Shrink to Fit?

by

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– August 1, 2006 –

Abstract: Emigration out of, and suburbanization within, Germany's East have left 1.2 million flats vacant. Housing policy's response is to encourage demolition of some 350,000 flats, to "stabilize housing markets". From the perspective of the paper's model, government pulls public housing down to push remaining housing's value up. This reduces welfare. Government should lower public housing rents instead, and only then consider demolition.

JEL - Classifications: R31, R21, R13
Keywords: Housing Markets, Vacant Housing, Demolition, Filtering

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1. Introduction

Emigration from, and suburbanization within, Germany's East have left 1.2 million flats vacant. This is one flat in six. Policy's response is to demolish 350,000 flats between 2001 and 2009. This policy may deserve closer analysis. Demolition not just takes place on a scale unseen yet; it is also widely regarded an important and useful precedent for how to deal with vacant housing imminent in societies with declining population. This paper asks two natural questions: Could adjusting housing's rents be preferable to adjusting housing's stock? And, given that we will find that rent adjustment *is* preferable: why has quantity adjustment been policy makers' choice nonetheless?

Policy makers offer two different types of motivation in support of the demolition underway. First, demolition is claimed to remove housing that is unlikely to be reoccupied anyway. Supply is merely shrunk to fit the level of demand foreseeable for the future. After all, those who have left for West Germany or East Germany's suburbs are unlikely to return. And permanently vacant housing is seen to generate negative externalities, and to be expensive to run. One might even add that demolition also has a strong role in the theory of urban redevelopment. Demolition may permit to develop alternative, more efficient urban land uses (e.g., Brueckner (2000)).

Only, it is difficult to see any of these rationales apply to East Germany's vacant housing. Most importantly, demand is not simply fixed. Substantial rent reductions should be able to generate significant increases in demand – even as the price elasticity of housing demand is low, and even if remigration into East Germany's cities were small. Demand increases would come not just from those living in, but also from those filtering towards, housing half-vacant today. Moreover, the fact that East Germany's average rents almost match those in Germany's West signals substantial leeway for lowering rents. Substantially lower rents should still be consistent with maintaining the existing housing stock.

Besides, nowhere has vacant housing been abandoned. Nor is there any evidence that vacant or half-vacant buildings have turned into hot spots of crime. Urban districts with large vacant housing are no crime-ridden ghettos. This, apart from its tremendous volume, makes East Germany's demolition very different from previous demolition policies seen elsewhere. We add that there is little post-demolition development. In fact, local governments often rezone the plots released, to explicitly rule out future residential redevelopment. Local governments also increasingly cut back the urban infrastructure that previously serviced the neighborhoods demolished now.

This leaves us with the second motivation for demolition given. This is demolition's expected contribution to the objective of ”stabilizing [East Germany's] housing markets” (BMVBS 2006, 11). This objective enjoys strong, unanimous, and astonishingly open support from all tiers of government. It also enjoys widespread acceptance among urban planners and economic geographers (e.g., Kabisch (2000), Jurczek/Köppen (2004)). The following quote illustrates what is meant by ”stabilization”. Commenting on recent signs that rents may have picked up, Karl-Heinz Daehre, secretary of housing in the East German state of Saxony-Anhalt, affirms: ”[Current housing policy's] clear focus on demolition has brought considerable relief to housing markets ...” (FAZ (2006)).

This paper argues that governments' ”stabilization objective” is misplaced, and should be replaced by the objective of maximizing urban welfare. By serving the ”stabilization objective” demolition is unlikely to maximize urban welfare. The paper's normative part shows that demolition reduces, while lowering rents would increase, welfare (Proposition 2). – At the same time this paper argues that taking government's declared ”stabilization objective” literally explains why demolition occurs nevertheless. In stylized fashion the paper's positive part recasts government's ”stabilization objective” as one of maximizing aggregate rents, and suggests that the tremendous demolition underway may simply be a means to maximize landlords' incomes (Proposition 1).

Both propositions' derivation hinges on the important institutional fact that more than one fourth of East Germany's rental housing is owned by local governments. Local governments' market shares are even larger in the segment of ”panel housing”, i.e., housing built between 1970 and 1990 by East Germany's then Socialist government. Assembled from prefabricated concrete slabs, panel housing blocks (or, interchangeably, Socialist housing blocks) are highly uniform in design, are between 5 to 11 storeys high, and comprise several dozen flats each. Socialist housing, today of below-average quality, is the segment where demolition proceeds fastest (e.g., Aehnelt (2004, 7), SAB (2006)). An estimated 100,000 Socialist flats had been demolished by the end of 2005 already.

Local governments may be able to set the rent for, and control the stock of, Socialist housing. But they exercise much less direct control over non-Socialist housing. This has led some to argue that local governments' power to interfere with market forces must be very limited (e.g., Maennig (2006)). Yet this view ignores the extent to which housing markets interact. Should local governments demolish, or raise rent in, Socialist housing, residents would partly filter up into greater housing qualities. This would drive up rents in greater qualities (Sweeney (1974)). Alternatively, should Socialist flats' rent fall, rents in higher qualities would tumble, too. From this perspective, local governments exercise considerable indirect control over non-Socialist housing. Socialist housing is the "corner stone" to East Germany's rent curve.

We separate local governments' decisions on Socialist housing's rent and Socialist housing's stock, by distinguishing between the short and the long run. In the short run a local government can only set Socialist housing's rent. In its pursuit of maximum net rents local government will not shift up the rent curve too far. Higher Socialist housing rent does raise rents in better quality segments, but it also reduces Socialist housing's tenant numbers. The optimum Socialist housing rent equates the marginal benefit associated with higher rents elsewhere with the marginal cost associated with falling tenant numbers in Socialist housing. But then Socialist housing's rent exceeds its marginal cost. Socialist housing's price is distorted.

In the long run local government disposes of an additional instrument, in being able to demolish part of the Socialist housing stock it owns. Now, vacant flats do not conveniently cluster. Instead, vacant flats are more or less uniformly distributed across the housing stock. Demolition requires uprooting those inhabiting the panel housing blocks that are about to be pulled down. Suppose that compensation permits those uprooted to move into next-higher, more expensive quality. Then movers trigger a cascade of successive filtering flows which drive up rent not just in next-higher quality, but rents throughout higher qualities (again, Sweeney (1974)). From this perspective, the population flows demolition induces seem as important as the loss in housing it imposes.

Long run demolition falls into two different classes. Demolition that is restricted to a single city not just drives higher segments' rents up, but also drives the mobile among higher segments' residents out. In contrast, simultaneous demolition drives higher segments' rents up without driving away the mobile among higher segments' residents. "Coordinated demolition" puts stronger pressure on higher segments than "isolated demolition". Coordinated demolition ultimately raises landlords' incomes by more than would isolated demolition. From this perspective, it is demolition's ubiquity (see map in BMVBS (2006, 24)) that achieves what both single-city-rent hikes and single-city-demolition cannot achieve: i.e., increase landlords' incomes further.

The paper's normative verdict on demolition is closely linked to the distortion on the market for Socialist housing mentioned earlier. In expelling Socialist housing tenants, demolition reduces the very number of those who value that segment by more than what it costs housing corporations to provide it. Coordinated demolition reduces (East) Germany's welfare. In contrast, reducing

Socialist flats' rent would attract residents from housing segments of greater quality whose individual benefit of living in Socialist housing would exceed the cost necessary to provide it. This is why a coordinated reduction of Socialist flats' rents would raise (East) Germany's welfare. Of course, ultimately substantial demolition may still be necessary, in that part of Socialist housing could be inefficiently located and may not be viable under market conditions (Bertaud/Renaud (1997)). But lower rents should come first.

Assuming that governments maximize landlords' incomes is a stylized representation of the idea that governments may be reluctant to let rents fall. The paper assumes, rather than explains, this objective. But here are three different types of motivation in its support (besides the simple fact mentioned that governments openly embrace this objective). First, it seems difficult to reconcile the excess of Socialist housing rent over marginal cost (documented below) with more benevolent models of local government behavior. Sometimes it is argued that housing corporations must meet a revenue constraint (GdW (2004)), and thus may be seen to resort to Ramsey prices. Yet legally local governments could certainly relax, or even abandon, this constraint if they really wanted to.

Second, East Germany's landlords have a strong incentive to overwhelm local governments, given the increases in all higher segments' rents they can expect from Socialist housing's demolition.[1] E.g., consider the apparent similarity between East Germany's demolition and the growth controls widely applied in Californian cities. From a landlord's perspective, demolition may in many ways do to a shrinking economy what growth controls do to a growing one. Either instrument keeps/pushes rents up, and hence benefits landlords. Recent explanations of growth controls focus on the possibility that local policy makers maximize an objective that includes net aggregate land rents (e.g., Brueckner (1998)) or even equals them (e.g., Helsely/Strange (1995)). This paper takes a similar approach to demolition.

And third, note that the *Kreditanstalt für Wiederaufbau* (KfW), a large federal government bank, has engaged in impressive lending to East Germany's housing suppliers. KfW's rehabilitation loans have benefitted every second East German dwelling, with nearly 40 billion Euro handed out between 1990 and 1999 alone (e.g., Reich (2000, 5)).[2] Federal government may prefer paying for demolition to the mass default on its loans which it must expect if rents fell. – To carry the idea of federal government involvement one step further, one might suspect

[1] In the case where demolition is not simultaneous, opposition is less forceful given that the more mobile among renters are likely to simply exit.

[2] One might even argue that it is Germany's federal government that is East Germany's single most important landlord.

that even West Germany's landlords should attempt to influence Germany's federal government. Given household mobility between West and East Germany (suppressed in this paper) and accounting for East Germany's substantial weight in overall housing, any reduction in Socialist housing's rents may reduce rents not just in East Germany's higher segments, but in West Germany's, too.

With its interest in an alternative to demolition, the paper attempts to inform housing policy. But with its analysis of filtering and migration it also intends to contribute to the literatures on demolition and on coordination in federal states. First, the paper extends Sweeney (1974)'s analysis of demolition in three ways. Demolition occurs in a context where households choose not just housing quality but residential location, too; demolition is not restricted to a single city but may take place in every city; and, demolition is analyzed also with respect to urban welfare. And second, the paper views demolition as yet another application of the literature on coordination of local decisions in federal states in the presence of household mobility (e.g., Keen/Marchand (1997)).

Part of the filtering literature is concerned with whether an addition of high quality housing eventually makes households in substandard housing filter up (e.g., Green/Malpezzi (2003, 159)). This concern has its mirror image in this paper's interest in whether, and to what effect, offering East Germany's abundant stock of low quality housing more cheaply would make households filter down. However, unlike the more detailed work that explicitly treats housing deterioration over time (e.g., Arnott/Davidson/Pines (1983) and Arnott/Braid (1997)), this paper's model is static. Finally, note how East Germany's transition contrasts with US cities' response to adverse shocks, where adjustment almost exclusively is via declining real estate prices, with virtually no demolition (see Glaeser/Gyourko (2005)).

Section 2 briefly outlines institutional aspects of East Germany's housing. Section 3 introduces households' housing quality choice; while Section 4 adds households' location choice. Section 5 assesses the comparative statics of changes in local governments' two instruments: Socialist flats' rent and stock. Section 6 outlines short-run equilibrium, departures from which are analyzed in section 7 on long run policies. Section 8 assesses policies from a welfare perspective. Section 9 gives a numerical example, and section 10 concludes.

2. A Brief Outline of East Germany's Housing

Table 1 provides rudimentary information on East Germany's housing. Roughly 3.3 million units of housing were built during the German Democratic Republic (GDR)'s reign (column 2), which lasted from 1949 to 1990. Among these, units built between 1970 and 1989 are this paper's Socialist housing. As mentioned, they are easily identified by their highly uniform slab type architecture. While the source underlying Table 1 does not disaggregate by building type, Hübl/Möller (1996, 14) put panel housing's total at an estimated 2.2 million units, i.e., at almost one third of East Germany's stock. Table 1 also illustrates the construction boom that followed East Germany's reunification with the West. Also owing to generous subsidization by federal government, more than a million new units of housing were built since 1991. Increased supply of greater housing qualities, joint with net emigration to Germany's West (e.g., Burda/Hunt (2001)) and strong suburbanization, are regarded by many as the "shock" that is responsible for the extent of vacant housing observed today.[3]

Table 1: East Germany's Housing Stock in 2002

Construction Year	Units (in Thousand)	Percent Vacant (in %)
– 1900	1,021	16.5
1901 – 1918	799	21.3
1919 – 1948	1,438	15.5
1949 – 1978	2,063	12.2
1979 – 1986	950	15.8
1987 – 1990	320	15.2
1991 – 2000	990	8.3
2001 – 2002	57	13.4
Total	7,638	14.4

Source: Federal Bureau of Statistics. For explanations see text.

Certainly vacant housing afflicts not just Socialist housing (column 3). Yet vacant flats seem particularly hard to justify in Socialist housing. Huge numbers of vacant Socialist flats contrast visibly with the observation that these flats were never offered particularly cheaply. In 2004, average monthly rent per square meter non-modernized Socialist housing was 5.4 Euro for tenants participating in the Socioeconomic Panel (SOEP).[4] Allowing for those roughly 2 Euro typically charged for property tax, water and garbage fees, gardening, etc. gives 3.4 Euro app. as our estimate of Socialist flats' net rent. (Corresponding figures in West Germany are 6.3 and 4.3 Euro, respectively, and hence hardly higher.) We

[3] An important question, not addressed in this paper, is whether continued subsidization and emigration can really be classified as a sudden, exogenous shock.

[4] Note that other, i.e. non-SOEP, sources arrive at very similar figures, e.g., GdW (2004), BBR (2004) and Klupp et al. (2004).

add that East Germany's Socialist housing rents display little variation across space.[5]

Next, high vacancy rates for Socialist housing seem particularly telling in view of its ownership. While many of East Germany's housing markets may be considered competitive, Socialist housing almost certainly may not. Jointly, housing corporations and housing cooperatives not just own close to one half of East Germany's rental housing (e.g., GdW (2004, 199)).[6] Housing corporations and housing cooperatives even own almost all of Socialist housing, given that earlier attempts at privatizing Socialist flats by and large failed. Housing corporations are almost always owned by local government. This motivates the paper's assumption that local government controls Socialist flats' rent in the short run[7], and that it even controls Socialist flats' stock and population in the long run.

Finally, this paper's particular interest in Socialist housing owes to the information conveyed by its homogeneity. Vacant Socialist flats are highly similar in quality to Socialist flats that are still occupied. It is argued by some that rents smaller than 3.4 Euro would not suffice to cover the operating cost of a square meter in the average vacant Socialist flat (e.g., Maennig (2006)). Yet, according to KfW, with its in-depth knowledge of housing corporations' balance sheets, operating profit on occupied non-modernized Socialist housing is positive (Steinert (2000, 11)).[8] Why should operating cost be higher for flats currently vacant than for highly similar, and profitable, flats occupied still? We infer that the operating cost of currently vacant Socialist housing must fall strictly short of current rent.

In fact, the notion that vacant flats are costly to run appears even more doubtful when accounting for the necessary distinction between average and marginal operating cost. The operating cost incurred when renting an *average* flat currently vacant cannot be the relevant concept. Any "small" reduction of Socialist flats' rent (as this paper's policy recommendation) would be concerned with the best, "marginal", third or fourth of vacant housing only. Those best vacant units should clearly exhibit an operating cost per square meter of less than 3.4 Euro.

[5]Again, see GdW (2004), BBR (2004) and Klupp et al. (2004). Consulting East German cities' rent statistics further confirms this impression.

[6]Unfortunately, GdW data are not disaggregated further. Anecdotal evidence suggests that housing corporations alone own substantially more than one fourth of East Germany's rental housing. For instance, the largest local housing corporation accounts for 37% of rental housing in Rostock and Cottbus, for 34% in Jena, and for 30% in Dessau.

[7]By Germany's laws governing the rental market landlords are not permitted to charge rents "substantially" above the local average. However, this paper's view is that if a particular landlord controls a large share of the local stock this restriction does not bind.

[8]Case studies also suggest that occupied non-modernized Socialist housing generates positive operating profit. E.g., Hübl/Möller (1996, 62), Simons (2001, section 4.2) and Bernt (2002, 14).

Current policy is entitled "Redeveloping East German Cities" ("*Stadtumbau Ost*"). Every second of East Germany's residents lives in a city with at least some demolition (BMVBS (2006, 24)). Total funding runs at 2.5 billion Euro. Most of it is assigned to pulling down housing and cutting back the infrastructure that previously serviced it, but some funding also is devoted to improving local amenities. Another 1.1 billion Euro of federal debt relief are available for housing corporations and housing cooperatives with vacancy rates in excess of 15%. For these actors, every square meter torn down roughly yields some 135 Euro/m^2 of public support (BMVBS (2006, 15)), amounting to substantially more than demolition's estimated cost of 40-50 Euro/m^2.[9]

3. Housing Quality Choice

Housing quality choice is at the heart of the model, given East German housing's heterogeneity in vintage, quality, architectural design and degree of public ownership. Let every East German city supply a range of $i = 1, ..., I$ different qualities of housing. Cities are small, open, and indexed by $j = 1, \ldots, J$, where J is a large number. A pair (i, j) indicates a housing segment's quality and location. The number of those living in city j's segment i is n_i^j, while the total number of city j's residents is given by $n^j = \sum_{i=1}^{I} n_i^j$. The total number of the urban system's residents is $N = \sum_{j=1}^{J} n^j$, and is fixed. Rent for a square meter of housing of quality i in city j is q_i^j.

Qualities are numbered in ascending order: 1 refers to the lowest, while I refers to the highest, quality.[10] Throughout the paper, 1 is non-modernized Socialist housing; 2 includes modernized Socialist housing as well as comparable non-Socialist multi-storey housing; and qualities 3 through I accommodate the wide variety of non-Socialist qualities. These are, e.g., refurbished pre 1900 villas, modern or modernized semi-detached single family homes, etc. Quality is measurable on a cardinal scale, with s_i denoting quality of the i-th segment. A given segment provides identical quality, irrespective of its location. I.e., housing quality pertains to the properties of the structure itself, rather than to features inherent to its environment (such as the landscape around, or man-made amenities near to, it).

A total of N/J households initially is born into each city. These households, or natives, are heterogenous with respect to a housing quality taste index θ

[9] We add that new legal instruments have been introduced, too. These facilitate remaining tenants' eviction from housing earmarked for demolition, as well as rezoning previously residential land.

[10] Arguably, housing built before 1918 yet not modernized recently may be of even lower quality than Socialist housing. Modifying the model to account for this would change none of its results but would make notation slightly more cumbersome.

and the moving cost m they incur when settling elsewhere in East Germany.[11] Specifically, natives to any city are distributed according to the joint c.d.f. $F(\theta, m) : [\theta', \theta''] \times [0, m'] \mapsto [0, 1]$, with $0 < \theta' < \theta''$ and $0 < m'$. So while households may change segment as well as location at will, tastes and/or mobility costs may convince them otherwise. F is assumed location-independent, and to exhibit strictly positive partial derivatives. Marginal distributions are denoted $F(\theta)$ and $F(m)$, and respective marginal densities are $f(\theta)$ and $f(m)$. Because we take θ and m to be independent, $F(\theta, m) = F(\theta)\, F(m)$.

In principle, this framework permits a formidable variety of simultaneous moves across quality and space. To illustrate, households from city j and currently living in that city's segment i could opt for segment $i-2$ in city $j+6$. To keep the multitude of potential moves tractable, however, we impose a restriction on inter city mobility popular in regional economics (e.g., Glaeser/Sacerdote/Scheinkman (1996)). We suggest that cities are arranged on a circle. Cities then are numbered consecutively from 1 to J in counterclockwise fashion. Households may move along this circle, but from their native city j to neighbor $j+1$ only.

All households have identical preferences. Let us focus on a given city's households, thus dropping the city index for the moment. Household utility in segment i is

$$U(\theta, s_i, h_i, x_i) = u(\theta,\, s_i) + r(h_i,\, s_i) + x_i \tag{1}$$

Utility depends on quality s_i, on floor space h_i, on the numéraire x_i as well as on the taste index θ. Utility is quasilinear in order to allow for the paper's applied focus. Besides $u_\theta, u_s, r_s, r_h > 0$ as well as $r_{hs} > 0 > r_{hh}$ we assume $u_{\theta s} > 0$. Hence any improvement in quality benefits large index households stronger than low index households. Put differently, subutility u exhibits strictly increasing differences with respect to (θ, s) (e.g., Carter (2003)).

Income w is exogenous and the same for every household. All residents are renters, because land in segment 1 is owned by local government while land in all other segments 2, ..., I is owned by absentee landlords.[12] A resident opting for quality i has budget constraint $x_i = w - q_i h_i$. Utility maximizing demand is found by solving $r_h(h_i, s_i) = q_i$ for h_i. Since r_h is strictly decreasing in h_i, its inverse exists and is also strictly decreasing. Inverting for h_i gives Marshallian demand for floor space $h(q_i, s_i)$, with $h_q(q_i, s_i) < 0$. Demand for floor space

[11] Arguably, East German households may emigrate to, and immigrate from, West Germany, too. From a stylized perspective, we assume that those who are mobile across regions have left already, leaving behind those who are mobile within East Germany only.

[12] From a stylized perspective, we ignore owner-occupied housing. Its share in East Germany was 32% in 2002 (BBR 2004, 83). But we briefly return to owner-occupiers in the paper's conclusions.

falls as rent rises. Due to the specific form of utility in (1), neither income nor household index affect housing demand. Maximum utility in quality s_i is $V(\theta, s_i, q_i) \equiv u(\theta, s_i) + v(s_i, q_i) + w$, where $v(s_i, q_i) = r(h(q_i, s_i), s_i) - q_i h(q_i, s_i)$. For reference, indirect utility's properties are collected in

Remark 1 (Indirect Utility Properties): (i) V exhibits strictly increasing differences with respect to θ and (s, q). (ii) $V_q(\theta, s_i, q_i) = v_q(s_i, q_i) = -h(q_i, s_i) < 0$, by the envelope theorem. (iii) $V_{\theta s} = u_{\theta s} > 0$, from the definition of V. (iv) $V_{\theta q} = 0$, from the definition of V.

The proof is in the Appendix. Households indifferent between two neighboring housing qualities i and $i+1$, where $i, i+1 \in \{1, ..., I\}$, are identified by setting

$$V(\theta_{i,i+1}, s_{i+1}, q_{i+1}) = V(\theta_{i,i+1}, s_i, q_i) \tag{2}$$

and solving for $\theta_{i,i+1}$. "Boundary households", or: boundaries, are also denoted $\theta_{i,i+1}(q_i, q_{i+1})$. Letting i run from 1 to $I-1$ identifies all boundaries, i.e., $\theta_{1,2}$, $\theta_{2,3}$, ... , $\theta_{I-1,I}$. For more flexible notation we put $\theta' = \theta_{0,1}$ as well as $\theta'' = \theta_{I,I+1}$. Of course, partial derivatives of these latter two (fixed) boundaries suggested by later notation (as, for instance, $\partial\theta_{0,1}/\partial q_0$ or $\partial\theta_{I,I+1}/\partial q_I$) are zero.

To illustrate the tradeoff confronting boundary household $\theta_{i,i+1}$, consider rewriting the indifference condition (2) as $u(\theta_{i,i+1}, s_{i+1}) - u(\theta_{i,i+1}, s_i) = v(s_i, q_i) - v(s_{i+1}, q_{i+1})$. Since the l.h.s. of this condition is strictly positive, so is its r.h.s. But then, by the properties of v, q_i must be strictly smaller than q_{i+1}. Higher quality commands higher rent. For a boundary household $\theta_{i,i+1}$ the extra in subutility from living in greater housing quality is offset by a loss in subutility due to consuming more expensive housing.

With the exception of $\theta_{0,1}$ and $\theta_{I,I+1}$, every boundary shifts with the two rents governing the segments it separates. Implicitly differentiating indifference condition (2) yields the signs of the corresponding two partial derivatives of $\theta_{i,i+1}$. Define $\delta_{i,i+1} = V_\theta(\theta_{i,i+1}, s_{i+1}, q_{i+1}) - V_\theta(\theta_{i,i+1}, s_i, q_i)$. Because $V_{\theta s} > 0$ and $V_{\theta q} = 0$ (Remark 1), we must have $\delta_{i,i+1} > 0$. But then

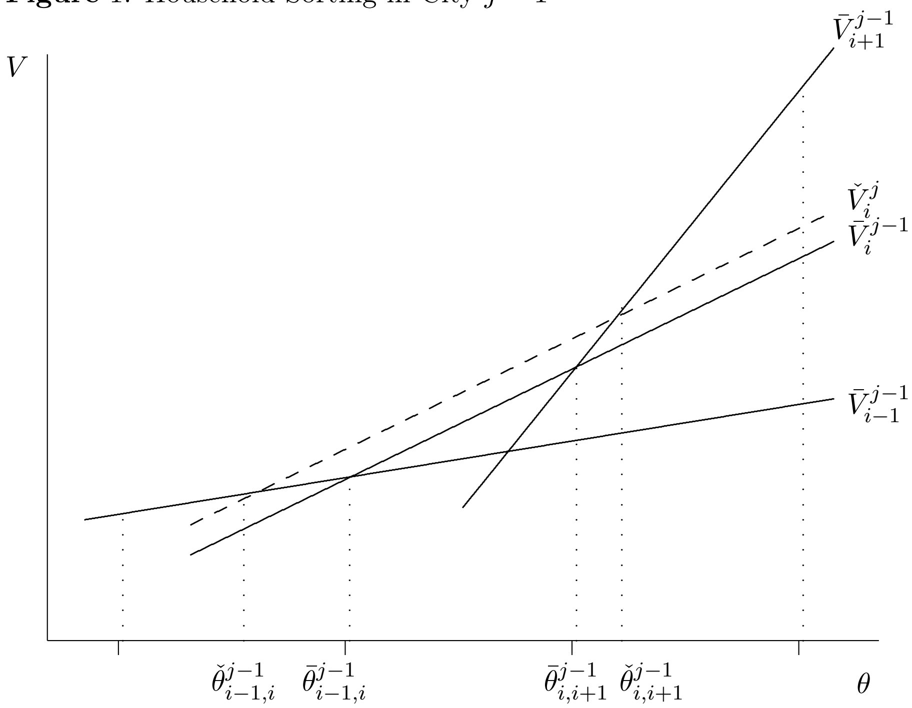
$$\frac{\partial\theta_{i,i+1}}{\partial q_i} = \frac{V_{q_i}(\theta_{i,i+1}, s_i, q_i)}{\delta_{i,i+1}} < 0 \quad \text{and} \quad \frac{\partial\theta_{i,i+1}}{\partial q_{i+1}} = -\frac{V_{q_{i+1}}(\theta_{i,i+1}, s_{i+1}, q_{i+1})}{\delta_{i,i+1}} > 0 \tag{3}$$

for $i = 1, ..., I-1$. Intuitively, raising segment i's rent reduces its "market share", while raising its neighbor's rent increases it. – The following Remark shows boundaries' use in describing households' optimal quality choice:

Remark 2 (Household Sorting By Quality): Households with taste index θ in the interval $[\theta_{i-1,i}, \theta_{i,i+1}]$ sort into housing of quality i, where $i = 1, ..., I$.

The proof is in the Appendix. But Figure 1 illustrates the Remark's sorting for city $j-1$, and for $u(\theta, s) = \theta \cdot s$. Indirect utilities in city $(j-1)$'s segments $i-1$, i and $i+1$ are illustrated as straight lines in (θ, V)-space. (Ignore the Figure's dashed line for the time being, as well as its intersections with either of the two straight lines of different slope.) Reflecting the increasing differences property, indirect utility for quality $i+1$ has a greater slope than that for quality i, etc. Boundaries between segments $(i-1, j-1)$ and $(i, j-1)$ and between segments $(i, j-1)$ and $(i+1, j-1)$ are shown as $\bar{\theta}^{j-1}_{i-1,i}$ and $\bar{\theta}^{j-1}_{i,i+1}$, respectively.

Figure 1: Household Sorting in City $j-1$



Existing housing in segment i, S_i, exhibits homogeneous quality. Only, its operating cost c varies across units. Ordering square meters by operating cost gives a strictly increasing function $c(S_i)$. Inverting it, and accounting for the fact that only housing with operating cost c below rent q will actually be offered, gives a strictly increasing housing supply function $S_i(q_i)$. This we assume to be differentiable, to exhibit $\partial S_i / \partial q_i > 0$ throughout its domain, and to be the same in every city. In our framework, rising rents expand extra supply only by drawing units into the market that were vacant previously. Changes in rent have no effect on construction, which is zero always.[13]

[13]This assumption simplifies the analysis. But it also reflects the observation that while current rents may exceed an existing square meter's operating cost (section 2), they may nonetheless be too low to pay for a new square meter's total cost. Also, subsidization of housing construction has essentially stopped. Housing construction in East Germany has dropped drastically.

4. Joint Residential Location and Housing Quality Choice

As a prelude to symmetric equilibrium, this section focuses on a symmetric allocation where rent and population for identical segments are the same in every city. Throughout this section, variables' values in a symmetric allocation are indicated by a bar. Due to symmetry,

$$\bar{q}_i^j = \bar{q}_i \ , \qquad \bar{n}_i^j = \bar{n}_i \qquad \text{and} \qquad \bar{n}^j = \bar{n} \qquad \text{for all } j, i \tag{4}$$

Then indirect utilities for given taste index θ and quality s_i are the same across cities, also, as are segment boundaries. I.e., $V(\theta, s_i, \bar{q}_i^j) = V(\theta, s_i, \bar{q}_i)$ for all j and i, and $\bar{\theta}_{i,i+1}^j \equiv \theta_{i,i+1}^j(\bar{q}_i^j, \bar{q}_{i+1}^j) = \theta_{i,i+1}^j(\bar{q}_i, \bar{q}_{i+1}) \equiv \bar{\theta}_{i,i+1}$ for all j and i. We conclude that $\bar{n}_i^j = \bar{n}^j \left(F(\bar{\theta}_{i,i+1}^j) - F(\bar{\theta}_{i-1,i}^j) \right)$.

Our first interest is in the impact small departures of q_i^j from $\bar{q}_i^j$ have on n_i^j. We need to distinguish two cases. On the one hand, for rents $\hat{q}_i^j$ above $\bar{q}_i^j$ we have $\hat{V}_i^j \equiv V(\theta, s_i, \hat{q}_i^j) < V(\theta, s_i, \bar{q}_i^{j+1}) \equiv \bar{V}_i^{j+1}$. Hence there must be migration from segment (i, j) to $(i, j+1)$. On the other hand, for rents $\check{q}_i^j$ below $\bar{q}_i^j$ we have $\check{V}_i^j \equiv V(\theta, s_i, \check{q}_i^j) > V(\theta, s_i, \bar{q}_i^{j-1}) \equiv V_i^{j-1}$. In this case there must be immigration from segment $(i, j-1)$, and possibly even other city $j-1$ segments, into (i, j). Given the asymmetry of migratory responses, the derivative of the segment population n_i^j with respect to q_i^j at $\bar{q}_i^j$ may depend on from where $\bar{q}_i^j$ is approached and, hence, need not exist. Fortunately, given our assumptions Remark 3 shows that this worry is not founded.

Remark 3 (Segment Population and Rent): The partial derivative of the number of consumers of segment i in city j, n_i^j, with respect to this segment's rent, q_i^j, at the initial symmetric allocation exists, and is equal to

$$\begin{aligned} \frac{\partial n_i^j(\bar{q}_i^j)}{\partial q_i^j} \quad &= \quad \bar{n}^j \, f(\bar{\theta}_{i,i+1}^j) \frac{\partial \theta_{i,i+1}^j(\bar{q}_i^j)}{\partial q_i^j} \quad - \quad \bar{n}^j \, f(\bar{\theta}_{i-1,i}^j) \frac{\partial \theta_{i-1,i}^j(\bar{q}_i^j)}{\partial q_i^j} \\ &+ \quad \bar{n}_i^j \;\; f(\bar{m}_i) \;\; \frac{\partial V(\bar{q}_i^j)}{\partial q_i^j} \qquad < \quad 0 \end{aligned} \tag{5}$$

where function arguments not our current focus are suppressed and $\bar{m}_i = 0$.

Again, for the proof see the Appendix. The derivative's three terms have an intuitive interpretation. Consider a one Euro rise in q_i^j. This rise, first, drives housing consumers away into rivalling segment $(i+1, j)$, with the magnitude of household loss indicated by the first term on the r.h.s. of (5). Second, it drives housing consumers away into rivalling segment $(i-1, j)$, as the second term on the r.h.s. of (5). And third, this rise makes consumers leave segment (i, j) for segment $(i, j+1)$, as captured by the third term on the r.h.s. of (5).

Alternatively, consider a one Euro drop in q_i^j. Then the third term on the r.h.s. of (5) instead represents the number of those attracted from segment $(i, j-1)$ to (i, j). Thus the emigration in the case of a marginal increase in q_i^j just equals the immigration in the case of a marginal decrease in q_i^j. This local property of the population in segment (i, j) is due to both, the symmetry of the initial allocation and the restriction that along-the-circle migration may be in one direction only. Moreover, in spite of the multitude of admissible moves the only relevant moves in response to a marginal increase or decrease in q_i^j are those from $(i, j-1)$ to (i, j), between $(i+1, j)$ or $(i-1, j)$ and (i, j), and from (i, j) to $(i, j+1)$.

Our next interest is in the effect a small departure in any of the other segments' prices q_1^j, ..., q_{i-1}^j, q_{i+1}^j, ..., q_I^j has on n_i^j. Rents in rivalling segments $(i-1, j)$ and $(i+1, j)$ do matter to demand for (i, j), as is seen from (2). But rents in remaining segments ..., $(i-3, j)$, $(i-2, j)$, $(i+2, j)$, $(i+3, j)$, ... do not matter. In particular, small rent changes in these latter segments certainly do not seduce any residents in city $j-1$ to switch into (i, j). Nor do they seduce any residents in (i, j) to leave for city $j+1$. Remark 4 sets out the only remaining relevant partial derivatives of n_i^j, with signs inferred from consulting (3).

Remark 4 (Segment Population and Neighboring Segments' Rents): The partial derivative of the number of consumers of segment i in city j, n_i^j, with respect to the next-higher segment's rent, q_{i+1}^j, at the initial symmetric allocation exists, and is equal to

$$\frac{\partial n_i^j(\bar{q}_{i+1}^j)}{\partial q_{i+1}^j} = \bar{n}^j f(\bar{\theta}_{i,i+1}^j) \frac{\partial \theta_{i,i+1}^j(\bar{q}_{i+1}^j)}{\partial q_{i+1}^j} > 0 \tag{6}$$

Likewise, the partial derivative of n_i^j with respect to the next-lower segment's rent, q_{i-1}^j, at the initial symmetric allocation exists, and equals

$$\frac{\partial n_i^j(\bar{q}_{i-1}^j)}{\partial q_{i-1}^j} = -\bar{n}^j f(\bar{\theta}_{i-1,i}^j) \frac{\partial \theta_{i-1,i}^j(\bar{q}_{i-1}^j)}{\partial q_{i-1}^j} > 0 \tag{7}$$

Again function arguments not our current focus are suppressed.

For the proof see the Apppendix. Since city j is small, rent changes induced in neighboring cities by manipulating q_i^j, q_{i-1}^j or q_{i+1}^j will not feed back into j. Demand spillovers into either of the two neighboring cities $j-1$ and $j+1$ travel along the – large – circle, either in clockwise or in counterclockwise fashion, to fade away eventually. They do not travel the entire circle so that there are no repercussions. With the derivation of derivatives complete, we no longer employ the bar notation. Derivatives' evaluation at a symmetric allocation is always implicitly understood. The population function for segment (i, j) henceforth is written $n_i^j(q_{i-1}^j, q_i^j, q_{i+1}^j)$, with its derivatives spelt out by (5), (6) and (7).

5. Rents, Filtering and Vacant Housing

This section introduces local government's instruments. In the short run local government controls q_1^j. In the long run it also controls the number of those Socialist housing tenants uprooted, and induced to move into segment i, z_i^j. In the short run $z_i^j = 0$ for all i. In the long run $dz_1^j < 0$ joint with $dz_i^j > 0$ for at least some $i > 1$. It seems reasonable to assume, and coincides with anecdotal evidence confirming, that local governments keep compensation payments as small as possible. I.e., local governments compensate for the loss in utility incurred from having to move into next-higher segment 2 only. Correspondingly we restrict ourselves to the case where $-\,dz_1^j = dz_2^j > 0$ joint with $dz_i^j = 0$ for all $i \geq 3$.[14]

While below (from section 6 on) the timing of instruments matters, this section explores the properties of changes in q_1^j and z_2^j in parallel fashion. Aggregation over individual housing demands for quality i in city j yields $H_i^j = (n_i^j\,(q_{i-1}^j, q_i^j, q_{i+1}^j) + z_i^j)\; h(q_i^j, s_i)$. Rents q_2^j, ..., q_I^j are determined by perfect competition, reflecting the strong role of diversified private ownership in higher segments. Rents in these latter segments must satisfy

$$\left(n_i^j(q_{i-1}^j, q_i^j, q_{i+1}^j) \,+\, z_i^j\right)\; h(q_i^j, s_i) \;=\; S(q_i^j) \qquad \text{for } i = 2, \ldots, I \tag{8}$$

This system of $I-1$ equations jointly determines $I-1$ rents. Differentiating system (8) with respect to the policy parameter $\alpha^j \in \{q_1^j, z_2^j\}$, inserting derivatives (5), (6) and (7), and rearranging gives the following equations (to be simplified shortly)

$$\begin{aligned}\left(\sigma_i^j + \varepsilon_i^j + \eta_i^j\right)\frac{\partial q_i^j}{\partial \alpha^j} \;&=\; n^j f(\theta_{i,i+1}^j)\,\frac{\partial \theta_{i,i+1}^j}{\partial q_i^j}\,\frac{\partial q_i^j}{\partial \alpha^j} \;-\; n^j f(\theta_{i-1,i}^j)\,\frac{\partial \theta_{i-1,i}^j}{\partial q_i^j}\,\frac{\partial q_i^j}{\partial \alpha^j}\\ &+\; n^j f(\theta_{i,i+1}^j)\,\frac{\partial \theta_{i,i+1}^j}{\partial q_{i+1}^j}\,\frac{\partial q_{i+1}^j}{\partial \alpha^j} \;-\, n^j f(\theta_{i-1,i}^j)\,\frac{\partial \theta_{i-1,i}^j}{\partial q_{i-1}^j}\,\frac{\partial q_{i-1}^j}{\partial \alpha^j}\\ &+\; \frac{\partial z_i^j}{\partial \alpha^j} \qquad \text{for } i = 2\ldots, I\end{aligned} \tag{9}$$

with σ_i^j, ε_i^j and η_i^j defined and explained in Table 2. Of course, $\partial z_i^j/\partial q_1 = 0$ for all i and j, given policy variables' mutual independence. And, $\partial z_i^j/\partial z_2^j = 0$ for $i = 3$, ..., I and for all j. Demolition's only direct effect on competitive segments is the one occurring in the second segment, i.e., $\partial z_2^j/\partial z_2^j = 1$.

[14] Aehnelt (2004) reports that close to 70% of those having to leave non-modernized Socialist housing (segment 1) to be bulldozed actually move into modernized Socialist housing (included in segment 2).

Table 2: Excess Housing Demand Changes Not Related to Filtering

Variable	Magnitude and Sign	Interpretation: Household equivalents of ...
σ_i^j	$-\, n_i^j f(\bar{m}_i) \frac{\partial V(q_i^j)}{\partial q_i^j} \; > \; 0$	... extra immigration into (i,j)
ε_i^j	$-\; n_i^j \frac{\partial h(q_i^j, s_i)}{\partial q_i^j} \frac{1}{h(q_i^j, s_i)} \; > \; 0$	... extra demand within (i,j) :
η_i^j	$\frac{\partial S(q_i^j)}{\partial q_i^j} \frac{1}{h(q_i^j, s_i)} \; > \; 0$	... extra housing supply (i,j) :

Consider, for instance, the effects of a one Euro decrease in q_i^j on market i, as collected on the first line of (9). First, this decrease attracts households from neighboring segments $(i+1, j)$ and $(i-1, j)$. Second, it attracts immigrants from segment $(i, j-1)$, of magnitude $\sigma_i^j(\partial q_i^j/\partial q_1^j)$. Third, it induces extra demand by those remaining in segment (i,j), and equivalent to what is required to house $\varepsilon_i^j(\partial q_i^j/\partial q_1^j)$ households. And fourth, it reduces supply in segment (i,j), by $\eta_i^j(\partial q_i^j/\partial q_1^j)$ units. The effects of a one Euro decrease in q_{i-1}^j or q_{i+1}^j are less varied. Either of these reductions pulls households away from (i,j), as indicated by the terms on the second line of the r.h.s. of (9).

Endogenous variables in (9) may be conveniently summarized by the $(I-1) \times 1$ vector $y^j = (\partial q_2^j/\partial \alpha^j \,, \ldots, \, \partial q_I^j/\partial \alpha^j)$. Let A^j be the $(I-1) \times (I-1)$ matrix of endogenous variables' coefficients. Generally, the coefficients of $\partial q_i^j/\partial \alpha^j$, $i = 2, \ldots, I$, are stacked into the $(i-1)$-th column of A^j:

$$\begin{array}{l} \\ \\ \\ \text{row } i-2 \\ \\ \text{row } i-1 \\ \\ \text{row } i \\ \\ \\ \\ \end{array} \left(\begin{array}{c} 0 \\ \vdots \\ 0 \\ n^j f(\theta_{i-1,i}^j) \frac{\partial \theta_{i-1,i}^j}{\partial q_i^j} \\ \\ n^j f(\theta_{i,i+1}^j) \frac{\partial \theta_{i,i+1}^j}{\partial q_i^j} \; - \; \left(\sigma_i^j + \varepsilon_i^j + \eta_i^j \right) \; - \; n^j f(\theta_{i-1,i}^j) \frac{\partial \theta_{i-1,i}^j}{\partial q_i^j} \\ \\ -n^j \; f(\theta_{i,i+1}^j) \frac{\partial \theta_{i,i+1}^j}{\partial q_i^j} \\ 0 \\ \vdots \\ 0 \end{array} \right) \tag{10}$$

where, to be sure, the first and the last column of A^j look slightly different.

The effects of a one Euro change in q_i^j on segments $i-1$, i and $i+1$ are found in column $(i-1)$'s three consecutive rows $i-2$, $i-1$, and i, respectively. The column's remaining entries are zero. Consult the element in column $i-1$ and row $i-1$, also shown in (10). This is a diagonal element of A^j. In absolute terms this (negative) element exceeds the sum of the two (positive) off-diagonal elements found in the column's rows $i-2$ and i. This property applies to any of A^j's columns. Thus A^j has a dominant diagonal.

Let b^j be the $(I-1) \times 1$ column vector containing the policy induced "demand shock" to the second segment

$$-\frac{\partial z_2^j}{\partial \alpha^j} + n^j f(\theta_{1,2}^j) \frac{\partial \theta_{1,2}^j}{\partial q_1^j} \frac{\partial q_1^j}{\partial \alpha^j} < 0 \tag{11}$$

in its first row, and exhibiting zeros elsewhere. This demand shock is strictly negative for both $\alpha \in \{q_1^j, z_2^j\}$. Equipped with this notation, (9) can more compactly be written as $A^j y^j = b^j$. Since A^j has a dominant diagonal it is non-singular (Takayama (1985), Theorem 4.C.1). Hence $(A^j)^{-1}$, and with it a solution to (9), exist. This solution is unique. Moreover, $(A^j)^{-1}$ has non-positive entries everywhere (Takayama (1985), Theorem 4.D.3, parts (I") and (III"), or Simon/Blume (1995), Theorem 8.14). Thus every component of the solution vector $(A^j)^{-1}b^j$ is simply the product of a non-positive number, taken from the first column of $(A^j)^{-1}$, and the "demand shock" (11). Then every element of the solution y^j is non-negative. In fact, this result can be sharpened further:

Remark 5 (Dominos): Derivatives of rents q_i^j with respect to policy variables q_1^j and z_2^j are strictly positive. Moreover,

$$\frac{\partial q_i^j}{\partial z_2^j} = \beta^j \frac{\partial q_i^j}{\partial q_1^j} > 0 \tag{12}$$

for $i = 2, \ldots, I$ and where $1/\beta^j = -n^j f(\theta_{1,2}^j) \; \partial\theta_{1,2}^j/\partial q_1^j > 0$. Expanding either policy parameter strictly increases rent in every higher segment.

Proof: See Appendix. Essentially Remark 5 is a variant of Sweeney (1974a)'s Lemma 7. This lemma applies to a more broadly defined set of preferences; while Remark 5 admits housing supply's endogenous adjustment as well as inter city migration. The underlying idea seems very intuitive. Much as in a sequence of dominos, both policies' bolstering of (reducing) demand for the second-lowest quality ultimately pushes up (pulls down) rent in every higher quality. An equally viable metaphor may be that of Socialist housing serving as a "pillar" or "corner stone" to housing in qualities towering above it. Should Socialist flats' rent give in, so would rents in every other segment. The entire rent curve would unravel.

Equilibrium segment boundaries are $\theta_{i,i+1}(q_i^j(\alpha^j), q_{i+1}^j(\alpha^j))$, and hence are composite functions of the policy variables. These equilibrium values we write as $t_{i,i+1}^j(\alpha^j)$. Differentiating $t_{i,i+1}^j(\alpha^j)$ with respect to α^j then gives the combined effect of α^j on the segment boundary:

$$\frac{\partial t_{i,i+1}^j}{\partial \alpha^j} = \frac{\partial \theta_{i,i+1}^j}{\partial q_i^j}\frac{\partial q_i^j}{\partial \alpha^j} + \frac{\partial \theta_{i,i+1}^j}{\partial q_{i+1}^j}\frac{\partial q_{i+1}^j}{\partial \alpha^j} \tag{13}$$

for $i = 1, \ldots, I-1$. Derivatives can thus be calculated by inserting boundaries' partial derivatives, from (3), as well as solutions for rent changes, from (12).

To infer boundary changes' signs we use (13) to rewrite (9) more efficiently:

$$\begin{aligned} n^j f(t_{i-1,i}^j)\frac{\partial t_{i-1,i}^j}{\partial \alpha^j} &= n^j f(t_{i,i+1}^j)\frac{\partial t_{i,i+1}^j}{\partial \alpha^j} \\ &- (\sigma_i^j + \varepsilon_i^j + \eta_i^j)\frac{\partial q_i^j}{\partial \alpha^j} + \frac{\partial z_i^j}{\partial \alpha^j} \end{aligned} \tag{14}$$

for $i = 2, \ldots, I$. This simpler version of (9) explicitly relates the "inflow" of households into segment (i,j) from $(i+1,j)$, as the first term on the r.h.s., to the stream of households "flowing out" of (i,j) into $(i-1,j)$, being the first term on the l.h.s. Inflows and outflows may be positive or negative. The following Remark 6 signs boundary changes, and hence filtering flows.

Remark 6 (Filtering): The signs of the derivatives of segment boundaries $t_{i,i+1}^j(\alpha^j)$ with respect to q_1^j and z_2^j are

$$\frac{\partial t_{i,i+1}^j}{\partial q_1^j}, \frac{\partial t_{i,i+1}^j}{\partial z_2^j} < 0 \quad \text{for } i = 2, \ldots, I-1 \tag{15}$$

$$-\frac{\partial t_{1,2}^j}{\partial q_1^j}, \frac{\partial t_{1,2}^j}{\partial z_2^j} > 0 \tag{16}$$

Every higher boundary $t_{2,3}^j$, $t_{3,4}^j$, ..., $t_{I-1,I}^j$ shifts down if q_1^j and z_2^j increase. However, $t_{1,2}^j$ only shifts down if q_1^j increases; while it shifts up if z_2^j increases.

Proof: See Appendix. An increase (decrease) in any boundary $\theta_{i,i+1}$ implies that households filter down from segment $i+1$ to segment i (up from i to $i+1$). Segment boundary changes and filtering flows point into opposite directions. Now, the recursive nature of (14) indicates that filtering flows cumulate. Start with the $i = 2$ version of (14), featuring the inflow from segment 3. Replace this inflow by making use of the $i = 3$ version of (14); and so forth. After successive substitution $I - 3$ times the net inflow into the hierarchy's lowest segment can be expressed as:

$$n^j\, f(t^j_{1,2})\, \frac{\partial t^j_{1,2}}{\partial \alpha^j} - \frac{\partial z^j_2}{\partial \alpha^j} \quad = \quad - \sum_{i=2}^{I} \left(\sigma^j_i + \varepsilon^j_i + \eta^j_i\right) \frac{\partial q^j_i}{\partial \alpha^j} \tag{17}$$

The expression on the r.h.s. mirrors the accumulation of filtering flows in housing's lowest quality. Suppose $dq_1 < 0$. By Remark 5, all rents fall. The immediate impact in (I, j) is the emergence of excess demand, due to extra demand within (I, j), to immigration from $(I, j - 1)$, and to a contraction of the supply to (I, j). Dissatisfied demand filters down to $(I - 1, j)$. Newcomers are joined by extra demand within $(I-1, j)$, immigrants originating in $(I-1, j-1)$, and a contraction of supply to $(I-1, j)$. Resulting excess demand in segment $(I-1, j)$ then filters down to $(I - 2, j)$. Etc. Metaphorically speaking, consecutive filtering flows *cascade* down the quality hierarchy.

Socialist flat occupancy is $H^j_1 = \left(n^j_1(q^j_1, q^j_2) - z^j_2\right) h(q^j_1, s_1)$. Differentiating, and making successive use of (5), (6), (7), (13) and (17), gives

Remark 7 (Socialist Flat Occupancy): The derivatives of Socialist flat occupancy with respect to q^j_1 and z^j_2 are

$$\frac{\partial H^j_1}{\partial q^j_1} \quad = \quad - \, h(q^j_1, s_1) \left(\sum_{i=2}^{I} (\sigma^j_i + \varepsilon^j_i + \eta^j_i) \frac{\partial q^j_i}{\partial q^j_1} + \sigma^j_1 + \varepsilon^j_1 \right) < 0 \tag{18}$$

$$\frac{\partial H^j_1}{\partial z^j_2} \quad = \quad - \, h(q^j_1, s_1) \left(\sum_{i=2}^{I} (\sigma^j_i + \varepsilon^j_i + \eta^j_i) \frac{\partial q^j_i}{\partial z^j_2} \right) < 0 \tag{19}$$

On the one hand, Socialist flats' employment would benefit from a reduction in Socialist rent: not just because of its immediate effect on existing consumers' and future immigrants' housing demands (captured by ε_1 and σ_1), but also because of the induced filtering inflow into Socialist housing (captured by the expression on the r.h.s. of (17)). This latter, indirect, effect is overlooked in current policy debate. On the other hand, Socialist flats' demolition not just reduces their vacant numbers. It also reduces the number of those in Socialist housing. This is why vacancy rates fall by much less than what was expected (e.g., SAB (2006)).

6. Symmetric Equilibrium

Local government maximizes $R^j = \sum_{i=1}^{I}(H_i^j q_i^j - \int_0^{H_i^j} c(\xi)\, d\xi\,)$, representing aggregate net rent.[15] In the short run local government controls q_1^j but needs to take $z_1^j = z_2^j = 0$ as given. Accounting for the equality $q_i^j = c(H_i^j)$ in competitive markets, maximization of R^j with respect to q_1^j implies setting

$$\frac{\partial R^j}{\partial q_1^j} = \sum_{i=1}^{I} H_i^j \frac{\partial q_i^j}{\partial q_1^j} + \left(q_1^j - c(H_1^j)\right) \frac{\partial H_1^j}{\partial q_1^j} \tag{20}$$

to zero. The first term on the r.h.s. represents the positive impact of a one Euro rent increase in Socialist housing, associated with its driving up rents in higher segments. The second term is the attendant reduction in Socialist flats' occupancy. Since marginal benefit is strictly positive, so must be marginal cost. Hence the markup of price over the expense from operating the most costly unit occupied, i.e., $(q_1^j - c(H_1^j)) > 0$.

Symmetric equilibrium is defined by: (i) symmetry with respect to rent and population (see (4)): (ii) households sorting into qualities $(1, j)$ through (I, j) for all j (see Remark 2); (iii) competitive housing market equilibrium in segments $(2, j)$ to (I, j) for all j (see (8)); and (iv) aggregate net rent maximization for all j (see (20)).[16] Note that the equilibrium's markup pricing for Socialist housing not just generates "involuntarily vacant housing" among Socialist flats but also, by sustaining higher rents in higher qualities, larger "voluntarily vacant housing" than would be observed if Socialist flats were priced at marginal cost.

Symmetric equilibrium serves as our point of departure for evaluating current policy. It echoes the fact that Socialist flats are currently priced above marginal cost, and it explains why apparently no individual city undercuts rents prevailing elsewhere in order to attract residents (section 2). Now, in the long run demolition is the extra instrument at city j's disposal. Suppose j is the only city to demolish ("isolated demolition"). By the envelope theorem, isolated demolition's effect is found by taking the first derivative of R^j with respect to z_2^j and evaluating at the equilibrium:

$$\frac{\partial R^j}{\partial z_2^j} = \sum_{i=2}^{I} H_i^j \frac{\partial q_i^j}{\partial z_2^j} + \left(q_1^j - c(H_1^j)\right) \frac{\partial H_1^j}{\partial z_2^j} \tag{21}$$

[15] Alternatively we could have assumed that local government maximizes a weighted average of aggregate net rent and consumer surplus, as in Brueckner (1998). But with household mobility this introduces the well-known difficulty of deciding on whether migrants' welfare should enter local government's objective (Mansoorian/Myers (1997)).

[16] The symmetric equilibrium's existence is assumed rather than shown. This follows a convention underlying much of the literature on tax competition (see Upmann/Ziad (2005)).

7. Policy Coordination

Let all local governments simultaneously deviate from the symmetric equilibrium in identical fashion. This has no impact on the inter city allocation of households. To distinguish between variable changes under the two different régimes, derivatives in the case of coordinated action carry the superscript ^{c}. Coordinated action shuts down interurban migration and, hence, wipes σ_i^j off all equations in (9). Define A^c as the coefficient matrix that results if σ_i^j is dropped from A^j. Clearly A^c retains the dominant diagonal property. Hence competitive rents' derivatives under coordinated action $y^c = ((\partial q_2/\partial\alpha)^c, \ldots, (\partial q_I/\partial\alpha)^c) = (A^c)^{-1}b$ are strictly positive, again. In fact, we can prove a stronger result:

Remark 8 (Effect of Coordination on Rents): Departing from symmetric equilibrium, a marginal increase in the policy parameter $\alpha \in \{q_1, z_2\}$ raises a given city j's rents in higher segments more strongly if it occurs simultaneously in all cities than if it occurs in j only. I.e.,

$$\left(\frac{\partial q_i}{\partial q_1}\right)^c > \frac{\partial q_i^j}{\partial q_1^j} > 0 \qquad \text{and} \qquad \left(\frac{\partial q_i}{\partial z_2}\right)^c > \frac{\partial q_i^j}{\partial z_2^j} > 0 \tag{22}$$

for all $i = 2, \ldots, I$.

The proof is in the Appendix, but the underlying idea seems intuitive enough. If coordinated action renders emigration entirely unattractive, housing demands in higher qualities respond less elastically to the rent increases triggered by rising Socialist flat rent or demolition. Higher segments' rents must rise more strongly if they are to make room for those Socialist housing tenants driven away by either policy.

Further information on coordinated action is found by setting σ_i^j to zero and replacing uncoordinated action derivatives $\partial q_i^j/\partial\alpha^j$ (Remark 5) by their coordinated action counterparts $(\partial q_i/\partial\alpha)^c$ (Remark 8). Making use of (3) and (22) in (13) then reveals that $\partial t_{1,2}^j/\partial\alpha^j < (\partial t_{1,2}/\partial\alpha)^c$. I.e., the l.h.s. of (17) is larger under coordinated action than under isolated action. But then so is its r.h.s. Employing this in (18) and (19) implies

$$\frac{\partial H_1^j}{\partial\alpha^j} < \left(\frac{\partial H_1}{\partial\alpha}\right)^c < 0 \tag{23}$$

Evacuation of Socialist housing due to either policy turns out to be smaller under coordinated action than under isolated action. It remains to inspect coordination's effect on R. Inserting the first inequality in (22) as well as the first inequality in (23) into (20) and (21) immediately implies

Proposition 1 (The Benefit of Coordination): Departing from symmetric equilibrium, an increase in either policy parameter $\alpha^j \in \{q_1^j, z_2^j\}$ raises urban rent income in city j by more if it occurs in all cities simultaneously than if it occurs in j only:

$$\left(\frac{\partial R}{\partial q_1}\right)^c > \frac{\partial R^j}{\partial q_1^j} = 0 \qquad \text{and} \qquad \left(\frac{\partial R}{\partial z_2}\right)^c > \frac{\partial R^j}{\partial z_2^j} \tag{24}$$

This is the paper's central positive result. Demolition's coordination raises demolition's return further. This return still need not be positive. (The policy simulation in Section 9 provides an example where it is positive.) Yet coordination makes its being positive more likely. From this perspective, coordinated demolition *may* be a successful means for pushing landlords' incomes even beyond those incomes attained in the initial equilibrium. – Proposition 1 also notes that local governments could alternatively have coordinated on further increases in rent instead. However, coordination may be more difficult with respect to rents. Increases in rents are reversible, while demolition is not.

8. Urban Welfare

Urban welfare in city j is given by an unweighted utilitarian welfare function W^j, which is the sum of net urban rent income R^j and consumer rent $C^j = n^j \sum_{i=1}^I \int_{t_{i-1,i}^j}^{t_{i,i+1}^j} \left(u(\theta, s_i) + v(s_i, q_i^j)\right) f(\theta)\, d\theta$. We focus on the effect of coordinated policies and hence drop the city index j. Differentiating C with respect to α, and employing (13), yields

$$\begin{aligned}\left(\frac{\partial C}{\partial \alpha}\right)^c &= \sum_{i=1}^{I} \left(F(t_{i,i+1}) - F(t_{i-1,i})\right) n\, v_q(s_i, q_i) \left(\frac{\partial q_i}{\partial \alpha}\right)^c \\ &+ \sum_{i=1}^{I} \left(u(t_{i,i+1}, s_i) + v(s_i, q_i)\right) n\, f(t_{i,i+1}) \left(\frac{\partial t_{i,i+1}}{\partial \alpha}\right)^c \\ &- \sum_{i=1}^{I} \left(u(t_{i-1,i}, s_i) + v(s_i, q_i)\right) n\, f(t_{i-1,i}) \left(\frac{\partial t_{i-1,i}}{\partial \alpha}\right)^c \end{aligned} \tag{25}$$

Expression (25) can be simplified greatly because successive terms in the sums on the second and third line of (25) cancel out. Pick the k-th term in the sum on the second line of (25). This is $(u(t_{k,k+1}, s_k) + v(s_k, q_k))\, n\, f(t_{k,k+1})\, (\partial t_{k,k+1}/\partial \alpha)^c$. Next, pick the $(k+1)$-th term in the sum on the third line of (25). This is $(u(t_{k,k+1}, s_{k+1}) + v(s_{k+1}, q_{k+1}))\, n\, f(t_{k,k+1})\, (\partial t_{k,k+1}/\partial \alpha)^c$. The two terms are exactly equal because any boundary household endowed with $t_{k,k+1}$ is indifferent between segments k and $k+1$ (see (2)). Since k was arbitrarily picked, the previous argument holds for any k. Ultimately the two sums on the second and third lines of (25) vanish. Finally, note that $v_q(s_i, q_i) = -h(q_i, s_i)$, by Remark 1. With these modifications the change in consumer rent reduces to

$$\left(\frac{\partial C}{\partial \alpha}\right)^c = -\sum_{i=1}^{I} H_i \left(\frac{\partial q_i}{\partial \alpha}\right)^c \; < \; 0 \tag{26}$$

which is strictly negative. However, the change in aggregate rent prominent in $(\partial C/\partial \alpha)^c$ is exactly offset by an equally sized change contained in $(\partial R/\partial \alpha)^c$, i.e., contained in

$$\left(\frac{\partial R}{\partial \alpha}\right)^c = \sum_{i=1}^{I} H_i \left(\frac{\partial q_i}{\partial \alpha}\right)^c + (q_1 - c(H_1)) \left(\frac{\partial H_1}{\partial \alpha}\right)^c \tag{27}$$

Urban welfare change is the sum of (26) and (27), and reduces to the simple expression given in equation (28), which can be signed using (18) and (19).

Proposition 2 (Coordination and Welfare): A coordinated policy generates a net change in the economy's welfare equal to:

$$\left(\frac{\partial W}{\partial \alpha}\right)^c = (q_1 - c(H_1)) \left(\frac{\partial H_1}{\partial \alpha}\right)^c \; < \; 0 \tag{28}$$

Coordinated demolition reduces, while a coordinated reduction of Socialist flats' rent raises, the economy's welfare.

This is the paper's central normative result. To illustrate, consider a small reduction in Socialist flats' rent q_1. That this increases welfare may seem puzzling at first sight, because average housing quality falls. And yet, households who filter down into lower qualities are compensated by corresponding reductions in rent. The only relevant, and positive, welfare effect that emerges is the gradual reoccupation of – the best part of – currently vacant Socialist housing.

9. Policy Simulation

This section illustrates the model's mechanics. Specifically we assume that $f(\theta) = 1$ on $[0, 1]$, and $f(m) = 1/d$ on $[0, d]$, where $d > 0$. There are five segments, with rents $q_1 = 5.5$, $q_2 = 6.0$, $q_3 = 6.5$, $q_4 = 7.0$ and $q_5 = 7.5$ Euro, respectively. Segment 1 is non-modernized Socialist housing. Segment boundaries are $\theta_{1,2} = 0.2$, $\theta_{2,3} = 0.4$, $\theta_{3,4} = 0.6$ and $\theta_{4,5} = 0.8$. Segment boundaries and rents chosen are unlikely to equal "true" boundaries and rents, but may represent a reasonable representation of East Germany's housing nonetheless. There are $J = 70$ cities, with $n = 100,000$ residents each. I.e., each segment i initially is chosen by $n_i = 20,000$ residents. The total inhabited stock is 7,000,000 dwellings, i.e., roughly equalling the true stock (Table 2). We assume that an additional 6,500 units are vacant within each segment, implying a vacancy rate of 25 %.

Let utility (1) for those living in segment i be $a_1\,\theta s_i - (h_i/a_3)^{-a_2} + x_i$, where $a_1, a_2, a_3 > 0$ are parameters to be determined. Housing demand becomes $h_i = a_3\,(q_i\,a_3/a_2)^{-\frac{1}{a_2+1}}$. We wish the price elasticity of housing to equal $-1/3$, i.e., an estimate well within the range of long run price elasticities typically found in the literature (e.g., Ermisch/Findlay/Gibb (1996)). Then a_2 must equal 2. Next, average floor space of East German SOEP households is 60 m^2 app. This translates into a restriction on a_3. For floor spaces to settle near 60 m^2 we need to set a_3 to 800. Then $h_1 = 61.5\,m^2$, $h_2 = 59.8\,m^2$, $h_3 = 58.2\,m^2$, $h_4 = 56.8\,m^2$ and $h_5 = 55.5\,m^2$. Indirect utility becomes $V(q_i, s_i, \theta) \approx a_1\,\theta\,s_i - (1.89)\,(800\,q_i)^{\frac{2}{3}} + w$.

Inserting indirect utilities V_i and observed boundaries $\theta_{i,i+1}$ into (2) permits us to infer unobservable quality increments $\delta_{i,i+1} = a_1(s_{i+1} - s_i)$. Inserting these in turn into (3), and recalling that $V_{q_i} = -h_i$, gives boundaries' partial derivatives. These depend on a_1. Setting a_1 to 0.1 gives sufficiently "plausible" partial derivatives of segment boundaries. For example, $\partial\theta_{1,2}/\partial q_1 = -0.04059$. I.e., reducing rent in Socialist housing by one Euro would attract 4,059 households from segment 2 to Socialist housing (ceteris paribus, see below).

We turn to ε_i, σ_i and η_i, defined in Table 2. We find that $\varepsilon_1 = 1,212$, $\varepsilon_2 = 1,111$, $\varepsilon_3 = 1,026$, $\varepsilon_4 = 952$ and $\varepsilon_5 = 889$ households. Further, rather than make assumptions on the supply functions we simply set $\eta_i = 1,000$ for all i. Finally, note that σ_i depends on d. We assume that $d = 200$. Then $\sigma_1 = 6,151$, $\sigma_2 = 5,975$, $\sigma_3 = 5,818$, $\sigma_4 = 5,676$ and $\sigma_5 = 5,547$.[17] To calibrate the model we require an increase in Socialist housing rent by one Euro to yield zero extra net revenue: $dq_1 = 1$. We calculate A (for a typical column see (10)), its inverse A^{-1}, $b'\,dq_1 = (-\,4059,\ 0,\ 0,\ 0)$ (with first row given in (11)), $y\,dq_1 = (A^{-1}\,b)\,dq_1$ as resultant changes in rents, and the change in occupied Socialist housing $(\partial H_1/\partial q_1)\,dq_1$ (given in (18)). Inserting results into the condition for optimum rent (20) permits us to solve for the markup consistent with local government's stipulated behavior. We find $q_1 - c(H_1) = 2.62$ Euro. Since $q_1 = 5.5$ Euro, $c(H_1) = 2.88$ Euro, which may seem a reasonable estimate.

We apply the calibrated model to a coordinated reduction in rent by one Euro first. Then $dq_1 = -1$ and $b'\,dq_1 = (4059,\ 0,\ 0\,,0)$. Setting $\sigma_i = 0$ for all i, and

[17] These figures may seem overly large. Yet note that 70 equally sized cities laid out in a circle over East Germany's fairly small territory would be quite close to each other. Given small distances, there should be many residents who will not tolerate paying one Euro extra in the long run.

repeating calculations using the parameter values found for a_1, a_2, a_3, $c(H_1)$ and d to identify A^c, $y^c\,dq_1$, etc. produces the figures in Table 3. A one Euro decrease in Socialist flats' rent induces rent reductions throughout higher segments (column 2). E.g., rent in the fourth segment still falls by 22 cents. These reductions trigger extra demand, and discourage supply (column 3). E.g., the 22 cent drop in the fourth segment induces extra demand for floor space equivalent to $\varepsilon_4\,dq_4$ =209 units, by those already in the fourth segment; while forcing $\eta_4\,dq_4 = 220$ marginal suppliers to the fourth segment to withdraw.

Columns 4 and 5 trace the filtering flows stimulated. A total of 379 households filter down from the fifth segment to the fourth. This inflow combines with the deficit of 429 housing units just identified. The resulting excess of 808 households turns away from the fourth, and into the third, segment. Filtering flows successively cumulate, with 2,312 households eventually switching from segment 2 to segment 1. There they unite with those 1,212 household equivalents representing extra demand by those already inhabiting Socialist flats. The example shows that filtering can be an important source of extra demand. Multiplying 3,524 households by $h_1 = 61.5\,m^2$ gives $dH_1 = 216,768\,m^2$ as the extent to which Socialist housing is reclaimed. Multiplying this area by the 2.62 Euro markup gives a monthly welfare gain of $dW = 568,857$ Euro.

Table 3: Coordinated Rent Reduction: Filtering and Welfare Gain

Qual. i	**Δ Rent** $\frac{\partial q_i}{\partial q_1}\,dq_1$	**Δ Demand** $(\varepsilon_i + \eta_i)\,\frac{\partial q_i}{\partial q_1}\,dq_1$	**Inflow** $\lvert nf(t_{i,i+1})\frac{\partial t_{i,i+1}}{\partial q_1}\,dq_1\rvert$	**Outflow** $\lvert nf(t_{i-1,i})\frac{\partial t_{i-1,i}}{\partial q_1}\,dq_1\rvert$
1	− 1.00	1,212	2,312	0
2	− 0.44	935	1,377	2,312
3	− 0.28	569	808	1,377
4	− 0.22	429	379	808
5	− 0.20	379	0	379
Change in Socialist Housing Occupancy dH_1 $[m^2]$				$216,768$
Change in Urban Welfare dW [Euro]				568,857

Scenario: Socialist rent reduction by 1 Euro in every city: $dq_1 = -1$ Euro.

Table 4 summarizes the results for the alternative scenario of coordinated demolition. Suppose that, to generate the planned total reduction in Socialist housing stock of 350,000 units, every city levels 5,000 units of Socialist housing. Suppose also that every block to be levelled is half-vacant only. Then 2,500 households need to be convinced to leave. Let each of these receive a payment sufficient to compensate for having to move into the second segment. Then $-dz_1 = dz_2 = 2500$, and $b'\, dz_2 = (-2500, 0, 0, 0)$. While Socialist flats' rent does not rise at all, rents in higher segments rise throughout (Column 2). For instance, rent in the second segment rises by 27 cents.

Rising rent in segment 2 makes some of its initial residents turn to segment 1 instead. Altogether 1,076 households filter down into the first segment. Set against those 2,500 households seduced to move upwards, this still leaves a net inflow of 1,424 households into the second segment (see Remark 6 and equation (17)). These fill (i) housing left behind by those contracting their housing consumption, $\varepsilon_2\, dq_2 = 303$ (column 3), (ii) housing offered by entrant landlords, $\eta_2\, dq_2 = 273$ (column 3), and (iii) housing deserted by those 848 households filtering further up (column 5). Etc. Filtering flows gradually subside as they near the highest, fifth segment. Floor space in Socialist housing no longer occupied equals the net outflow of 1,424 households lost to the second segment, multiplied by 61.5 m^2. This gives $dH_1 = -87,586\, m^2$. Multiplying by the markup indicates a welfare "gain" equal to $-229,849$ Euro (cf. Proposition 2).

Table 4: Coordinated Demolition: Filtering Flows and Welfare Loss

Qual. i	**Δ Rent** $\frac{\partial q_i}{\partial z_2}\, dz_2$	**Δ Demand** $(\varepsilon_i + \eta_i)\, \frac{\partial q_i}{\partial z_2}\, dz_2$	**Net Inflow**	**Net Outflow**
1	0.00	0	0	1,424
2	0.27	– 576	1,424	848
3	0.17	– 350	848	498
4	0.14	– 264	498	234
5	0.12	– 234	234	0
Change in Net Rent Income dR [Euro]				$588,140$
Change in Socialist Housing Occupancy dH_1 [m^2]				$-87,586$
Change in Urban Welfare dW [Euro]				$-229,849$

Scenario: Demolition of 5,000 Socialist flats in every city, with 2,500 households compensated for moving into the second segment: $dz_2 = 2500$.

Knocking down 5,000 units of half-vacant Socialist housing reduces vacant Socialist housing by less, i.e., by 3,576 units only. This explains why relief in vacancies lags flat numbers demolished (SAB (2006)). Table 4 also reports that urban net rent income R rises, by $588,140$ Euro. Coordinated demolition indeed pushes landlords' incomes beyond those pocketed in the initial rent-setting equilibrium (cf. Proposition 1). We add that isolated demolition, occurring in a single city only, would have generated a change in net rent income of $-25,653$ Euro, i.e., a loss. In the example indeed it is demolition's coordination that makes demolition worthwhile.

From a cost-benefit perspective, the rent reduction not undertaken is demolition's opportunity cost. Demolition's true cost is $798,706$, rather than just $229,849$, Euro.[18] Multiplying by 12 months, and aggregating across all 70 cities, gives a yearly true cost of 671 million Euro. Applying a "social discount rate" of 4%, assuming that Socialist housing would last for another 30 years and that all demolition occurs today, indicates an aggregate true welfare cost of app. 12.1 billion Euro. Adding the immediate cost of demolition, as roughly captured by those 2.5 billion Euro subsidies towards demolition (section 2), points to a total of 14.6 billion Euro foregone. Of course, these estimates are proportional, and hence are strongly sensitive, to the estimate of the markup.

10. Conclusions

The paper takes literally local governments' declared objective of "stabilizing housing markets" when looking for an explanation of current demolition's tremendous volume. Demolition, coordinated demolition in particular, may indeed serve the "stabilization objective". Demolition uproots residents inhabiting the blocks that are about to be torn down. These subsequently filter up to, and push up rents in, higher segments. Demolition's ubiquity reinforces this effect. Ultimately, coordinated demolition may indeed "stabilize" landlords' incomes.

The paper also argues that the stated objective of "stabilizing housing markets" is misplaced. Local governments' pursuing this objective by raising panel housing rents in the past may have contributed to the tremendous stock of vacant housing in the first place. Ironically, vacant housing now is used to justify the current policy of bulldozing not only vacant housing but also housing that is inhabited still. The paper argues that it is price, rather than quantity, adjustment that would contribute to East German cities' redevelopment.

[18] As always, all results hinge on whether the derivatives employed are sufficiently good approximations of the model's non-linear functions in the vicinity of equilibrium.

Appendix

Proof of Remark 1:

(i) Consider comparing the maximum utilities that result for two distinct combinations of quality and rent (s', q') and (s'', q''), where $s', s'' \in \{s_1, ...s_I\}$ and $q', q'' \in \mathbb{R}_+$. Suppose $s'' > s'$ and $q'' > q'$. With u exhibiting strictly increasing differences with respect to θ and s, we have $u(\theta'', s'') - u(\theta', s'') \gtreqless u(\theta'', s') - u(\theta', s')$ for $\theta'' \gtreqless \theta'$. Adding and subtracting $w + v(q'', s'')$ on this inequality's l.h.s., and adding and subtracting $w + v(q', s')$ on its r.h.s., gives

$$\begin{aligned} V(\theta'', s'', q'') - V(\theta', s'', q'') &\gtreqless V(\theta'', s', q') - V(\theta', s', q') \\ &\text{for all } \ \theta'' \gtreqless \theta' \ \text{ and } \ (s'', q'') > (s', q') \end{aligned}$$

Put differently, indirect utility V exhibits strictly increasing differences with respect to θ and (s, q). Proofs of (ii) to (iv) are indicated in the Remark. □

Proof of Remark 2:[19]

Given Remark 1, $V(\theta_{i,i+1}, s_{i+1}, q_{i+1}) - V(\theta, s_{i+1}, q_{i+1}) \gtreqless V(\theta_{i,i+1}, s_i, q_i) - V(\theta, s_i, q_i)$ for all $\theta \lesseqgtr \theta_{i,i+1}$. Inserting the indifference condition (2), and also spelling out the resulting pair of inequalities out for segment $i-1$, translates into

$$V(\theta, s_{i+1}, q_{i+1}) \gtreqless V(\theta, s_i, q_i) \quad \text{for all } \theta \gtreqless \theta_{i,i+1} \tag{29}$$

$$V(\theta, s_i, q_i) \gtreqless V(\theta, s_{i-1}, q_{i-1}) \quad \text{for all } \theta \gtreqless \theta_{i-1,i} \tag{30}$$

Hence residents with a taste index θ in the interval $[\theta_{i-1,i},\, \theta_{i,i+1}]$ prefer segment i to either of i's neighbors $i-1$ and $i+1$.

In fact, residents in $[\theta_{i-1,i},\, \theta_{i,i+1}]$ prefer i to *every* other quality. Consider, for example, a much better segment s_k, with $k > i+1$. Specifying the pair of inequalities in (30) for $i = k$ gives $V(\theta, s_k, q_k) \gtreqless V(\theta, s_{k-1}, q_{k-1})$ for all $\theta \gtreqless \theta_{k-1,k}$. Since residents in $[\theta_{i-1,i}, \theta_{i,i+1}]$ exhibit $\theta < \theta_{k-1,k}$, they are more strongly attracted to segment $k-1$ than to k. Applying this argument to successively lower segments until segment $i+1$ is reached, shows that $V(\theta, s_k, q_k) < V(\theta, s_{k-1}, q_{k-1}) < ... < V(\theta, s_i, q_i)$ for all residents with θ in $[\theta_{i-1,i}, \theta_{i,i+1}]$. A similar argument applies to the case where $k < i-1$. □

Proof of Remark 3:

We divide the proof into three steps. First we derive the right derivative of n_i^j with respect to q_i^j at $\bar{q}_i^j$, then we derive its left derivative, then we compare the two. (i) Consider a rent $\hat{q}_i^j$ which strictly exceeds $\bar{q}_i^j$. Resulting variable values are denoted $\hat{\theta}_{i,i+1}^j = \theta_{i,i+1}(\hat{q}_i^j, \bar{q}_{i+1}^j)$, etc. Further, define

$$\hat{m}_i = \bar{V}_i^{j+1} - \hat{V}_i$$

[19] The proof merely is an application of the comprehensive treatment in Topkis (1998).

as the pull that segment $(i, j+1)$ exerts on those initially in segment (i, j). Surely only those with taste parameter θ in $[\hat{\theta}^j_{i-1,i}\,,\,\hat{\theta}^j_{i,i+1}]$ and mobility cost m beyond $\hat{m}_i$ will remain in segment (i, j). In this case, and given independence between m and θ, $\hat{n}^j_i$ simply equals $\bar{n}^j(F(\hat{\theta}^j_{i,i+1}) - F(\hat{\theta}^j_{i-1,i}))(1 - F(\hat{m}_i))$. In contrast, $\bar{n}^j_i$ equals $\bar{n}^j\,(F(\bar{\theta}^j_{i,i+1}) - F(\bar{\theta}^j_{i-1,i}))$.

Now consider the ratio $(\hat{n}^j_i - \bar{n}^j_i)/(\hat{q}^j_i - \bar{q}^j_i)$. This ratio can be expanded into the following expression

$$\begin{aligned} & \bar{n}^j\,\frac{F(\hat{\theta}^j_{i,i+1}) - F(\bar{\theta}^j_{i,i+1})}{\hat{\theta}^j_{i,i+1} - \bar{\theta}^j_{i,i+1}} \cdot \frac{\hat{\theta}^j_{i,i+1} - \bar{\theta}^j_{i,i+1}}{\hat{q}^j_i - \bar{q}^j_i} \cdot (1 - F(\hat{m}_i)) \\ - & \bar{n}^j\,\frac{F(\hat{\theta}^j_{i-1,i}) - F(\bar{\theta}^j_{i-1,i})}{\hat{\theta}^j_{i-1,i} - \bar{\theta}^j_{i-1,i}} \cdot \frac{\hat{\theta}^j_{i-1,i} - \bar{\theta}^j_{i-1,i}}{\hat{q}^j_i - \bar{q}^j_i} \cdot (1 - F(\hat{m}_i)) \\ - & \bar{n}^j\,\frac{F(\hat{m}_i) - F(\bar{m}_i)}{\hat{m}_i - \bar{m}_i} \cdot \frac{\hat{m}_i - \bar{m}_i}{\hat{q}^j_i - \bar{q}^j_i} \cdot (F(\bar{\theta}^j_{i,i+1}) - F(\bar{\theta}^j_{i-1,i})) \end{aligned} \tag{31}$$

where $\bar{m}_i = 0$.

By the implicit function theorem, $\theta_{i,i+1}$ is a differentiable, and hence continuous, function of $\hat{q}^j_i$ for rents $\hat{q}^j_i$ sufficiently close to $\bar{q}^j_i$. Then as we let $\hat{q}^j_i$ approach $\bar{q}^j_i$, $\hat{\theta}_{i,i+1}$ approaches $\bar{\theta}_{i,i+1}$. Since $F(\theta)$ is differentiable, then the first ratio on the first line of (31) converges to $f(\bar{\theta}^j_{i,i+1})$.

Taking limits of all other terms in (31) and applying similar arguments eventually gives the right derivative of n^j_i with respect to q^j_i at $\bar{q}^j_i$:

$$\begin{aligned} \lim_{\hat{q}^j_i \to \bar{q}^{j+}_i} \quad \frac{\hat{n}^j_i - \bar{n}^j_i}{\hat{q}^j_i - \bar{q}^j_i} \quad = & \quad \bar{n}^j\, f(\bar{\theta}^j_{i,i+1}) \frac{\partial \theta^j_{i,i+1}(\bar{q}^j_i\,,\bar{q}^j_{i+1})}{\partial q^j_i} \\ - & \quad \bar{n}^j\, f(\bar{\theta}^j_{i-1,i}) \frac{\partial \theta^j_{i-1,i}(\bar{q}^j_{i-1}, \bar{q}^j_i)}{\partial q^j_i} \\ + & \quad \bar{n}^j \left(F(\bar{\theta}^j_{i,i+1}) - F(\bar{\theta}^j_{i-1,i}) \right) f(\bar{m}_i)\, \frac{\partial V(\bar{q}^j_i)}{\partial q^j_i} \end{aligned} \tag{32}$$

(ii) Consider a rent $\check{q}^j_i$ strictly below $\bar{q}^j_i$. Resulting variable values are $\check{\theta}^j_{i,i+1} = \theta_{i,i+1}(\check{q}^j_i\,, \bar{q}^j_{i+1})$, etc. Given $\check{q}^j_i < \bar{q}_i$, segment (i, j) clearly is more attractive than segments of comparable quality elsewhere. We decompose $\check{n}^j_i$ into natives and immigrants. The ratio of the change in natives to the change in rent can be written as

$$\bar{n}^j\,\frac{\left(F(\bar{\theta}^j_{i,i+1}) - F(\bar{\theta}^j_{i-1,i})\right) - \left(F(\check{\theta}^j_{i,i+1}) - F(\check{\theta}^j_{i-1,i})\right)}{\bar{q}^j_i - \check{q}^j_i}$$

Letting $\check{q}_i^j$ approach $\bar{q}_i^j$ gives the derivative of natives' numbers with respect to q_i^j at $\bar{q}_i^j$, i.e.,

$$\bar{n}^j\, f(\bar{\theta}^j_{i,i+1})\,\frac{\partial \theta^j_{i,i+1}(\bar{q}_i^j)}{\partial q_i^j} \;-\; \bar{n}^j\, f(\bar{\theta}^j_{i-1,i})\,\frac{\partial \theta^j_{i-1,i}(\bar{q}_i^j)}{\partial q_i^j} \tag{33}$$

Next we turn to immigrants to (i,j). Figure 1 (in the text) points to which natives from city $j-1$ may be attracted to (i,j). The Figure shows maximum utility in the three segments $(i-1,j-1)$, $(i,j-1)$ and $(i+1,j-1)$, as well as maximum utility in competing segment (i,j), as functions of the taste index θ. Define

$$\check{m}_i = \check{V}_i^j - \bar{V}_i^{j-1}$$

as the pull that segment (i,j) exerts on those initially in segment $(i,j-1)$. On the one hand, the unknown number of those wanting to emigrate to (i,j), $d\,E$, certainly is smaller than $\bar{n}^{j-1}(F(\check{\theta}^{j-1}_{i,i+1}) - F(\check{\theta}^{j-1}_{i-1,i})\,)F(\check{m}_i)$. On the other hand, this number certainly is greater than $\bar{n}^{j-1}(F(\bar{\theta}^{j-1}_{i,i+1}) - F(\bar{\theta}^{j-1}_{i-1,i}))\,F(\check{m}_i)$. I.e.,

$$\frac{\bar{n}^{j-1}(F(\bar{\theta}^{j-1}_{i,i+1}) - F(\bar{\theta}^{j-1}_{i-1,i}))\,F(\check{m}_i)}{\check{q}_i^j - \bar{q}_i^j} \;\le\; \frac{d\,E}{\check{q}_i^j - \bar{q}_i^j} \;\le\; \frac{\bar{n}^{j-1}(F(\check{\theta}^{j-1}_{i,i+1}) - F(\check{\theta}^{j-1}_{i-1,i}))\,F(\check{m}_i)}{\check{q}_i^j - \bar{q}_i^j}$$

The first and the last term in this series of inequalities converge to the same expression as $\check{q}_i^j$ approaches $\bar{q}_i^j$. Hence so does the middle term, by the "squeezing rule". The derivative of the number of migrants from $j-1$ to j thus is the limit of either the first or the last term in the preceding series of inequalities, and hence can be derived as:

$$\bar{n}^{j-1}\left(F(\bar{\theta}^{j-1}_{i,i+1}) - F(\bar{\theta}^{j-1}_{i-1,i})\right) f(\bar{m}_i)\frac{\partial V(\bar{q}_i^j)}{\partial q_i^j} \tag{34}$$

Adding (33) and (34) gives the left derivative of n_i^j with respect to q_i^j at $\bar{q}_i^j$:

$$\begin{aligned}
\lim_{\check{q}_i^j \to \bar{q}_i^{j-}} \frac{\bar{n}_i^j - \check{n}_i^j}{\bar{q}_i^j - \check{q}_i^j} \;&=\; \bar{n}^j\, f(\bar{\theta}^j_{i,i+1})\,\frac{\partial \theta^j_{i,i+1}(\bar{q}_i^j, \bar{q}_{i+1}^j)}{\partial q_i^j} \\
&-\; \bar{n}^j\, f(\bar{\theta}^j_{i-1,i})\,\frac{\partial \theta^j_{i-1,i}(\bar{q}_{i-1}^j, \bar{q}_i^j)}{\partial q_i^j} \\
&+\; \bar{n}^{j-1}\left(F(\bar{\theta}^{j-1}_{i,i+1}) - F(\bar{\theta}^{j-1}_{i-1,i})\right) f(\bar{m}_i)\frac{\partial V(\bar{q}_i^j)}{\partial q_i^j}
\end{aligned} \tag{35}$$

(iii) Comparing (35) with (32) shows that the derivative at $\bar{q}_i^j$ exists, if the initial allocation is symmetric. □

Proof of Remark 4:

The proof is similar to that of Remark 3. □

Proof of Remark 5:

We show $\partial q_2^j/\partial q_1^j > 0$ first. Assume, to the contrary, that $\partial q_2^j/\partial q_1^j = 0$. (We can exclude the case where $\partial q_2^j/\partial q_1^j < 0$. This would be inconsistent with the fact given in the text that $\partial q_i/\partial q_1 \geq 0$ must hold for all $i = 2, ..., I$.) If setting $i = 2$, choosing $\alpha = q_1$, and dropping the city index, equation (9) becomes

$$- (\sigma_2 + \varepsilon_2 + \eta_2) \frac{\partial q_2}{\partial q_1} \; + \; nf(t_{2,3}) \frac{\partial t_{2,3}}{\partial q_1} \; + \; \left(-nf(\theta_{1,2}) \frac{\partial \theta_{1,2}(q_1, q_2)}{\partial q_2} \frac{\partial q_2}{\partial q_1} \right) = n\, f(\theta_{1,2}) \frac{\partial \theta_{1,2}(q_1, q_2)}{\partial q_1} \tag{36}$$

where $\partial t_{2,3}/\partial q_1$ is shorthand for

$$\frac{\partial t_{2,3}}{\partial q_1} = \frac{\partial \theta_{2,3}(q_2, q_3)}{\partial q_2} \frac{\partial q_2}{\partial q_1} + \frac{\partial \theta_{2,3}(q_2, q_3)}{\partial q_3} \frac{\partial q_3}{\partial q_1} \tag{37}$$

The combined effects transmitted by changes in demand, migration as well as housing supply, are contained in the first term on the l.h.s. of (36). This term must be zero. After all, $\partial q_2/\partial q_1 = 0$ by assumption.

Next, filtering vis-à-vis the third segment, as the second term on the l.h.s. of (36), must be non-negative. After all, $\partial q_3/\partial q_1 \geq 0$, as established in the text, while $\partial q_2/\partial q_1 = 0$, by assumption. Hence $\partial t_{2,3}/\partial q_1 \geq 0$, by inserting (3) into (37).

Finally, the third term on the l.hs. of (36) must be zero, given the assumption that $\partial q_2/\partial q_1 = 0$. Hence the l.h.s. of (36) is non-negative. This contradicts the fact that the r.h.s. of (36) is strictly negative (see (3)). We conclude that $\partial q_2/\partial q_1 > 0$. Repeating this argument for successively higher quality segments reveals that $\partial q_i/\partial q_1$ for all $i = 2, \ldots, I$. The proof for showing that $\partial q_i/\partial z_2 > 0$ for all $i = 2, \ldots, I$ is similar. □

Proof of Remark 6:

(i) We focus on the case where $\alpha = q_1$ first. The I-th segment version of (14) reads

$$nf(t_{I-1,I}) \frac{\partial t_{I-1,I}}{\partial q_1} = - (\sigma_I + \varepsilon_I + \eta_I) \frac{\partial q_I}{\partial q_1} \tag{38}$$

Following Remark 5 the r.h.s. of (38) is strictly negative. Then so must be its l.h.s. Thus $\partial t_{I-1,I}/\partial q_1 < 0$.

Next, consider the $(I-1)$-th segment version of (14):

$$\begin{aligned} nf(t_{I-2,I-1})\frac{\partial t_{I-2,I-1}}{\partial q_1} &= nf(t_{I-1,I})\frac{\partial t_{I-1,I}}{\partial q_1} \\ &+ \left(-\sigma_{I-1}-\varepsilon_{I-1}-\eta_{I-1}\right)\frac{\partial q_{I-1}}{\partial q_1} \end{aligned} \tag{39}$$

As just seen, the first term on the r.h.s. of (39) is strictly negative. Given $\partial q_{I-1}/\partial q_1 > 0$ (Remark 5), the second term on the r.h.s. of (39) is strictly negative, also. But then so must be the l.h.s. of (39). Thus, $\partial t_{I-2,I}/\partial q_1 < 0$. Proceeding in this fashion towards successively lower qualities will show that derivatives of all boundaries (except for $t_{0,1}$) with respect to q_1 are strictly negative.

(ii) Now focus on the case where $\alpha = z_2$. Analysis of derivatives of boundaries with respect to z_2 is identical to the analysis in the previous paragraph for $i = 3, \ldots I$. These derivatives are all strictly negative, too.

Treating the case of $\partial t_{1,2}/\partial z_2$ is even simpler. The $i = 2$ version of (13) reduces to:

$$\frac{\partial t^j_{1,2}}{\partial z^j_2} = \frac{\partial \theta^j_{1,2}}{\partial q^j_2}\frac{\partial q^j_2}{\partial z^j_2}$$

which is strictly positive. □

Proof of Remark 8:

The proof is by contradiction. Assume that Remark 8 is not true. Then there must exist a segment $i \in \{2, \ldots, I\}$ for which both

$$\left(\frac{\partial q_i}{\partial \alpha}\right)^c \leq \frac{\partial q^j_i}{\partial \alpha^j} \qquad \text{and} \qquad \left(\frac{\partial q_{i-1}}{\partial \alpha}\right)^c \geq \frac{\partial q^j_{i-1}}{\partial \alpha^j} \tag{40}$$

hold.

Let us explore i's upper neighbor $i+1$. If $i+1 \leq I$ and if $(\partial q_{i+1}/\partial\alpha)^c \leq \partial q^j_{i+1}/\partial\alpha^j$ then we continue by investigating $i+2$. Else we stop. Suppose we stop after l steps, where $l \geq 0$. Then our procedure constructs a "connected sequence" $M = i, i+1, \ldots, i+l$ of adjacent quality segments, of length $l+1$.

Consider M's lower neighbor $i-1$ first. By definition of M, this segment either enjoys a stronger rent rise (if $i > 2$), or the same rent rise (if $i = 2$), under coordination than under isolated action. Hence $(\partial t_{i-1,i}/\partial\alpha)^c \leq \partial t^j_{i-1,i}/\partial\alpha^j$, by inspection of (13).

Likewise, consider M's upper neighbor $i+l+1$ next. If $i+l+1 > I$ then this upper neighbor does not exist. Then $(\partial t_{i+l,i+l+1}/\partial\alpha)^c = \partial t^j_{i+l,i+l+1}/\partial\alpha^j = 0$.

Alternatively, if $i + l + 1 \leq I$ then this upper neighbor exists, and by M's definition experiences a strictly stronger rent rise under coordination. I.e.,

$$\left(\frac{\partial q_{i+l+1}}{\partial \alpha}\right)^c > \frac{\partial q^j_{i+1+l}}{\partial \alpha^j}$$

Hence $(\partial t_{i+l,i+l+1}/\partial\alpha)^c > \partial t^j_{i+l,i+l+1}/\partial\alpha^j$. To summarize both cases in one statement, $(\partial t_{i+l,i+l+1}/\partial\alpha)^c \geq \partial t^j_{i+l,i+l+1}/\partial\alpha^j$ always.

Recursive substitution in (14) l times gives

$$\begin{aligned} nf(t^j_{i-1,i})\left(\frac{\partial t^j_{i-1,i}}{\partial \alpha^j}\right) &= -\sum_{k=0}^{l}\left(\sigma^j_{i+k} + \varepsilon^j_{i+k} + \eta^j_{i+k}\right)\left(\frac{\partial q^j_{i+k}}{\partial \alpha^j}\right) \\ &\quad + nf(t^j_{i+l,i+l+1})\left(\frac{\partial t^j_{i+l,i+l+1}}{\partial \alpha^j}\right) + \gamma_i \qquad (41) \end{aligned}$$

in the uncoordinated case, and

$$\begin{aligned} nf(t^j_{i-1,i})\left(\frac{\partial t_{i-1,i}}{\partial \alpha}\right)^c &= -\sum_{k=0}^{l}\left(\varepsilon^j_{i+k} + \eta^j_{i+k}\right)\left(\frac{\partial q_{i+k}}{\partial \alpha}\right)^c \\ &\quad + nf(t^j_{i+l,i+l+1})\left(\frac{\partial t_{i+l,i+l+1}}{\partial \alpha}\right)^c + \gamma_i \qquad (42) \end{aligned}$$

in the coordinated case, with $\gamma_i = (\partial z^j_2/\partial\alpha^j) = (\partial z_2/\partial\alpha)^c$ if $i = 2$ and zero otherwise.

Now we make use of the rankings of boundary changes under coordination and under isolated action just derived. Subtracting the l.h.s. of (42) from the l.h.s. of (41) gives a non-negative number. Subtracting the r.h.s. of (42) from the r.h.s. of (41) gives a strictly negative number, given our assumption (40). Either the first or the second equation cannot be satisfied. This contradicts the model's assumptions. □

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