A positive theory of lending of last resort *

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Abstract. This paper aims at explaining why central banks provide lending of last resort services to individual commercial banks. We argue that LOLR transactions are motivated by the macroeconomic objective of stabilizing the economy. By providing LOLR transactions the central bank lowers the probability that banks go bankrupt, reduces uncertainty on the money multiplier and thus lowers inflation and output variance. The model also explains why central banks tend to support only large banks and the effects of possible moral hazard behavior in consequence of LOLR guarantees. Interestingly enough, moral hazard contributes to a higher degree of macroeconomic stability but also higher stabilization costs.

Keywords: lender of last resort, too-big-to-fail, moral hazard, macroeconomic stability

JEL code: E50, E58

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1. Introduction

While there is anecdotal evidence of lender of last resort[1] transactions from a large number of countries and occasions,[2] formal attempts to explain why central banks provide LOLR services have been remarkably rare (Goodhart and Huang, 2005). Recently, some attempts have been made to fill this gap. While Freixas, Parigi and Rochet (2004) and Cordella and Levy Yeyati (2003) argue that LOLR transactions are motivated by the attempt to correct market failures, the models by Rochet and Vives (2004) and Bartolini and Drazen (2004) focus on the question of why commercial banks might be reluctant to make use of LOLR services. These models provide fruitful insights from a microeconomic perspective. However, Goodhart and Huang (2005) argue that micro models fail to notice that the primary motive for providing LOLR transactions are macro rather than micro concerns.[3] We agree with this view and aim at developing a positive theory of lending of last resort in this paper. The model we present stands in the tradition of Goodhart and Huang's (2005) work in so far as the basic motive for providing LOLR services is similar. However, in order to study the macroeconomic consequences of providing LOLR, we model the economy in a more detailed way. We basically argue that fractional reserve banking systems are prone to liquidity crises whenever the public suddenly changes its preferences towards holding more high-powered money. This sudden change in preferences might be triggered by the closure of commercial banks. Whenever commercial banks are closed this results in uncertainty on the future money multiplier, thereby making monetary policy more difficult and causing welfare decreasing fluctuations of inflation and output around their target levels. In such a setting, LOLR transactions can contribute to a lower variability of both inflation and output.[4]

[1] A lender of last resort (LOLR) stands ready to inject high-powered money into the banking system whenever a bank is solvent but suffers from temporary liquidity problems. See Bagehot (1873) for a classical view on the LOLR function.

[2] See e.g. the evidence reported in Humphrey and Keleher (1984), Miron (1986), Bordo (1986), Eichengreen and Portes (1987), Bordo (1990) and Dowd (1999).

[3] Freixas, Parigi and Rochet (2004) themselves basically agree with this point of view.

[4] One might be tempted to argue that failures of commercial banks could be prevented by implementing adequate systems of bank regulation and supervision or private safety nets. Although numerous countries implemented such systems, many central banks seem to be willing to provide individual LOLR services at least to large commercial banks. Thus, these instruments often seem to be insufficient to

While LOLR services are a "free lunch" when these services can be provided at no cost, we show that the situation is different when at least unsuccessful LOLR transactions are costly. In this case central banks have an incentive to save only the large banks, while the small banks are closed, an empirical phenomenon which is well known as the "too-big-to-fail doctrine". As a consequence, a welfare maximizing central bank will accept higher inflation and output variability when facing such LOLR costs.

Furthermore we study the effects of potential moral hazard behavior due to a LOLR guarantee. We show that large banks react by increasing their risky investments (although to different extents) while a fraction of small banks behave inversely in order to profit from the LOLR option. As a consequence, central banks will save more banks than in the absence of moral hazard. This will lead to a lower variability of inflation and output and thus a higher degree of macroeconomic stability. However, we also show that this implies higher stabilization costs for central banks.

The paper is organized as follows: the basic macroeconomic model is explained in section 2. In section 3 we work out the basic macroeconomic rationale for LOLR transactions for the case of a homogenous banking sector and costless LOLR transactions and show the implications for inflation and output. In section 4 we allow for a heterogenous banking sector and take costs of LOLR transactions explicitly into account. Section 5 deals with the moral hazard issue resulting from an explicit or implicit LOLR guarantee. The paper closes with a summary of the main arguments and some conclusions.

prevent central banks from intervening in the banking sector. This might be due to inefficiencies of the implemented systems of bank regulation and supervision. However, even under well constructed systems of bank regulation bank failures may occur. Thus, a positive theory of lending of last resort is even necessary when additional measures to stabilize the banking system are in use. However, an analysis of the interactions between banking regulation and supervision, private safety nets and LOLR services is beyond the intentions of this paper.

2. The basic model

2.1. Macroeconomic framework

We make use of a standard AS-AD model as macroeconomic framework.[5] Aggregate demand is given by

$$\pi = m - y, \tag{1}$$

where π is inflation, m is money supply and y is aggregate output. The model is expressed in logs. Since we use a static framework we do not use time indices in the following. Aggregate supply is given by

$$\pi = \pi^e + y \tag{2}$$

with π^e being inflation expectations.

Using equations (2) and (1) we can calculate equilibrium output as

$$y = \frac{m - \pi^e}{2} \tag{3}$$

and equilibrium inflation as

$$\pi = \frac{m + \pi^e}{2}. \tag{4}$$

We assume wages to be bargained collectively by unions and that unions form rational expectations on future inflation, i.e.

$$\pi^e = E[\pi] = E[m]. \tag{5}$$

2.2. Central Bank

Following a standard money multiplier approach[6] we assume the money supply to be the product of base money and a money multiplier. Thus, we can express money supply in logs as

$$m = b + \omega, \tag{6}$$

[5] See e.g. Gärtner (2003).
[6] See e.g. Walsh (2000), p. 404-406.

with b being the log of base money and ω the log of the money multiplier. In the following we assume for simplicity that the central bank is capable of controlling base money perfectly[7] via open market operations.[8]

The money multiplier depends on various factors. In the simplest case the multiplier depends on (i) the reserve requirement ratio, (ii) the ratio of excess reserves banks hold with the central bank and (iii) the currency-to-deposit ratio. In the following were are primarily concerned with the currency-to-deposit ratio. With respect to the currency-to-deposit ratio we assume that the public is willing to hold a certain fraction of its liquid funds in cash.

2.3. Banking sector

We assume the banking sector to consist of N banks collecting deposits from the private sector and making investing decisions.[9] Banks are supposed to be risk neutral and solely interested in maximizing (expected) profits. For the moment we assume banks to be homogeneous. However, because we relax this assumption throughout the paper we already introduce a bank-index i with $i = 1, 2, ..., N$. Furthermore we assume for simplicity that banks do not own any capital on their own.

Banks have to decide how to invest their deposits D_i, collected from the private sector. We assume that they can choose between two projects: a safe and a risky one. The safe project has a return on capital of $r_1 > 1$ with certainty. The risky project is assumed to be described by the following production function[10]

$$Q_i(C_i, \lambda_i) = (\beta_0 + \lambda_i) \cdot C_i - \beta_1 \cdot {C_i}^2 \tag{7}$$

with C_i being the capital invested into the risky project. The parameters β_0 and β_1 are assumed to be positive. In order to ensure that at least some investment into the risky

[7] This assumption can be relaxed without a material effect on the derived results.
[8] We consider the case of a closed economy here. Thus, we neglect possible restrictions for monetary policy which could result from the choice of the exchange rate system.
[9] One might justify the existence of these banks by the assumption of indivisibilities of the projects or informational advantages of banks.
[10] See Krugman (1998).

project is profitable, we further assume $\beta_0 + E[\lambda_i] > r_1$. The variable λ_i is a bank-specific random variable, inducing some uncertainty into the bank's investment decision.[11]

We assume that banks receive their returns on the two projects in the end of the period. Banks receive a signal on the status of the risky project throughout the project period, although the initial investment decision is irreversible.

For simplicity we also assume the cost of capital and transaction costs to be zero. Let γ_i be the share of deposits which bank i invests into the risky project ($C_i = \gamma_i \cdot D_i$). We can then write bank i's profit P_i as

$$P_i = (1 - \gamma_i) \cdot D_i \cdot r_1 + Q_i(C_i) - D_i. \tag{8}$$

The bank's instrument variable is γ_i. Maximizing the bank's expected profit requires

$$\gamma_i^* = \frac{\beta_0 + E[\lambda_i] - r_1}{2 \cdot \beta_1 \cdot D_i}. \tag{9}$$

In the following we denote optimally chosen values of γ or C with asterisks. Using $C_i = \gamma_i \cdot D_i$ we can calculate the capital a bank invests into the risky project as

$$C_i^* = \frac{\beta_0 + E[\lambda_i] - r_1}{2 \cdot \beta_1}. \tag{10}$$

It is easy to see that the amount bank i invests into the risky project is independent from the amount of deposits D_i it collected. In equilibrium the marginal expected return of the risky project equals the risk-free rate r_1, provided that γ_i is chosen optimally and D_i is sufficiently large to make the optimal investment in the risky project possible (which we assume in the following).

We will now turn to the question of under what circumstances a bank i becomes illiquid. Illiquidity implies that a bank is not able to repay all deposits D_i.[12] Thus, bank i is liquid as long as λ_i does not fall below the critical realization of λ fulfilling

$$D_i = (1 - \gamma_i) \cdot D_i \cdot r_1 + Q(C_i). \tag{11}$$

[11] We assume $E[\lambda_i]$ and $\sigma_{\lambda,i}$ to be identical for all banks i.

[12] Without external help illiquidity leads to insolvency with certainty. We discuss this distinction later.

After inserting equation (7) in equation (11), making use of $C_i = C^*$ and $\gamma_i = \gamma_i^*$ and solving for λ_i we end up with

$$\lambda_i^{crit} = \frac{2 \cdot \beta_1 \cdot (1 - r_1)}{\beta_0 + E[\lambda_i] - r_1} \cdot D_i + \frac{r_1 + E[\lambda_i] - \beta_0}{2}. \quad (12)$$

Thus, for all λ_i falling short of λ_i^{crit} bank i becomes illiquid. Obviously, λ_i^{crit} depends negatively[13] on the amount of deposits bank i collected. Since we assume in this section that all banks have collected the same amount of deposits, λ_i^{crit} is identical for all banks.

For our following calculations it is useful to state the random variable λ_i more precisely. For simplicity, we assume λ_i to be generated by the following bivariate random process:

$$\lambda_i = \begin{cases} \overline{\lambda} & \text{with probability } p \\ \underline{\lambda} & \text{with probability } 1 - p \end{cases} \quad (13)$$

We further assume $\underline{\lambda} < \lambda_i^{crit}$ and $\overline{\lambda} > \lambda_i^{crit}$ so that bank i becomes illiquid if the bad state occurs ($\lambda_i = \underline{\lambda}$) and stays healthy in the good state ($\lambda_i = \overline{\lambda}$).[14] Thus, two outcomes of the project are possible (high success, low success). As outlined earlier the banks receive a signal $Z_i \in \{\overline{Z}, \underline{Z}\}$ on the success of the project throughout the project period.

Given the bank receives the signal $Z_i = \overline{Z}$, the project will always end with high success ($\overline{\lambda}$) and thus earn enough profits to pay back all deposits in the end of the project period. Thus, the bank faces no illiquidity problems in this case and will always survive the period.

In the case where the bank receives the signal $Z_i = \underline{Z}$ of low success ($\underline{\lambda}$) the profits are insufficient to pay back all deposits and the bank is in danger of becoming illiquid and thus insolvent in the end of the period. However, we assume that there is a non-zero probability that the project can nevertheless be finished with high success (in this case the return would be $Q(C_i^*, \overline{\lambda})$) whenever the bank invests some additional capital into the project after getting the bad signal $Z = \underline{Z}$. We assume that the necessary additional capital is a function of the capital already invested into the project

$$C_i^+ = F \cdot C_i = F \cdot C^* \quad (14)$$

[13] This conclusion results from $\beta_0 + E[\lambda_i] > r_1$ and $r_1 > 1$.

[14] In order to ensure that the production function always delivers positive values we assume $\underline{\lambda} > -\beta_0$. It should be noted that the following expositions can be transferred to more complex distributions of λ. The results do not change qualitatively as long as there is at least one state of the world in which the bank goes bankrupt.

with $0 < F \leq 1$. However, when the project in fact turns out to be of low success in the end of the period, which is assumed to happen with probability h, the bank is insolvent and thus has to be closed. We assume that the remaining funds of the bank are equally distributed among the bank's depositors in the case of a bank closure. The same procedure applies for the case in which the bank refrains (or has to refrain) from investing additional capital into the project. Whenever the projects turn out to be of high success after an injection of additional capital it will survive, which happens with probability $1 - h$.

Up to now we did not discuss the origin of the necessary additional funds. One possibility would be to save a portion of the initially collected deposits for the case of emergency. However, this would cause opportunity costs. In order to minimize these costs a bank would obviously reduce the sum invested in the safe project. For simplicity we assume that the opportunity costs are higher than the expected surplus from the emergency liquidity, i.e.

$$F \cdot C_i^* \cdot r_1 > (1-p) \cdot (1-h) \cdot Q(C_i^*, \overline{\lambda}). \tag{15}$$

In this case banks do not hold any emergency liquidity. Thus, a bank would have to borrow the amount of C_i^+ from a third party as, for example, the central bank. We will discuss the incentive of the central bank to provide this capital later in this paper.

Finally, we assume that $\overline{\lambda}$ is large enough that bank i is able to pay back both deposits and borrowed capital, i.e.

$$D_i + F \cdot C_i^* < Q(C_i^*, \overline{\lambda}) + (1-\gamma_i^*) \cdot D_i \cdot r_1. \tag{16}$$

2.4. Private sector

As outlined earlier, the private sector holds its liquid assets in cash and in bank deposits. For simplicity we assume that all depositors are identical, i.e. they have the same preferred cash-to-deposits ratio, the same wealth and thus the same amount of deposits in banks. It seems to be reasonable to assume that the preferred cash-to-deposits ratio remains unchanged (i.e. $\omega = 0$) as long as no bank is closed. However, this is hardly the case when banks are closed (Goodhart and Huang, 2005). At least depositors holding deposits with the failing banks will likely decrease the share of the wealth they held in the form of bank deposits. We therefore assume that a bank closure causes the public to move out of deposits into cash, thereby increasing the cash-to-deposits ratio and lowering the money multiplier.[15] More precisely, we assume the following relationship

$$\omega = -\sum_{i=0}^{n} I_i \cdot \epsilon_i \tag{17}$$

with

$$\epsilon_i = \sum_{k=1}^{K_i} \varepsilon_{i,k}$$

and

$$\varepsilon_{i,k} = \begin{cases} 0 & \text{with probability } 0.5 \\ 1 & \text{with probability } 0.5 \end{cases} \tag{18}$$

with n being the number of banks which got into liquidity troubles (with $0 \leq n \leq N$). I_i is a dummy variable with the value $I_i = 1$, if bank i goes bankrupt and with $I_i = 0$, if bank i survives. The variable $\varepsilon_{i,k}$ is a random variable capturing the reaction of depositor k on the closure of bank i. The variable ϵ_i then captures the amount by which the money multiplier decreases when bank i is closed.

We assume that all $\epsilon_{i,k}$ are uncorrelated. As a consequence, the expectation of the aggregate reaction on the closure of bank i is increasing in the number of depositors (and thus in the amount of deposits) of the bank , i.e.

$$\frac{\partial E[\epsilon_i]}{\partial K_i} > 0. \tag{19}$$

Moreover, the variance of the aggregate reaction on the closure of a bank is increasing in the number of depositors of the bank, i.e.

$$\frac{\partial \sigma_{\epsilon_i}}{\partial K_i} > 0. \tag{20}$$

[15] Obviously, one could also introduce contagion effects into the model by allowing that even depositors of initially healthy banks increase their cash-to-deposit ratio. Since doing so would complicate the calculations without having a qualitative effect on the results, we refrain from introducing contagion effects, here.

Of course, for the case of a homogeneous banking sector the expected aggregate reaction of the money multiplier on a bank closure is identical for every single bank i.

3. A macroeconomic justification of the central bank's LOLR function

We now turn to the question of why central banks provide LOLR services. We show that the motive for LOLR transactions is the fear that closing down an illiquid bank might induce excessive money multiplier uncertainty thereby increasing the variability of inflation and output in the economy. For the moment we assume that LOLR transactions carry no costs, regardless of whether they are successful or not.

In order to do so we start out with an analysis of the Nash equilibrium for the case where the central bank provides no LOLR services. We then study the case where the central bank has the possibility of acting as a lender of last resort and compare the results.

3.1. Nash equilibrium without central bank's LOLR function

In a first step we study the behavior of the commercial banks and the central bank when the central bank has no LOLR function. The sequence of events for this case is shown in Figure 1.

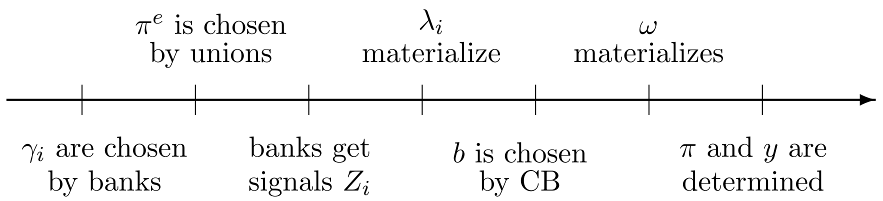

Figure 1. Sequence of events without LOLR function

In the first stage commercial banks collect deposits from the private sector and invest in the two available projects. Afterwards, unions build their (rational) inflation expectations and use these expectations in wage negotiations. In a next step, commercial banks receive a signal on the status of the risky project.[16] Then the project is realized, i.e. the λ_i are determined. The banks with the bad outcome of the project are closed while the others survive. Then, the central bank determines base money via open market operations. Finally, the shutdown of banks causes the public to partially move out of deposits into cash thereby lowering the money multiplier. Inflation and output are determined.

In order to calculate the Nash equilibrium, we start out with the optimization problem of the central bank. The central bank minimizes its loss function given by

$$L = \frac{1}{2} \cdot (\pi - \pi^*)^2 + \frac{1}{2} \cdot y^2. \tag{21}$$

According to this loss function the central bank tries to achieve the optimal inflation rate π^* while fixing the output on its natural level $y^n = 0$.[17] We also assume that the central bank's loss function is identical to the social loss function.

Inserting equation (4) for inflation and (3) for output in the central bank's loss function yields

$$L = \frac{1}{2} \cdot \left(\frac{m + \pi^e}{2} - \pi^* \right)^2 + \frac{1}{2} \cdot \left(\frac{m - \pi^e}{2} \right)^2. \tag{22}$$

We now can derive the optimal money supply m^* by differentiating the loss function with respect to m and solving for m:

$$m^* = \pi^*. \tag{23}$$

However, the central bank cannot determine the money supply directly but only the monetary base. The optimal choice of base money can be calculated using equation (6) and implies:

$$b = m^* - \omega. \tag{24}$$

[16] Of course, this signal is of limited use in the case where the central bank provides no LOLR services. Due to the high opportunity costs of holding cash, banks hold no excess reserves and thus cannot afford the necessary capital on their own.

[17] The results remain unchanged when the central bank's loss function only takes deviations of inflation from its target rate into account.

While the central bank knows the optimal money supply m^* there is uncertainty about the magnitude of the money multiplier. Thus, the central bank has to form an expectation on ω. Without a central bank with a LOLR function, illiquid banks always go bankrupt. In this case equation (17) simplifies to

$$\omega = -\sum_{i=1}^{n} \epsilon_i \tag{25}$$

because of $I_i = 1$ for $i = 1$ to n. The mathematical expectation of the money multiplier, given the knowledge about the number of illiquid banks n, is

$$E[\omega] = -\sum_{i=1}^{n} E[\epsilon_i]. \tag{26}$$

Inserting equation (26) and equation (23) into the expectation of equation (24) yields the optimal monetary base

$$b^* = \pi^* + \sum_{i=1}^{n} E[\epsilon_i]. \tag{27}$$

In order to calculate the actual money supply we insert equation (27) and (25) into equation (6):

$$m = \pi^* + \sum_{i=1}^{n} \left(E[\epsilon_i] - \epsilon_i\right). \tag{28}$$

Following equation (5) expected inflation is equal to the expectation of equation (28) and thus

$$\pi^e = E[\pi] = E[m] = \pi^*. \tag{29}$$

Using equations (4) (28) and (29) we can calculate actual inflation as

$$\pi = \pi^* + \frac{1}{2} \cdot \sum_{i=1}^{n} \left(E[\epsilon_i] - \epsilon_i\right). \tag{30}$$

Obviously, the variance of the actual inflation rate is driven by deviations of ϵ_i and $E[\epsilon_i]$ and thus σ_{ϵ_i}. For $\sum_{i=1}^{n} \epsilon_i = \sum_{i=1}^{n} E[\epsilon_i]$ actual inflation equals the optimal inflation rate π^*. Similarly we can calculate the actual output as

$$y \;=\; \frac{1}{2} \cdot \sum_{i=1}^{n} \left(E[\epsilon_i] - \epsilon_i\right). \tag{31}$$

For $\sum_{i=1}^{n} \epsilon_i < \sum_{i=1}^{n} E[\epsilon_i]$ for all i the output exceeds its natural level, for $\sum_{i=1}^{n} \epsilon_i = \sum_{i=1}^{n} E[\epsilon_i]$ the natural level will be achieved.

We can also calculate the social loss for the case of a central bank without LOLR function by inserting equation (31) for output and equation (30) for inflation into the loss function (21). Doing so yields

$$L = \frac{1}{4} \cdot \left(\sum_{i=1}^{n} \left(E[\epsilon_i] - \epsilon_i \right) \right)^2 . \tag{32}$$

Due to the homogeneity assumption of banks we can calculate the expected social loss as

$$E[L] = \frac{1}{4} \cdot (E[n])^2 \cdot \sigma_{\epsilon}^2 . \tag{33}$$

3.2. Equilibrium with central bank's LOLR function

We now turn to the case where the central bank has a LOLR function and thus is allowed to support banks in illiquidity troubles. Again we start out with the sequence of events shown in Figure 2. In stage 1 commercial banks again collect deposits from the private sector and invest them in the two available projects. Afterwards unions build their (rational) inflation expectations and use these expectations in wage negotiations. In the next step banks receive signals on the status of the risky project. Those commercial banks getting a signal of low success will then ask the central bank for liquidity assistance, since they have already invested all deposits in the two projects.[18] The central bank has then to decide whether it provides LOLR services.[19] Neither the commercial bank nor the central bank knows at this stage whether the project will be finished with high or low success. Thus, the central bank has no information on the status of the bank (illiquid or insolvent).[20] In a next step the project success materializes and banks turn out to

[18] For simplicity we assume that the central bank can also observe signals on the projects states. Thus, there is no information asymmetry between commercial banks and central bank.

[19] We assume that the central bank has enough resources to be able to provide LOLR services and to carry out the necessary open market operations. Whenever this would not be the case the central bank could simply not provide LOLR services.

[20] Goodhart (1987,1995) argues that a clear-cut distinction between illiquidity and insolvency is often impossible because banks requiring LOLR assistance are already under suspicion of being insolvent.

be solvent or insolvent. In the latter case they are closed. Afterwards, the central bank determines base money via open market operations. Finally, whenever a bank turns out to be insolvent, the public partially moves out of deposits into cash thereby lowering the money multiplier. Inflation and output are determined.

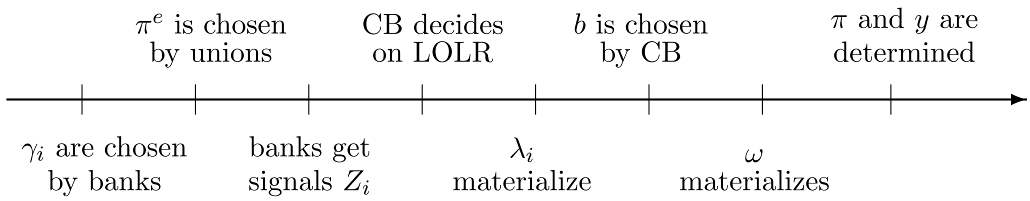

Figure 2. Sequence of events with LOLR function

With LOLR function, the central bank can now save illiquid banks by providing liquidity assistance. Thus, money supply is now given by

$$m \;=\; b - h \cdot \sum_{i=1}^{n} \epsilon_i. \tag{34}$$

Using equation (23) we can then derive optimal base money as

$$b^* = \pi^* + h \cdot \sum_{i=1}^{n} E[\epsilon_i]. \tag{35}$$

Substituting optimal base money for b in equation (34) yields the actual money supply

$$m \;=\; \pi^* + h \cdot \sum_{i=1}^{n} (E[\epsilon_i] - \epsilon_i). \tag{36}$$

The expected money supply is

$$E[m] \;=\; \pi^*. \tag{37}$$

Using $\pi^e \;=\; E[m]$ we can calculate the actual inflation rate by inserting equation (37) and equation (36) in equation (4), i.e.

$$\pi \;=\; \pi^* + \frac{h}{2} \cdot \sum_{i=1}^{n} (E[\epsilon_i] - \epsilon_i). \tag{38}$$

Similarly we can calculate the actual output as

$$y = \frac{h}{2} \cdot \sum_{i=1}^{n} (E[\epsilon_i] - \epsilon_i). \tag{39}$$

As in the case without a LOLR function, the actual inflation rate depends on ϵ_i.[21] However, due to $h < 1$ the variance of both, inflation and output, is lower now. Thus, the central bank can reduce inflation and output variability when making use of LOLR transactions thereby also reducing the expected social loss. In order to show this we first calculate the social loss for the case of a central bank with LOLR function by inserting equation (39) for output and equation (38) for inflation into the loss function (21). Doing so yields

$$L = \frac{h^2}{4} \cdot \left(\sum_{i=1}^{n} (E[\epsilon_i] - \epsilon_i) \right)^2. \tag{40}$$

Expected social loss under a central bank with a LOLR function is then given by

$$E[L] = \frac{h^2}{4} \cdot (E[n])^2 \cdot \sigma_\epsilon^2 \tag{41}$$

and thus lower as when having a central bank without a LOLR function (see equation (33)) as long as $h < 1$.

4. The "too-big-to-fail doctrine" under LOLR

While the model presented in the previous section provides a reasonable macroeconomic foundation for the LOLR function of central banks, it fails to explain why central banks do not always support illiquid banks. To make the model more realistic we modify two of the earlier made assumptions. First, we give up the assumption of a completely homogeneous banking sector. More precisely, we now allow that banks differ in their sizes, i.e. the amount of deposits they collect, and thus in the number of depositors.[22] However, we

[21] Note that inflation variance is identical for the case of $h = 1$, i.e. all insolvent banks turn out to be bankrupt ex post. For $h = 0$ inflation variance is zero since all illiquid banks can be saved by liquidity assistance in this case.

[22] Since it is not the aim of this paper to explain the structure of the banking sector we stick to the assumption that bank sizes are given exogenously.

stick to the assumption that $E[\lambda_i]$ and $\sigma_{\lambda,i}$ are identical for all banks i. Second, we take into account that providing liquidity assistance might trigger opportunity costs.[23] These costs arise because the funds used for LOLR transactions cannot be used for alternative purposes. The opportunity costs of LOLR transactions depend (i) on the amount of provided liquidity and (ii) on the interest rate on alternative investments.

We already showed (see equation (10)) that in equilibrium all banks invest the same amount of deposits in the risky project.[24] Thus, the amount of funds necessary to provide LOLR services to a certain bank is the same for all banks. LOLR transactions have typically a very short time-horizon. We can thus neglect the opportunity costs of successful LOLR transactions. However, when the central bank decides to provide LOLR services and the LOLR transaction fails, the central bank has to bear opportunity costs. We assume that these costs C_i^O for bank i are a function of the amount of liquidity provided,[25] i.e.

$$C_i^O := I_i \cdot J_i \cdot F \cdot C^* \tag{42}$$

with J_i being a dummy variable taking the value 1 if the central bank provides LOLR services and 0 if not.

The sequence of events does not differ from the one already described in the previous section. However, due to the fact that banks can differ in their sizes now, the effects of bank closures on the money multiplier differ from bank to bank. While the expected decrease of the money multiplier is low when a small bank fails, the expected effect is considerably stronger when a major bank has to be closed. This is due to the fact that

[23] Goodhart and Huang (2005) argue that central banks face reputation costs when unsuccessfully trying to save banks. However, it is somewhat questionable why these costs should occur since the central bank has no possibility to find out in advance whether the project will end up with the good or the bad outcome.

[24] We should remember that this result is independent of the homogeneity assumption we made in the previous section as long as even small banks have enough deposits to make the optimal investment into the risky project feasible.

[25] Note that the case in which the commercial bank's profit from the investment into the safe project is sufficient to pay back (parts of) the LOLR credit is irrelevant. Since the central bank's LOLR transactions are solely motivated by preventing uncertainty on the money multiplier, these banks are never supported by LOLR transactions. Since the central bank can observe a bank's size, it can also judge the extent to which the bank in fact needs LOLR services.

large banks have more depositors. Thus more individuals are affected by a bank closure of a large bank.

Unlike the case discussed in the previous section, providing LOLR services is not a "free lunch" if unsuccessful LOLR transactions cause opportunity costs. In order to find out under what circumstances the central bank will provide LOLR services, we compare expected losses when providing LOLR and when not.[26]

The loss resulting from a possible closure of bank i can be written as

$$L_i = \frac{1}{2} \cdot (\pi - \pi^*)^2 + \frac{1}{2} \cdot y^2 + C_i^O. \tag{43}$$

Whenever the central bank decides not to provide LOLR, the last term of this expression yields zero (due to $J_i = 0$ for all i). This case is identical to the case analyzed in section 3.1 with $n = 1$. Substituting the equilibrium inflation rate from equation (30) and equilibrium output from equation (31) into the loss function yields

$$L_i^{NLOLR} = \frac{1}{4} \cdot (E[\epsilon_i] - \epsilon_i)^2 . \tag{44}$$

We then can calculate the expected loss for the case where the central bank provides no LOLR services as

$$E\left[L_i^{NLOLR}\right] = \frac{1}{4} \cdot \sigma_{\epsilon_i}^2. \tag{45}$$

We now turn to the case where the central bank decides to provide LOLR. The loss of bank i can be calculated by substituting equilibrium inflation from equation (38) and output from equation (39) in equation (43), i.e.

$$L_i^{LOLR} = \frac{h^2}{4} \cdot (E[\epsilon_i] - \epsilon_i)^2 + C_i^O. \tag{46}$$

With LOLR ($J_i = 1$ for all i) the chance to save bank i is $1 - h$ and expected opportunity costs can be written as

$$E[C_i^O] = h \cdot F \cdot C_i^*. \tag{47}$$

[26] In order to do so we can simply add up the individual losses since we assumed that the λ_i are uncorrelated.

Thus, the expected loss when providing LOLR is

$$E[L_i^{LOLR}] \;=\; \frac{h^2}{4} \cdot \sigma_{\epsilon_i}^2 + h \cdot F \cdot C_i^*. \tag{48}$$

The central bank will decide to provide liquidity assistance if the expected loss resulting from this option is lower than when restraining from it, i.e.

$$\begin{aligned} E[L_i^{NLOLR}] \;&>\; E[L_i^{LOLR}] \\ \Leftrightarrow \sigma_{\epsilon_i} \;&>\; \sqrt{\frac{4 \cdot h \cdot F \cdot C_i^*}{1-h^2}}. \end{aligned} \tag{49}$$

Obviously, the decision to provide LOLR assistance depends on σ_{ϵ_i}, i.e. the standard deviation of the shock determining the reaction of depositors on the failure of bank i and thus on the size of the bank. Large banks cause large uncertainty when being closed and thus will be saved. Small banks get no LOLR services and will be closed when encountering liquidity troubles. This implication of the model is in line with the observation that governments or central banks are not willing to accept failures of large financial intermediaries while they often do not intervene in the case of smaller ones. This phenomenon is well-known as the so-called "too-big-to-fail doctrine".[27] The model also implies that the critical size of a bank depends negatively on the probability of a successful LOLR transaction $1 - h$.

If the central bank follows a too-big-to-fail doctrine, inflation and output will vary to a larger extent as is the case where LOLR transactions are costless. In order to calculate the inflation rate and output, we define the bank size σ_{ϵ_i}, which equals equation (49), as critical bank size σ_{crit}. Furthermore we sort the n banks in liquidity crises by size such that $\sigma_{\epsilon_1} < \sigma_{\epsilon_2} ... < \sigma_{\epsilon_n}$. Thus, we end up with two different types of banks: small banks with $\sigma_{\epsilon_i} < \sigma_{crit}$ and large banks with $\sigma_{\epsilon_i} \geq \sigma_{crit}$. While the large banks will be saved by LOLR the small banks will not.

[27] As large banks have a higher chance of being saved, this may trigger bank mergers in order to become too big to fail. See Hunter and Wall (1989) and Boyd and Graham (1991) for a discussion of this aspect. Benson, Hunter and Wall (1995) and Penas and Unal (2000) find supporting empirical evidence for this hypothesis. While in fact bank mergers have been an often observed phenomenon throughout the last decade it is somewhat unlikely that the too-big-to-fail doctrine alone leads to an excessive concentration of the banking sector due to possible diseconomies of scale.

Let n_{crit} be the number of small banks and $n - n_{crit}$ the number of large banks in liquidity crises. We can then calculate actual inflation with LOLR, opportunity costs and different bank sizes as

$$\begin{aligned} \pi &= \frac{n_{crit}}{n} \cdot \left[\pi^* + \frac{1}{2} \cdot \sum_{i=1}^{n_{crit}} (E[\epsilon_i] - \epsilon_i) \right] + \frac{n - n_{crit}}{n} \cdot \left[\pi^* + \frac{h}{2} \cdot \sum_{i=n_{crit}}^{n} (E[\epsilon_i] - \epsilon_i) \right] \\ \Leftrightarrow \pi &= \pi^* + \frac{1}{2} \cdot \frac{n_{crit}}{n} \cdot \sum_{i=1}^{n_{crit}} (E[\epsilon_i] - \epsilon_i) + \frac{n - n_{crit}}{n} \cdot \frac{h}{2} \cdot \sum_{i=n_{crit}}^{n} (E[\epsilon_i] - \epsilon_i). \qquad (50) \end{aligned}$$

Similarly for output we get

$$y = \frac{n_{crit}}{n} \cdot \frac{1}{2} \cdot \sum_{i=1}^{n_{crit}} (E[\epsilon_i] - \epsilon_i) + \frac{n - n_{crit}}{n} \cdot \frac{h}{2} \cdot \sum_{i=n_{crit}}^{n} (E[\epsilon_i] - \epsilon_i). \qquad (51)$$

Whether the expected loss under a central bank with a LOLR function is lower than without finally depends on the magnitude of the opportunity costs of unsuccessful LOLR transactions. Whenever at least one bank is large enough to be saved, the expected loss under LOLR is lower than without LOLR. Since the expected loss under LOLR and without LOLR is identical for the case that no single bank is considered to be too-big-to-fail, providing the central bank with a LOLR function is still optimal.

5. Macroeconomic consequences of moral hazard under LOLR

In the previous sections we implicitly assumed that the central bank's decision to provide LOLR services has no influence on the investment decisions of commercial banks. Thus, we dealt with the case where moral hazard behavior is totally absent. One might justify this assumption by assuming that the central bank successfully conceals its LOLR activities. However, it seems to be more reasonable to assume that commercial banks are aware of the LOLR function of the central bank. In the following we study how the knowledge about the central bank's incentives to provide LOLR services to certain commercial banks affects the behavior of banks and what consequences this behavior has for the variability of inflation and output.

Up to now we have not taken into account that the investment decision of banks that know that they will be saved by the central bank is different from the one of banks that know that they will get no support in the case of a liquidity shortage. As it was shown earlier, the investment decision of a bank crucially depends on $E[\lambda_i]$. Whenever a bank knows that it will not be supported by LOLR transactions we have

$$E[\lambda_i]^{NMH} = p \cdot \overline{\lambda} + (1-p) \cdot \underline{\lambda}. \tag{52}$$

However, when a bank knows that the central bank will try to save the bank in the case of emergency troubles, the expectation on λ_i changes to

$$E[\lambda_i]^{MH} = p \cdot \overline{\lambda} + (1-p) \cdot \left((1-h) \cdot \overline{\lambda} + h \cdot \underline{\lambda}\right). \tag{53}$$

Due to $\overline{\lambda} > \underline{\lambda}$ we have

$$\overline{\lambda} = \underline{\lambda} + \tilde{\lambda} \tag{54}$$

with $\tilde{\lambda} > 0$. Thus, we can rewrite equation (52) as

$$E[\lambda_i]^{NMH} \;=\; p \cdot \overline{\lambda} + (1-p) \cdot \underline{\lambda} + (1-p) \cdot (1-h) \cdot \tilde{\lambda}. \tag{55}$$

Obviously, banks that know they will be saved have a higher $E[\lambda_i]$ ceteris paribus than those which are not saved.[28] Since, according to equation 10, the optimal amount to be invested into the risky project positively depends on $E[\lambda_i]$ banks knowing they will receive LOLR assistance invest more into the risky project than those that can not expect to be supported by the central bank, i.e.

$$C^{*,MH} = \frac{\beta_0 + E[\lambda_i]^{MH} - r_1}{2 \cdot \beta_1} > C^{*,NMH}. \tag{56}$$

This sort of moral hazard behavior increases in the probability $1-h$ that a LOLR transaction will succeed in saving the bank.

However, not only the optimal amount of capital to be invested into the risky project but also the expected profit of a bank depends on $E[\lambda_i]$. Inserting equation (7) into equation (8) and taking the mathematical expectation yields

$$E[P_i] = (r_1 - 1) \cdot D_i + (\beta_0 + E[\lambda_i] - r_1) \cdot C_i - \beta_i \cdot C_i^2. \tag{57}$$

[28] More precisely, the expectation of λ_i increases by $(1-p) \cdot (1-h) \cdot \tilde{\lambda}$ compared to the case where a bank receives no LOLR services. It is easy to see that both cases are equivalent for the case where LOLR is never successful, i.e. $h = 1$.

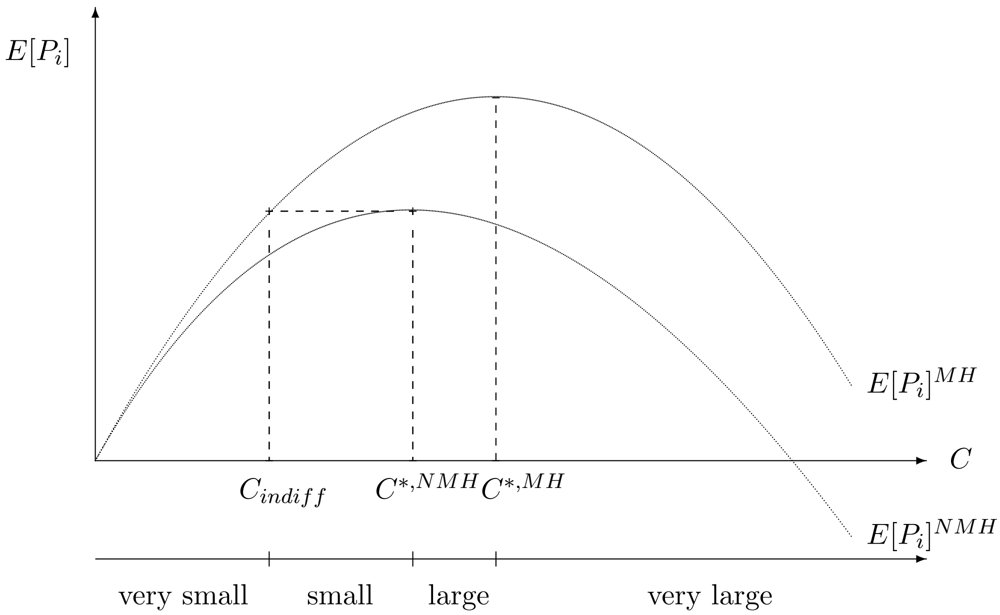

Figure 3. Expected profits of bank i with and without LOLR

In Figure 3 we show a plot of the two expected profit functions of a commercial bank with LOLR assistance and thus moral hazard ($E[P]^{MH}$) and without ($E[P]^{NMH}$) for alternative levels of capital invested into the risky project C. The difference between the two curves is equal to the expected value of the central banks' LOLR transactions.

We already showed in the previous section that the central bank's decision to provide LOLR services to a certain bank i depends on the size of the bank (D_i) and the expected opportunity costs of saving the bank (which is a function of the capital invested into the risky project C_i). For every single commercial bank i we can thus calculate a critical sum of risky investments C_i^{crit} that may not be exceeded to get LOLR assistance in the case of liquidity shortages.

In order to study how the presence of moral hazard influences the behavior of commercial banks, it is useful to subdivide banks by bank size into four subgroups. The results are summarized in Table I.

The first group ("very large banks") consists of banks fulfilling $C_i^{crit} > C^{*,MH}$. These banks know that they are large enough to receive LOLR services even when increasing their risky exposure to $C^{*,MH}$. Since doing so increases their expected profit, they will show moral hazard behavior thereby increasing the central bank's costs of unsuccessful LOLR transactions.

The second group ("large banks") is defined by $C^{*,NMH} < C_i^{crit} < C^{*,MH}$. While these banks are large enough to be saved without showing moral hazard behavior they know that they will not be saved when investing $C^{*,MH}$ into the risky project. These banks maximize their expected profits by choosing $C_i = C_i^{crit}$ and thus show moderate moral hazard behavior. Doing so guarantees that these banks will be supported in the case of liquidity shortages.

The third group ("small banks") is defined by $C_{indiff} < C_i^{crit} < C^{*,NMH}$ with C_{indiff} being the lowest value fulfilling

$$E[P(C_{indiff})]^{MH} = E[P(C^{*,NMH})]^{NMH}. \quad (58)$$

Commercial banks fulfilling this condition would be too small to be rescued when investing $C^{*,NMH}$ into the risky project. However, these small banks can increase their expected profits by decreasing their risky exposures to the level C_i^{crit}, which guarantees that the banks will receive LOLR in emergency cases. Thus, not only large and very large banks react on the provision of LOLR services but small banks do as well.

The last group ("very small banks") consists of banks for which $C_i^{crit} < C_{indiff}$ holds true. These very small banks have no incentive to decrease their risky exposure because their expected profit from investing $C_i = C^{*,NMH}$ is always larger than the one under LOLR. Thus, only very small banks do not alter their behavior when LOLR is provided by the central bank and thus show no moral hazard behavior.

Thus, moral hazard behavior in consequence of the central bank's LOLR function has two interesting macroeconomic implications.

First, moral hazard leads to more bank rescues than in the case where no moral hazard is present. This is due to the fact that with moral hazard in equilibrium all banks fulfilling $C_i^{crit} > C_{indiff}$ (i.e. all small, large and very large banks) are supplied with LOLR transactions whereas without moral hazard only those banks fulfilling $C_i^{crit} >$

Table I. Comparison of optimal investment in risky project with and without LOLR.

	moral hazard impossible		moral hazard possible	
Bank size	C_i	receives LOLR	C_i	receives LOLR
very small	$C^{*,NMH}$	no	$C^{*,NMH}$	no
small	$C^{*,NMH}$	no	C_i^{crit}	yes
large	$C^{*,NMH}$	yes	C_i^{crit}	yes
very large	$C^{*,NMH}$	yes	$C^{*,MH}$	yes

$C^{*,NMH}$ (large and very large banks) receive LOLR services (with $C_{indiff} < C^{*,NMH}$).[29] In consequence more banks can be expected to be saved, thus leading to a lower expected variance of inflation and output.

Second, expected costs of LOLR transactions increase as a consequence of two effects. On the one hand, large and very large banks now engage in moral hazard behavior and invest more capital into the risky project. On the other hand, the number of banks that are provided with LOLR services increases. Since LOLR transactions do not always save the supported banks, the expected costs from unsuccessful LOLR transactions will thus increase.

6. Conclusions

In this paper we argue that central banks' observed behavior to serve as a lender of last resort is motivated by the macroeconomic objective of stabilizing output and inflation around their target levels. By providing LOLR transactions, the central bank intends to lower the probability that banks go bankrupt, thereby decreasing uncertainty on the money multiplier and inflation and output variance. When at least unsuccessful LOLR transactions cause costs, the central bank has an incentive to save only large banks, a

[29] Remember that C_i^{crit} denotes the maximal amount of capital bank i with deposits D_i can invest into the risky project without having to fear not to be saved by the central bank via LOLR transactions. Without moral hazard, all banks invest the same capital $C^{*,NMH}$ into the risky project. All banks for which $C_i^{crit} > C^{*,NMH}$ holds true are saved. With moral hazard, even small banks are saved since they decrease their exposure in risky investments.

well-documented empirical phenomenon known as "too-big-to-fail doctrine". It is often argued that providing LOLR services leads to serious moral hazard behavior on the side of commercial banks, making LOLR transaction less useful. We show that in fact large and very large banks react on the provision of LOLR services via increasing their risky exposures. However, small banks also have an incentive to decrease their investments into risky projects in order to profit from LOLR services. In equilibrium banks' reactions on the provision of LOLR contribute to a higher degree of macroeconomic stability but also higher to stabilization costs.

It should be noted that it is an interesting feature of the presented model that the central bank should not engage in attempts to conceal its role as a lender of last resort. While it is not easy to conceal LOLR transactions succeeding in doing so would cause excessive macroeconomic instability.

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