A theory of industry location and residence choice

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Abstract

We present a new economic geography model in which the location of industry and residences are jointly determined. Individuals choose their place of residence and place of work independently, that is, they may work in their region of residence or commute to the other region. The model produces either dispersion or agglomeration of residences depending on the strength of demand and supply linkages, a competition effect in industrial production, and housing market congestion. Interestingly, we find that commuting leads to more agglomeration of residences when trade costs are either sufficiently low or high.

JEL Classification: R12, R23

Key words: agglomeration, geographical mobility, industry location, migration, commuting

1 Introduction

Most workers today commute to work, many over long distances. For instance, in Germany about 17 % of commuters commute more than 25 km one-way and 5 % commute more than 50 km (Statistisches Bundesamt, 2005). Comparing OECD data (OECD, 2000) with results of a study commissioned by the European Commission (MKW, 2001) reveals that commuting activity is even higher in Japan, the UK, Canada or Australia as compared to the European Union. There is further evidence that not only the number of interregional commuters, but also commuting distances have increased over time, at least in the Netherlands (see Vermeulen, 2003). Cross-border commuting within Europe has also increased (see MKW, 2001; Mathä and Wintr, 2007). Obviously, the advantages of living in city A and working in B must outweigh the costs of commuting. Interestingly enough, there is little systematic analysis of this topic, at least as far as theory is concerned.

This paper examines geographical mobility in a two-region model with agglomeration effects. The basic idea is that individuals can choose their residence and place of work independently. In the model, individuals consume industrial goods in their region of residence, and the price index of these goods negatively depends on the number of firms. Individuals also consume housing, whose price rises with the agglomeration of residences. The wage rate of mobile workers is also influenced by the number of workers. Commuting occurs from the low-wage region to the high-wage region such that in (an interior) equilibrium wages net of commuting costs are equalized. The extent of net commuting depends upon the level of commuting costs and trade freeness.

The model without commuting (i.e. with prohibitively high commuting costs) has equilibria with dispersion for low and high trade freeness and either partial or full agglomeration for intermediate levels of freeness. Long distance commuting alters these results: When trade is sufficiently closed, the large region (in terms of resident population) has a lower wage than the smaller one, and commuting therefore takes place from the large to the small region. Conversely, with relatively free trade, the larger region has a higher wage and therefore mobile workers commute from the small to the large region.

What then are the effects of commuting on the agglomeration of residences? Intuitively, one might think that individuals' residence choices lead to less agglomeration than without commuting, since with commuting one can live in small cities where housing is cheap and work in large cities where wages are high. However, this turns out not to be correct. In fact we find that residences are more agglomerated than without commuting, in general. This result is best thought of in terms of the interplay of the four fundamental location forces of the model: market size (the "demand linkage"), the local price index of consumer goods (the "supply linkage"), local competition (the "competition effect") and congestion in the housing market. The location of industry and residences is simultaneously determined by these forces in the underlying model without commuting and reflects the relative strength of these forces at various stages of trade integration: at low levels of trade freeness, the dispersive effect of local competition is dominant whereas at high levels of trade freeness the location decision is determined by housing prices, again implying a dispersed outcome. At intermediate ranges of trade freeness, the supply linkage and the demand linkage induce an agglomeration of economic activity.

With commuting, this simultaneity is broken. The commuting decision is guided by the wage differential across locations which itself only depends on market size and the state of local competition. The residence choice is directly affected only by the local prices of goods and housing. The other two forces affect the choice of residences only indirectly, namely through the anticipation of the decision at the commuting stage. Intuitively, the supply linkage is then the dominant force at low levels of trade freeness. Hence, commuting fosters agglomeration when trade is relatively closed. At high levels of trade freeness, on the other hand, the residence choice is dominated by the dispersive effect of housing prices in the model with commuting, as well. At intermediate ranges of trade freeness the balance of forces is more complex. Without commuting, the demand linkage is usually a force which strongly fosters agglomeration. With commuting, however, market size exerts only an indirect effect on the residence choice. We show that the threshold level of trade freeness at which full agglomeration is just sustained is lower with commuting than without. Hence, the finding that commuting fosters agglomeration is reversed at a range of intermediate trade costs.

Wie view the contributions of the paper as threefold. First, we provide a unified theory of geographical mobility, where the location of industry and residence are jointly determined. Second, we view our analysis as a contribution in the ongoing research program of the new economic geography which seeks to understand the fundamental forces of the location of economic activity. In particular, we are able to shed light on the question how the market equilibrium's propensity to agglomerate is affected by commuting. Third, and related to the previous point, our analysis builds a bridge between two well-known models widely used in the new economic geography (see Baldwin et al, 2003): the footloose-entrepreneur model of Forslid and Ottaviano (2003) which is the basis of the residence choice in our analysis, and the footloose-capital model of Martin and Rogers (1995). In fact, our commuting model is isomorphic to the footloose capital model: instead of capital, however, in our model it is workers who move. As in the footloose capital model where capital owners spend their income at their place of residence, commuters also spend their income where they live.

The theoretical literature on long-distance commuting is still scant. The traditional urban model of a monocentric city looks at commuting within cities. Applications of *intraurban* commuting to NEG models include Krugman and Livas Elizondo (1995), Tabuchi (1998) and Murata and Thisse (2005). There is also a literature on non-monocentric cities where jobs are dispersed, but again, it is generally assumed that while individuals may have long or short commutes, they commute within cities (see, e.g., Fujita and Ogawa, 1982; White, 1988). Zax (1994) even argued that workplace and residence relocations are substitutes in intra-regional mobility, but are complements in inter-regional mobility (i.e., workers who relocate between regions also tend to change jobs). But due to new high-speed trains long-distance commuting costs were substantially reduced over the last decades at least in Japan and Europe. Perfect complementarity of quits and long distance moves does no longer exist. Hence, there is need for an analysis of simultaneous job and residence locations in an interregional framework. Ogura (2005) analyzes intercity commuting in a model where cities may impose growth controls. His argument is that growth controls increase wages which attracts commuters from other cities. In addition, search theory models have been applied to analyze job and residential mobility outside a standard urban economics framework (for an overview see Van Ommeren, 2000).

This paper proceeds as follows. We present the model in the next section. The choice of workplace and residence is analyzed in section 3, and the last section concludes the paper.

2 The model

Our model extends the model of Pflüger and Südekum (2006) to include long-distance commuting. The model is a variant of the 'footloose entrepreneur' model of Forslid and Ottaviano (2003), which itself is an (almost) analytically solvable variant of Krugman (1991) agglomeration model. In contrast to the Forslid-Ottaviano model, the Pflüger-Südekum model has a quasilinear upper-tier utility function (see also Pflüger, 2004) and an additional congestion force in the form of a fixed housing supply.

The economy consists of two regions called home (H) and foreign (F) which are ex ante symmetric in terms of preferences, technology, trade costs and labor endowments. Each region is endowed with L units of land. There are two sectors. A manufacturing sector (X) characterized by increasing returns, monopolistic competition and iceberg trade costs produces a composite of manufacturing varieties. A perfectly competitive sector labelled agriculture (A) produces a homogeneous good with constant returns to scale. The A-good is traded without costs. This good is produced in both countries and is taken as the numéraire, i.e., its price is normalized to one.

Furthermore, there are two types of workers, skilled and unskilled labor. While unskilled labor is interregionally immobile, it is mobile between sectors, i.e., it can be employed in agriculture as well as manufacturing. Skilled workers on the other hand are mobile between regions but are employed only in manufacturing. The number of skilled workers may also be thought as the supply of human capital. Skilled workers are also referred to as entrepreneurs (see below). The number (i.e., mass) of immobile unskilled workers in each region is denoted by N and the total number (mass) of mobile workers is H^w. We will define $\rho \equiv N/H^w$ as the relative number of immobile workers.

Mobile skilled workers may either commute (at a round trip cost of $t \geq 0$) or migrate costlessly from one region to the other. Commuters buy the composite good completely in their region of residence.[1] Thus commuters earn the wage of the region to which they commute, while consuming land and other goods at home.

[1] An extension would let mobile workers consume also at their place of work.

Each consumer has a quasi-linear utility function given by

$$U = C_A + \mu \ln C_X + \eta \ln C_L - [\mu (\ln \mu - 1) + \eta (\ln \eta - 1)], \quad \text{with } C_X = \left(\int_0^M x_i^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

where $\sigma > 1$. C_A is consumption of the numéraire good, C_L is land consumption, C_X is consumption of the manufacturing aggregate as in Dixit and Stiglitz (1977), x_i is the quantity of variety i, M is the mass of varieties produced in the manufacturing sector and σ is the elasticity of substitution between any two manufacturing varieties. For an individual with net income y, the budget constraint is

$$C_A + \int_0^M p_i x_i di + QC_L = y, \tag{2}$$

where p_i denotes the consumer price of variety i and Q the price of land. Utility maximization leads to the demand functions x_i, C_X , C_L and C_A and indirect utility V:

$$x_i = \mu P^{\sigma-1} p_i^{-\sigma}, \quad C_X = \mu/P, \quad C_L = \eta/Q, \quad C_A = y - \mu - \eta, \tag{3}$$

$$V = y - \mu \ln P - \eta \ln Q, \tag{4}$$

$$\text{where } P \equiv \left(\int_0^M p_i^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} \tag{5}$$

is the perfect CES price index.

The numéraire good is produced with labor as the only input under a linear technology: $X_A = N_A$, where N_A is labor input and X_A is output. Perfect competition leads to marginal cost pricing in this sector. By implication, the agricultural wage is equal to the marginal product of labor, i.e. one.

Industrial firms produce with a non-homothetic technology. Each firm requires one unit of human capital and c units of unskilled labor for each unit of output produced. Since each firm produces with one skilled worker, skilled workers are also called entrepreneurs. Total costs of a firm which produces variety i are $\pi + cX_i$, where π is the wage of a skilled worker and X_i is output of this firm.

Transport costs are of the iceberg type: only $1/\tau$ of a unit bought in the other region can be used for consumption, with $\tau \geq 1$. Hence, indicating producer prices by a hat, in the region where the variety is produced we have $p_i = \hat{p}_i$ and in the other region $p_j = \tau \hat{p}_j$.

In the Dixit-Stiglitz model of monopolistic competition mill pricing is optimal. Since the model is symmetric, we will henceforth look at the home region only, the corresponding expressions for the foreign region being completely analogous. Profits of firm i in home are

$$\Pi_i^H = (\hat{p}_i - c)X_i^H + (\hat{p}_i - c)\tau X_i^F - \pi^H, \tag{6}$$

where total output is $X_i = X_i^H + \tau X_i^F$. Maximizing producer profits gives the optimal producer price of each variety:

$$\hat{p}_i = \hat{p} = \frac{c\sigma}{\sigma - 1}. \tag{7}$$

We normalize marginal costs to $c = (\sigma-1)/\sigma$, which gives $\hat{p} = 1$ and, hence, the consumer price index of the manufacturing good in home is

$$P^H = (s_K + \phi(1 - s_K))^{\frac{1}{1-\sigma}}, \tag{8}$$

where s_K stands for the share of skilled workers that is employed in the home region, and $1 - s_K$ is the foreign region's share. $\phi \equiv \tau^{1-\sigma}$ stands for the degree of trade freeness with $0 < \phi \leq 1$. Since each firm requires one unit of human capital, the number of varieties is equal to the number of employed skilled workers: $M_H = s_K H^w$, where M_H is the number of varieties produced in home. The world endowment H^w has been normalized to one.

We can now complete the characterization of the short-run equilibrium of the model. In the monopolistic competition framework, competition drives pure profits to zero and the reward to human capital is then equal to the operating profit. Using $\hat{p} = 1$, equations (3) and (8), and denoting the number of skilled workers living in home by s_L, aggregate local demand for variety i is

$$X_i^H = \frac{\mu(\rho + s_L)}{s_K + \phi(1 - s_K)}. \tag{9}$$

This equation displays the difference between our model and models without commuting: in the latter, residents are also workers (or entrepreneurs) while in our model, skilled workers need not reside at their place of work. If individuals commute from F to H we have $s_K > s_L$, i.e., the share of domestic firms exceeds the share of skilled workers living in home.

Using (9) and applying the zero profit condition to (6), we find the wage of a skilled worker employed in the home region:[2]

$$\pi^H(s_L, s_K, \phi) = \frac{\mu}{\sigma}\left(\frac{\rho + s_L}{s_K + \phi(1 - s_K)} + \frac{\phi\,(\rho + 1 - s_L)}{1 - s_K + \phi s_K}\right). \tag{10}$$

The wage differential across the two locations is of particular relevance in the ensuing analysis of commuting. It is given by

$$\Delta\pi(s_L, s_K, \phi) = \frac{\mu(1-\phi)}{\sigma}\left(\frac{\rho + s_L}{s_K + \phi(1 - s_K)} - \frac{\rho + 1 - s_L}{1 - s_K + \phi s_K}\right). \tag{11}$$

where $\Delta\pi(s_L, s_K, \phi) \equiv \pi^H(s_L, s_K, \phi) - \pi^F(s_L, s_K, \phi)$.

The price of land is determined by the market equilibrium condition. Letting L be the fixed supply of land, supply must equal aggregate demand, or:

$$L = \frac{\eta(\rho + s_L)H^w}{Q^H}.$$

We assume a three-stage model. At the first stage, individuals choose their place of residence. The migration equation is given by the following myopic ad hoc dynamic equation which is standard in agglomeration models with mobile labor (Ottaviano and Thisse, 2004):

$$\dot{s}_L = \Delta V(s_L, \phi) s_L (1 - s_L),$$

where $\Delta V(s_L, \phi) \equiv V^H(s_L, \phi) - V^F(s_L, \phi)$, so that the share of skilled workers in Home increases when skilled labor realizes higher utility in H than in F. A spatial equilibrium arises at $s_L \in (0, 1)$ when $\Delta V(s_L, \phi) = 0$, or at $s_L = 0$ when $\Delta V(0, \phi) \leq 0$, or at $s_L = 1$ when $\Delta V(1, \phi) \geq 0$.

At the second stage, skilled workers choose whether or not to commute, for given residences. This will determine the allocation of entrepreneurs across regions, for given s_L. Their choice is based upon a comparison of commuting costs and the entrepreneurial wage differential across locations $\Delta\pi(s_L, s_K, \phi)$. Commuting follows the myopic adjustment dynamics which is familiar from the footloose capital model (see Martin and Rogers, 1995;

[2] Since our focus is on the allocation of workers and residences between regions in a process of trade integration which is symbolized by the trade freeness parameter ϕ, we suppress the dependence of wages on the other parameters for convenience.

Baldwin et al, 2003; Ottaviano and Thisse, 2004):

$$\begin{aligned}
\dot{s}_K &> 0 \quad \text{if} \quad \Delta\pi(s_L, s_K, \phi) > t \quad \text{and} \quad s_K < 1 && (12)\\
\dot{s}_K &< 0 \quad \text{if} \quad \Delta\pi(s_L, s_K, \phi) < -t \quad \text{and} \quad s_K > 0 && (13)\\
\dot{s}_K &= 0 \quad \text{otherwise} && (14)
\end{aligned}$$

When the skilled wage in H minus commuting costs exceeds the foreign skilled wage, additional entrepreneurs commute from F to H, as shown by (12). The converse case is in equation (13). If the entrepreneurial wage differential is smaller in absolute terms than the level of commuting costs, no additional commuting will occur as shown by (14). From $\Delta\pi(s_K, s_K, \phi)$ (which simply is $\Delta\pi(s_L, s_K, \phi)$ evaluated at $s_L = s_K$) the direction of commuting flows can be obtained. If $\Delta\pi(s_K, s_K, \phi) > t$ $(< -t)$, in equilibrium $s_K > (<) s_L$. If $-t \leq \Delta\pi(s_K, s_K, \phi) \leq t$, no commuting will occur. Hence, there is a band of inaction, very similar to the no delocation band in the footloose capital model with relocation costs (see Baldwin et al, 2003).

In an interior equilibrium where some individuals commute from F to H we must have

$$\Delta\pi(s_L, s_K, \phi) = t, \tag{15}$$

and likewise if commuting occurs from H to F it must be true that

$$\Delta\pi(s_L, s_K, \phi) = -t. \tag{16}$$

Finally, at the third stage, individuals work, buy and sell, for given residence and commuting patterns. This gives the short-run equilibrium we have just described, for given s_L, s_K.

As usual, we solve the game backwards. Using the short-run industry equilibrium and taking the location of residences as given, we therefore start by describing the equilibrium at the commuting stage. Afterwards, we analyze the residence choice stage.

3 Location and commuting choice

3.1 Prohibitive commuting costs

As a benchmark, consider the case where commuting costs are prohibitively high. If $t \rightarrow \infty$, commuting will not occur and the equilibrium of the model corresponds to the equilibrium in the original Pflüger-Südekum model. Individuals then live and work in the same region and $s_K = s_L$.

For very low or very high trade costs, the only stable equilibrium is the symmetric equilibrium. There are two bifurcation points where a symmetric equilibrium loses (the break point ϕ_B^1) and regains (the redispersion point ϕ_B^2) local stability, respectively.[3] These bifurcations are 'sub-critical pitchforks' (see e.g. (Grandmont, 1988) for an elaboration). As the level of trade freeness is gradually increased beyond the break point, two stable equilibria emerge, where one region contains an increasing share of industry. At ϕ_S^1 (where $V^H = V^F$ for $s_L = 0$ or $s_L = 1$) all manufacturing activity just happens to be in one of the two regions. As the degree of trade freeness is increased further, partial agglomeration re-emerges at ϕ_S^2 and a smooth redispersion sets in. Hence, $0 < \phi_B^1 < \phi_S^1 < \phi_S^2 < \phi_B^2 < 1$.[4]

The location equilibria are determined by the trade-off between the model's agglomeration and dispersion forces, as is well known from other new economic geography models. First, there is a supply linkage. The region with the higher share of the mobile factor has a larger manufacturing sector, and therefore a lower manufacturing price index. Second, there is a demand linkage. Increasing the share of the mobile factor in one region implies a larger local market. This raises the relative profitability of this market and the skilled wage differential $\Delta\pi$. A stabilizing (dispersive) effect derives from the fact that, shifting firms from the foreign to the domestic region increases competition for given expenditures on domestic products, while lowering competition in the other region. Finally, there is a fourth effect, a further dispersion force, deriving from rising relative housing prices with rising relative resident population, which is independent of trade costs.

These forces will play an important role in the model with commuting as well. In par-

[3] Analytical details of this and the other results of this section are derived in Pflüger and Südekum (2006) and summarized in Appendix A.

[4] Again, see Pflüger and Südekum (2006).

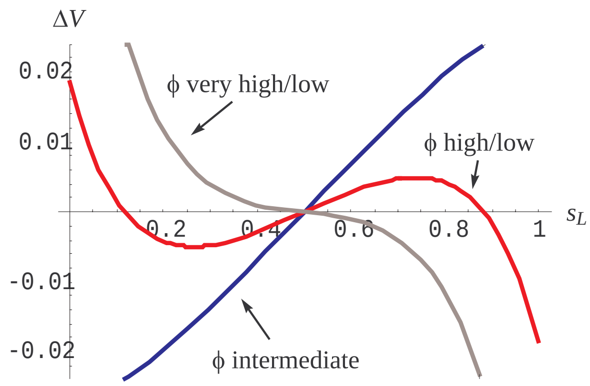

Figure 1: ΔV without long-distance commuting

ticular, we will repeatedly use the fact that the sum of the demand linkage and competition effect is positive (negative) if and only if $\phi > (<) \hat{\phi} \equiv \rho/(1+\rho)$, since $d\Delta\pi(s_K, s_K, \phi)/ds_K > (=, <)\, 0 \;\Leftrightarrow\; \phi > (=<)\, \hat{\phi}$: when trade is relatively free (closed) the demand linkage is stronger (weaker) than the competition effect and, by implication, the larger (smaller) region has the higher entrepreneurial wage rate.[5] Note that the threshold $\hat{\phi}$ rises with the share of the immobile unskilled, rho, since it is the local demand of these which strengthens the (dispersive force) of the competition effect.

Figure 1 shows the difference in utility for varying s_L and ϕ. The parameters are $\eta = 0.2$, $\mu = 0.5$, $\sigma = 3$, and $\rho = 1$. The bifurcation diagram is shown in Figure 2.

3.2 Equilibrium with zero commuting costs

We are now interested in how location equilibria are affected by the possibility of commuting. If individuals commute in equilibrium, we have to distinguish between agglomeration of jobs and agglomeration of residences. We will proceed by first describing in detail the commuting equilibrium, i.e., for given residence choice we identify the pattern of commuting.

[5]It follows from the analysis in Pflüger and Südekum (2006) that $\phi_S^1 < \hat{\phi} < \phi_S^2$.

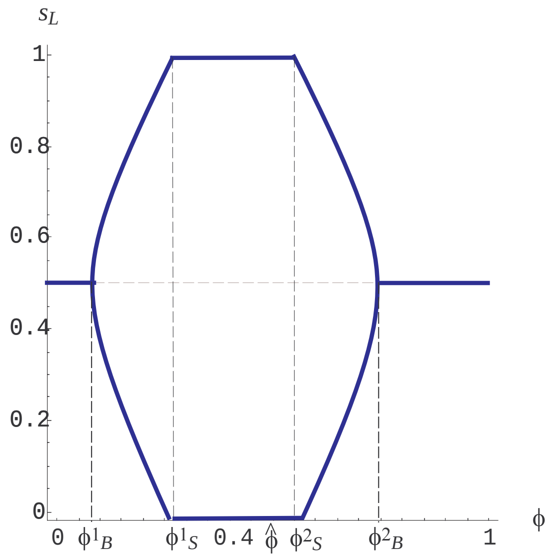

Figure 2: Bifurcation diagram without long-distance commuting

3.2.1 Commuting equilibrium

Since differences in net wages determine commuting patterns, commuting always occurs in one direction only. The pattern of commuting depends on the level of trade freeness which crucially affects the entrepreneurial wage differential across locations. In our analysis in the previous section, we have already identified the critical level $\hat{\phi} \equiv \rho/(1+\rho)$. If the level of trade freeness exceeds (falls short of) this threshold, the wage differential without commuting, $\Delta\pi^o \equiv \Delta\pi(s_L, s_L, \phi)$, is positive (negative) in the larger region. Hence, workers commute from the small to the large region (vice versa) when $\phi > \hat{\phi}$ ($\phi < \hat{\phi}$).

The incentive to commute, the commuting equilibrium and the entrepreneurial wage differential which results in such a commuting equilibrium (and which is relevant for the location equilibrium at the first stage) are illustrated in two part figure 3. Red curves depict the case where the level of trade freeness falls short of the critical level $\phi < \hat{\phi}$. The case where the level of trade freeness exceeds this critical level is illustrated by blue curves.

Start with the case of high trade freeness ($\phi > \hat{\phi}$). The blue line in the upper panel of figure 3 shows that $\Delta\pi(s_L, s_L, \phi) > (<)0$ for $s_K = s_L > (<)1/2$ and, hence, there is commuting into (out of) the domestic region. If the share of residents s_L in the domestic region is neither too large nor too small, an interior commuting equilibrium obtains in which the wage differential is eliminated. From (11), we can solve for the equilibrium number of entrepreneurs in home, $\tilde{s}_K$, as a function of the skilled population living in home, s_L. For this equilibrium we have $\Delta\pi(s_L, \tilde{s}_K, \phi) = 0$:

$$\tilde{s}_K(s_L, \phi) = \frac{1+\phi}{(1-\phi)(1+2\rho)} s_L - \frac{\phi - (1-\phi)\rho}{(1+2\rho)(1-\phi)}. \tag{17}$$

This relationship is depicted by the (linear) upward-sloping part of the blue curve in the lower panel of figure 3. If the domestic share of residents exceeds (falls short of) the threshold $\overline{s}_L$ ($\underline{s}_L$), everyone works in H (F) and a wage differential remains: when everyone works in home, $\Delta\pi(s_L, 1, \phi) > 0$ and conversely, when everyone works in foreign, $\Delta\pi(s_L, 0, \phi) < 0$. The threshold levels of an interior equilibrium at the third stage if $\phi > \hat{\phi}$ are

$$\overline{s}_L = \frac{1 + (1-\phi)\rho}{1+\phi} > \frac{1}{2} \quad \text{and} \quad \underline{s}_L = \frac{\phi - (1-\phi)\rho}{1+\phi} < \frac{1}{2}. \tag{18}$$

Note that these threshold levels depend on ϕ. If $\phi = 1$, wages are independent of the location choice. Hence, s_K is indeterminate (and the blue curve in the lower panel of figure 3 vertical).

Turning to the case of low trade freeness ($\phi < \hat{\phi}$), the red line in the upper panel of figure 3 indicates that $\Delta\pi(s_L, s_L, \phi) > (<) 0$ for $s_L = s_K < (>) 1/2$ and mobile workers commute into (out of) the domestic region for $s_L = s_K < (>) 1/2$ thereby eroding any wage differential. The commuting equilibrium is depicted by the red upward-sloping line which is analytically given by (17). Figure 3 reveals that corner solutions ($s_K = 0$ or $s_K = 1$) are ruled out when $\phi < \hat{\phi}$.

Drawing these two cases together, the mass of mobile workers in the home region is given by:

$$s_K = \max[0, \min[\tilde{s}_K(s_L, \phi), 1]] \tag{19}$$

for $\phi < 1$ and $s_K \in [0, 1]$ for $\phi = 1$.

Observe that the relationship between s_K and s_L expressed in equations (17) and (19) and depicted in the lower panel of figure 3 is always non-negative for all levels of (non-complete) trade freeness. Increasing the share of skilled residents in the domestic economy has a positive effect on the domestic industry share (less/more than one-for-one for low/high

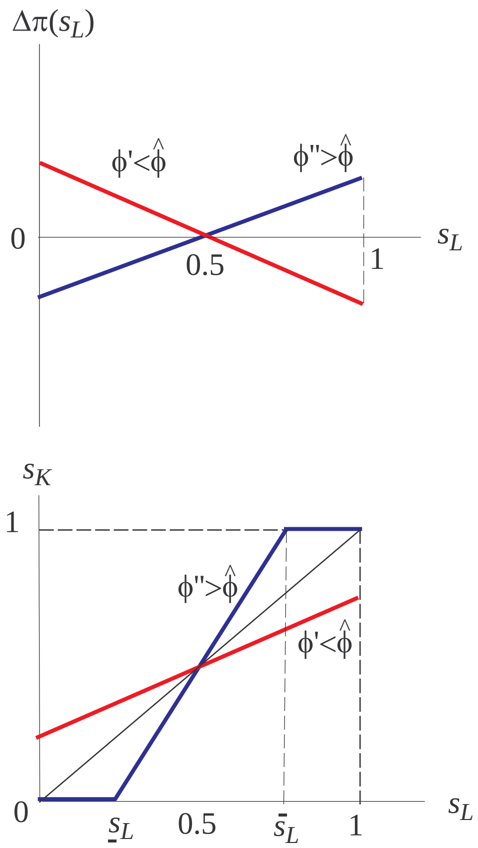

Figure 3: Commuting pattern with $t = 0$

levels of trade freeness) in the commuting stage unless the domestic economy already houses all of industry.

3.2.2 Location equilibrium

Ultimately, we are interested in the effect of commuting on the location of residences. Therefore, we now describe the effect of commuting on the location equilibrium. In particular, we ask whether commuting leads to more or less agglomeration of residences than what we find in a world without long-distance commuting.

Let us rewrite the utility of an individual choosing to live in H as

$$V^H = \max\{\pi^H, \pi^F\} - \mu \ln P^H - \eta \ln Q^H. \tag{20}$$

An analogous expression holds for an individual choosing to live in F. Equation (20) shows that the choice of residence will affect the price level of manufactured goods and the price of housing but not the wage, since costless commuting means the individual will work in the high-wage region regardless of her residence.

Therefore, the utility difference of a resident between living at home or in foreign can be written as:

$$\Delta V(s_L, \phi) = \mu \ln\left(\frac{P^F}{P^H}\right) + \eta \ln\left(\frac{Q^F}{Q^H}\right) \tag{21}$$

$$= \frac{\mu}{1-\sigma} \ln\left(\frac{1 - s_K + \phi s_K}{s_K + \phi(1 - s_K)}\right) + \eta \ln\left(\frac{\rho + 1 - s_L}{\rho + s_L}\right). \tag{22}$$

with $s_K = s_K(s_L, \phi)$ given by (19). The first term on the right of (21) shows the effect of the difference in the price levels between the regions, and the second term shows the effect of differences in housing prices. Again, note that the difference in wages disappears from the utility differential (21), since commuting means the individual will earn the same wage (namely, $\max\{\pi^H, \pi^F\}$) regardless of her place of residence.

Commuting changes the utility difference and therefore migration incentives through its effect on the price index of industrial goods (the housing market congestion does not depend on the mass of commuters since by assumptions firms occupy no space).

Equation (21) shows the fundamental forces of agglomeration and dispersion in the model with commuting. Since differences of profits have disappeared from this equation, the usual demand linkage and competition effects of agglomeration running through the differences in profits are no longer operative. We are therefore left with the differences in housing prices and the supply linkage running through the price indices of industrial goods.

Looking at relative housing prices first, from (21) we have

$$\frac{d\ln(Q^F/Q^H)}{ds_L} = -\frac{1+2\rho}{(\rho+1-s_L)(\rho+s_L)} < 0. \tag{23}$$

The housing market effect always deters agglomeration and is independent of trade freeness.[6]

Proceeding likewise, from (21) we find the effect of varying s_L on relative manufacturing prices. If $\phi = 1$, consumer good prices are uniform across regions and independent of industry locations and residence choices: $P^F/P^H = 1$. Otherwise:

$$\frac{d\ln(P^F/P^H)}{ds_L} = \frac{d\ln(P^F/P^H)}{ds_K}\frac{ds_K}{ds_L} \tag{24}$$

$$= \frac{-(1-\phi^2)}{(1-\sigma)(1-s_K+\phi s_K)(s_K+\phi(1-s_K))}\frac{ds_K}{ds_L}. \tag{25}$$

Using (17), we get

$$\frac{d\ln(P^F/P^H)}{ds_L} = -\frac{(1+2\rho)}{(1-\sigma)(\rho+1-s_L)(\rho+s_L)} > 0 \tag{26}$$

The cost-of-living effect would seem to depend on ϕ. As shown by (25), increasing the number of firms in H lowers the relative price index in H and fall in relative prices is decreasing in ϕ. However, this effect is just offset by the effect of a larger population on the number of firms through commuting, as shown by (26): when ϕ is low, prices decrease strongly with the number of firms but the number of firms increases less than proportionately with population since commuting occurs from the large to the small region; conversely with high ϕ prices decrease less with the number of firms but commuting implies that the number of firms increases more than proportionately with population.

[6] This follows because of quasilinear utility which implies that the demand for housing does not depend on income.

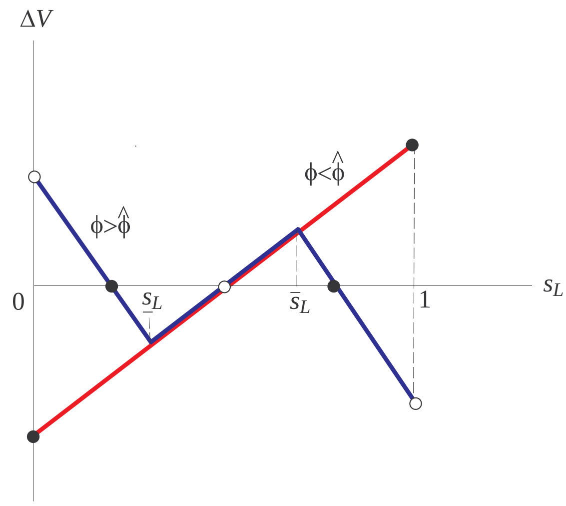

Figure 4: Equilibrium with zero commuting costs

Indeed, in the interior commuting equilibrium, the price effect is independent of the degree of trade freeness and (21) simplifies to

$$\Delta V(s_L, \phi) = \left(\frac{\mu}{1-\sigma} + \eta\right) \ln\left(\frac{\rho + 1 - s_L}{\rho + s_L}\right). \tag{27}$$

Hence, we get

$$\frac{d\left(\Delta V(s_L, \phi)\right)}{ds_L} = -\left(\frac{\mu}{1-\sigma} + \eta\right) \frac{1 + 2\rho}{(\rho + 1 - s_L)(\rho + s_L)} > 0, \tag{28}$$

where the inequality follows from the fundamental parameter restriction imposed in Pflüger and Südekum (2006) to restrict the congestion force associated with the housing sector (see Appendix A).

Interestingly, we find that the utility differential ΔV slopes upward in the interior commuting equilibrium regardless of the level of trade freeness (below $\phi = 1$). Since this holds, in particular, at the symmetric allocation $s_L = 1/2$, we have:

Proposition 1 *When commuting costs are zero, a symmetric equilibrium is always unstable provided that trade costs are positive.*

Figure 4 shows the utility differential and location equilibria with commuting for small and large ϕ.[7] Symmetry ($s_L = 1/2$, and, therefore, $s_K = 1/2$) is always an equilibrium, i.e. $V^H(1/2, \phi) = V^F(1/2, \phi)$. This location equilibrium is unstable for all non-zero levels of trade freeness, however, as we have just shown.

An interior commuting equilibrium always obtains for $\phi < \hat{\phi}$. Hence, the utility difference ΔV slopes upwards for all values $s_L \in [0, 1]$. The red line depicts this case and reveals that we have a unique unstable symmetric equilibrium and two stable equilibria with full agglomeration of residences.

An interior commuting equilibrium also obtains for $1 > \phi > \hat{\phi}$ as long as $\underline{s}_L \leq s_L \leq \overline{s}_L$ (recall that $s_K \in (0, 1)$ in this case). Hence, as depicted by the blue line, the utility difference must also slope upwards in the vicinity of the symmetric equilibrium. However, at the point where everyone works in, say, H, an increase in the resident population has no more effect on the working population (this corresponds to $s_L > \overline{s}_L$ in the lower panel in Figure 3).

Setting $s_K = 1$ in (21) and differentiating for $\phi < 1$ w.r.t. s_L gives:

$$\left.\frac{d\Delta V(s_L, \phi)}{ds_L}\right|_{s_L > \overline{s}_L} = -\frac{\eta(1 + 2\rho)}{(\rho + s_L)(\rho + 1 - s_L)} < 0. \tag{29}$$

Hence, the ΔV curve has a kink at $\overline{s}_L$ and slopes downward for $s_L > \overline{s}_L$. As ϕ gets larger, the utility differential must eventually get negative for large s_L when trade is almost completely free. This is intuitive since when trade is completely free, location plays no role for the agglomeration of firms and the prices of industrial goods. The only effect of increasing population is then increased congestion in the housing market, which implies that as $\phi \to 1$, ΔV must be negative at $s_L = 1$.

Hence, we can conclude that with sufficiently free trade a stable equilibrium with a partial agglomeration of residences emerges. By analogous reasoning, the same holds true at the point when everyone works in F.

Lemma 1 *(i) There exists a level of trade freeness $\bar{\phi}$, with $\bar{\phi} > \hat{\phi}$, such that $\Delta V(1, \bar{\phi}) = 0$. (ii) For $1 > \phi > \bar{\phi}$ and $s_L > 1/2$ ($s_L < 1/2$), there exists a unique s_L, with $s_L < 1$ ($s_L > 0$),*

[7] ΔV ist strictly concave for $s_L \in (0, 1/2)$ and strictly convex for $s_L \in (1/2, 1)$. For our set of parameters, the curve is close to linear.

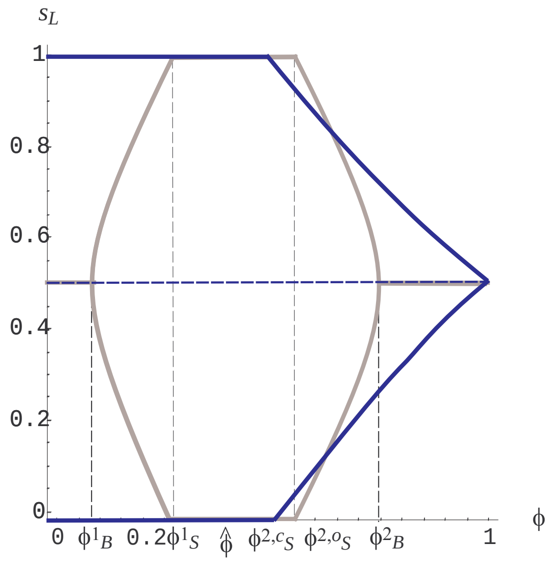

Figure 5: Bifurcation diagram with zero commuting costs

such that $\Delta V(s_L, \phi) = 0$ *and* $d\Delta V(s_L/\phi)/ds_L < 0$. *(iii) Furthermore,* $\Delta V(1/2, 1) = 0$ *and* $d\Delta V(1/2, 1)/ds_L < 0$.

Proof. See Appendix B. ■

Lemma 1 directly implies the following result:

Proposition 2 *Suppose commuting costs are zero. Then, (i) if* $\phi < \bar{\phi}$*, there exist an unstable symmetric equilibrium and two stable equilibria with full agglomeration, one with* $s_L = 1$ *and one with* $s_L = 0$*; (ii) if* $1 > \phi > \bar{\phi} > \hat{\phi}$*, there exist an unstable symmetric equilibrium and two stable asymmetric equilibria with partial agglomeration of residences, one with* $s_L > 1/2$ *and one with* $s_L < 1/2$*; (iii) if* $\phi = 1$*, there exists a unique stable symmetric equilibrium.*

Figure 5 shows the bifurcation diagram, comparing the case of zero commuting costs (dark lines) with the case of prohibitive costs (light lines).

Proposition 2 together with (19) implies the following for the location of industry:

Corollary 1 *With zero commuting costs, when* $\phi < \hat{\phi}$*, there is partial agglomeration of*

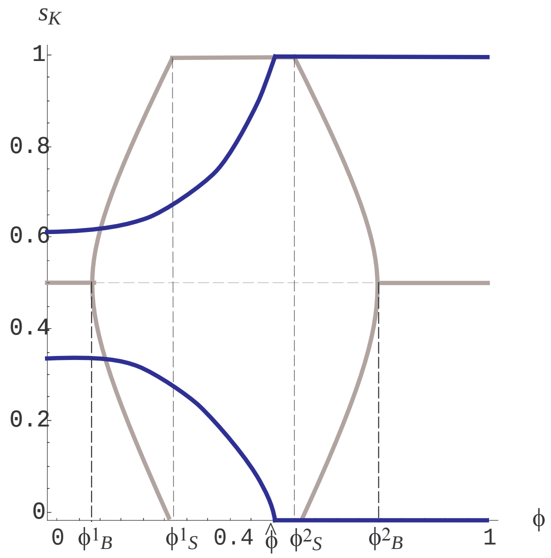

Figure 6: Bifurcation diagram (s_K)

firms, with either $1/2 < s_K < 1$ *or* $1/2 > s_K > 0$*; and when* $1 > \phi \geq \hat{\phi}$*, there is complete agglomeration of firms with either* $s_K = 1$ *or* $s_K = 0$*.*

The bifurcation diagram for the location of industry with and without commuting is shown in Figure 6.

Figure 5 shows that for $\phi < \hat{\phi}$, we have unambiguously more agglomeration of residences with commuting. This is immediately clear, since only full agglomeration can be a stable equilibrium in this case.

For $1 > \phi > \hat{\phi}$, we find more agglomeration with commuting with sufficiently free trade. However, we find that with commuting, partial agglomeration emerges at lower trade freeness than without commuting. Hence, the effect of commuting on agglomeration depends on the level of trade freeness.

We prove in the Appendix that (a) the 'sustain' point ϕ_2^S – i.e., the point where we have full agglomeration of residences (and jobs) in H – is lower with than without commuting,[8]

[8]The intuition is as follows: the sustain point with commuting equalises the dispersion force of the housing sector with the agglomeration force of the cost-of-living effect evaluated at $s_K = 1$. Without commuting, these two forces have the same strength (since $s_K = s_L = 1$) but, since $\phi_S^{2,c} > \hat{\phi}$, nominal wages are higher in H than in F when $s_L = 1$. Therefore, $\Delta V^o(1, \phi_S^{2,c}) > 0$.

while (b) in the commuting case the 'break' point ϕ_2^B – i.e., the point where the symmetric equilibrium becomes unstable – equals one with commuting and thus exceeds the corresponding point in the no-commuting case. The indices c, o here refer to the equilibrium with and without commuting respectively.

Lemma 2 *We have* $\hat{\phi} < \phi_S^{2,o} < \phi_S^{2,c} < \phi_B^{2,o} < \phi_B^{2,c} = 1$.

Proof. See Appendix B. ■

Furthermore, partial agglomeration with commuting leads to a higher share of mobile workers in the partial core than without commuting:

Lemma 3 *Define* $s_L^{P,j} > 1/2$ *as the share of entrepreneurs in an equilibrium with partial agglomeration (in home) for* $j = c, o$. *Then for all* $\phi_S^{2,c} < \phi < \phi_S^{2,o}$, *we have* $s_L^{P,c} < s_L^{P,o}$, *while for* $\phi > \phi_S^{2,o}$, *we have* $s_L^{P,c} > s_L^{P,o}$.

Proof. See Appendix B. ■

Therefore, we have shown the following:

Proposition 3 *Comparing commuting and no-commuting equilibria, we have more agglomeration of residences in the commuting case when trade freeness is low or high, while there is less agglomeration for intermediate trade freeness, that is,* $s_L^c \leq s_L^o$ *for all* $\phi \in [\phi_S^{2,c}, \phi_S^{2,o}]$ *and* $s_L^c \geq s_L^o$ *for* $\phi < \phi_S^{2,c}$ *and* $\phi > \phi_S^{2,o}$.

3.3 Equilibrium with positive commuting costs

We now analyze the case of positive commuting costs, proceeding like we did in the last subsection. However, since the algebra of the case with positive commuting costs is more involved, we state our main result without proof and rely on graphical representations instead.

3.3.1 Commuting equilibrium

The pattern of commuting again depends on the difference in skilled wages. However, due to commuting costs, no commuting occurs unless the wage differential is large enough. In particular, if $0 < \Delta\pi(s_L, s_L, \phi) < t$ or $0 > \Delta\pi(s_L, s_L, \phi) > -t$, no one will commute (in cases where commuting would occur without commuting costs).

The incentive to commute and the commuting equilibrium in such an equilibrium are portrayed in two part figure 7. This figure is analogous to Figure 3 and, in addition, takes the fixed commuting costs into account. These are depicted by the horizontal lines t and $-t$ in the upper panel.

Start with the case of low trade freeness ($\phi < \hat{\phi}$) depicted by the red lines in figure 7. As in the case of zero commuting costs, no corner solutions can occur. There is a 'band of inaction' where no commuting occurs when $s_L \in (\underline{s}_L''', \overline{s}_L''')$ where the wage differential without commuting (denoted $\Delta\pi^o$ in the following) is less than t in absolute terms (for the thresholds see Appendix C). Outside this band, the mass of mobile workers $s_K(s_L, \phi)$ solves $\Delta\pi(s_L, s_K) = t$ (when commuting occurs from F to H) or $\Delta\pi(s_L, s_K) = -t$ (when commuting occurs from H to F).[9] Hence, the mass of mobile workers in the domestic region is given by $s_K(s_L)$ when $|\Delta\pi^o| > t$ and the wage differential is t. When $|\Delta\pi^o| < t$, the mass of mobile workers in the domestic region is given by s_L and the wage differential is $\Delta\pi^o$.

The case of high trade freeness ($\phi > \hat{\phi}$) is depicted by the blue lines. The upper panel shows that commuting is inhibited when the domestic share of residents is in the interval $\underline{s}_L', \overline{s}_L'$. Hence, $s_K = s_L$ as shown in the lower panel.

Outside this band of inaction, the analysis is much the same as in the case of zero commuting costs. For the mass of mobile workers in the domestic region we obtain:

$$s_K = \begin{cases} s_L & \text{if } \ |\Delta\pi^o| < t \\ \max[0, \min[\tilde{s}_K, 1]] & \text{if } \ |\Delta\pi^o| \geq t \end{cases} . \tag{30}$$

The analytical relationships $\tilde{s}_K = s_K(s_L, \phi)$ in (30) which result in interior commuting equilibria are derived from $\Delta\pi(s_L, s_K, \phi) = t$ (commuting from F to H) and from

[9] The analytical relationships $s_K = s_K(s_L, \phi)$ which result in interior commuting equilibria and the threshold levels $\underline{s}_L', \overline{s}_L'$ are derived in Appendix C.

$\Delta\pi(s_L, s_K, \phi) = -t$ (commuting from H to F), respectively. These relationships and the thresholds $\overline{s}''_L$ and $\underline{s}''_L$ which indicate the borderline values of the share of domestic residents at which corner solutions obtain in the commuting equilibrium, are analytically stated in Appendix C).

The entrepreneurial wage differential relevant for the residence choice (see below) corresponding to the solution of the commuting equilibrium is:

$$\Delta\pi = \begin{cases} \Delta\pi^o & \text{if} \quad |\Delta\pi^o| < t \\ \text{sign}[\Delta\pi^o]t & \text{if} \quad |\Delta\pi^o| \geq t \end{cases}. \tag{31}$$

Inside the band of inaction where $|\Delta\pi^o| < t$ the entrepreneurial wages correspond to their values without commuting. Outside, the wage differential is equalized at t (when $\Delta\pi > 0$) or $-t$ (when $\Delta\pi < 0$). This holds for an interior equilibrium. Again, the relevant wage differential also equals t or $-t$ for a corner equilibrium: If everyone works in H (which occurs when $\Delta\pi > 0$), say, the wage obtained when living in H is π^H and that when living in F is $\pi^H - t$, hence the differential t. Similarly, when everyone works in F ($\Delta\pi < 0$), the wage differential is $-t$.

At an interior commuting equilibrium, increasing s_L increases the share of firms at home s_K more than (less than) proportionately depending on whether $\phi > (<)\hat{\phi}$.

This completes our discussion of the equilibrium at the commuting stage.

3.3.2 Location equilibrium

As in the previous subsection, we analyze the effect of commuting on the utility differential of skilled workers. The utility differential which guides the location choice, takes the entrepreneurial wage differential and the equilibrium share of firms $s_K(s_L, \phi)$ in the commuting equilibrium into account (i.e. (30)). We now have:

$$\Delta V(s_L, \phi) = \frac{\mu}{1-\sigma} \ln \left(\frac{1 - s_K + \phi s_K}{s_K + \phi(1 - s_K)} \right) + \eta \ln \left(\frac{\rho + 1 - s_L}{\rho + s_L} \right) + \text{sign}[\Delta\pi^o] \min\{|\Delta\pi^o|, t\}, \tag{32}$$

with $s_K = s_K(s_L)$ as given by equations (A.10) and (A.14) of Appendix C.

From the analysis of the second stage around $s_L = 1/2$ we know that, due to the presence of commuting costs, no commuting occurs (i.e. $s_K = s_L$) and the entrepreneurial wage

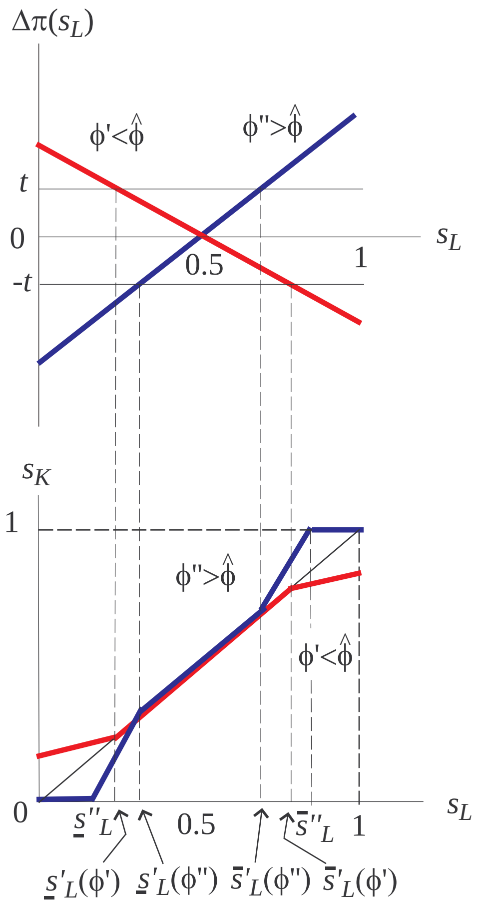

Figure 7: Commuting with $t > 0$

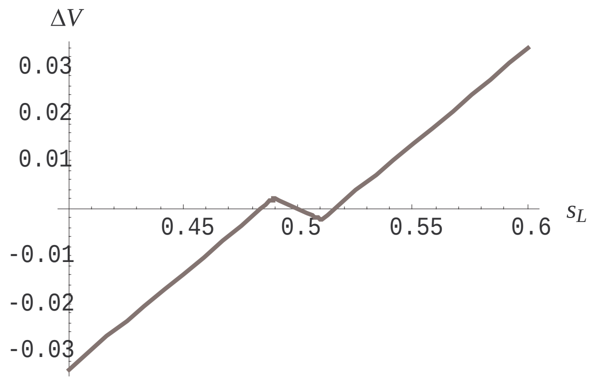

Figure 8: ΔV with $t > 0$ and $\phi < \phi_B^1$.

differential is the same as with prohibitive commuting costs for all levels of trade freeness. Hence, long-distance commuting has no effect on the local stability of the symmetric equilibrium. This is depicted in figure 10, which superimposes the bifurcation diagram with positive commuting costs on the bifurcation diagram with prohibitive commuting costs (i.e. figure 2). This implies, of course, that the symmetric equilibrium is stable with commuting if and only if it is stable without commuting.

To explore the stability characteristics of other possible equilibria, we have to analyze the utility differential when we are outside the band on inaction. As we know from the commuting stage, the degree of trade freeness now matters.

Start with low levels of trade freeness, i.e. $\phi < \hat{\phi}$. For $s_L \notin (\underline{s}_L''', \overline{s}_L''')$, an interior equilibrium must obtain, i.e., we have to take into account that $s_K = s_K(s_L, \phi)$ as given by (30). By the same argument as in the previous subsection, when $s_K = \tilde{s}_K$, it can be shown that ΔV must be upward sloping. An example for $\phi < \phi_B^1$ is shown in Figure 8 (The parameter values are $\eta = 0.2$, $\mu = 0.5$, $\sigma = 2$, $\rho = 1$ and $\phi = 0.02$.).[10]

When $\phi > \hat{\phi}$, we have to consider again that there may not be an interior solution for s_K when individuals commute. As discussed before, around $s_L = 1/2$ we always have the same utility differential as without commuting, since here the commuting cost is too large relative to the gain in net wages so that no one commutes. Then, when individuals commute, say, from F to H (which occurs when $s_L > 1/2$), the utility differential slopes upward until

[10]Here and below, the critical levels ϕ_B^1 and so on refer to the no-commuting case.

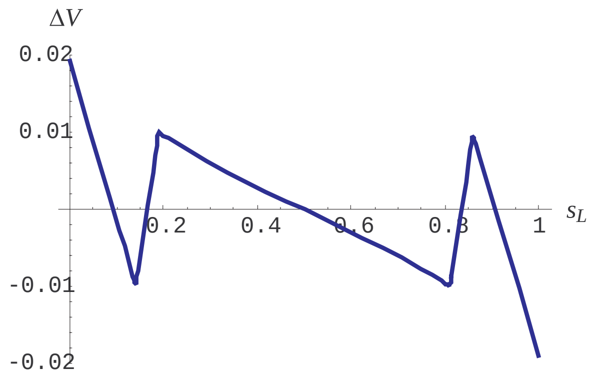

Figure 9: ΔV with $t > 0$ and $\phi > \phi_B^2$.

$\tilde{s}_K = 1$ when it starts to slope downward. Figure 9 shows an example when $\phi > \phi_B^2$. There are now three stable equilibria, one symmetric and two asymmetric. In addition, there are two unstable asymmetric equilibria with partial agglomeration. Furthermore, there is a level of trade freeness $\check{\phi}$ (with $\phi_B^2 > \check{\phi} > \hat{\phi}$) where the two stable equilibria with partial agglomeration fall just within the band of inaction.

Summarizing this discussion, we have:

Proposition 4 *Suppose commuting costs are positive but low. Then, (i) if $\phi > \bar{\bar{\phi}}$, there is a unique stable symmetric equilibrium, if $\bar{\bar{\phi}} > \phi > \phi_B^2$, there is a stable symmetric equilibrium and two stable asymmetric equilibria, if $\phi_B^2 > \phi > \check{\phi}$, there are four stable asymmetric equilibria, and if $\check{\phi} > \phi > \hat{\phi}$, there are two stable asymmetric equilibria; (ii) if $\phi < \phi_B^1 < \hat{\phi}$, there is a stable symmetric equilibrium and two stable equilibria with full agglomeration where $s_L = 0$ or $s_L = 1$, and if $\phi_B^1 < \phi < \hat{\phi}$, there are two stable equilibria with full agglomeration where $s_L = 0$ or $s_L = 1$.*

The corresponding bifurcation diagram is shown in Figure 10.

It is instructive to compare the equilibria with different commuting costs. With prohibitive commuting costs, we have the familiar bell-shaped bifurcation diagram of Pflüger and Südekum (2006). In the extreme case of zero commuting costs, the symmetric equilibrium is destabilized by commuting except at high levels of trade freeness where the location choice is dominated by housing costs. With positive (but low) commuting costs,

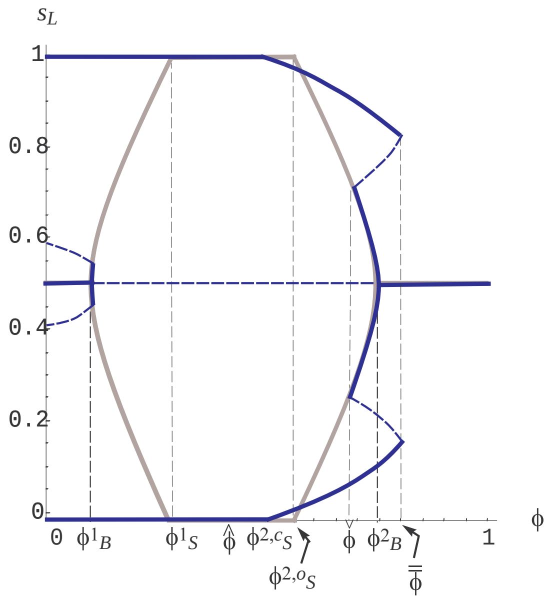

Figure 10: Bifurcation diagram with positive commuting costs

the symmetric equilibrium regains its stability characteristics. However, further equilibria with partial or full agglomeration of location now emerge.

4 Conclusion

This paper has examined commuting in an economic geography model. Even though commuting means that individuals can live where housing is cheap (small towns) and commute to work where wages are high (big cities), we find that commuting leads to more agglomeration of residences, in general. In effect, since more commuters also mean more firms in the region where they work, industrial goods prices are lower there which reinforces the supply linkage.

The paper's insights may be used to think about policies for regions where individuals commute to work in other regions. Examples may include Luxemburg, East Germany, the Mezzogiorno, some regions in Central and Eastern Europe and elsewhere. In particular, at first sight one might think that subsidizing commuting would be beneficial for these regions since it allows those workers to stay at their place of residence. However, our model shows that instead a commuting subsidy might lead to even more individuals leaving such a region.

We should mention, however, that as in all NEG models, the results are driven by some particular assumptions of the model, and that changing these results may alter our conclusions. For instance, other agglomeration models may yield different predictions about the relation between population size, size of the workforce and wages. Also, we would get somewhat different results if we allowed commuters to spend some of their income at their place of work. We believe, however, that our model is a useful starting place for analyzing the joint determination of work and residence choices in an economic geography framework.

Appendix

A The model without commuting

Here we briefly present some parameter restrictions taken from Pflüger and Südekum (2006). In order to obtain two real bifurcation levels, $1 + 4\sigma(\sigma - 1)[1 - \gamma(\sigma - 1)] > 0$ has to be fulfilled, where $\gamma = \eta/\mu$. A sufficient condition to fulfill this requirement which we assume to hold is $1 - \gamma(\sigma - 1) > 0$. In order to rule out that the agglomerative forces become so strong that the symmetric equilibrium is unstable even at infinite trade costs, $[\rho/(\rho-1)-2\rho](2\rho+1)/\sigma < \gamma$ has to be satisfied (no-black-hole condition). A sufficient condition is $2\rho > \sigma/(\sigma-1)$. Both sectors are only active after trade if $\rho\sigma/[(2\rho+1)(\sigma-1)] > \mu$.

B Zero commuting costs: Proofs

Proof of Lemma 1. (i) If $\phi = \hat{\phi}$, $\Delta V(1, \phi) > 0$. For $\phi > \hat{\phi}$ and $s_L > \bar{s}_L$,

$$\Delta V(s_L, \phi) = \left(\frac{\mu}{1-\sigma}\right) \ln(\phi) + \eta \ln\left(\frac{\rho + 1 - s_L}{\rho + s_L}\right) + \Delta\pi(s_L, 1).$$

If $\phi = 1$, we get immediately

$$\Delta V(1, 1) = \eta \ln\left(\frac{\rho}{1+\rho}\right) < 0.$$

Since $\Delta V(1, \phi)$ is continuous and decreasing in ϕ, there exists a unique $\bar{\phi} > \hat{\phi}$, such that $\Delta V(1, \phi) = 0$. (ii) Let $\phi > \hat{\phi}$. Obviously, $\Delta V(\bar{s}_L, \phi) > 0$. Let $s_L > \bar{s}_L$. $\Delta V(s_L, \phi)$ is strictly concave. Hence, if $\phi > \bar{\phi}$, there exists exactly one s_L, with $1 > s_L > \bar{s}_L$, such that $\Delta V(s_L, \phi) = 0$. This s_L fulfills $d\Delta V(s_L, \phi)/ds_L < 0$. A similar argument holds true for $s_L < 1/2$. (iii) $\Delta V(s_L, 1) = \eta \ln\left(Q^F/Q^H\right)$. The unique solution of $\Delta V(s_L, 1) = 0$ is $s_L = 1/2$, where $d\Delta V(1/2, 1)/ds_L < 0$. ■

Proof of Lemma 2. Note first that from the previous analysis, we know that the analogues of $\phi_1^{B,o}$ and $\phi_1^{S,o}$ do not exist, i.e., for $\phi < \hat{\phi}$ only full agglomeration is an equilibrium. Second, for the 'sustain points' we find $\phi_S^{2,c} < \phi_S^{2,o}$: We know $s_K(1, \hat{\phi}) = 1$. From (21), we have

$$\Delta V(1,\phi) = \frac{\mu}{1-\sigma}\ln\phi + \eta\ln\hat{\phi}, \tag{A.1}$$

and therefore

$$\phi_S^{2,c} = \hat{\phi}^{\eta(\sigma-1)/\mu} > \hat{\phi}. \tag{A.2}$$

The difference

$$\Delta V^o(s_L,\phi) - \Delta V^c(1,\phi) = \frac{\mu}{\sigma}(1-\phi)\left(1+\rho-\frac{\rho}{\phi}\right) \tag{A.3}$$

is positive at $\phi = \phi_S^{2,c}$ since $\phi = \phi_S^{2,c} > \hat{\phi}$ (and thus $s_K(1,\phi) = 1$), which establishes that $\phi_S^{2,o} > \phi_S^{2,c}$.

Finally, the 'break point' $\phi_B^{2,o}$ solves $d\Delta V(s_L, \phi_B^{2,o})/ds_L = 0$. With commuting, the symmetric equilibrium is never stable for $\phi < 1$. At $\phi = 1$, however, we have $\Delta V^c(1,1) = \Delta V^o(1,1) = 0$. ■

Proof of Lemma 3 (sketch). We show that (i) when there is partial agglomeration, we have that there is a unique level of ϕ, say $\tilde{\phi}$ such that $s_L^{P,c}(\phi) = s_L^{P,o}(\phi)$, and (ii) $s_L^{P,c}(\phi) > s_L^{P,o}(\phi)$ when $\phi > \tilde{\phi}$. For part (i), define the firm share under partial agglomeration with commuting, $s_L^{P,c} = \hat{s}_L(\phi)$ which solves

$$\Delta V^c(\hat{s}_L(\phi),\phi) = 0, \tag{A.4}$$

and likewise, the firm share without commuting, $s_L^{P,o} = \tilde{s}_L(\phi)$ which solves

$$\Delta V^o(\tilde{s}_L(\phi),\phi) = 0. \tag{A.5}$$

Differentiation gives

$$\frac{\partial s_L^{P,o}}{\partial\phi} = -\frac{\partial\Delta V^o/\partial\phi}{\partial\Delta V^o/\partial s_L}, j = c, o. \tag{A.6}$$

Differentiating (A.7) and using (A.6) gives a expression for

$$\frac{d\Delta V^c(\tilde{s}_L(\phi),\phi)}{d\phi} = \frac{\partial\Delta V^c(\tilde{s}_L(\phi),\phi)}{\partial\phi} + \frac{\partial\Delta V^c(\tilde{s}_L(\phi),\phi)}{\partial s_L}\frac{\partial\tilde{s}_L(\phi)}{\partial\phi} \tag{A.7}$$

which is exceedingly complex. However, evaluating the expression at $\hat{\phi}$ and 1, we find

$$\frac{d\Delta V^c(\tilde{s}_L(\hat{\phi}),\hat{\phi})}{d\phi} = \mu(1+\rho)\left(\frac{1}{\rho(1-\sigma)} + \frac{\eta\hat{\phi}^{\mu/(\eta(1-\sigma))}(1+\rho)(\hat{\phi}^{2\mu/(\eta(\sigma-1))}-1)}{(1+2\rho)\sigma(\mu+\eta(1-\sigma))}\right) < 0 \tag{A.8}$$

$$\frac{d\Delta V^c(\tilde{s}_L(1),1)}{d\phi} = \frac{\mu}{1-\sigma} < 0. \tag{A.9}$$

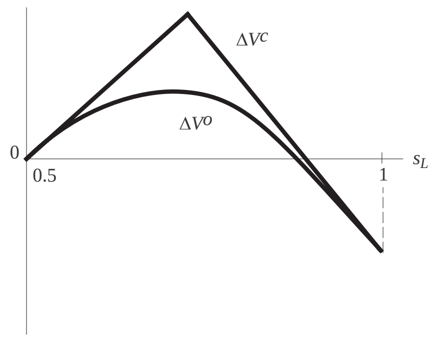

Figure 11:

Since $\frac{d\Delta V^c(\tilde{s}_L(\phi),\phi)}{d\phi}$ is monotone in ϕ **(to be shown)**, we know that there is a unique $\tilde{\phi}$ where $\Delta V^c(\hat{s}_L(\phi),\phi) = \Delta V^o(\tilde{s}_L(\phi),\phi) = 0$.

(ii) For $\phi > \tilde{\phi}$, we can show that ΔV^c and ΔV^o have the shape shown in Figure 11, so that $s_L^{P,c} > s_L^{P,o}$ when $\phi > \tilde{\phi}$. Conversely, since $\tilde{\phi}$ is unique, $s_L^{P,c} < s_L^{P,o}$ when $\phi < \tilde{\phi}$. ■

C Positive commuting costs

From $\Delta\pi(s_L, s_K, \phi) = -t$, one obtains

$$\begin{aligned} s_K &= \frac{1}{2} - \frac{1}{2(1-\phi)\sigma t}\Big\{\mu(1+2\rho)(1-\phi) \\ &\quad -\sqrt{(1-\phi)^2\mu^2(1+2\rho)^2 - 2\mu(1-\phi^2)\sigma(1-2s_L)t + \sigma^2t^2(1+\phi)^2}\Big\}. \end{aligned} \tag{A.10}$$

Setting s_K equal to 0 and s_L, respectively, yields

$$\underline{s}_L'' = \frac{\mu(1-\phi)[\phi-(1-\phi)\rho] - \phi\sigma t}{\mu(1-\phi^2)}, \tag{A.11}$$

$$\begin{aligned} \underline{s}_L' &= \frac{1}{2} + \frac{1}{2(1-\phi)\sigma t}\Big\{2\mu[\phi-(1-\phi)\rho] \\ &\quad -\sqrt{4\mu^2[\phi-(1-\phi)\rho]^2 + (1+\phi)^2\sigma^2t^2}\Big\}, \end{aligned} \tag{A.12}$$

$$\begin{aligned} \overline{s}_L''' &= \frac{1}{2} + \frac{1}{2(1-\phi)\sigma t}\Big\{2\mu[\phi-(1-\phi)\rho] \\ &\quad +\sqrt{4\mu^2[\phi-(1-\phi)\rho]^2 + (1+\phi)^2\sigma^2t^2}\Big\}. \end{aligned} \tag{A.13}$$

From $\Delta\pi(s_L, s_K, \phi) = t$, one obtains

$$s_K = \frac{1}{2} + \frac{1}{2(1-\phi)\sigma t}\Big\{\mu(1+2\rho)(1-\phi) - \sqrt{(1-\phi)^2\mu^2(1+2\rho)^2 + 2\mu(1-\phi^2)\sigma(1-2s_L)t + \sigma^2 t^2(1+\phi)^2}\Big\}. \quad (A.14)$$

Setting s_K equal to 1 and s_L, respectively, yields

$$\overline{s}''_L = \frac{\mu(1-\phi)[1+(1-\phi)\rho] + \phi\sigma t}{\mu(1-\phi^2)}, \quad (A.15)$$

$$\overline{s}'_L = \frac{1}{2} - \frac{1}{2(1-\phi)\sigma t}\Big\{2\mu[\phi-(1-\phi)\rho] - \sqrt{4\mu^2[\phi-(1-\phi)\rho]^2 + (1+\phi)^2\sigma^2 t^2}\Big\}, \quad (A.16)$$

$$\underline{s}'''_L = \frac{1}{2} - \frac{1}{2(1-\phi)\sigma t}\Big\{2\mu[\phi-(1-\phi)\rho] + \sqrt{4\mu^2[\phi-(1-\phi)\rho]^2 + (1+\phi)^2\sigma^2 t^2}\Big\}. \quad (A.17)$$

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