defined by

$$\overline{H} := \{(\lambda^s L^s, \mu^s K^s(\lambda^s), \gamma^d X^d(\lambda^s)) \mid (\lambda^s, \mu^s, \gamma^d) \in [0,1]^3\}$$

The set $\overline{H}$ is an eight-sided body in a three-dimensional space, called L-K-Y-space with $\lambda^s L^s$ as the L-coordinate, $\mu^s K^s(\lambda^s)$ as the K-coordinate and $\gamma^d X^d(\lambda^s)$ is the Y-coordinate (figure 2).

There is a one-to-one correspondance between the unit cube $[0,1]^3$ in $\lambda^s - \mu^s - \gamma^d$-space and the set $\overline{H}$ in L-K-Y-space. With the knowledge of L we can determine λ^s unambiguously because of the linear relationship $L(\lambda^s)$. Since $K(\lambda^s, \mu^s)$ and $Y(\lambda^s, \gamma^d)$ depend linearly on μ^s and γ^d, respectively, the injectivity is proved. The locus of the transaction offers is represented by the curve E.

A partition of $\overline{H}$ according to the values of the variables λ^s, μ^s and γ^d, more precisely

$$\overline{H} := \bigcup_{i=1}^{8} \overline{H}^i \quad \text{with}$$

[1]Henceforth a list of arguments exhibits only those variables of which the effects are to be investigated explicitly. Moreover we will only indicate variables with a subscript t, if the timefactor is important for the analysis.

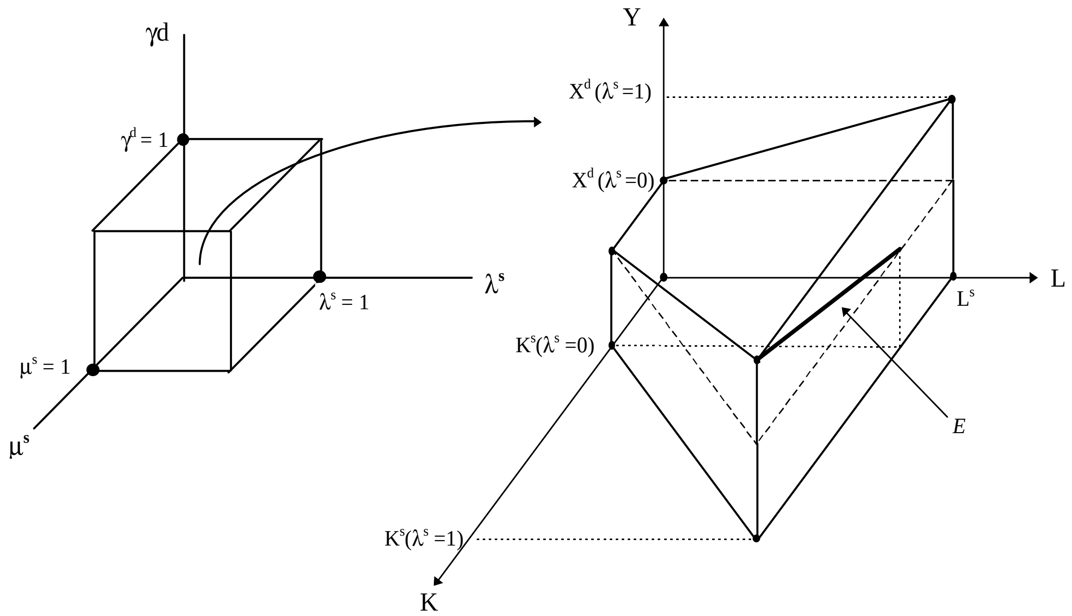


figure 2

$$
\begin{aligned}
\overline{H}^1 &:= \{(\lambda^s L^s, \mu^s K^s(\lambda^s), \gamma^d X^d(\lambda^s)) \mid \lambda^s \in [0,1[,\ \mu^s \in [0,1[,\ \gamma^d = 1\,\} \\
&= \overline{H}\Big|_{\lambda^s<1,\mu^s<1,\gamma^d=1;} \qquad \overline{H}^2 := \overline{H}\Big|_{\lambda^s<1,\mu^s<1,\gamma^d<1}; \\
\overline{H}^3 &:= \overline{H}\Big|_{\lambda^s<1,\mu^s=1,\gamma^d=1}; \qquad \overline{H}^4 := \overline{H}\Big|_{\lambda^s<1,\mu^s=1,\gamma^d<1}; \\
\overline{H}^5 &:= \overline{H}\Big|_{\lambda^s=1,\mu^s<1,\gamma^d=1}; \qquad \overline{H}^6 := \overline{H}\Big|_{\lambda^s=1,\mu^s<1,\gamma^d<1}; \\
\overline{H}^7 &:= \overline{H}\Big|_{\lambda^s=1,\mu^s=1,\gamma^d=1}; \qquad \overline{H}^8 := \overline{H}\Big|_{\lambda^s=1,\mu^s=1,\gamma^d<1}.
\end{aligned}
$$

is depicted in figure 3.

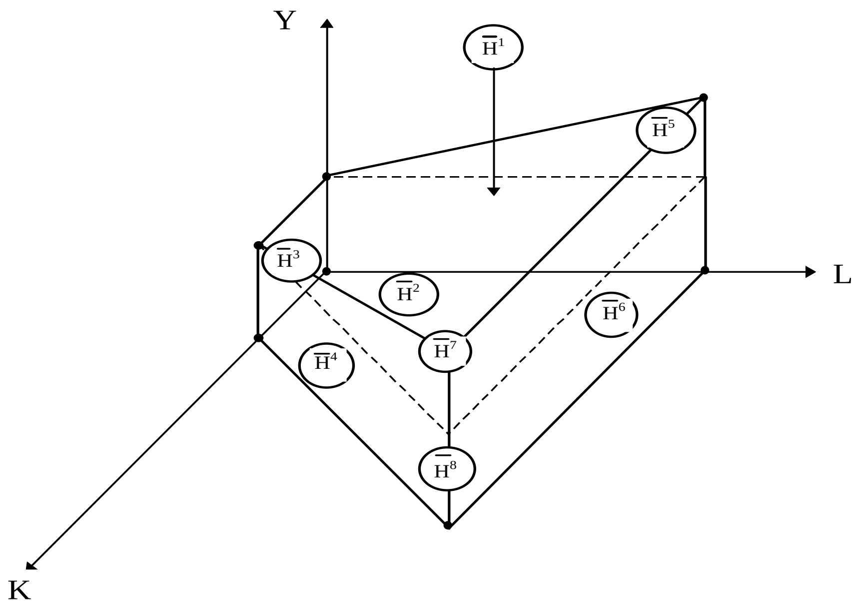


figure 3

Taking into account the government demand and the old consumers' aggregate demand, the total aggregate demand of commodities becomes

$$Y_t^d = h\left[\lambda_t^s \frac{w_t}{p_t} L^s + \frac{(1-tax)\Pi_t}{p_t} + \frac{K_{t-1} r_{t-1}}{p_t}\right] + G + \frac{S_t}{p_t}$$

$$= Y^d(\lambda_t^s, w_t, p_t, \Pi_t, K_{t-1}r_{t-1}, S_t).$$

The locus of the expectations of the whole consumption sector is an eight-sided body which results from $\overline{H}$ through an extension of the vertical edges of $\overline{H}$ by the amount $S/p + G$.

The Production Sector

We assume there are m identical firms which are informed of the prevailing prices p_t, w_t, r_t and the quantity signals expected by the whole production sector. Concerning the labour market they reckon with an all-or-nothing rationing as well as the consumers do. With the quota $\lambda_t^d = \min\{L^s/L_t^d, 1\}$ the rationing mechanism becomes

$$l_t = \begin{cases} l_t^d & \text{with probability} \quad \lambda_t^d \\ 0 & \text{with probability} \quad 1-\lambda_t^d. \end{cases}$$

Therefore a firm's demand for labour is (completely) satisfied with probability λ_t^d. If it is rationed on the labour market, it will refrain from producing during the current period. This also implies that it will not acquire capital goods and therefore not run the risk to make losses during the current period because of a financing of a capital stock.

Regarding the capital transactions the producers expect a quota $\mu_t^d = \min\{K_t^s/K_t^d, 1\}$ where K_t^d denotes the aggregate capital demand. Then the individual firm's rationing is given by

$$k_t = \begin{cases} k_t^d & \text{with probability} \quad \varepsilon\,\mu_t^d \\ v_t k_t^d & \text{with probability} \quad 1-\varepsilon\,\mu_t^d \end{cases}$$

where we indicate the actual transaction by k_t and the effective capital demand by k_t^d. The structural parameter $\varepsilon \in [0,1]$ is constant. The coefficient v_t is defined as

$$v_t = \frac{\mu_t^d - \varepsilon\mu_t^d}{1-\varepsilon\mu_t^d}$$

and determines the transaction level in the case of rationing. From this follows that the expected value of a producer's capital transaction is $\mu_t^d k_t^d$.

On the commodity market a firm faces a similar rationing mechanism:

$$y_t = \begin{cases} y_t^s & \text{with probability} \quad \sigma\,\gamma_t^s \\ d_t y_t^s & \text{with probability} \quad 1-\sigma\,\gamma_t^s, \end{cases}$$

with the constant parameter $\sigma \in [0,1]$, the signal $\gamma_t^s = \min\{Y_t^d/Y_t^s, 1\}$ and the coefficient d_t defined as

$$d_t = \frac{\gamma_t^s - \sigma\gamma_t^s}{1-\sigma\gamma_t^s}.$$

From this follows that the probability for complete sales of the effective goods supply is $\gamma_t^s\sigma$. The expected value of y_t is $\gamma_t^s y_t^s$.

Regarding the technology a typical producer employs, it is given by an atemporal production function of the Cobb-Douglas-type:

$$f(l,k) = l^a k^b \qquad \text{with } \ a+b<1.$$

As his objective function is expected profit, his problem becomes

$$\max_{l,k} \mu_t^d\varepsilon \left[\gamma_t^s f(l,k) - \frac{w_t}{p_t} l - \frac{r_t}{p_t} k\right] + (1-\mu_t^d\varepsilon)\left[\gamma_t^s f(l,v_t k) - \frac{w_t}{p_t} l - \frac{r_t}{p_t} v_t k\right].$$

Note that the rationing parameter λ_t^d is no argument of the profit function because of the assumed 0/1-rationing on the labour market. The solutions are the effective labour demand

$$l_t^d = (\gamma_t^s)^{\frac{1}{1-a-b}} \left(\frac{b\,p_t}{r_t}\right)^{\frac{b}{1-a-b}} \left(\frac{a\,p_t}{w_t}\right)^{\frac{1-b}{1-a-b}} (\mu_t^d)^{\frac{1-b}{1-a-b}} [\varepsilon + v_t^{b-1}(1-\varepsilon)]^{\frac{1}{1-a-b}} \tag{3}$$

and the effective capital demand

$$k_t^d = (\gamma_t^s)^{\frac{1}{1-a-b}} \left(\frac{b\,p_t}{r_t}\right)^{\frac{1-a}{1-a-b}} \left(\frac{a\,p_t}{w_t}\right)^{\frac{a}{1-a-b}} (\mu_t^d)^{\frac{a}{1-a-b}} [\varepsilon + v_t^{b-1}(1-\varepsilon)]^{\frac{1}{1-a-b}}.$$

The partial derivatives of the effective factor demands with respect to γ^s are positive:

$$\frac{\partial l^d}{\partial \gamma^s} > 0, \qquad \frac{\partial k^d}{\partial \gamma^s} > 0. \tag{4}$$

This means that the factor demands decrease with increasing expected rationing on the commodity market. Variations of μ_t^d yield

$$\frac{\partial l^d}{\partial \mu^d} > 0 \quad \text{with} \quad \lim_{\mu^d \to 0} l^d = (\gamma^s)^{\frac{1}{1-a-b}} \left(\frac{b\,p}{r}\right)^{\frac{b}{1-a-b}} \left(\frac{a\,p}{w}\right)^{\frac{1-b}{1-a-b}} (1-\varepsilon)^{\frac{b}{1-a-b}}, \tag{5}$$

$$\frac{\partial k^d}{\partial \mu^d} < 0 \quad \text{with} \quad \lim_{\mu^d \to 0} k^d = \infty. \tag{6}$$

Note that an increase in rationing on the capital market induces the producer to lower his labour demand but to raise his capital demand. Moreover the two limiting values are of interest. If the producer is rationed to zero on the capital market he will still maintain a positive labour demand whereas his capital demand will go to infinity. [6]

For the effective aggregate labour demand follows

$$L_t^d = L^d(\gamma_t^s, \mu_t^d, w_t, p_t, r_t) = m\, l^d(\gamma_t^s, \mu_t^d, w_t, p_t, r_t) \tag{7}$$

The aggregate effective capital demand consists only of the sum of the individual capital demands of the $\lambda_t^d m$ producers whose labour demand has been satisfied:

$$K_t^d = K^d(\lambda_t^d, \mu_t^d, \gamma_t^s, w_t, p_t, r_t) = m\, \lambda_t^d k^d(\mu_t^d, \gamma_t^s, w_t, p_t, r_t).$$

The aggregate goods supply is composed of the production of the $m\, \lambda_t^d \mu_t^d \varepsilon$ producers who could employ the optimal quantities of their production factors and the rest of the producers who had to work with a rationed capital stock:

$$Y_t^s = Y^s(\lambda_t^d, \mu_t^d, \gamma_t^s, w_t, p_t, r_t) = m\, \lambda_t^d[\mu_t^d \varepsilon (l_t^d)^a (k_t^d)^b + (1 - \mu_t^d \varepsilon)(l_t^d)^a (v_t k_t^d)^b].$$

Now it is possible to define the locus of the aggregate transaction offers

$$F := \{(L^d(\mu^d, \gamma^s), K^d(\lambda^d, \mu^d, \gamma^s), Y^s(\lambda^d, \mu^d, \gamma^s)) \mid (\lambda^d, \mu^d, \gamma^s) \in [0,1]^3\},$$

as well as the locus of the expected aggregate transaction for varying rationing signals

$$\overline{F} := \{(\lambda^d L^d(\mu^d, \gamma^s), \mu^d K^d(\lambda^d, \mu^d, \gamma^s), \gamma^s Y^s(\lambda^d, \mu^d, \gamma^s)) \mid (\lambda^d, \mu^d, \gamma^s) \in [0,1]^3\}$$

with $\quad L = \lambda^d L^d(\mu^d, \gamma^s), \quad K = \mu^d K^d(\lambda^d, \mu^d, \gamma^s), \quad Y = \gamma^s Y^s(\lambda^d, \mu^d, \gamma^s).$

Analogously to the consumption sector we partition $\overline{F}$ with regard to the values of the corresponding quantity signals λ^d, μ^d and γ^s:

$$\overline{F} := \bigcup_{i=1}^{8} \overline{F}^i \quad \text{with}$$

[6] In this context the question arises whether a firm's production costs are always covered even if its goods supply is rationed. In the BCW-model it could be checked that a producer will never get into debt if the production function is $f(l) = al^b$ and in addition the time-independent conditions $a > 0$ and $0 \leq b \leq (1 - \sigma)$ are fullfilled. In our model under the assumption of $f(l,k) = l^a k^b$ the financing of the input factors is not always ensured but it depends on the values of γ_t^s and μ_t^d. For details see Förster [1998]. Since this problem occurs only in extreme situations, e.g. when $\mu^d \to 0$, it will be left out of consideration in our further discussions.

$$\overline{F}^1 := \overline{F}\Big|_{\lambda^d=1,\mu^d=1,\gamma^s\le 1;} \qquad \overline{F}^2 := \overline{F}\Big|_{\lambda^d=1,\mu^d=1,\gamma^s=1};$$
$$\overline{F}^3 := \overline{F}\Big|_{\lambda^d=1,\mu^d\le 1,\gamma^s\le 1}; \qquad \overline{F}^4 := \overline{F}\Big|_{\lambda^d=1,\mu^d\le 1,\gamma^s=1};$$
$$\overline{F}^5 := \overline{F}\Big|_{\lambda^d\le 1,\mu^d=1,\gamma^s\le 1}; \qquad \overline{F}^6 := \overline{F}\Big|_{\lambda^d\le 1,\mu^d=1,\gamma^s=1};$$
$$\overline{F}^7 := \overline{F}\Big|_{\lambda^d\le 1,\mu^d\le 1,\gamma^s\le 1}; \qquad \overline{F}^8 := \overline{F}\Big|_{\lambda^d\le 1,\mu^d\le 1,\gamma^s=1}.$$

The locus $\overline{F}$ is represented in the L-K-Y-space by a ray which starts in the origin (figure 4). In general, each point lying on this ray is related to several combinations of λ^d, γ^s and μ^d. This fact, which is due to the Cobb-Douglas production function, implies that there is no one-to-one correspondence between the locus $\{\lambda^d, \mu^d, \gamma^s | (\lambda^d, \mu^d, \gamma^s) \in [0,1]^3\}$ and $\overline{F}$. In appendix 1 we supply evidence of this assertion and derive relations between K, L and Y which are now used to rewrite $\overline{F}^i$ as

$$\overline{F}^1 = \overline{F}^3 = \overline{F}^5 = \overline{F}^6 = \overline{F}^7 = \overline{F}^8 := \left\{ \left(L, \frac{w\,b}{r\,a} L, \frac{w}{a\,p} L \right) \Big| \, 0 \le L \le L^d(1,1) \right\},$$
$$\overline{F}^4 := \left\{ \left(L, \frac{w\,b}{r\,a} L, \frac{w}{a\,p} L \right) \Big| \, L^d(\mu^d = 0, 1) \le L \le L^d(\mu^d = 1, 1) \right\},$$
$$\overline{F}^2 := \{(L^d(1,1), K^d(1,1,1), Y^s(1,1,1))\}.$$

Therefore the locus $\overline{F}^i$ represents the whole ray except for $i = 2$ and $i = 4$. Due to (5) the ray $\overline{F}^4$ does not start from the origin but at $L^d(\mu^d = 0, \gamma^s = 1)$ and goes to the final point of the ray $\overline{F}$. Moreover the locus of the expectations in the case $\lambda^d = \mu^d = \gamma^s = 1$ is only a point on the ray. Taking into account the derivatives (4), (5), $\overline{F}^2$ has to be the final point of the ray.

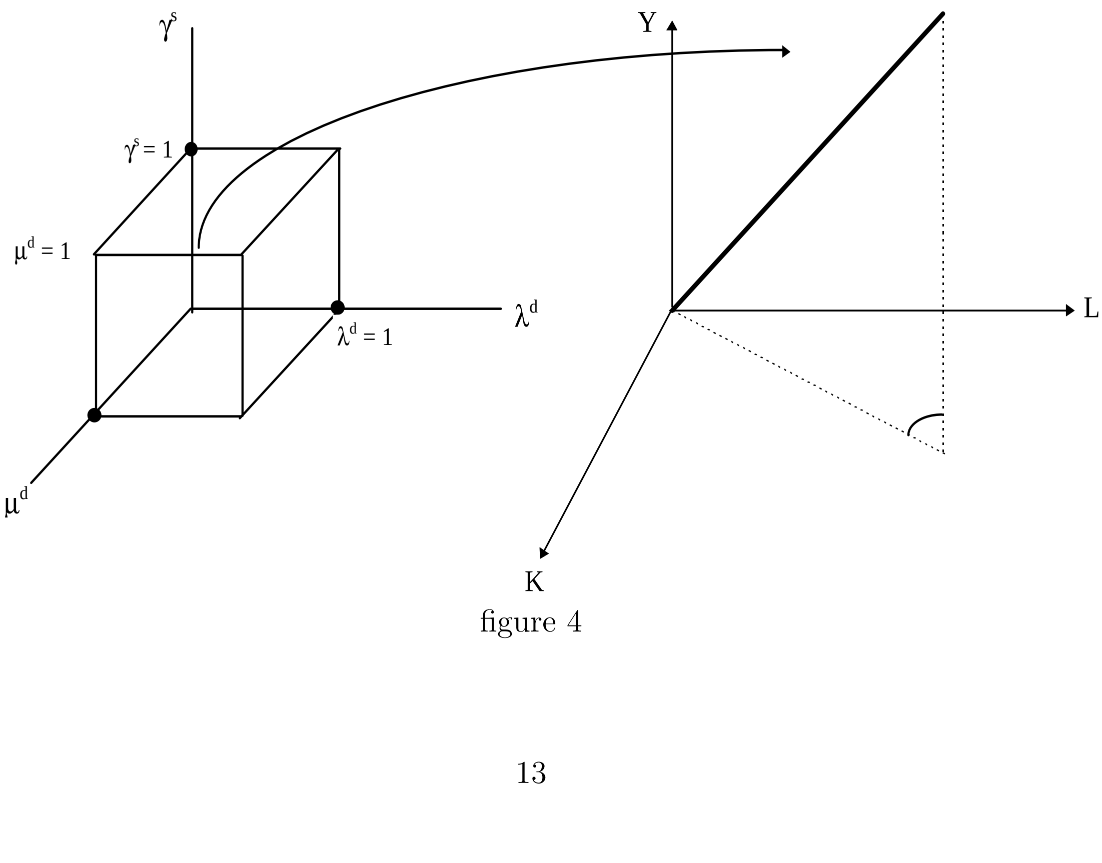


figure 4

A three-dimensional parametric representation of all possible aggregate transaction offers of the production sector has turned out to be quite difficult because a certain level of L^d, K^d and Y^s can be explained by various constellations of λ^d, μ^d and γ^s. For this reason we will restrict ourselves to illustrating the transaction offers only when we determine graphically as an example the temporary equilibrium systems 1 and 6.

2.2 Temporary Feasible States

For any period t we can now describe a feasible allocation as a temporary equilibrium with rationing as follows.

Definition: Given the real wage w_t/p_t, the real interest rate r_t/p_t, the real net profit $(1 - \text{tax})\Pi_t/p_t$, the real consumption of the old agents S_t/p_t, the real payment of interest $(K_{t-1}r_{t-1})/p_t$, and the public expenditures G, a list of quantity disequilibrium signals $(\lambda_t^s, \lambda_t^d, \mu_t^s, \mu_t^d, \gamma_t^s, \gamma_t^d, \delta_t, \beta_t) \in [0,1]^8$ is a temporary equilibrium with stochastic rationing and allocation $(\overline{L}_t, \overline{K}_t, \overline{Y}_t)$ if the following equations are satisfied:

$$
\begin{aligned}
&(1) \quad \overline{L}_t = \lambda_t^s L^s = \lambda_t^d L^d(\mu_t^d, \gamma_t^s),\\
&(2) \quad \overline{K}_t = \mu_t^s K^s(\lambda_t^s) = \mu_t^d K^d(\lambda_t^d, \mu_t^d, \gamma_t^s),\\
&(3) \quad \overline{Y}_t = \gamma_t^s Y^s(\lambda_t^d, \mu_t^d, \gamma_t^s) = \gamma_t^d X^d(\lambda_t^s) + \delta_t \frac{S_t}{p_t} + \beta_t G,\\
&(4) \quad (1-\lambda_t^s)(1-\lambda_t^d) = 0, \qquad (1-\mu_t^s)(1-\mu_t^d) = 0,\\
&\qquad\;\; (1-\gamma_t^s)(1-\gamma_t^d) = 0, \qquad \gamma_t^d(1-\delta_t) = 0 \qquad \delta_t(1-\beta_t) = 0.
\end{aligned}
\tag{8}
$$

Conditions (1) to (3) are saying that a temporary equilibrium obtains when the economic agents express, in response to the expectations of the rationing signals they hold, effective supplies and demands which give rise to expected transactions that confirm these signals; thus no agent has a reason to change his proposed action. According to equations (4) not more than one side can be rationed. Moreover, if demand rationing occurs in the commoditiy market, the young consumers are rationed first. After their complete rationing, the demand of the old consumers and, at last, the demand of the government will be restricted.

We distinguish eight different types of temporary equilibria, called systems, depending on the prevailing quantity quotas. Their distinctive feature is that rationing takes place in all three markets. Apart from that there still exist 19 border cases to which the Walrasian Equilibrium belongs as well. In a border case at least one market is in equilibrium. In the following table 1 we depict the above-mentioned eight systems. From this it is also evident that the border cases can be seen as particular cases of the systems.

Since in a state of equilibrium the agents' expectations are confirmed, an equilibrium is graphically determined at the point of intersection of the loci of

	1	2	3	4	5	6	7	8
λ_t^s	$< \mathbf{1}$	$< \mathbf{1}$	$< \mathbf{1}$	$< \mathbf{1}$	$= 1$	$= 1$	$= 1$	$= 1$
λ_t^d	$= 1$	$= 1$	$= 1$	$= 1$	$\leq \mathbf{1}$	$\leq \mathbf{1}$	$\leq \mathbf{1}$	$\leq \mathbf{1}$
μ_t^s	$< \mathbf{1}$	$< \mathbf{1}$	$= 1$	$= 1$	$< \mathbf{1}$	$< \mathbf{1}$	$= 1$	$= 1$
μ_t^d	$= 1$	$= 1$	$\leq \mathbf{1}$	$\leq \mathbf{1}$	$= 1$	$= 1$	$\leq \mathbf{1}$	$\leq \mathbf{1}$
γ_t^s	$\leq \mathbf{1}$	$= 1$	$\leq \mathbf{1}$	$= 1$	$\leq \mathbf{1}$	$= 1$	$\leq \mathbf{1}$	$= 1$
γ_t^d	$= 1$	$< \mathbf{1}$	$= 1$	$< \mathbf{1}$	$= 1$	$< \mathbf{1}$	$= 1$	$< \mathbf{1}$
δ_t	$= 1$	$\leq \mathbf{1}$	$= 1$	$\leq \mathbf{1}$	$= 1$	$\leq \mathbf{1}$	$= 1$	$\leq \mathbf{1}$
β_t	$= 1$	$\leq \mathbf{1}$	$= 1$	$\leq \mathbf{1}$	$= 1$	$\leq \mathbf{1}$	$= 1$	$\leq \mathbf{1}$

table 1

the expected transactions of the consumption- and the production sector. Let us recall figure 3 revealing the partition of $\overline{H}$ and imagine in addition the ray $\overline{F}$. If $\overline{F}$ intersects the locus $\overline{H}^i$ then the corresponding system i occurs.

As examples we want to analyse systems 1 and 6 which coincide with the well-known regimes of Keynesian Unemployment and Repressed Inflation, respectively, in the temporary equilibrium models without a capital market.

System 1

A temporary equilibrium of type 1 is determined by a triple of parameters $(\lambda_t^s, \mu_t^s, \gamma_t^s)$, such that:

$$\overline{L}_t = \lambda_t^s L^s = L^d(1, \gamma_t^s),$$
$$\overline{K}_t = \mu_t^s K^s(\lambda_t^s) = K^d(1, 1, \gamma_t^s),$$
$$\overline{Y}_t = \gamma_t^s Y^s(1, 1, \gamma_t^s) = X^d(\lambda_t^s) + \frac{S_t}{p_t} + G.$$

Apart from the border case $\gamma^s = 1$ system 1 is characterized by an excess supply in all three markets. Taking into account the definition of the loci $\overline{H}^1$ and $\overline{F}^1$ a triple $(\overline{L}_t, \overline{K}_t, \overline{Y}_t)$ is an allocation of system 1 if [7]

$$(\overline{L}_t, \overline{K}_t, \overline{Y}_t) \in$$
$$[\{(\lambda^s L^s, \mu^s K^s(\lambda^s), \gamma^d X^d(\lambda^s)) \mid \lambda^s \in [0,1[,\ \mu^s \in [0,1[,\ \gamma^d = 1\} + \{(0, 0, \frac{S_t}{p_t} + G)\}] \cap$$
$$\{(\lambda^d L^d(\mu^d, \gamma^s), \mu^d K^d(\lambda^d, \mu^d, \gamma^s), \gamma^s Y^s(\lambda^d, \mu^d, \gamma^s)) \mid \lambda^d = 1, \mu^d = 1, \gamma^s \in [0,1]\},$$

that is

$$(\overline{L}_t, \overline{K}_t, \overline{Y}_t) \in [\overline{H}_t^1 + \{(0, 0, \frac{S_t}{p_t} + G)\}] \cap \overline{F}_t^1.$$

The intersection of these loci is depicted in figure 5. Note that the concave curve which starts in the origin and goes to the final point of the ray represents

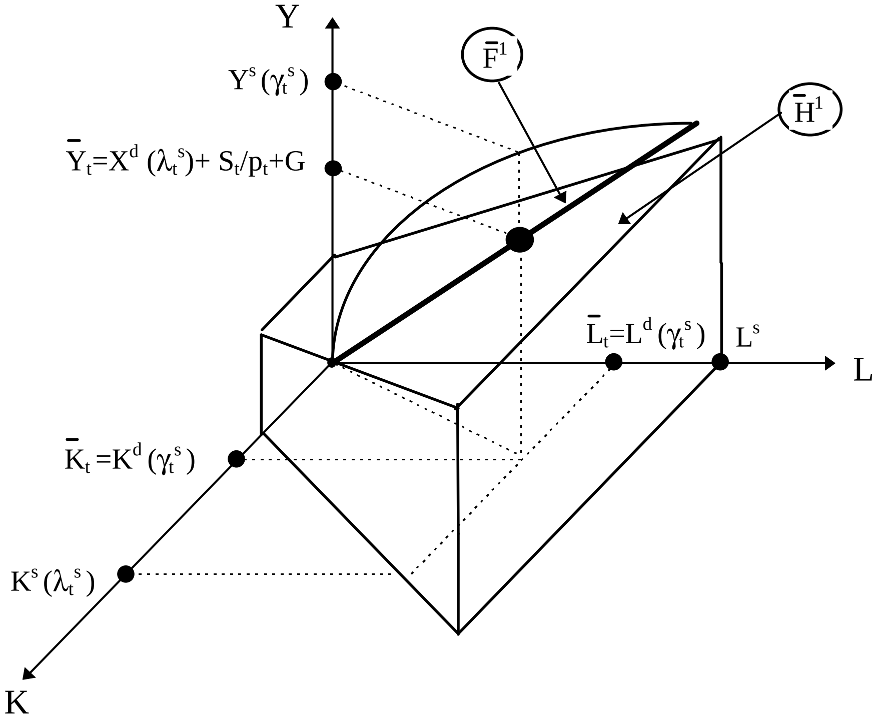


figure 5

the locus of the transaction offers of the production sector. In the border case $\gamma^s = 1$ the ray would end exactly in the area $\overline{H}^1$.

The figure reveals that the consumers' supplies are $L^s > \overline{L}_t$ and $K_t^s > \overline{K}_t$ whereas their commodity demand $X_t^d + S_t/p_t + G = \overline{Y}_t$ is completely satisfied. The firms get their factor demands $L_t^d = \overline{L}_t$ and $K_t^d = \overline{K}_t$, but they have to be contented with the goods sales $\overline{Y}_t < Y_t^s$. The resulting quotas $\lambda_t^s = \overline{L}_t/L^s$, $\mu_t^s = \overline{K}_t/K_t^s$, $\gamma_t^s = \overline{Y}_t/Y_t^s$ and $\lambda_t^d = \mu_t^d = \gamma_t^d = 1$ are such that, if expected by the agents at exactly these values, lead them to announce the corresponding transaction offers (L^s, K_t^s, Y_t^d) and (L_t^d, K_t^d, Y_t^s). Therefore in that case all expectations are confirmed and no agent has a reason to change his transaction offer. It must be stressed that the effective aggregate supplies of rationed agents exceed the actual transactions. This can be traced back to the uncertainty the agents are confronted with concerning the amount they can trade.

As we mentioned before, the advantage of the stochastic approach is that it provides a reliable indicator for the size of the agents' dissatisfaction with the exchanged quantities. This indicator can be used for the adjustment of prices. In fact, consider the ratio of the excess supply and the aggregate supply of the respective market. Concerning the goods market we obtain $1 - \gamma_t^s$. According

[7] $\overline{H}^1 + \{(0, 0, S_t/p_t + G)\}$ means that each point of $\overline{H}^1$ is shifted by $(0, 0, S_t/p_t + G)$.

to the derivative

$$\frac{d(1-\gamma^s)}{d\overline{Y}} = -\frac{1}{Y^s + \gamma^s \dfrac{\partial Y^s}{\partial \gamma^s}}$$

and the fact that $\partial Y^s/\partial\gamma^s$ is positive the rationing quota $1-\gamma^s$ decreases with increasing sales $\overline{Y}$. Analogously $1-\lambda_t^s$ and $1-\mu_t^s$ are appropriate measures of the size of rationing in the labour market and the capital market, respectively.

System 6

In a state of equilibrium of type 6 there is an excess in labour and goods demand and rationing of capital supply. In the border case belonging to system 6 the labour market is in equilibrium. Hence it is described by a triple $(\lambda_t^d, \mu_t^s, \gamma_t^d)$ such that

$$\begin{aligned} \overline{L}_t &= L^s = \lambda_t^d L^d(1,1), \\ \overline{K}_t &= \mu_t^s K^s(1) = K^d(\lambda_t^d, 1, 1), \\ \overline{Y}_t &= Y^s(\lambda_t^d, 1, 1) = \gamma_t^d X^d(1) + \delta_t \frac{S_t}{p_t} + \beta_t G. \end{aligned}$$

This implies

$$\lambda_t^d = \frac{L^s}{L^d(1,1)}, \qquad \mu_t^s = \frac{K^d(\lambda_t^d,1,1)}{K^s(1)}, \qquad \gamma_t^d = \frac{\overline{Y}_t - \delta_t S_t/p_t - \beta_t G}{X^d(1)}.$$

First of all we consider the case $\delta_t\beta_t = 1$. Recalling the definition of the loci $\overline{H}^6$ and $\overline{F}^6$, a triple $(\overline{L}_t, \overline{K}_t, \overline{Y}_t)$ is an allocation of system 6 if

$$\begin{aligned} &(\overline{L}_t, \overline{K}_t, \overline{Y}_t) \in \\ &[\{(\lambda^s L^s, \mu^s K^s(\lambda^s), \gamma^d X^d(\lambda^s)) \mid \lambda^s = 1,\, \mu^s \in [0,1[,\, \gamma^d \in [0,1[\,\} + \{(0,0,\frac{S_t}{p_t} + G)\}] \cap \\ &\{(\lambda^d L^d(\mu^d,\gamma^s), \mu^d K^d(\lambda^d,\mu^d,\gamma^s), \gamma^s Y^s(\lambda^d,\mu^d,\gamma^s)) \mid \lambda^d \in [0,1]\,, \mu^d = 1, \gamma^s = 1\}, \\ &\qquad = [\overline{H}_t^6 + \{(0,0,\frac{S_t}{p_t} + G)\}] \cap \overline{F}_t^6. \end{aligned}$$

System 6 is illustrated in figure 6. In the border case $\lambda^s = \lambda^d = 1$ the ray would end in the area $\overline{H}^6$. If $\delta_t\beta_t < 1$ obtained, $\overline{Y}_t$ would be smaller than the amount $S_t/p_t + G$ demanded by the old consumers and the government. In the L-K-Y-space the corresponding point of intersection would lie below the broken line.

In contrast to the systems 1 and 6 just described, in an equilibrium of type 3, 5, 7 or 8 the transaction offers are not all uniquely determined by the values of the aggregate transactions $(\overline{L}_t, \overline{K}_t, \overline{Y}_t)$. As an example we show this for

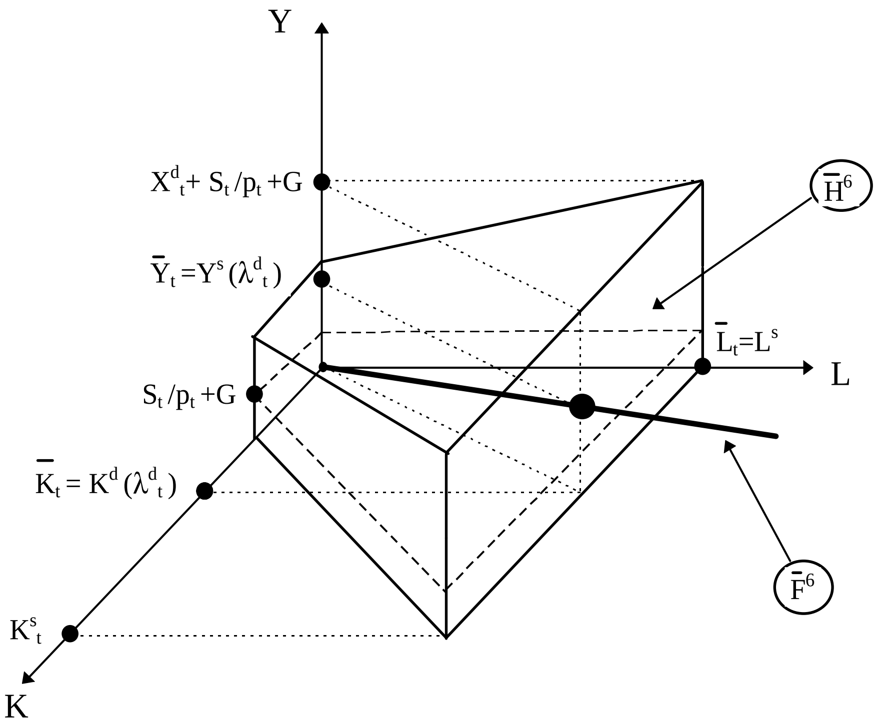


figure 6

system 3. In that case there is rationing of labour supply, capital demand and goods demand. This is expressed by a triple $(\lambda_t^s, \mu_t^d, \gamma_t^s)$ such that

$$\begin{aligned}
\overline{L}_t &= \lambda_t^s L^s = L^d(\mu_t^d, \gamma_t^s), \\
\overline{K}_t &= K^s(\lambda_t^s) = \mu_t^d K^d(1, \mu_t^d, \gamma_t^s), \\
\overline{Y}_t &= \gamma_t^s Y^s(1, \mu_t^d, \gamma_t^s) = X^d(\lambda_t^s) + \frac{S_t}{p_t} + G.
\end{aligned}$$

The actual labour employment $\overline{L}_t$ coincides with the aggregate labour demand L_t^d. Due to (3) several combinations of μ^d and γ^s give rise to the same transaction offer L_t^d. Hence with $\overline{K}_t < K^d(\mu^d, \gamma^s)$ and $\overline{Y}_t < Y^s(\mu^d, \gamma^s)$ the capital demand $K^d(\mu^d, \gamma^s)$ and the goods supply $Y^s(\mu^d, \gamma^s)$ are not uniquely determined. In fact we can define the set of all potential transaction offers compatible with $(\overline{L}_t, \overline{K}_t, \overline{Y}_t)$ as

$$\begin{aligned}
\{(L^d(\mu^d, \gamma^s), K^d(\mu^d, \gamma^s), Y^s(\mu^d, \gamma^s))| \quad & L^d(\mu^d, \gamma^s) = \overline{L}, \;\; \mu^d K^d(\mu^d, \gamma^s) = \overline{K}, \\
& \gamma^s Y^s(\mu^d, \gamma^s) = \overline{Y}, \;\; (\mu^d, \gamma^s) \in [0,1]^2\}.
\end{aligned}$$

In particular this shows that system 3 admits as well the two border cases $(\lambda^s < 1, \mu^d < 1, \gamma^s = \gamma^d = 1)$ and $(\lambda^s < 1, \mu^s = \mu^d = 1, \gamma^s < 1)$. It is worth bearing in mind that the systems 3, 5, 7 and 8 are represented in the L-K-Y-space by the intersection of the ray with an edge or a corner.

The existence of a temporary equilibrium is in contrast to the BCW-model not always ensured. This can be traced back to the Cobb-Douglas-production function and the form of the rationing mechanism in the capital market. However, nonexistence is possible in system 4 only. In our numerical simulations we overcome this problem by adjusting the interest rate in direction of its market clearing value until an equilibrium exists. If an equilibrium is achieved, it will always be unique.

2.3 The BCW-model

We mentioned in the introduction that the BCW-model is incorporated in our model. This can easily be understood by inspecting the equations (8) determining the equilibrium allocation in all three markets and by explicating the functions $L^d(\cdot)$, $K^s(\cdot)$, $K^d(\cdot)$, $Y^s(\cdot)$ and $X^d(\cdot)$:

1. $\lambda_t^s L^s =$

$$\lambda_t^d n'(\gamma_t^s)^{\frac{1}{1-a-b}} \left(\frac{b\,p_t}{r_t}\right)^{\frac{b}{1-a-b}} \left(\frac{a\,p_t}{w_t}\right)^{\frac{1-b}{1-a-b}} (\mu_t^d)^{\frac{1-b}{1-a-b}} [\varepsilon + v_t^{b-1}(1-\varepsilon)]^{\frac{1}{1-a-b}},$$

2. $$\mu_t^s(1-h)\left[\lambda_t^s \frac{w_t}{p_t} L^s + \frac{(1-tax)\Pi_t}{p_t} + \frac{K_{t-1}r_{t-1}}{p_t}\right] = \tag{9}$$

$$\mu_t^d n' \lambda_t^d (\gamma_t^s)^{\frac{1}{1-a-b}} \left(\frac{b\,p_t}{r_t}\right)^{\frac{1-a}{1-a-b}} \left(\frac{a\,p_t}{w_t}\right)^{\frac{a}{1-a-b}} (\mu_t^d)^{\frac{a}{1-a-b}} [\varepsilon + v_t^{b-1}(1-\varepsilon)]^{\frac{1}{1-a-b}},$$

3. $\gamma_t^s n' \lambda_t^d\, [\mu_t^d \varepsilon\, l^a k^b + (1-\mu_t^d\varepsilon) l^a (v_t k)^b] =$

$$\gamma_t^d h \left[\lambda_t^s \frac{w_t}{p_t} L^s + \frac{(1-tax)\Pi_t}{p_t} + \frac{K_{t-1}r_{t-1}}{p_t}\right] + \delta \frac{S_t}{p_t} + \beta_t G.$$

If we then consider the case $\lim_{b\to 0}$ and assume $K_{t-1}r_{t-1} = 0$ for the initial period, we obtain the equations for the BCW-model: [8]

$$(1) \qquad \lambda_t^s L^s = \lambda_t^d n'(\gamma^s)^{\frac{1}{1-a}} \left(\frac{a\,p_t}{w_t}\right)^{\frac{1}{1-a}},$$

$$(2) \qquad \gamma_t^s n' \lambda_t^d l^a = \gamma_t^d h \left[\lambda_t^s \frac{w_t}{p_t} L^s + \frac{(1-tax)\Pi_t}{p_t}\right] + \delta \frac{S_t}{p_t} + \beta_t G.$$

We have left out the equation for the capital market because the capital demand becomes zero. Note that in an equilibrium this transaction offer is only compatible with the quantity signal $\mu_t^d = 1$. Consequently the state of Keynesian Unemployment in the BCW-model coincides with regime 1 in our model. Moreover the Classical Unemployment regime, the Repressed Inflation regime and the Underconsumption regime correspond to system 2, system 6 and system 5, respectively. Other types of equilibria can never occur in the BCW-model.

[8] When we investigate the dynamic behaviour of the BCW-model by means of numerical simulations we have to make the assumption of a downward interest rate rigidity in addition to $b = 0$. This restriction is necessary for technical reasons.

2.4 Representation of the Equilibrium Regimes in the *p*-*w*-*r*-Space

The unambiguous determination of an equilibrium allocation $(\overline{L}, \overline{K}, \overline{Y})$ makes it possible to partition a price-wage-interest (p-w-r-) space into subspaces in such a way that all p-w-r-combinations of a subspace give rise to a certain equilibrium state. This has been done with the help of a graphic software for the following numerical example:

$a = 0.5$	$b = 0.25$	$m = 200$	$L^s = 800$	$\Pi = 150$
$S = 300$	$\text{tax} = 0.2$	$\text{G} = 20$	$h = 0.474.$	

The market is in the Walrasian Equilibrium when

$p^* = 1.5$	$r^* = 0.75$	$w^* = 0.375.$

The decomposition which is shown in figure 7 is generally valid, since it could be proved in Förster [1998] that the values of the parameters do not affect the qualitative appearance of the border curve and the border areas.

We obtain the result that the graphical counterpart of systems 1, 2, 4 and 6 are subspaces. The regimes 3, 5 and 8 are degenerated to border areas and regime 7 even shrinks to a curve. Moreover the border cases G1 ($\lambda^s < 1, \mu^s < 1, \gamma^s = \gamma^d = 1$), G4 ($\lambda^s < 1, \mu^s = \mu^d = 1, \gamma^d < 1$) and G6 ($\lambda^s = \lambda^d = 1, \mu^s < 1, \gamma^d < 1$) separate regimes 1 and 2, regimes 2 and 4 and regimes 2 and 6, respectively. The marked black point represents the p-w-r-triple which generates the Walrasian state. Moreover all border areas and the border curve touch each other at the Walrasian point. The existence problem is not considered in the three-dimensional chart because otherwise the view would be reduced. But it should be kept in mind that not all p-w-r-combinations represented by points lying in the subspace 4 really lead to an equilibrium.[9][10]

In the following we want to demonstrate by means of system G6 how a border area is determined. The calculation of all the other border areas including the border curve and the proof of the degeneration of the systems 3, 5, 7 and 8 in the p-w-r-space to areas and a curve, respectively, are given in Förster [1998]. Starting point are the equations to determine the allocation in state G6:

[9] Due to the employed software it was convenient to depict the lists of data instead of the corresponding characteristic functions of the systems. Though the effect is that the areas in the graphic do not border directly on each other and therefore small gaps emerge.

[10] The arrows will be interpreted in the next section.

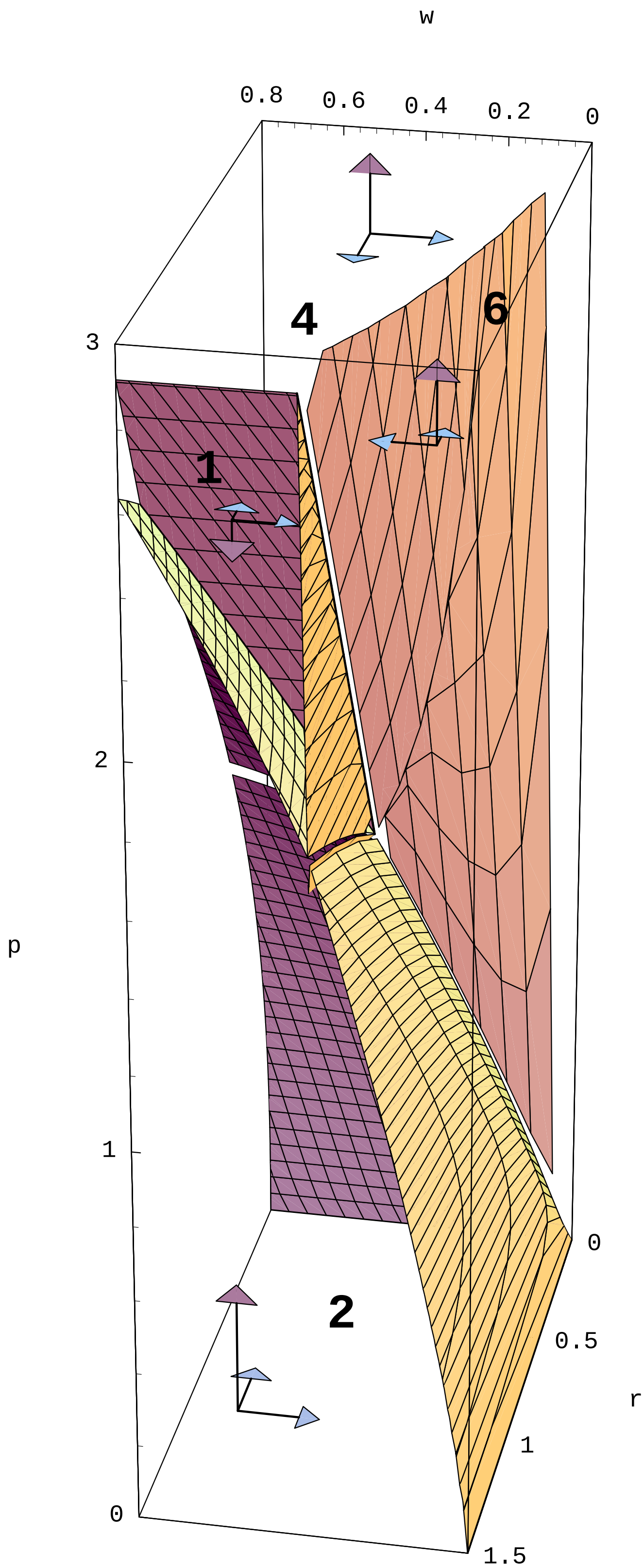


figure 7

$$L^s = L^d(1,1,w,p,r),$$
$$\mu^s K^s(1,w,p) = K^d(1,1,w,p,r),$$
$$Y^s(1,1,w,p,r) = \gamma^d X^d(1,w,p) + \delta\frac{S}{p} + \beta G.$$

Taking the first equation and replacing $L^d(1,1,w,p,r)$ by means of (3) and (7), we get, after some transformations, the following function for r which represents an area in the p-w-r-space:

$$r = \frac{m^{\frac{1-a-b}{b}}\, a^{\frac{1-b}{b}}\, b\, p^{\frac{1}{b}}}{(L^s)^{\frac{1-a-b}{b}}\, w^{\frac{1-b}{b}}}. \tag{10}$$

Taking into account (19), the situation in the goods market is described by

$$\frac{w}{a\,p}\,L < X^d(1,w,p) + \frac{S}{p} + G.$$

Using (2) for $X^d(1,w,p)$ and $\overline{L} = L^s$ we rewrite the inequality as

$$p > \frac{w\,L^s\left(\frac{1}{a} - h\right) - h\left[(1-tax)\Pi + K_{t-1}r_{t-1}\right] - S}{G}.$$

Now we consider the rationing in the capital market. From relation (20) we obtain

$$\frac{w\,b}{r\,a}\,L < K^s(1,w,p).$$

Substituting of L by L^s and $K^s(1,w,p)$ by (1) we get

$$\frac{w}{r}\frac{b}{a}L^s < (1-h)\left(\frac{w}{p}L^s + \frac{(1-tax)\Pi + K_{t-1}r_{t-1}}{p}\right).$$

Replacement of r by (10) and rearrangement of terms yields

$$p > \frac{w^{\frac{1}{1-b}}(L^s)^{\frac{1-a}{1-b}}}{m^{\frac{1-a-b}{1-b}}\, a^{\frac{1}{1-b}}\left[(1-h)w\,L^s + (1-h)((1-tax)\Pi + K_{t-1}r_{t-1})\right]^{\frac{b}{1-b}}}. \tag{11}$$

Observe that the two inequalities for p determine the range of definition of the border area.

2.5 Theoretical Considerations about Dynamics

So far our analysis of the economy has been essentially static. For a given vector $(w_t/p_t,\ r_t/p_t,\ (1-tax)\Pi_t/p_t,\ S_t/p_t,\ K_{t-1}r_{t-1}/p_t)$ we have defined a temporary equilibrium with quantity rationing and described the corresponding allocation $(\overline{L}, \overline{K}, \overline{Y})$. Now the intriguing question arises how the economy develops over time. To answer this we have to investigate the dynamic behaviour of the state variables $(w_t, p_t, r_t, \Pi_t, K_{t-1}r_{t-1}, S_t)$ which, in addition to time, depend as well as on the parameters $(a, b, h, G, tax, L^s, m)$. It will be useful to express the state variables in real terms as follows:

$$\alpha_t = \frac{w_t}{p_t}, \qquad r'_t = \frac{r_t}{p_t}, \qquad s_t = \frac{S_t}{p_t}, \qquad \pi_t = \frac{\Pi_t}{p_t}, \qquad z_t = \frac{K_{t-1}r_{t-1}}{p_t}.$$

Concerning price adjustment we adopt the usual assumption that prices fall (rise) when there is an excess of demand (excess of supply). This can be expressed by

$$\begin{array}{llll} p_{t+1} > p_t \iff \gamma_t^d < 1, & & p_{t+1} < p_t \iff \gamma_t^s < 1, \\ w_{t+1} > w_t \iff \lambda_t^d < 1, & & w_{t+1} < w_t \iff \lambda_t^s < 1, \\ r_{t+1} > r_t \iff \mu_t^d < 1, & & r_{t+1} < r_t \iff \mu_t^s < 1. \end{array}$$

The arrows in figure 7 indicate the directions of change in price, wage and interest for the regimes 1, 2, 4 and 6.[11] As we mentioned before, prices are adjusted according to the strength of rationing which, in the stochastic approach, can be measured in a satisfactory manner by means of the quantity signals. For the numerical simulations the adjustment functions are specified in the following way:[12]

$$\begin{array}{llll} p_{t+1} = (\gamma_t^s)^{\psi_1}\, p_t, & \text{if } \gamma_t^s < 1, & p_{t+1} = \left(\frac{\gamma_t^d + \delta_t + \beta_t}{3}\right)^{-\psi_2} p_t, & \text{if } \gamma_t^d < 1, \\ w_{t+1} = (\lambda_t^s)^{\nu_1}\, w_t, & \text{if } \lambda_t^s < 1, & w_{t+1} = (\lambda_t^d)^{-\nu_2}\, w_t, & \text{if } \lambda_t^d < 1, \\ r_{t+1} = (\mu_t^s)^{o_1}\, r_t, & \text{if } \mu_t^s < 1, & r_{t+1} = (\mu_t^d)^{-o_2}\, r_t, & \text{if } \mu_t^d < 1. \end{array}$$

The nonnegative parameters $\psi_1, \psi_2, \nu_1, \nu_2, o_1$ and o_2, determine the velocities of adjustment. If, for example, ν_1 is set to zero, we will simulate downward wage rigidity. From the above real adjustment functions we derive the ones for the real wage:

$$\alpha_{t+1} = \frac{(\lambda_t^s)^{\nu_1}}{(\gamma_t^s)^{\psi_1}}\,\alpha_t \qquad \text{if } (\overline{L}, \overline{K}, \overline{Y}) \in \text{system } 1,$$

$$\alpha_{t+1} = \frac{(\lambda_t^s)^{\nu_1}}{\left(\dfrac{\gamma_t^d + \delta_t + \beta_t}{3}\right)^{-\psi_2}}\,\alpha_t \quad \text{if } (\overline{L}, \overline{K}, \overline{Y}) \in \text{system } 2 \cup \text{system } 4,$$

[11] Figure 6 reveals that starting from system 1 or system 2 the economy approaches the Walrasian state in all probability. In system 4 and 6 the economy tends to move away from the Walrasian equilibrium because of the prevailing price adjustment. Concerning systems 3, 5 and 7 it could be shown in Förster [1998] that the economy will always turn into system 1 although the corresponding quantity signals and consequently the price, wage and interest adjustments are not uniquely determined. Regarding regime 8 it cannot be clarified if there is a change to system 4 and system 6, respectively.

[12] Following Bignami, Colombo and Weinrich (2000), a more general formulation is given in appendix 2.

$$\alpha_{t+1} = \frac{(\lambda_t^d)^{-\nu_2}}{\left(\dfrac{\gamma_t^d + \delta_t + \beta_t}{3}\right)^{-\psi_2}} \alpha_t \quad \text{if } (\overline{L}, \overline{K}, \overline{Y}) \in \text{system 6},$$

and the real interest:

$$r'_{t+1} = \frac{(\mu_t^s)^{o_1}}{(\gamma_t^s)^{\psi_1}} r'_t \qquad \text{if } (\overline{L}, \overline{K}, \overline{Y}) \in \text{system 1},$$

$$r'_{t+1} = \frac{(\mu_t^s)^{o_1}}{\left(\dfrac{\gamma_t^d + \delta_t + \beta_t}{3}\right)^{-\psi_2}} r'_t \quad \text{if } (\overline{L}, \overline{K}, \overline{Y}) \in \text{system 2} \cup \text{system 6},$$

$$r'_{t+1} = \frac{(\mu_t^s)^{-o_2}}{\left(\dfrac{\gamma_t^d + \delta_t + \beta_t}{3}\right)^{-\psi_2}} r'_t \quad \text{if } (\overline{L}, \overline{K}, \overline{Y}) \in \text{system 4}.$$

For the growth rate of the price level $\theta_t = p_{t+1}/p_t$ we get:

$$\theta_t = (\gamma_t^s)^{\psi_1} \qquad \text{if } (\overline{L}, \overline{K}, \overline{Y}) \in \text{system 1},$$

$$\theta_t = \left(\frac{\gamma_t^d + \delta_t + \beta_t}{3}\right)^{-\psi_2} \quad \text{if } (\overline{L}, \overline{K}, \overline{Y}) \in \text{system 2} \cup \text{system 4} \cup \text{system 6}.$$

The adjustment equations for the state variables π_t, z_t and s_t result directly from the structure of the model. Since π_t and z_t depend on the actual transactions in the three markets, we derive first the functions to determine $\overline{L}_t, \overline{K}_t$ and $\overline{Y}_t$. For this we use the two-dimensional projections of the loci $\overline{H}$ and $\overline{F}$ on the L-Y-plane and L-K-plane, respectively. This is possible because through the projection no information gets lost about the loci. Consider the L-Y-plane in figure 8. We define a function

$$\Gamma_t^Y(L) := h\,\alpha_t L + h(1 - tax)\pi_t + h\,z_t + G + s_t$$

which represents for $0 \le L < L^s$ the projection of $\overline{H}^1 + \{(0, 0, s_t + G)\}$[13] and $\overline{H}^3 + \{(0, 0, s_t, G)\}$ and for $L = L^s$ the projection of $\overline{H}^5 + \{(0, 0, s_t + G)\}$ and $\overline{H}^7 + \{(0, 0, s_t, G)\}$. In sum, it is the locus of the transactions L and Y expected by the consumers for the case $\lambda^s \le 1$ and $\gamma^d = 1$. Moreover we define a function

$$\Delta_t^Y(L) := \frac{1}{a}\alpha_t L$$

which is according (19) in the range $0 \le L \le L^d(1, 1)$ the projection of the ray $\overline{F}$ on the L-Y-plane. Since $h < 1 < 1/a$ there exists $\tilde{L} > 0$ such that $\Gamma_t^Y(\tilde{L}) = \Delta_t^Y(\tilde{L})$. More precisely we obtain

[13] Henceforth we shall write the projection of the loci $\overline{H}^i + \{(0, 0, s_t + G)\}$ as $\overline{H}_p^i$ and $\overline{F}$ as $\overline{F}_p$.

$$\widetilde{L} = \frac{h\left[(1-tax)\pi_t + z_t\right] + G + s_t}{\alpha_t\left(\dfrac{1}{a} - h\right)} \quad =: \; \widetilde{\mathcal{L}}\,(\alpha_t, \pi_t, z_t, s_t). \tag{12}$$

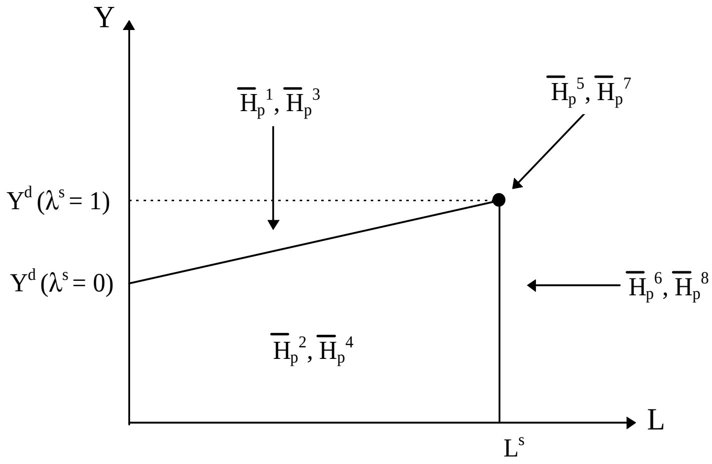


figure 8

Now we turn to the L-K-plane in figure 9. We define a function $\Gamma_t^K(L)$ as

$$\Gamma_t^K(L) := (1-h)\alpha_t L + (1-h)(1-tax)\pi_t + (1-h)z_t$$

which, in the range $0 \le L < L^s$, coincides with the loci $\overline{H}_p^3$ and $\overline{H}_p^4$ and for $L = L^s$ with $\overline{H}_p^7$ and $\overline{H}_p^8$ (where $\overline{H}_p^i$ denotes the projection of the locus $\overline{H}^i \subset \Re^3$ into the L-K-plane). Hence it is the locus of the consumers' expectations L and K for the case $\lambda^s \le 1$ and $\mu^s = 1$. Furthermore, according to (20) we define a function

$$\Delta_t^K(L) := \frac{\alpha_t}{r_t'}\frac{b}{a}L$$

which denotes in the range $0 \le L \le L^d(1,1)$ the projection of the ray $\overline{F}$ on the L-Y-plane. Consider L^* such that $\Gamma_t^K(L^*) = \Delta_t^K(L^*)$. If it exists, it is given by

$$L^* = \frac{(1-h)\left[(1-tax)\pi_t + z_t\right]\, r_t'}{\alpha_t\left(\dfrac{b}{a} - (1-h)\, r_t'\right)} \quad =: \; \mathcal{L}^*(\alpha_t, r_t', \pi_t, z_t). \tag{13}$$

L^* is positive if $b/a - (1-h)r_t' > 0$.

Concerning the L-Y-plane various parameter constellations can either cause the ray $\overline{F}_p$ to cross $\overline{H}_p^1$, $\overline{H}_p^3$ (including $\overline{H}_p^5$, $\overline{H}_p^7$) or $\overline{H}_p^6$, $\overline{H}_p^8$ or to remain inside the rectangle (now the index p indicates projection into the L-Y-plane). In the first case the employment at the point of intersection would be $\widetilde{L}$, in the second L^s and in the third $L^d(1,1,\alpha_t,r_t')$. Analogous considerations for the

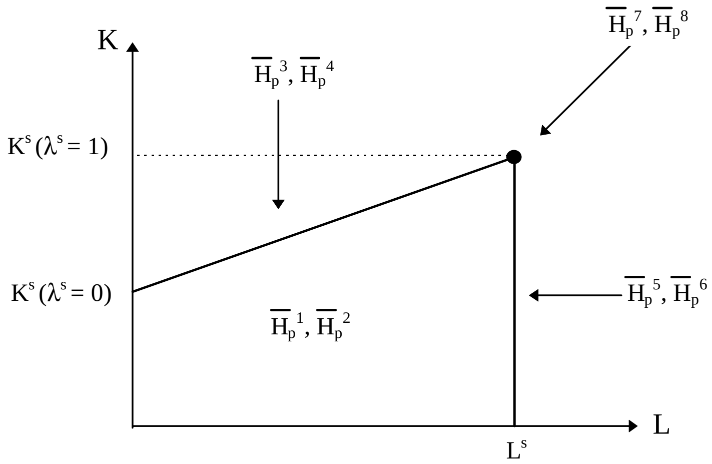


figure 9

L-K-plane yield that the employment can be L^s, $L^d(1, 1, \alpha_t, r'_t)$ or L^*. The equilibrium level of employment is determined by means of both projections. It is then evident that $\overline{L}_t$ is given by

$$\overline{L}_t = min\{\widetilde{\mathcal{L}}\,(\cdot), \mathcal{L}^*(\cdot), L^s, L^d(1, 1, \alpha_t, r'_t)\} =: \mathcal{L}(\alpha_t, r'_t, \pi_t, z_t, s_t). \tag{14}$$

If an equilibrium exists, the capital transaction $\overline{K}_t$ and the goods transaction $\overline{Y}_t$ result from

$$\begin{aligned} \overline{K}_t &= \Delta_t^K(\overline{L}_t) =: \mathcal{K}(\alpha_t, r'_t, \pi_t, z_t, s_t) \\ \overline{Y}_t &= \Delta_t^Y(\overline{L}_t) =: \mathcal{Y}(\alpha_t, r'_t, \pi_t, z_t, s_t). \end{aligned} \tag{15}$$

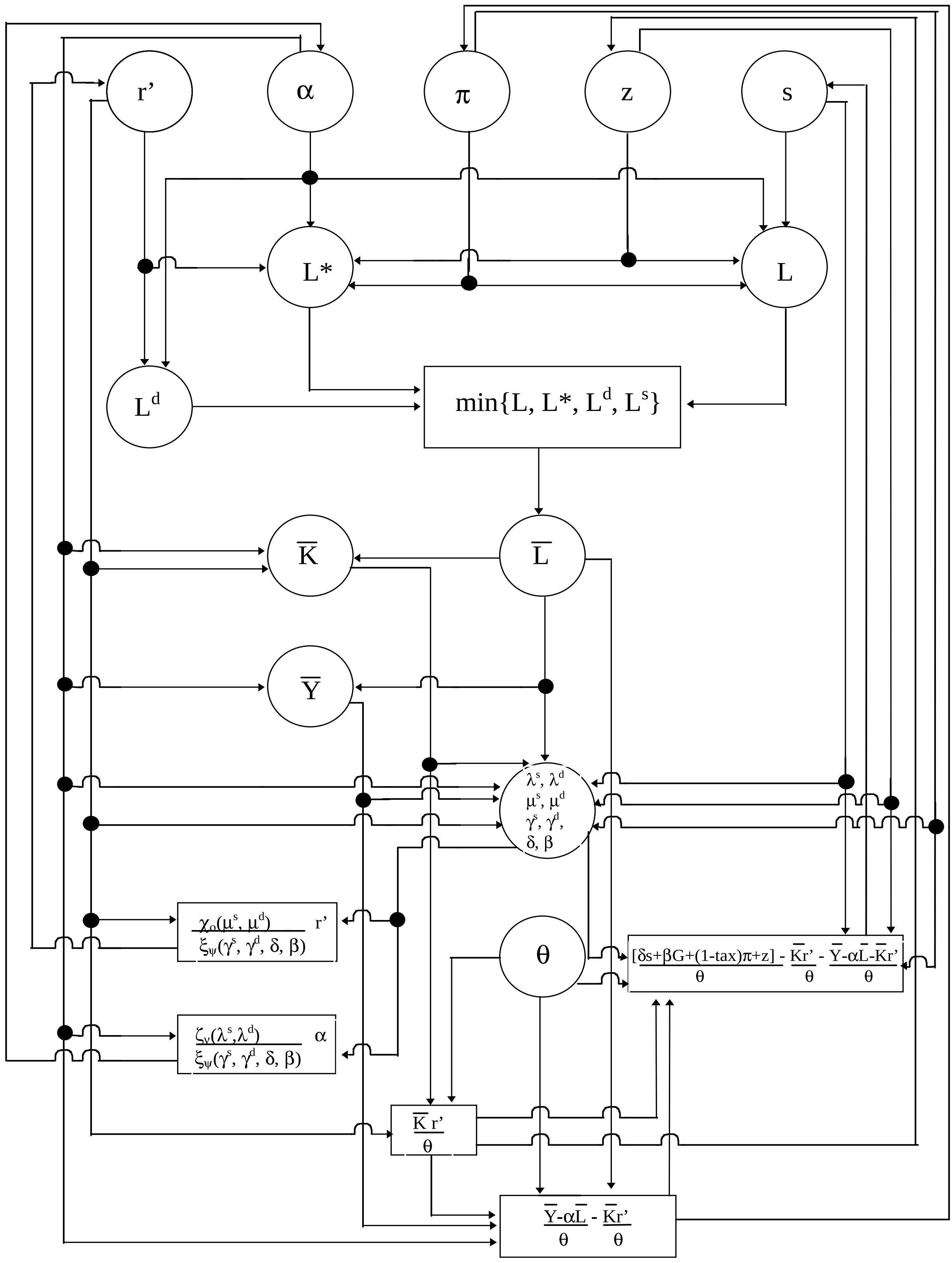
Now the development of s_t, z_t and π_t is described by the following adjustment equations:

$$\begin{aligned} s_{t+1} = {} & \frac{1}{p_{t+1}}[(1 - tax)\Pi_t + K_{t-1}r_{t-1} + w_t\overline{L}_t - p_t\overline{X}_t] = \\ & \frac{1}{p_{t+1}}[(1 - tax)\Pi_t + K_{t-1}r_{t-1} + w_t\overline{L}_t - p_t\overline{Y}_t + \delta_t S_t + \beta_t p_t G] = \\ & \frac{1}{\theta_t}(\delta_t s_t + \beta_t G + (1 - tax)\pi_t + z_t) - \pi_{t+1} - z_{t+1}, \\ z_{t+1} = {} & \mathcal{K}(\cdot)\, r'_t \frac{1}{\theta_t}, \end{aligned}$$

$$\pi_{t+1} = \frac{1}{\theta_t}\, [\mathcal{Y}(\cdot) - \alpha_t\, \mathcal{L}(\cdot) - r'_t\, \mathcal{K}(\cdot)]\,.$$

The dynamic behaviour of the economy is given by the sequence $\{(\alpha_t, r'_t, \pi_t, z_t, s_t)\}_{t=1}^{\infty}$ where the state variables satisfy the corresponding adjustment equations.

A distinctive feature of our model is its dynamic feedback structure which is illustrated in figure 10 in form of a flowchart. Note that variables are written in circles and functions in boxes.



r'
α
π
z
s
L*
L
L^d
min{L, L*, L^d, L^s}
$\overline{K}$
$\overline{L}$
$\overline{Y}$
λ^s, λ^d μ^s, μ^d γ^s, γ^d, δ, β
$\frac{\chi_o(\mu^s, \mu^d)}{\xi_\psi(\gamma^s, \gamma^d, \delta, \beta)}$ r'
θ
$\frac{[\delta s+\beta G+(1\text{-tax})\pi+z]}{\theta} - \frac{\overline{K}r'}{\theta} - \frac{\overline{Y}-\alpha\overline{L}-\overline{K}r'}{\theta}$
$\frac{\zeta_\nu(\lambda^s,\lambda^d)}{\xi_\psi(\gamma^s, \gamma^d, \delta, \beta)}$ α
$\frac{\overline{K}\,r'}{\theta}$
$\frac{\overline{Y}-\alpha\overline{L}}{\theta} - \frac{\overline{K}r'}{\theta}$

figure 10

2.6 A Modification: The Model with Permanent Capital Market Clearing

In this section we consider our model under the alternative assumption of permanent capital market clearing. The prerequisite for this is that for any price-wage-combination there exists an interest rate which clears the capital market in accordance with agents' expectations. In appendix 3 we prove that this is true. Under these circumstances, at the end of a period the interest rate is not adjusted any more according to the rationing situation in the capital market but instead it is set at its market clearing value for the subsequent period.

The interest rate depends on the prevailing system or, which is equivalent, on the point of intersection of the curve $\mathcal{L}^*(r')$ with $f(r')$. To determine the equilibrium rate r'_t we define a function $\widetilde{\mathcal{R}}$ which is obtained by equating $\mathcal{L}^*(\alpha_t, \pi_t, z_t, r'_t)$ to L^s,

$$\widetilde{r} = \frac{L^s \alpha_t \, b}{a\,(1-h)\,((1-tax)\pi_t + z_t) + a\,(1-h) L^s \alpha_t} =: \widetilde{\mathcal{R}}\,(\alpha_t, \pi_t, z_t),$$

and a function $\mathcal{R}^*$ which results from the equation $\mathcal{L}^*(\alpha_t, \pi_t, z_t, r'_t) = \widetilde{\mathcal{L}}\,(\alpha_t, \pi_t, z_t, s_t)$ as

$$r^* = \frac{b\,[h\,((1-tax)\pi_t + z_t) + G + s_t]}{(1-h)((1-tax)\pi_t + z_t) + (a - a\,h)\,(G + s_t)} =: \mathcal{R}^*(\pi_t, z_t, s_t).$$

The equation $\mathcal{L}^*(\alpha_t, \pi_t, z_t, r'_t) = L^d(1, 1, \alpha_t, r'_t)$ cannot be solved explicitly for r'_t. Its solution, here denoted $r^\diamond$, is nevertheless given implicitly by the equation

$$\frac{(1-h)\,((1-tax)\pi_t + z_t)\,(r^\diamond)^{\frac{1-a}{1-a-b}}}{\alpha_t \left(\dfrac{b}{a} - (1-h)\,r^\diamond\right)} - n'\, b^{\frac{b}{1-a-b}}\, a^{\frac{1-b}{1-a-b}}\, \alpha^{\frac{-1+b}{1-a-b}} = 0.$$

The equilibrium interest is then defined by

$$r'_t = \text{Min}\{\widetilde{\mathcal{R}}\,(\alpha_t, \pi_t, z_t), \mathcal{R}^*(\pi_t, z_t, s_t), \mathcal{R}^\diamond(\alpha_t, \pi_t, z_t)\} =: \mathcal{R}(\alpha_t, \pi_t, z_t, s_t).$$

With this variant of the model the interest rate is a function of all the other state variables of the same period, for which the adjustment equations derived in the previous section still hold true.

3 Simulations

The dynamical systems of the model with credit rationing, called model I, and the model with permanent capital market clearing, called model II, are not tractable analytically. This is due to the piecewise definition of the adjustment equations and the necessity of calculating feasible allocations and the corresponding quantity signals. From (14) and (15) the functions $\mathcal{L}(\cdot)$, $\mathcal{K}(\cdot)$ and $\mathcal{Y}(\cdot)$ to determine $(\overline{L}, \overline{K}, \overline{Y})$ are nonlinear and in some cases the rationing signals can only be determined with numerical methods. Hence the use of numerical simulations is unavoidable to gain an insight into the dynamic behaviour of both models. We use the computer program MACRODYN which has been developed by V. Böhm, M. Lohmann and other members of the research group on macro dynamics of the University of Bielefeld to analyse dynamic macroeconomic models. MACRODYN consists of the programs and the numerical tools for the investigation of systems like the ones presented here. In particular we show below time series, bifurcation diagrams and cartograms. Concerning the implementation of model I and model II, the reader is refered to appendix 4.

Carrying out the numerical simulations for model I requires to assume values for the following parameters: production function parameters (a,b), price adjustment speed downward and upward (psi1,psi2), wage adjustment speed downward and upward (nu1,nu2), interest rate adjustment speed downward and upward (omikron1, omikron2), initial values of the state variables (real wage alfa0, real money stock mreal0, profit level pg0, real interest rate rreal0, real interest z0), labour supply (Ls), exponent of the utility function (h), public expenditures (G), proportional tax rate (tax), number of producers (m), number of iterations (simd), structural parameter of the rationing mechanism in the capital market which is relevant to the producer (epsilon), parameter to fix the strength of the manual interest rate adjustment (f). The simulations for model II are based on the same parameter set except for the parameters omikron1, omikron2, epsilon, f and rreal0.

Our numerical investigations focus on the role of the capital market for the dynamic development of the economy. Doing this, three different scenarios can be simulated: credit rationing (model I), permanent capital market clearing (model II) and the absence of a capital market (BCW-model). Moreover we are interested in the existence of Non-Walrasian steady states.

First of all we start from the following Walrasian equilibrium set:

a = 0.5	b = 0.25	psi1 = 0.1	psi2 = 0.1	nu1 = 0.1
nu2 = 0.1	o1 = 0.1	o2 = 0.1	m = 200	h = 0.474
pg0 = 100	alfa0 = 0.25	mreal0 = 200	rreal0 = 0.5	z0 = 100
Ls = 800	G = 20	tax = 0.2	$\varepsilon = 0.5$	f = 1.2

and we consider a reduction in the value of the initial real money stock to mreal0 = 100. In figure 11 and 12 we present time series of capital, output and employment for model I and model II. It is obvious from them that the economy returns faster to its Walrasian state in the case of permanent capital market clearing than in the case of credit rationing. The difference is particularly significant in the capital market. In model II the Walrasian level of capital transactions is reached after a transitional phase of about 60 periods whereas in model I not until 80 periods.

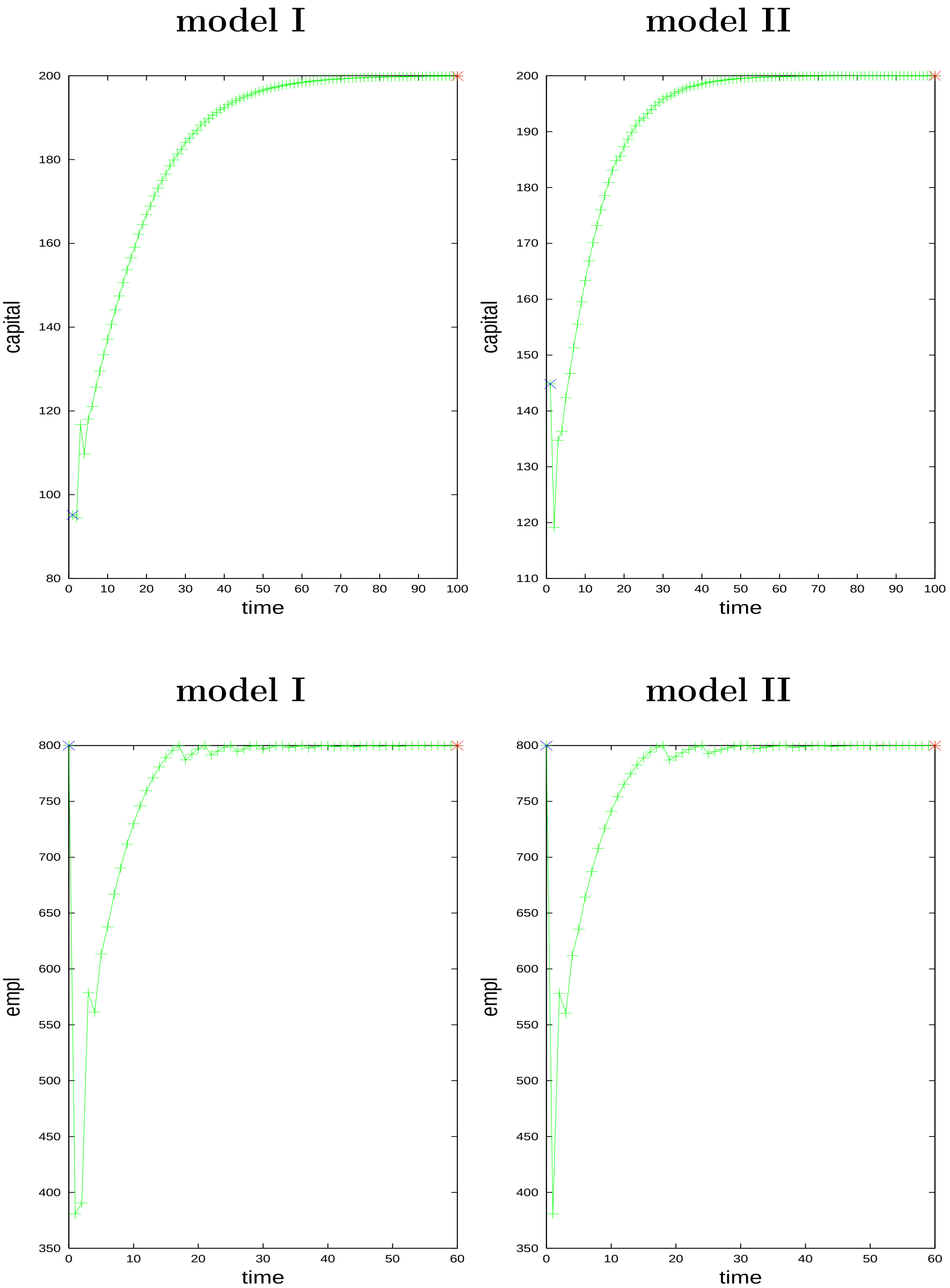


figure 11

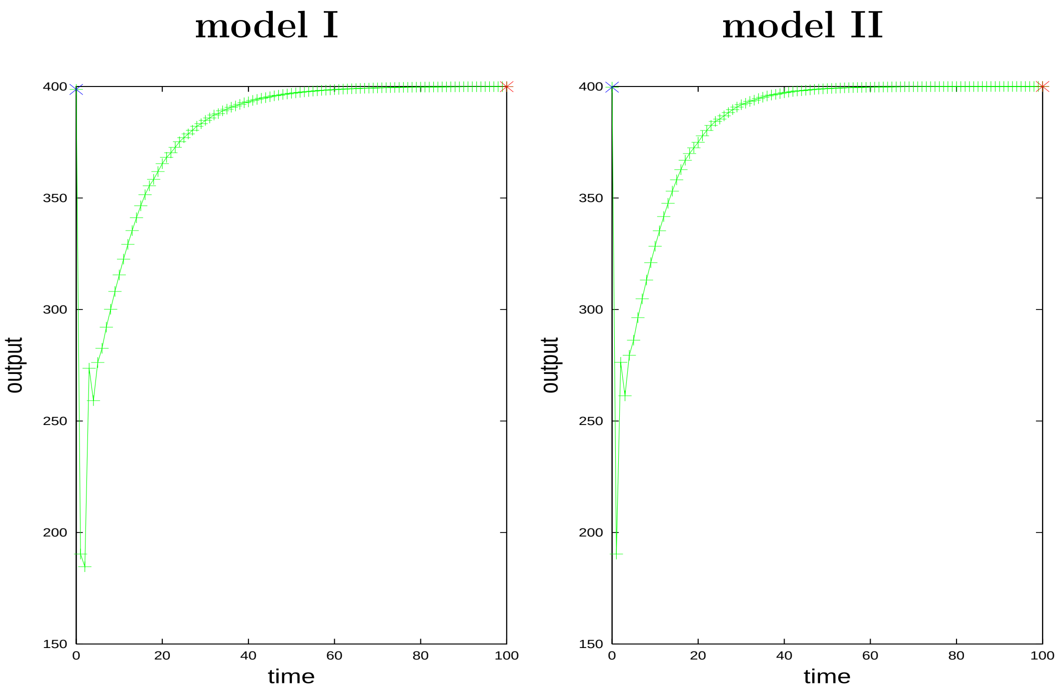


figure 12

Next we carry out a bifurcation analysis. With this tool we can investigate to what extent a variation of a parameter for the same initial values causes a change in the qualitative behaviour of a dynamical system. We confine ourselves to considering the effects of varying the government demand within an interval of values which are economically meaningful. The results for model I and model II are shown in figure 13 and 14, respectively.

The bifurcation diagram for alfa in figure 13 shows convergence toward a fixed point for values of G lower than 20, whereas for higher values it displays complex behaviour. The fixed points portray steady states in system 1, that is stationary states with Keynesian Unemployment. This is confirmed by the bifurcation diagrams for employment and teta. The bifurcation diagram for alfa in figure 14 shows only fixed points. At values of G lower than 20 the points can be identified with the help of the bifurcation diagrams for employment and teta as steady states in system 3. These are stationary states with Keynesian Unemployment and capital market clearing. Values of G higher than 20 lead to steady states in system 8.

Now the intriguing question arises how the qualitative behaviour of the dynamical systems will change if the importance of the capital market is reduced more and more until in the end the transition to the BCW-model is performed. To answer this we use a new and powerful tool, developed by Lohmann and Wenzelburger [1996], to detect cycles and non-cyclic behaviour by variation of two parameters. We consider in addition to a change in G a decrease of b

model I

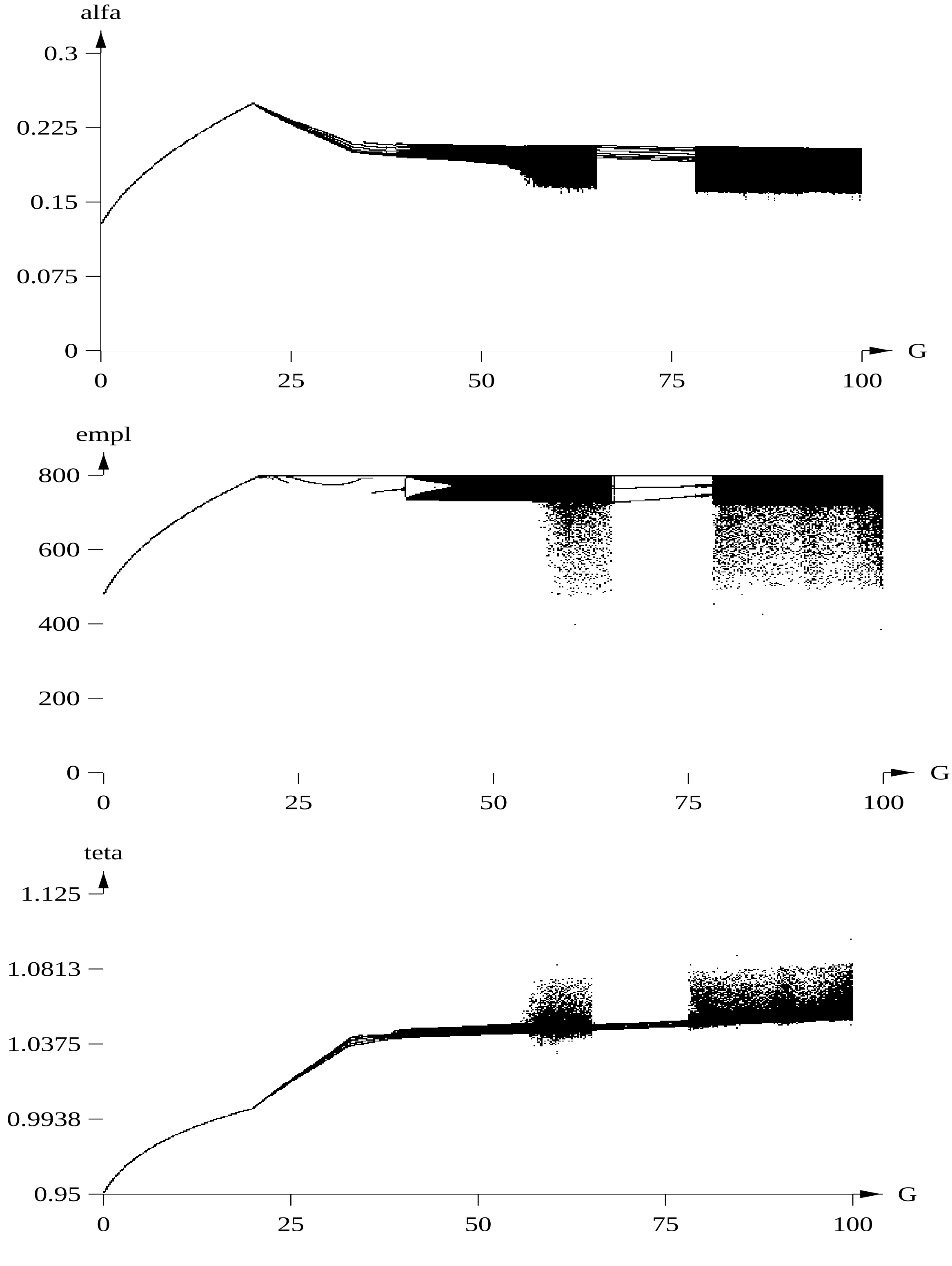


figure 13

model II

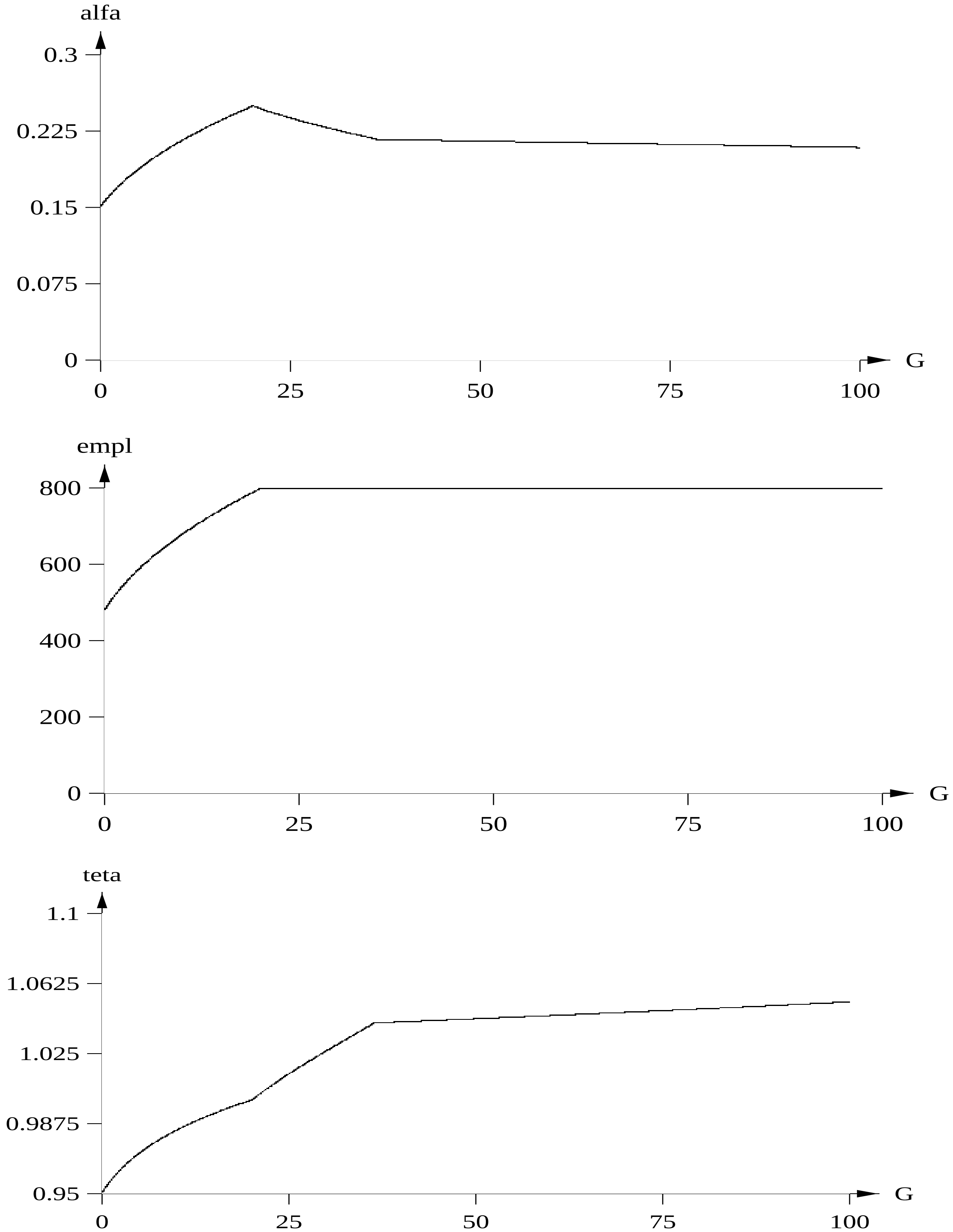


figure 14

from 0.25 to 0.0001.[14] The outcomes for model I and model II are visualized in so-called cartograms (figure 13). Note that the second cartogram for model I holds for a tax rate of 0.5. In these cartograms the colour attached to each point (b,G) reflects a certain type of dynamics. The range of colours informs then about the detecting cycles in the respective cartograms.

The cartograms for model I supply evidence of fixed points, cycles of different order and non-cyclic behaviour. Moreover, for b close to the value 0.0001 which simulates the BCW-model, the cartograms display only fixed points. It is obvious that convergence behaviour increases for a tax rate of 0.5. From the cartogram for model II it can be seen that all colours except magenta have vanished. This means permanent capital market clearing gives only rise to convergence to fixed points. In order to find out if this result is more generally valid we turn to the following numerical example:

a = 0.85	b = 0.01	psi1 = 0.4	psi2 = 0.4	nu1 = 0
nu2 = 0.4	o1 = 0.00001	o2 = 0.00001	m = 100	h = 0.5
pg0 = 15	alfa0 = 0.85	mreal0 = 46.25	rreal0 = 0.5	z0 = 0
Ls = 100	G = 7.5	tax = 0.5	$\varepsilon = 0.5$	f = 1.2

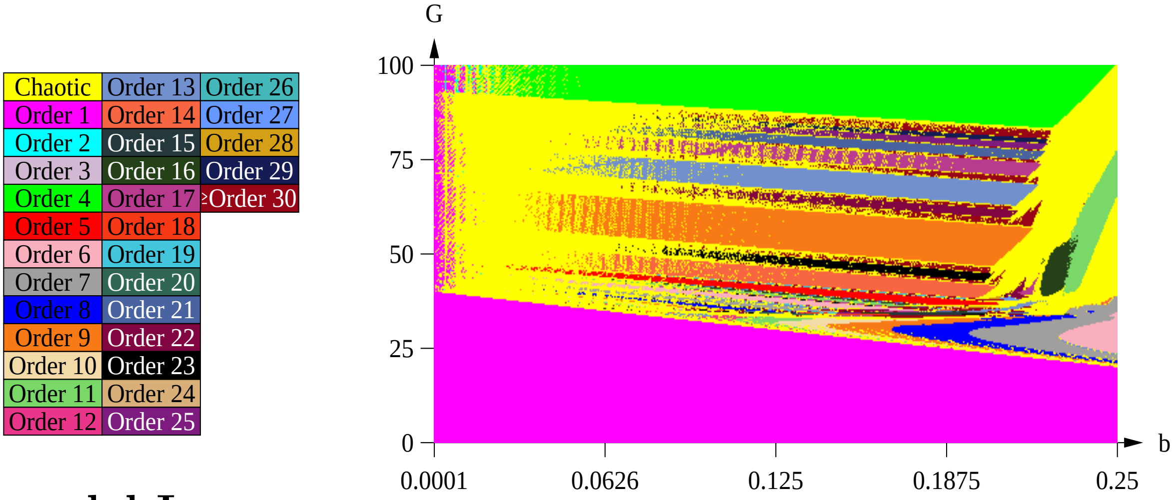
Now we consider in the cartograms shown in figure 16 the effects of changes in both government policy parameters for a fixed value of b. The cartograms reveal that for the BCW-model and model II dynamic phenomena like cycles and non-cyclic behaviour are possible as well. Nevertheless the regular behaviour in model II predominates in comparison with model I.

4 Concluding Remarks

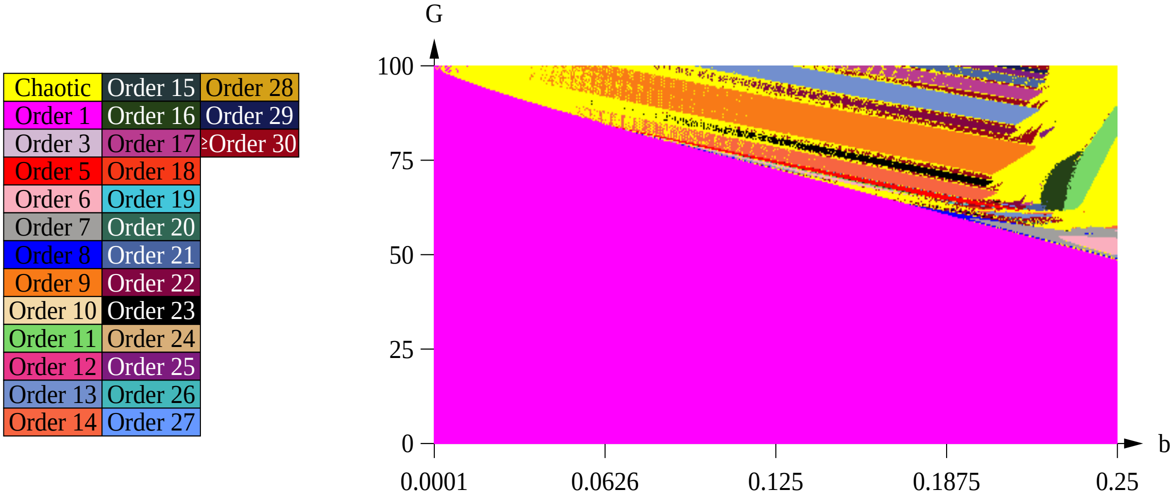
The goal of our paper was to clarify the relevance of the capital market for the economic situation. We have presented a new line of approach to the problem of credit rationing by introducing a dynamic macroeconomic model with three distinctive features. First it is founded on microeconomic principles since the transaction offers of the consumers and the producers are the explicit outcomes of their optimization behaviour. Regarding the second, transactions take place in every period which is not dependent on whether the prices are at their market clearing values. For describing a consistent allocation in each period we employ the temporary equilibrium concept with quantity rationing. Finally agents are confronted with stochastic rationing mechanisms in all three markets. Due to this assumption we obtain an appropriate measure for the strength of rationing which is decisive for the price adjustments. Within a de-

[14]For technical reasons we cannot set b exactly to zero.

model I



model I



model II

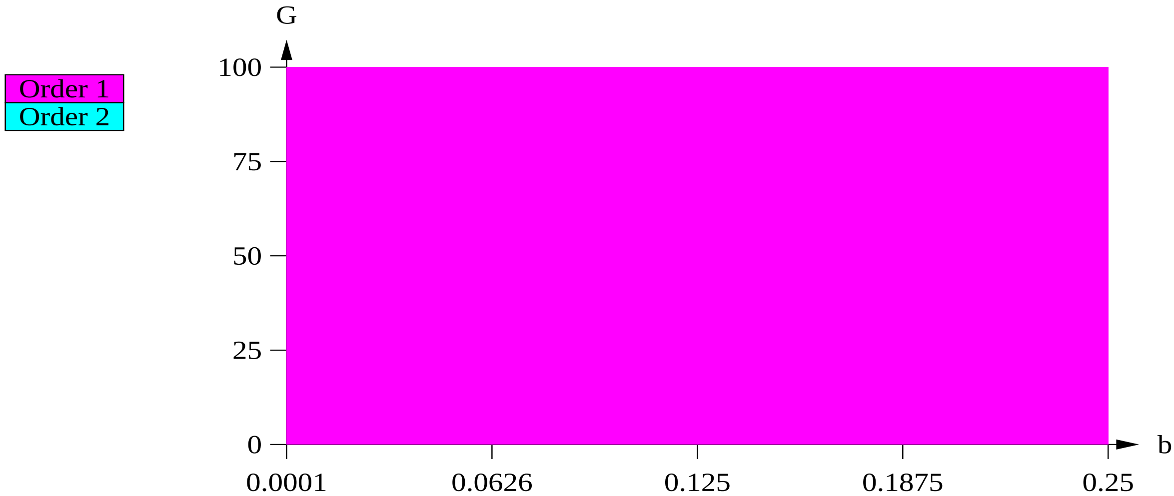
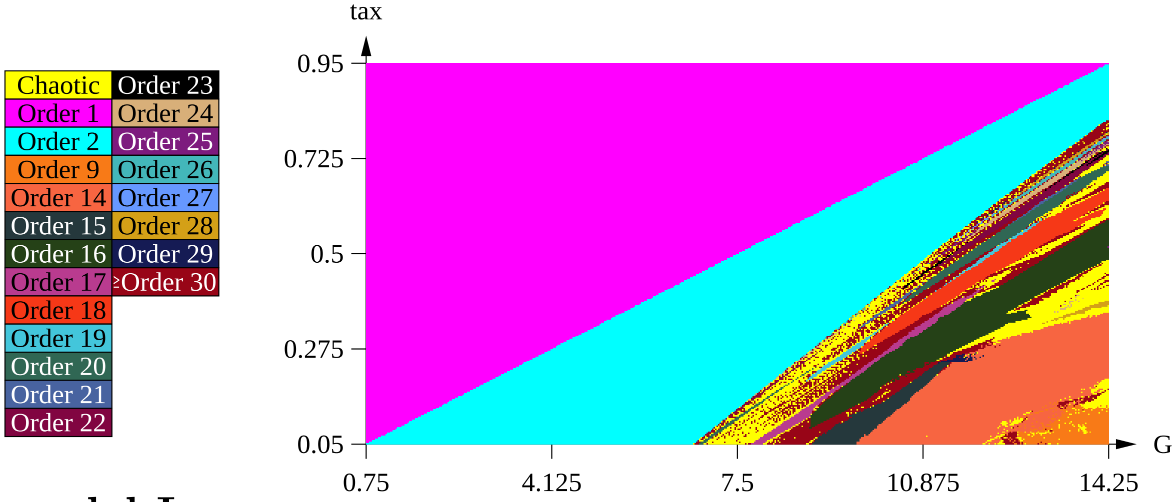
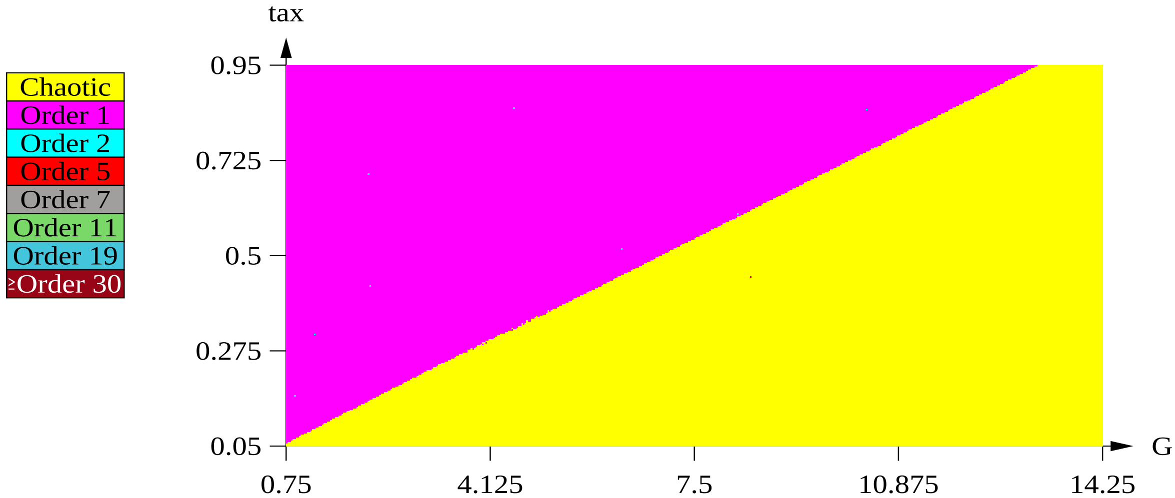


figure 15

BCW-model



model I



model II

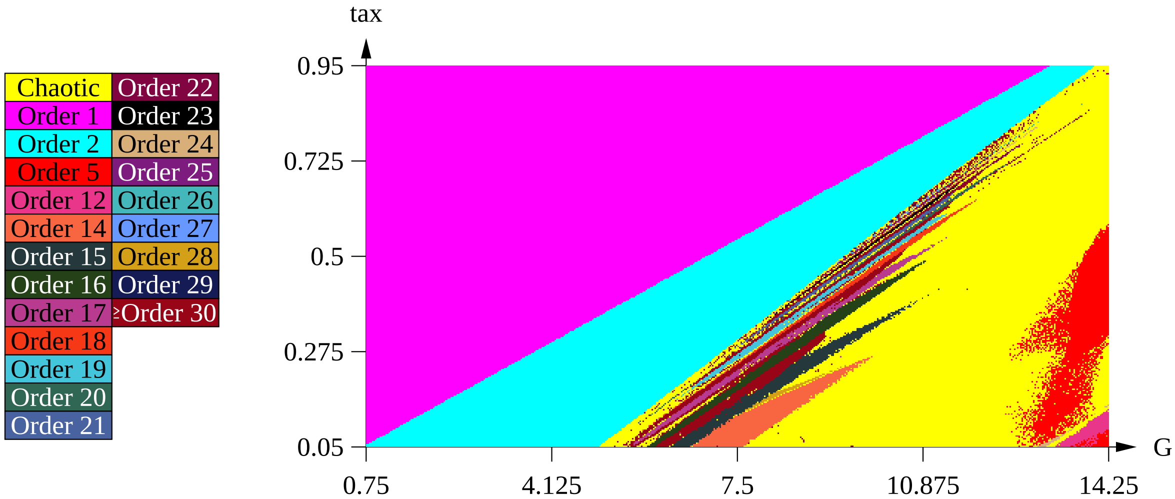


figure 16

terministic framework it would be difficult to do justice to this fact without generating new conceptional problems. The model without a capital market presented by Bignami, Colombo and Weinrich (2000) served as a basis for our model and moreover it is included as a special case in it. In addition it could be proved that the structure of our model allows a modification to the effect that the capital market is always cleared. Thus we could investigate our central problem under the different circumstances of permanent capital market clearing, of credit rationing and of the absence of a capital market.

For our model with credit rationing we presented a decomposition of the price-wage-interest-space (p-w-r-space) into subspaces which correspond to certain equilibrium regimes. We realized that four of eight systems degenerate to an area or even to a curve. Moreover with this graphical tool and the theoretical considerations on dynamics we gained first insights in the dynamic behaviour of the economy. We restricted ourselves to partition the p-w-r-space for the model possessing the highest complexity. Nevertheless it is worth noting that analogous static analysis have been carried out for the other two model variants as well which can be seen in Förster (1998).

Valuable information about the model's dynamic behaviour results from the numerical simulations. It was not possible to perform an analytical investigation since the respective dynamical systems are stepwise defined. The cartograms give evidence of fixed points, cyclic and non-cyclic behaviour for all three model variants. The ones for the model with credit rationing stand out from the others because they indicate a distinctive complex behaviour. Moreover, the bifurcation analysis provides the evidence of steady states with credit rationing. This contradicts the widely-spread opinion that slowly adjusting interest rates generate only temporary credit rationing, and strongly motivates further research in this field.

A promising improvement to be tackled in forthcoming studies is of course the modelling of the capital market. For the moment it is structured in a simple way in order to ensure the tractability of the model. In detail, we intend to introduce a bank and bonds in order to investigate the impact of credit rationing on the efficiency of monetary policy. Apart from that an analytical investigation of steady states and a more complete computational analysis of the dynamic behaviour of the models would be of interest.

Appendix 1

We want to prove that, with the production function $f(l,k) = l^a k^b$, the correspondance between the locus $\{(\lambda^d, \mu^d, \gamma^s) | (\lambda^d, \mu^d, \gamma^s) \in [0,1]^3\}$ and the locus of the aggregate expected transactions of the producers $\overline{F}$ as defined in the main text is not one-to-one. To start with, the following relationships are immediate:

$$\frac{\partial f(l,k)}{\partial l} = \frac{a}{l} f(l,k), \qquad f(l,k) = \frac{\partial f(l,k)}{\partial l}\frac{a}{l}, \qquad f(l, v\,k) = \frac{\partial f(l, v\,k)}{\partial l}\frac{a}{l}. \quad (16)$$

On the commodity market the producers expect transactions at a rate of

$$Y = \gamma^s Y^s(\lambda^d, \mu^d, \gamma^s) = \gamma^s m \lambda^d [\mu^d \varepsilon f(l^d, k^d) + (1 - \mu^d \varepsilon) f(l^d, v\,k^d)].$$

Taking into account equation (16) yields

$$Y = m\lambda^d \frac{l^d}{a} \left(\gamma^s \mu^d \varepsilon \frac{\partial f(l^d, k^d)}{\partial l^d} + (1 - \mu^d \varepsilon)\gamma^s \frac{\partial f(l^d, v\,k^d)}{\partial l^d} \right). \quad (17)$$

Maximization of the expected profit

$$g(l,k) = \mu^d \varepsilon \gamma^s f(l,k) + (1 - \mu^d \varepsilon)\gamma^s f(l, v\,k) - \frac{w}{p}\,l - \frac{r}{p}\,\mu^d k$$

leads to a first order condition, which we write as

$$\mu^d \varepsilon \gamma^s \frac{\partial f(l,k)}{\partial l} + (1 - \mu^d \varepsilon)\gamma^s \frac{\partial f(l, v\,k)}{\partial l} = \frac{w}{p}. \quad (18)$$

Substituting this in equation (17), we get

$$Y = m\lambda^d l^d \,\frac{1}{a}\frac{w}{p} = \frac{w}{p\,a}\,L. \quad (19)$$

Similar calculations for the production factor capital result in

$$Y = \gamma^s Y^s(\lambda^d, \mu^d, \gamma^s) = \frac{r}{p\,b}\,\mu^d K^d(\lambda^d, \mu^d, \gamma^s) = \frac{r}{p\,b}\,K \implies K = \frac{w\,b}{r\,a}\,L. \quad (20)$$

(19) and (20) show that $\overline{F}$ is a ray (in $\Re^3$) and hence it cannot be isomorphic to the cube $[0,1]^3$.

Appendix 2

We consider the price adjustment function

$$p_{t+1} = \xi_\psi(\gamma_t^s, \gamma_t^d, \delta_t, \beta_t)\, p_t$$

and define $\psi = (\psi_1, \psi_2)$, with ψ_1 and ψ_2 as the downward and upward adjustment speed, respectively. Taking into account the equations $\gamma_t^d(1 - \delta_t) = 0$ and $\delta_t(1 - \beta_t) = 0$ (condition 4 of definition 8), we assume

$$\xi_\psi(1,1,1,1) = 1, \quad \forall \psi,$$

$$\begin{aligned}
\xi_\psi(\gamma_t^s,\gamma_t^d,\delta_t,\beta_t) < 1 &\iff \gamma_t^s < 1, &&\forall\psi_1 > 0,\\
\xi_\psi(\gamma_t^s,\gamma_t^d,1,1) > 1 &\iff 0 < \gamma_t^d < 1, &&\forall\psi_2 > 0,\\
\xi_\psi(\gamma_t^s,0,\delta_t,1) > 1 &\iff 0 < \delta_t < 1, &&\forall\psi_2 > 0,\\
\xi_\psi(\gamma_t^s,0,0,\beta_t) > 1 &\iff \beta_t < 1, &&\forall\psi_2 > 0.
\end{aligned}$$

The wage adjustment function is given by

$$w_{t+1} = \zeta_\nu(\lambda_t^s,\lambda_t^d)\,w_t$$

with $\nu = (\nu_1,\nu_2)$ and

$$\begin{aligned}
&\zeta_\nu(1,1) = 1, \quad \forall\nu,\\
&\zeta_\nu(\lambda_t^s,\lambda_t^d) < 1 \iff \lambda_t^s < 1, \quad \forall\nu_1 > 0,\\
&\zeta_\nu(\lambda_t^s,\lambda_t^d) > 1 \iff \lambda_t^d < 1, \quad \forall\nu_2 > 0.
\end{aligned}$$

Analogously for the interest adjustment mechanism holds

$$r_{t+1} = \chi_o(\mu_t^s,\mu_t^d)\,r_t,$$

with $o = (o_1,o_2)$ and

$$\begin{aligned}
&\chi_o(1,1) = 1, \quad \forall o,\\
&\chi_o(\mu_t^s,\mu_t^d) < 1 \iff \mu_t^s < 1, \quad \forall o_1 > 0,\\
&\chi_o(\mu_t^s,\mu_t^d) > 1 \iff \mu_t^d < 1 \quad \forall o_2 > 0.
\end{aligned}$$

The families of functions $\{\xi_\psi\}_{\psi\geq 0}$, $\{\zeta_\nu\}_{\nu\geq 0}$ and $\{\chi_o\}_{o\geq 0}$ are assumed to possess the following properties:

(I) $\xi_\psi,\zeta_\nu,\chi_o$ are differentiable and

$$\frac{\partial\xi_\psi}{\partial\gamma_t^s} \geq 0\,, \quad \frac{\partial\xi_\psi}{\partial\gamma_t^d} \leq 0\,, \quad \frac{\partial\xi_\psi}{\partial\delta_t} \leq 0\,, \quad \frac{\partial\xi_\psi}{\partial\beta_t} \leq 0\,,$$

$$\frac{\partial\zeta_\nu}{\partial\lambda_t^s} \geq 0\,, \quad \frac{\partial\zeta_\nu}{\partial\lambda_t^d} \leq 0\,, \quad \frac{\partial\chi_o}{\partial\mu_t^s} \geq 0\,, \quad \frac{\partial\chi_o}{\partial\mu_t^d} \leq 0\,.$$

(II) $\psi_1' > \psi_1$ implies

$\xi_{\psi_1'}(\gamma_t^s,\gamma_t^d,1,1) < \xi_{\psi_1}(\gamma_t^s,\gamma_t^d,1,1)$, $\forall$ $(\gamma_t^s,\gamma_t^d,1,1)$ such that $\gamma_t^s < 1, \gamma_t^d = 1$

and $\psi_2' > \psi_2$ implies

$\xi_{\psi_2'}(\gamma_t^s,\gamma_t^d,\delta_t,\beta_t) > \xi_{\psi_2}(\gamma_t^s,\gamma_t^d,\delta_t,\beta_t)$, $\forall$ $(\gamma_t^s,\gamma_t^d,\delta_t,\beta_t)$ such that $\gamma_t^s = 1, \gamma_t^d < 1$;

$\nu_1' > \nu_1$ implies

$\zeta_{\nu_1'}(\lambda_t^s,\lambda_t^d) < \zeta_{\nu_1}(\lambda_t^s,\lambda_t^d)$, $\forall$ $(\lambda_t^s,\lambda_t^d)$ such that $\lambda_t^s < 1, \lambda_t^d = 1$

and $\nu_2' > \nu_2$ implies

$\zeta_{\nu_2'}(\lambda_t^s,\lambda_t^d) > \zeta_{\nu_2}(\lambda_t^s,\lambda_t^d)$, $\forall$ $(\lambda_t^s,\lambda_t^d)$ such that $\lambda_t^s = 1, \lambda_t^d < 1$;

$o_1' > o_1$ implies

$\chi_{o_1'}(\mu_t^s, \mu_t^d) < \chi_{o_1}(\mu_t^s, \mu_t^d), \ \forall \ (\mu_t^s, \mu_t^d)$ such that $\mu_t^s < 1, \mu_t^d = 1$

and $o_2' > o_2$ implies

$\chi_{o_2'}(\mu_t^s, \mu_t^d) > \chi_{o_2}(\mu_t^s, \mu_t^d), \ \forall \ (\mu_t^s, \mu_t^d)$ such that $\mu_t^s = 1, \mu_t^d < 1$.

(III) $\xi_0(\cdot) \equiv 1; \quad \zeta_0(\cdot) \equiv 1; \quad \chi_0(\cdot) \equiv 1.$

Note that this general formulation contains the functions specified in the main text as a particular case.

Appendix 3

We want to supply evidence for the existence of a market clearing interest rate. First of all we have to clarify in which systems an equilibrium in the capital market can occur and how the equilibrium level of employment is determined.

In system 3 the constellation $\mu^s = \mu^d = 1$ can be associated with rationing of labour and goods supply $(\lambda^s < 1, \gamma^s < 1)$ or with rationing of labour but an equilibrium in the goods market $(\lambda^s < 1, \gamma^s = \gamma^d = 1)$. If we consider the projections in figures 8 and 9, a state of type 3 is given when there is an intersection of $\overline{H}_p^3$ with $\overline{F}_p^3$ in the L-Y-plane and, at the same employment level, an intersection of $\overline{H}_p^3$ with $\overline{F}_p^3$ in the L-K-plane. Taking into account equation (14) for $\overline{L}_t$ and the definitions of $\widetilde{L}$ and L^* it follows from the projections that in the case $(\lambda^s < 1, \gamma^s < 1)$ the equilibrium level of employment is $\overline{L}_t = L^* = \widetilde{L}$. Moreover, if we assume $(\lambda^s < 1, \gamma^s = \gamma^d)$ in both diagrams, the respective projection $\overline{F}_p^3$ must end in the corresponding $\overline{H}_p^3$, and thus the equilibrium level is $\overline{L}_t = L^* = \widetilde{L} = L^d(1, 1, \alpha_t, r_t')$.

In regime 4 capital market clearing is possible in connection with $(\lambda^s < 1, \gamma^d < 1)$. In this particular case the projection $\overline{F}_p^4$ in the L-K-plane ends in $\overline{H}_p^4$. Then the resulting equilibrium employment is $\overline{L}_t = L^* = L^d(1, 1, \alpha_t, r_t')$.

System 7 includes several cases with an equilibrium in the capital market. Specifically these are $(\lambda^d < 1, \gamma^s = \gamma^d = 1)$, $(\lambda^s = \lambda^d = 1, \gamma^s < 1)$, $(\lambda^d < 1, \gamma^s < 1)$ and $(\lambda^s = \lambda^d = 1, \gamma^s = \gamma^d = 1)$. According to the two-dimensional projections we obtain the equilibrium employment $\overline{L}_t = L^* = \widetilde{L} = L^s$ which equals $L^d(1, 1, \alpha_t, r_t')$ in the Walrasian state.

The temporary equilibria with capital market clearing $(\lambda^d < 1, \gamma^d < 1)$ and $(\lambda^s = \lambda^d = 1, \gamma^d < 1)$ belong to system 8. The equilibrium level of employment amounts to $\overline{L}_t = L^* = L^s$ and $\overline{L}_t = L^* = L^s = L^d(1, 1, \alpha_t, r_t')$, respectively. The latter arises if in the L-Y-plane $\overline{F}_p^8$ ends in $\overline{H}_p^8$ and in the L-K-plane $\overline{F}_p^8$ ends in the corner point $\overline{H}_p^8$.

From these considerations we reach the conclusion that, due to uniqueness of equilibrium, capital market clearing necessitates the equality of $\overline{L}_t$, L^* and at least one of L^s, $L^d(1,1,\alpha_t,r'_t)$ and $\widetilde{L}$. The question therefore is whether there always exists a real interest rate at which L^* coincides with the minimum of $L^s, \widetilde{L}$ and $L^d(1,1,\alpha_t,r'_t)$ for all possible values of all other state variables and parameters.

In order to answer this we have to investigate next the functions $L^d(r')$, L^s and $\widetilde{\mathcal{L}}$ with respect to their dependence on r'.[15] L^s is constant, $\widetilde{\mathcal{L}}$ is according to (12) independent of r' and from (3,7) $L^d(r')$ decreases with an increasing interest rate. We define a function $f(r')$ as

$$f(r') = \text{Min}\{L^s, \widetilde{\mathcal{L}}, L^d(r')\}.$$

This function is (weakly) monotonically decreasing and satisfies $\lim_{r'\to\infty} f(r') = 0$. By (13) the function $\mathcal{L}^*(r')$ starts in the origin and is strictly monotonically increasing. Therefore there must be an intersection of $f(r')$ with $\mathcal{L}^*$ at a r'_0 (figure 17) which proves the existence of a market clearing interest.[16]

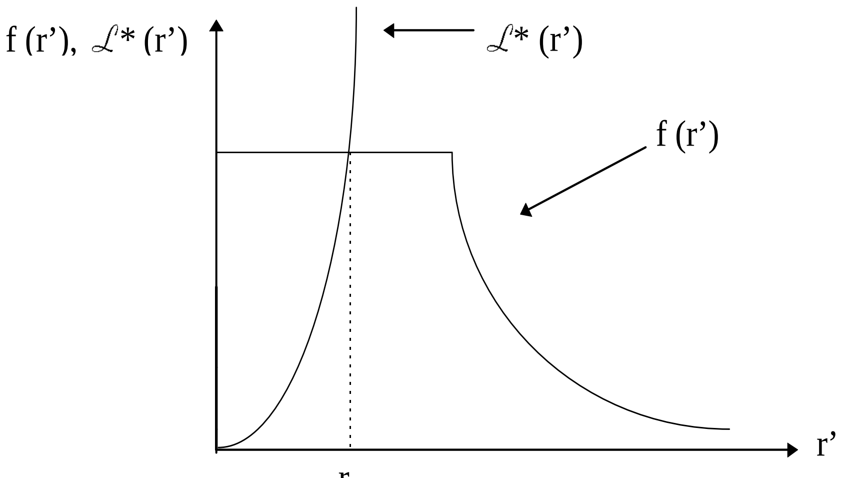


figure 17

Appendix 4

The theoretical considerations on dynamics of model I in section 2.5 revealed that the adjustment equations depend on the prevailing type of equilibrium. For the numerical simulations we have therefore to formulate an algorithm which is also capable to determine this type. Since the equilibrium level of employment is uniquely determined by the minimum of $\widetilde{L}$, L^*, L^s and $L^d(\cdot)$,

[15]All other variables are assumed to be constant and therefore they do not appear in the lists of arguments.

[16]This figure shows only the qualitative behaviour of the curves.

the following procedure provides a comparison of the variables: [17]

if	$\widetilde{L} \leq L^s$	and	$\widetilde{L} \leq L^d(\cdot)$	and	$\widetilde{L} < L^*$,	then	system 1	(5),
if	$\widetilde{L} \leq L^s$	and	$\widetilde{L} \leq L^d(\cdot)$	and	$\widetilde{L} \geq L^*$,	then	system 4	$(3, 7, \mathrm{W})$,
if	$\widetilde{L} \leq L^s$	and	$\widetilde{L} > L^d(\cdot)$	and	$L^d(\cdot) < L^*$,	then	system 2,	
if	$\widetilde{L} \leq L^s$	and	$\widetilde{L} > L^d(\cdot)$	and	$L^d(\cdot) \geq L^*$,	then	system 4,	
if	$\widetilde{L} > L^s$	and	$L^s \leq L^d(\cdot)$	and	$L^s < L^*$,	then	system 6,	
if	$\widetilde{L} > L^s$	and	$L^s \leq L^d(\cdot)$	and	$L^s \geq L^*$,	then	system 4	(8),
if	$\widetilde{L} > L^s$	and	$L^s > L^d(\cdot)$	and	$L^d(\cdot) < L^*$,	then	system 2,	
if	$\widetilde{L} > L^s$	and	$L^s > L^d(\cdot)$	and	$L^d(\cdot) \geq L^*$,	then	system 4.	

The degenerated cases 3, 5, 7 and 8 including the Walrasian state (W) are not explicitly determined but assigned to system 1 and system 4, respectively.[18] This simplification is justified by the fact that these states can be seen as special cases of the latter systems and moreover will almost surely never occur in the numerical simulations.

Now we can formulate the system specific equations for the allocation, the quantity signals and the state variables:[19]

System 1

Employment level: $\overline{L}_t = \widetilde{L}_t$; output level: $\overline{Y}_t = \dfrac{\alpha_t}{a}\,\overline{L}_t$; capital stock:

$\overline{K}_t = \dfrac{\alpha_t}{r'_t}\dfrac{b}{a}\,\overline{L}_t$; rationing signals' values: $\lambda_t^d = 1$, $\mu_t^d = 1$, $\gamma_t^d = 1$, $\lambda_t^s = \dfrac{\overline{L}_t}{L^s}$,

$$\mu_t^s = \frac{\overline{K}_t}{(1-h)(\alpha_t\,\overline{L}_t + (1-tax)\pi_t + z_t)},\quad \gamma_t^s = \frac{(\overline{Y}_t)^{1-a-b}}{(n')^{1-a-b}\left(\dfrac{a}{\alpha_t}\right)^a\left(\dfrac{b}{r'_t}\right)^b};$$

[17]Concerning the calculation of $\widetilde{L}$, L^*, L^s and $L^d(\cdot)$ it must be considered that after (13) L^* is only positive for $b/a - (1-h)r'_t > 0$. If this condition is not fulfilled in the programm, L^* receives the value $L^s + 1$, in order to exclude L^* as minimum.

[18]Regime 5 can be understood as a particular case of system 1 if we take into account that the triple $(\lambda^d = 1, \mu^s < 1, \gamma^s < 1)$ also generates a state of type 5. Considering the cases $(\lambda^s < 1, \mu^d < 1, \gamma^s = 1)$, $(\lambda^d = 1, \mu^d < 1, \gamma^s < 1)$ and $(\lambda^d = 1, \mu^d < 1, \gamma^d < 1)$ characterizing system 3, 7 and 8, respectively, the assignment to system 4 is reasonable.

[19]The respective adjustment equations for price, wage and inflation rate can be taken from section 2.5.

profit level: $\pi_{t+1} = \frac{1}{\theta_t}(\overline{Y}_t - \alpha_t \overline{L}_t - r'_t \overline{K}_t)$; interest: $z_{t+1} = \frac{\overline{K}_t r'_t}{\theta_t}$;

money stock adjustment: $s_{t+1} = \frac{1}{\theta_t}(s_t + G + (1 - tax)\pi_t + z_t) - \pi_{t+1} - z_{t+1}$;

System 2

$$\overline{L}_t = L^d_t;\ \overline{Y}_t = \frac{\alpha_t}{a}\overline{L}_t;\ \overline{K}_t = \frac{\alpha_t}{r'_t}\frac{b}{a}\overline{L}_t;\ \lambda^d_t = 1,\quad \mu^d_t = 1,\qquad \gamma^s_t = 1,$$

$$\lambda^s_t = \frac{\overline{L}_t}{L^s},\ \mu^s_t = \frac{\overline{K}_t}{(1-h)(\alpha_t \overline{L}_t + (1 - tax)\pi_t + z_t)},$$

$$\text{if}\,\overline{Y}_t < G, \text{ then}: \beta_t = \frac{\overline{Y}_t}{G},\quad \delta_t = 0,\quad \gamma^d_t = 0;$$

$$\text{if}\,\overline{Y}_t \geq G, \text{ but}\,\overline{Y}_t < G + s_t, \text{ then}: \beta_t = 1,\quad \delta_t = \frac{\overline{Y}_t - G}{s_t},\quad \gamma^d_t = 0;$$

$$\text{if}\,\overline{Y}_t \geq G + s_t, \text{ then}: \beta_t = 1,\quad \delta_t = 1,\quad \gamma^d_t = \frac{\overline{Y}_t - s_t - G}{h(\alpha_t \overline{L}_t + (1 - tax)\pi_t + z_t)};$$

$$\pi_{t+1} = \frac{1}{\theta_t}(\overline{Y}_t - \alpha_t \overline{L}_t - r'_t \overline{K}_t);\ z_{t+1} = \frac{\overline{K}_t r'_t}{\theta_t};$$

$$s_{t+1} = \frac{1}{\theta_t}(\delta_t s_t + \beta_t G + (1 - tax)\pi_t + z_t) - \pi_{t+1} - z_{t+1};$$

System 4

$$\overline{L}_t = L^*_t;\ \overline{Y}_t = \frac{\alpha_t}{a}\overline{L}_t;\ \overline{K}_t = \frac{\alpha_t}{r'_t}\frac{b}{a}\overline{L}_t;\ \lambda^d_t = 1,\ \mu^s_t = 1,\ \gamma^s_t = 1,$$

$$\lambda^s_t = \frac{\overline{L}_t}{L^s},\ \mu^d_t = \frac{-C^2 + 1 - \varepsilon}{\varepsilon} + 2C^2 - 2\sqrt{\frac{C^2}{\varepsilon} - \frac{C^4}{\varepsilon} - C^2 + C^4}$$

with $C = \dfrac{(\overline{K}_t)^{1-a-b}}{(n')^{1-a-b}\left(\frac{b}{r'_t}\right)^{1-a}\left(\frac{a}{\alpha_t}\right)^{a}}$, [20]

[20]The quantity signal μ^d can only be solved analytically for certain values of the parameter b of the production function. The equation for μ^d which is given above, holds for $b = 0.5$ and on condition $(1-\varepsilon)^b \leq C \leq 1$. In our program μ^d is calculated for any value of b by means of the numerical method Regula Falsi.

$$\text{if}\,\overline{Y}_t < G,\ \text{then}: \beta_t = \frac{\overline{Y}_t}{G};\ \ \delta_t = 0;\ \ \gamma_t^d = 0;$$

$$\text{if}\,\overline{Y}_t \geq G,\ \text{but}\,\overline{Y}_t < G + s_t,\ \text{then}: \beta_t = 1;\ \ \delta_t = \frac{\overline{Y}_t - G}{s_t};\ \ \gamma_t^d = 0;$$

$$\text{if}\,\overline{Y}_t \geq G + s_t,\ \text{then}: \beta_t = 1;\ \ \delta_t = 1;\ \ \gamma_t^d = \frac{\overline{Y}_t - s_t - G}{h(\alpha_t\,\overline{L}_t + (1 - tax)\pi_t + z_t)};$$

$$\pi_{t+1} = \frac{1}{\theta_t}(\overline{Y}_t - \alpha_t\,\overline{L}_t - r_t'\,\overline{K}_t);\ \ z_{t+1} = \frac{\overline{K}_t\,r_t'}{\theta_t};$$

$$s_{t+1} = \frac{1}{\theta_t}(\delta_t s_t + \beta_t G + (1 - tax)\pi_t + z_t) - \pi_{t+1} - z_{t+1};$$

System 6

$$\overline{L}_t = L^s;\ \ \overline{Y}_t = \frac{\alpha_t}{a}\,\overline{L}_t;\ \ \overline{K}_t = \frac{\alpha_t}{r_t'}\frac{b}{a}\,\overline{L}_t;\ \ \lambda_t^s = 1;\ \ \mu_t^d = 1;\ \ \gamma_t^s = 1;$$

$$\lambda_t^d = \frac{L^s}{L_t^d};\ \ \mu_t^s = \frac{\overline{K}_t}{(1-h)(\alpha_t\,\overline{L}_t + (1 - tax)\pi_t + z_t)};$$

$$\text{if}\,\overline{Y}_t < G,\ \text{then}: \beta_t = \frac{\overline{Y}_t}{G};\ \ \delta_t = 0;\ \ \gamma_t^d = 0;$$

$$\text{if}\,\overline{Y}_t \geq G,\ \text{but}\,\overline{Y}_t < G + s_t,\ \text{then}: \beta_t = 1;\ \ \delta_t = \frac{\overline{Y}_t - G}{s_t};\ \ \gamma_t^d = 0;$$

$$\text{if}\,\overline{Y}_t \geq G + s_t,\ \text{then}: \beta_t = 1;\ \ \delta_t = 1;\ \ \gamma_t^d = \frac{\overline{Y}_t - s_t - G}{h(\alpha_t\,\overline{L}_t + (1 - tax)\pi_t + z_t)};$$

$$\pi_{t+1} = \frac{1}{\theta_t}(\overline{Y}_t - \alpha_t\,\overline{L}_t - r_t'\,\overline{K}_t);\ \ z_{t+1} = \frac{\overline{K}_t\,r_t'}{\theta_t};$$

$$s_{t+1} = \frac{1}{\theta_t}(\delta_t s_t + \beta_t G + (1 - tax)\pi_t + z_t) - \pi_{t+1} - z_{t+1}$$

For the implementation of model II we have formulated an algorithm to determine the temporary equilibrium states which consists of a comparison of the interest rates $\widetilde{r}$, r^* and $r^\diamond$.[21] In model II regime 7 including the Walrasian state represents a degenerated case which we assign to system 8.

[21] Since the value of $r^\diamond$ cannot analytically be determined we employ again the numerical method Regula Falsi.

if $\tilde{r} \le r^*$	and $\tilde{r} \le r^\diamond$,	then	system 8	(system 7, W),
if $\tilde{r} \le r^*$	and $\tilde{r} > r^\diamond$,	then	system G4,	
if $\tilde{r} > r^*$	and $r^* \le r^\diamond$,	then	system 3,	
if $\tilde{r} > r^*$	and $r^* > r^\diamond$,	then	system G4.	

In the following we give the equations for the analysis of systems 3, 8 and G4 which occur in model II.

System 3

real interest rate: $r'_t = r^\diamond$; employment level:

$$\overline{L}_t = \frac{(1-h)[(1-tax)\Pi_t + K_{t-1}\, r_{t-1}]\, r_t}{\dfrac{w_t\, p_t\, b}{a} - (1-h)\, w_t\, r_t}; \text{ output level: } \overline{Y}_t = \frac{\alpha_t}{a}\, \overline{L}_t;$$

capital stock: $\overline{K}_t = \dfrac{\alpha_t}{r'_t}\dfrac{b}{a}\, \overline{L}_t$; rational signals' values: $\lambda^d_t = 1$; $\mu^d_t = \mu^s_t = 1$;

$$\gamma^d_t = 1;\ \lambda^s_t = \frac{\overline{L}_t}{L^s};\ \gamma^s_t = \frac{(\overline{Y}_t)^{1-a-b}}{(n')^{1-a-b}\left(\dfrac{a}{\alpha_t}\right)^a \left(\dfrac{b}{r'_t}\right)^b};\ \text{inflation rate: } \theta_t = (\gamma^s_t)^{\psi_1};$$

profit level: $\pi_{t+1} = \dfrac{1}{\theta_t}(\overline{Y}_t - \alpha_t\, \overline{L}_t - r'_t\, \overline{K}_t)$; interest: $z_{t+1} = \dfrac{\overline{K}_t\, r'_t}{\theta_t}$;

money stock adjustment: $s_{t+1} = \dfrac{1}{\theta_t}(s_t + G + (1-tax)\pi_t + z_t) - \pi_{t+1} - z_{t+1}$;

real wage adjustment: $\alpha_{t+1} = \dfrac{(\lambda^s_t)^{\nu_1}}{(\gamma^s_t)^{\psi_1}}\, \alpha_t$;

System 8

$$r'_t = \tilde{r};\ \overline{L}_t = \frac{(1-h)[(1-tax)\Pi_t + K_{t-1}\, r_{t-1}]\, r_t}{\dfrac{w_t\, p_t\, b}{a} - (1-h)\, w_t\, r_t};$$

$$\overline{Y}_t = \frac{\alpha_t}{a}\, \overline{L}_t;\ \overline{K}_t = \frac{\alpha_t}{r'_t}\frac{b}{a}\, \overline{L}_t;\ \lambda^s_t = 1;\ \mu^d_t = \mu^s_t = 1;\ \gamma^s_t = 1;\ \lambda^d_t = \frac{\overline{L}_t}{L^d_t}$$

if $\overline{Y}_t < G$, then : $\beta_t = \dfrac{\overline{Y}_t}{G}$; $\delta_t = 0$; $\gamma^d_t = 0$;

$$\text{if}\,\overline{Y}_t \geq G, \text{ but}\overline{Y}_t < G + s_t, \text{ then}: \beta_t = 1;\ \ \delta_t = \frac{\overline{Y}_t - G}{s_t};\ \ \gamma_t^d = 0;$$

$$\text{if}\,\overline{Y}_t \geq G + s_t, \text{ then}: \beta_t = 1;\ \ \delta_t = 1;\ \ \gamma_t^d = \frac{\overline{Y}_t - s_t - G}{h(\alpha_t\,\overline{L}_t + (1-tax)\pi_t + z_t)};$$

$$\theta_t = \left(\frac{\gamma_t^d + \delta_t + \beta_t}{3}\right)^{-\psi_2};\ \ \pi_{t+1} = \frac{1}{\theta_t}(\overline{Y}_t - \alpha_t\,\overline{L}_t - r'_t\,\overline{K}_t);\ \ z_{t+1} = \frac{\overline{K}_t\,r'_t}{\theta_t};$$

$$s_{t+1} = \frac{1}{\theta_t}(\delta_t s_t + \beta_t G + (1-tax)\pi_t + z_t) - \pi_{t+1} - z_{t+1};$$

$$\alpha_{t+1} = \frac{\left(\frac{\gamma_t^d + \delta_t + \beta_t}{3}\right)^{\psi_2}}{(\lambda_t^d)^{\nu_2}}\,\alpha_t;$$

System G4

$$r'_t = r^*;\ \ \overline{L}_t = \frac{(1-h)[(1-tax)\Pi_t + K_{t-1}\,r_{t-1}]\,r_t}{\frac{w_t\,p_t\,b}{a} - (1-h)\,w_t\,r_t};$$

$$\overline{Y}_t = \frac{\alpha_t}{a}\,\overline{L}_t;\ \ \overline{K}_t = \frac{\alpha_t}{r'_t}\frac{b}{a}\,\overline{L}_t;\ \ \lambda_t^d = 1;\ \ \mu_t^d = \mu_t^s = 1;\ \ \gamma_t^s = 1;\ \ \lambda_t^s = \frac{\overline{L}_t}{L^s};$$

$$\text{if}\,\overline{Y}_t < G, \text{ then}: \beta_t = \frac{\overline{Y}_t}{G};\ \ \delta_t = 0;\ \ \gamma_t^d = 0;$$

$$\text{if}\,\overline{Y}_t \geq G, \text{ but}\overline{Y}_t < G + s_t, \text{ then}: \beta_t = 1;\ \ \delta_t = \frac{\overline{Y}_t - G}{s_t};\ \ \gamma_t^d = 0;$$

$$\text{if}\,\overline{Y}_t \geq G + s_t, \text{ then}: \beta_t = 1;\ \ \delta_t = 1;\ \ \gamma_t^d = \frac{\overline{Y}_t - s_t - G}{h(\alpha_t\,\overline{L}_t + (1-tax)\pi_t + z_t)};$$

$$\theta_t = \left(\frac{\gamma_t^d + \delta_t + \beta_t}{3}\right)^{-\psi_2};\ \ \pi_{t+1} = \frac{1}{\theta_t}(\overline{Y}_t - \alpha_t\,\overline{L}_t - r'_t\,\overline{K}_t);\ \ z_{t+1} = \frac{\overline{K}_t\,r'_t}{\theta_t};$$

$$s_{t+1} = \frac{1}{\theta_t}(\delta_t s_t + \beta_t G + (1-tax)\pi_t + z_t) - \pi_{t+1} - z_{t+1};$$

$$\alpha_{t+1} = (\lambda_t^s)^{\nu_1}\left(\frac{\gamma_t^d + \delta_t + \beta_t}{3}\right)^{\psi_2}\alpha_t$$

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