General Equilibrium and the Emergence of (Non) Market Clearing Trading Institutions

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ABSTRACT. We consider a pure exchange economy, where for each good several trading institutions are available, only one of which is market-clearing. The other feasible trading institutions lead to rationing. To learn on which trading institutions to coordinate, traders follow behavioral rules of thumb that are based on the past performances of the trading institutions. Given the choice of institutions, market outcomes are determined by an equilibrium concept that allows for rationing. We find that full coordination on the market-clearing institutions without any rationing is a stochastically stable outcome, independently of the characteristics of the alternative available institutions. We also find, though, that coordination on other, non-market-clearing institutions with rationing can be stochastically stable.

The theory of general equilibrium is built upon two main assumptions. First, for given prices economic agents behave rationally. Second, prices are determined by the market clearing condition. This paper is part of a research line which attempts to explore to which extent are these assumptions necessary and/or justified.

Let us focus on market clearing. While not incorporated into general equilibrium models, actual market exchanges typically take place within clear institutional frameworks that range from the bazaar to the continuous double auction market. A huge variety of different markets institutions is used, and often different trading institutions exist for even for the trade of the same good. Furthermore, there is a lot of empirical and experimental evidence (see e.g. Holt [6], Plott [10], and Ockenfels and Roth [9]) showing that the specific rules of a trading institution affect the transactions' prices and the market outcomes. Often prices are biased towards one market side (see e.g. Plott [10] for the experimental investigation of the posted offer institutions) or otherwise constrained (e.g. by price rigidities), so that market-clearing is not always obtained. In this case some market participant face *rationing*. The question arises whether such an inefficient market institution can survive in the long run, or whether for efficiency reasons it would be replaced by a trading institution that promotes market clearing.

If the market outcomes at alternative market institutions differ, agents face a coordination problem in their institutional choice. A rational trader might opt for an institution that does not lead to market clearing outcomes and that does not maximize the gains of trade, simply because all other traders do so, and because trade at an inefficient institution is still better than having no trading partner at all. Borrowing from the literature of learning in games, we postulate a dynamic learning model for the institutional choice and ask whether traders will learn to coordinate on market-clearing institutions within a general equilibrium framework. When choosing among the trading institutions traders rely on certain behavioral "rules of thumb" for learning. We find this approach reasonable because due to the complexity of the institution evaluation problem faced by the agents, enriched by the coordination issue, rational learning seems implausible.

In a previous paper ([3]) we posed a similar question within a partial equilibrium buyers-sellers model with a single homogenous good. There we found that, for general learning processes, a market-clearing institution is always active in the long run, independently of the characteristics of the alternative, available institutions. This positive result has to be qualified by the additional finding that, in general, there might be alternative surviving institutions. Of course, in that model we abstracted from all general equilibrium effects. In particular, the identities of buyers and sellers were a priori fixed, so that e.g. a seller could never become a buyer.

In this paper, we account for the general equilibrium feedback effects of the institutional choice. Furthermore, the general equilibrium framework also allows us to endogenize whether a particular trader is a buyer of a seller of a particular good. To do so, we consider a pure exchange economy with K goods where for each good several trading institutions are available. For each good there is always one market-clearing institution. Furthermore, there is a numeraire good for which only one market clearing institution is feasible. At the beginning of the game every trader chooses for each good (but the numeraire good) at which market institution he wants to trade this good. Thus, each good can be distinguished according to the institution at which it is traded. After the choices of the institutions, trade is conducted according to the following rule: For each trader, for every good, and for every institution supply and demand is derived under the additional condition that supply and demand of the goods from the unchosen institutions is zero. This gives us the unrationed supply and demand functions for every trader, for every good, and for every institution. This in turn determines the market clearing prices for each good for each institution. Non-market clearing institutions are characterized by a parameter $r \neq 1$, which determines the amount of rationing at this institution. The amount of rationing determines the actual, non-market clearing price, and the actually realized trades. The model is closed by the residual trade of the numeraire good conducted on its market clearing institution.

Of course, the modelling device leading to our equilibrium concept is borrowed from the framework of an economy with rationing as developed by Dreze [4]. Our concept differs from that of Dreze in two respects. First, we directly model rationing, whereas Dreze focused on price rigidities. Second, traders do not take rationing into account when deriving their (unrationed) excess demand function. Rather, the actual demands are determined using the unrationed ones and the rationing schemes. So agent do not forecast rationing in advance, but through learning realized rationing affects agents' institution choice in the following period.

In order to model the learning process we use the stochastic stability techniques brought into the economics literature by Kandori, Mailath, and Rob [7] and Young [12] to analyze coordination games with players learning on which equilibrium to coordinate. Specifically, we axiomatize the traders' learning process to model the basic idea that they switch from one institution to a different one if they observe that the current-period results (prices and traded quantities as resulting from rationing) are better for them.

Conceptually, our analysis is also related to a recent literature which shows the stability properties of perfectly competitive behavior in learning models with boundedly rational agents, hence. Alós-Ferrer and Ania [2] show that, in any game where payoffs depend only on the own strategy and an aggregate of all strategies, and those payoffs present strategic substitutability (as e.g. in Cournot oligopolies or rent-seeking games), the profile where agents maximize payoffs taking the resulting aggregate as given is *evolutionarily stable*[1] and the only long-run outcome in imitation-based learning dynamics. Such stability result for aggregate-taking outcomes can be considered as a learning-based foundation for perfectly competitive behavior. The approach of this paper is complementary to the one of Alós-Ferrer and Ania in that we do not consider the stability of outcomes by themselves, but rather the stability of market institutions which channel those outcomes.

As a result, we show that the "Walrasian" constellation, where traders coordinate exclusively on market-clearing institutions (one per good), is *always* stochastically stable. This generalizes our previous result to a general equilibrium framework. We then proceed to show that, in general, certain non-market-clearing institutions also survive.

1. The Game Form

There are $i = 1, ..., N$ traders and $k = 1, ..., K$ commodities, plus a numeraire commodity $k = 0$. Each trader is characterized by excess demand functions

$$x_k^i\left(p^i\right)$$

where $p^i \in \mathbb{R}_+^K$ is the price vector with which trader i is confronted. Prices are measured in units of the numeraire good.

We consider a very regular economy. The excess demand functions are assumed to fulfill the following properties:

(A0) for all $i = 1, ..., N$, $k = 1, ..., K$,

(i) $x_k^i\left(p^i\right)$ is differentiable and strictly decreasing in p_k^i;
(ii) there exists an $a > 0$ such that for all $p^i \in \mathbb{R}_+^K$, $x_k^i\left(p^i\right) > -a$;
(iii) there exists a $p^i \in \mathbb{R}_+^K$, such that $x_k^i\left(p^i\right) < 0$;
(iv) if $p^{in} \to p^i$ where $p^i \neq 0$ and $p_k^i = 0$, then $x_k^i\left(p^{in}\right) \to \infty$;
(v) for all $k \neq l$, $\frac{\partial x_k^i\left(p^i\right)}{\partial p_l} > 0$.

A0(ii) is equivalent to the assumption that no trader can sell short. A0(iii) implies that each trader is endowed with a positive amount of every good. A0(iv) is fulfilled whenever the underlying preferences are strongly monotone, and A0(v) assumes that the goods are gross substitutes.

For each good $k \neq 0$ there exists a finite, nonempty set Z_k of institutions at which this good can be traded. In the first stage of the game, each trader decides for each

[1] We are referring here to the finite-population concept due to Schaffer [11]. In contrast to the infinite-population counterpart, an evolutionarily stable strategy in the former sense does not, in general, correspond to a Nash equilibrium.

good the institution at which he wants to trade. Hence, the strategy set of a every trader i is given by

$$S^i = \prod_{k=1}^{K} Z_k$$

Given a strategy profile $s \in S = \prod_{i=1}^{N} S^i$, denote by $N(s,z)$ the set of players who have chosen to trade good $k \neq 0$ at institution $z \in Z_k$. We say that an institution z is *nonempty* given s if $|N(s,z)| \geq 1$, i.e. there are at least one trader at z. The set of all nonempty institutions given s is denoted by $A(s)$. An institution z is *empty* if it is not nonempty.

Institution z is characterized by a rationing parameter $r_z > 0$. Let $Z = \{0\} \cup \left(\cup_{k=1}^{K} Z_k\right)$.

Let $\widetilde{x}_z^i$ denote the realized excess demand for trader i, i.e.

$$\widetilde{x}_z^i(p,s) = \begin{cases} r_z \cdot x_k^i\left(p^i\right) & \text{if} \quad r_z \leq 1 \text{ and } x_k^i\left(p^i\right) \geq 0 \\ x_k^i\left(p^i\right) & \text{if} \quad r_z \leq 1 \text{ and } x_k^i\left(p^i\right) \leq 0 \\ \frac{1}{r_z} \cdot x_k^i\left(p^i\right) & \text{if} \quad r_z \geq 1 \text{ and } x_k^i\left(p^i\right) \leq 0 \\ x_k^i\left(p^i\right) & \text{if} \quad r_z \geq 1 \text{ and } x_k^i\left(p^i\right) \geq 0 \end{cases}$$

where $z = z(s,i,k) \in Z_k$ is such that $i \in N(s,z)$ and $p_k^i = p_{z(s,i,k)}$. The realized excess demand for the numeraire is given by

$$\widetilde{x}_0^i(p,s) = -\sum_{k=1}^{K} p_{z(s,i,k)} \widetilde{x}_{z(s,i,k)}^i(p,s).$$

The interpretation is as follows. At every institution z for commodity k where a trader is active, he wants to trade $x_k^i\left(p^i\right)$. There are, however, institutions where one market side is rationed. If $r_z < 1$, buyers are rationed and can realize only r_z times their intended trades, whereas sellers face no restriction. If $r_z > 1$, sellers can only realize $\frac{1}{r_z}$ of their intended trades. If $r_z = 1$, neither market side is rationed and we say that institution z is *fully competitive*. We assume that, for each good k there is one fully competitive, Walrasian institution $w_k \in Z_k$ such that $r_{w_k} = 1$. Commodity 0 is used as the medium of exchange at all institutions for all other commodities and, therefore, there is no rationing for this commodity. This closes the model[2]. Note that the realized excess demand functions also fulfill the properties of A0. In order to close the model, the residual trade is conducted with the numeraire good on its market clearing institution.

In equilibrium, prices are determined in such a way that markets "clear" *taking rationing into account.*

Definition 1. *Given a vector $r = (r_z)_{z\in Z}$ and a strategy profile s, an (r,s)-equilibrium is given by a price vector $p^* = (p_z^*)_{z\in Z}$ such that, for all $k \neq 0$ and for all $z \in Z_k$,*

(i) $\sum_{i\in N(s,z)} \widetilde{x}_z^i(p^*,s) = 0$,

(ii) $\sum_{i=1,\dots N} \widetilde{x}_0^i(p^*,s) = 0$.

[2]This idea of a numeraire good which is traded without rationing is taken from [4].

Lemma 1. *Assume A0. For every $r = (r_z)_{z\in Z}$ and $s \in S$, there exists a unique (r,s)-equilibrium with strictly positive equilibrium prices at every nonempty institution.*

Proof. Note first that every trader trades every good at exactly one institution. Hence, we can redefine the economy so that the goods of different nonempty institutions are regarded as different goods (independently of whether they are physically the same). Formally, for every $r = (r_z)_{z\in Z}$ and for every $s \in S$, there are $i = 1, ..., N$ traders and $z = 1, ..., |A(s)|$ commodities, plus a numeraire commodity $z = 0$. For each trader i, each commodity z, and each price vector $p \in \mathbb{R}_+^{|A(s)|}$, the individual excess demand functions $\widehat{x}_z^i(p)$ are given by:

$$\begin{aligned} \widehat{x}_z^i(p) &= \begin{cases} \widetilde{x}_z^i(p,s) & \text{if } i \in N(s,z) \\ 0 & \text{otherwise} \end{cases} \\ \widehat{x}_0^i(p) &= \widetilde{x}_0^i(p,s) \end{aligned}$$

The aggregate excess demand functions $\widehat{x}_z^i(p)$ of the redefined economy are given by:

$$\widehat{x}_z(p) = \sum_{i=1}^{N} \widehat{x}_z^i(p)$$

Since the realized excess demand functions $\widetilde{x}_z^i(p,s)$ fulfill the properties of A0, and since for each commodity z of the redefined economy there exists least one trader i such that $i \in N(s,z)$, the aggregate excess demand functions $\widehat{x}_z(p)$ also fulfill the properties of A0. Furthermore, the definition of the realized excess demand of the numeraire guarantees that Walras Law holds. Hence, standard arguments imply existence of an Walrasian equilibrium of the redefined economy. Obviously, equilibrium prices and allocations of the redefined economy are an (r,s)-equilibrium of the original economy. Furthermore A0(iv) implies that equilibrium prices at the nonempty institutions are strictly positive. And finally, A0(v) implies the uniqueness of the Walrasian equilibrium, which translates into the uniqueness of the (r,s)-equilibrium. ■

The described model corresponds to a game form where strategies model the choice of institutions, and outcomes correspond to actual allocations of goods among traders. If demands are derived from maximization of standard utility functions, a natural way to complete this game form to a game would be simply to take payoffs to be given by the corresponding utilities derived from the allocations. We decided, however, to use a more general approach in order to allow for non-maximizing behavior.

2. The Learning Model

Behavior of traders will be modelled through two elements. First, whether a trader is able to revise his choice at all in a given period, and second, how will he choose the institutions he wants to trade at if given the opportunity to revise his previous choice. The revision probabilities will be described in Section 2.2.

2.1. The Choice of Institutions. How choices are revised is captured by behavioral rules, which can in general be expressed as mappings

$$B^i : S \to \Delta S^i,$$

i.e. given that the current strategy profile is given by s', $B^i(s')(s^i)$ denotes the probability that trader i will choose the combination of institutions prescribed in s^i next period, for any arbitrary s^i. Note that we allow each trader to correlate institution choices for different goods. Given an institution $z \in Z_k$, denote further by $B^i_k(s')(z)$ the probability that trader i will choose institution z for good k next period. That is, B^i_k is the marginal probability distribution derived from B^i, but B^i need not be equal to the product of its marginal distributions.

We assume that traders receive or make use of far less information than that contained in the whole strategy profile s. Specifically, they observe, for every nonempty institution, only the price and the rationing parameter. Formally, denote by

$$I(s) = \left[A(s), (p_z(s), r_z)_{z\in A(s)}\right]$$

the information available to traders. We assume that

$$B^i\left(s_1\right) = B^i\left(s_2\right) \text{ whenever } s_1^i = s_2^i \text{ and } I(s_1) = I(s_2).$$

This means that, when taking a decision, traders only take their previous decision, prices and rationing of nonempty institutions into account.

We make the following assumptions on the behavioral rules of traders:

(A1) For every strategy profile s, every good k, and every institution $z \in A(s) \cap Z_k$,

(i) if $i \in N(s, z)$ with $\widetilde{x}^i_k(s) \geq 0$ and there exists $z' \in Z_k$ with $r_{z'} \geq r_z$ (i.e. buyers are more rationed at z than at z', if at all rationed) and $p_{z'} \leq p_z$, or

(ii) if $i \in N(s, z)$ with $\widetilde{x}^i_k(s) \leq 0$ and there exists $z' \in Z_k$ with $r_z \leq r_{z'}$ (i.e. sellers are more rationed at z than at z', if at all rationed) and $p_{z'} \geq p_z$,

then the probability that agent i leaves institution z is strictly positive, i.e.

$$B^i_k(s)(z) < 1.$$

The intuition for A1 is straightforward. Take the case of a buyer who at a given institution z. He observes that buyers at another institution z' are less rationed (or not rationed at either institution) and pay a strictly lower price. Of course, a myopic buyer will not expect to become a seller if he switches to z' (by A0). Now A1 states that the buyer wants to switch either to z' with at least some probability, or to some other (maybe even better) third institution. Hence, he does not want to stay with certainty at z. The same intuition holds for a seller if he observes a higher price at another institution where sellers are less rationed.

One particular implication of A1 is that, if a trader is confronted with the same rationing in two different institutions (e.g. if he is not rationed in either institution), he does not stay at the institution with the worse price.

We want to extend the reasoning above to institutions where the trader is not too much rationed. Take a situation where a trader at institution z is not rationed. He observes another institution z' where he would received a better price, but at the cost of some rationing. Provided that the rationing is moderate compared to the price difference, he wants to switch. This reasoning is formalized by the following assumption:

(A1*) For every strategy profile s, every good k, and every institution $z \in A(s) \cap Z_k$:

(i) Take $p' < p$. Then there exists a $\underline{r}(p', p) < 1$ such that: if $i \in N(s, z)$ with $\widetilde{x}_k^i(s) \geq 0, p_z = p$, $r_z \geq 1$ and there exists $z' \in Z_k$ with $p_{z'} \leq p'$ and $r_{z'} \geq \underline{r}(p', p)$, then
$$B_k^i(s)(z) < 1$$
i.e. the probability that agent i leaves institution z is strictly positive.

(ii) Take $p' > p$. Then there exists a $\overline{r}(p', p) > 1$ such that: if $i \in N(s, z)$ with $\widetilde{x}_k^i(s) \leq 0, p_z = p$, $r_z \leq 1$ and there exists $z' \in Z_k$ with $p_{z'} \geq p'$ and $r_{z'} \leq \underline{r}(p', p)$, then
$$B_k^i(s)(z) < 1$$
i.e. the probability that agent i leaves institution z is strictly positive.

These assumptions can also be fulfilled by simpler behavioral rules such that traders only take into account, for nonempty institutions, the actual prices, and which market side is rationed (if any), but not the exact amount of rationing. Formally, we say that a behavioral rule B^i for trader i is *elementary* if

$$B^i(s_1) = B^i(s_2) \text{ whenever } s_1^i = s_2^i \text{ and } I^E(s_1) = I^E(s_2).$$

where
$$I^E(s) = \left[A(s), (p_z(s), sign\ \ (r_z - 1))_{z \in A(s)}\right]$$
is the elementary information traders can use.

Under the assumptions above, we obtain the following result for general (not necessarily elementary) behavioral rules:

Lemma 2. *Assume A1. Given any strategy profile $s \in S$ such that, for a good k, both the institution w_k and another, not fully competitive $z \in Z_k$ are nonempty, there exists at least agent $i \in N(s, z)$ such that*

$$B_k^i(s)(z) < 1$$

for all $i \in N(s, z)$ in that market side, i.e. the probability that those agents leave institution z is strictly positive.

Proof. Suppose $r_z < 1$. This implies that, at z, (weak) sellers are not rationed. If sellers wants to leave institution z, the proof is completed. If some seller wants to stay at institution z with certainty, then by A1(ii) it follows that $p_{w_k} < p_z$. Since buyers are rationed at institution z but there is no rationing at w_k, A1(i) implies that all (weak) buyers have positive probability to leave institution z. The proof for $r_z > 1$ is analogous. ■

(A2) For every strategy profile s, every good k, and every two institutions $z, z' \in Z_k$, we have that, if z is nonempty and z' is empty under s, then

$$B_k^j(s)\left(z'\right) = 0 \text{ for all } j \in N(s,z).$$

Intuitively, if the demand function of a trader prescribes to trade in a given, nonempty institution, it is because the trader prefers trading over no trading. Hence, a switch from an nonempty institution (where he is allowed to trade to at least some extent) to an empty institution (where he will not be able to trade) will never benefit him and therefore he will never switch to that empty institution. Alternatively, this assumption can be interpreted as an information constraint: empty institutions are not even observed, hence they are not perceived as alternatives.

Assumptions (A1), (A1*), and (A2) can be obtained from a simple behavioral rule which prescribes to compare the currently observed outcomes at all the nonempty trading institutions and switch with positive probability to those yielding the best outcomes, according to the own utility function. Such a rule is myopic in two respects. First, agents do not take into account the fact that switching from one institution to another affects the market outcome. Second, in making such simple, virtual utility comparisons, agents neglect the feedback effects that changes in the market outcome for one good has in the outcome for other goods.

2.2. Revision Opportunities. When can agents revise their choices? It is common in learning models to explicitly introduce some inertia allowing for the possibility that not all agents are able to revise strategies simultaneously. Different specifications of how revision opportunities arrive give rise to different dynamics and often affect the results. Rather than adopting a specific formulation, here, we postulate a general class of dynamics encompassing the standard examples (and many others), which are then reviewed below. See Alós-Ferrer [1] for a discussion.

Let $E(i,s)$ denote the event that agent i receives revision opportunity when the current state is s, and let $E^*(i,s) \subseteq E(i,s)$ denote the event that agent s is the only agent receiving revision opportunity in s.

Assumption D: $\Pr(E^*(i,s)) > 0$ for every agent i and state s.

Notice that assumption D implies that $\Pr(E(i,s)) > 0$, i.e. every agent has strictly positive probability of being able to revise at any given state.

Assumption D is rather general. It is fulfilled by the standard models considered in the literature of learning in games. In these models, revision opportunities are either modelled through independent probabilities (a case we call independent inertia) or assumed to arrive in an asynchronous way (also called non-simultaneous learning).

Independent Inertia. There is an exogenous, independent (across traders and periods) probability $0 < 1 - \rho < 1$ such that the agent does not get revision opportunity in a given state (inertia). Obviously, $\Pr(E^*(i,s)) = \rho(1-\rho)^{N-1} > 0$ for any agent i, hence verifying D.

Non-simultaneous Learning. Each period, only one agent (i.e. either a buyer or a seller) is (randomly) selected and allowed to revise his strategy. Hence, $\Pr(E^*(i,s)) = \frac{1}{N}$ for any trader i, verifying D.

The specification above allows for more general learning processes than those described by independent inertia or non-simultaneous learning. Since the revision probability $\Pr(E(i,s))$ is a function of the state s, it might depend e.g. on the difference of payoffs between different institutions (so that unsatisfied traders are more likely to revise), or on idiosyncratic characteristics of the currently chosen institution.

2.3. Learning Dynamics. Assumption D and the behavioral rules B^i define a stationary Markov chain on the (finite) state space S. Given two states $s, s' \in S$, denote by $P^0(s,s')$ the probability of transition from s to s' in one period for the learning process. The transition matrix of the process is given by $P^0 = \left[P^0(s,s')\right]_{s,s' \in S}$.

To fully characterize the learning dynamics, it is useful to summarize the basic results of Markov chains. An *absorbing set*[3] is a minimal subset of states which, once entered, is never abandoned. An *absorbing state* is an element which forms a singleton absorbing set, i.e. s is absorbing if and only if $P^0(s,s) = 1$. States that are not in any absorbing set are called *transient*.

Every absorbing set of a Markov chain induces an *invariant distribution*, i.e. a distribution over states $\mu \in \Delta(S)$ which, if taken as initial condition, would be reproduced in probabilistic terms after updating (more precisely, $\mu \cdot P = \mu$). The invariant distribution induced by an absorbing set $A \subseteq S$ has support A. The set of all possible invariant distributions of the process is the convex hull of the invariant distributions associated to the absorbing sets. By the Ergodic Theorem, the invariant distribution associated to a given absorbing set describes the time-average behavior of the system once (and if) it gets into that class. That is, $\mu(s)$ is the limit of the average time that the system spends in state s, along any sample path that eventually gets into the corresponding absorbing set. If, additionally, the absorbing set is *aperiodic*,[4] then the associated invariant distribution describes also the long-run probabilities of the states in the class, $\lim_{T\to\infty} q \cdot P^T = \mu$ for all probability distributions q whose support is contained in the absorbing set. This result is referred to as the Fundamental Theorem of Markov Chains.

A Markov chain is *ergodic* if it has a unique absorbing set. The (unique) invariant distribution constitutes the long-run prediction for such a process, since it represents the limit behavior of the process independently of initial conditions. If the process is not ergodic, then several invariant distributions exist, describing the long-run behavior along different sample paths, i.e. the prediction depends on the initial conditions.

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[3] Also called recurrent communication class or limit set.

[4] Loosely speaking, an absorbing set is aperiodic if it contains no deterministic non-trivial cycles. A sufficient condition for aperiodicity is that for some state s in the set, $P(s,s) > 0$. Note also that any absorbing state is aperiodic.

Obviously, by A2, full coordination at exactly one institution per good is an absorbing state. Hence, there is a multiplicity of absorbing sets. In order to select among them, and following the literature, we proceed to study stochastic stability. The dynamics is enriched with a perturbation in the form of experiments (or mistakes) as follows. With an independent probability $\varepsilon > 0$, each trader, in each round, might experiment (or make a mistake or mutate), and simply pick a new combination of institutions at random,[5] independently of other considerations.

The dynamics with experimentation is called *perturbed* learning process. Its transition matrix is denoted by $P\varepsilon$. Since experiments make transitions between any two states possible, the perturbed process has a single absorbing set formed by the whole state space (such processes are called *irreducible*). Hence, the perturbed process is ergodic. The corresponding (unique) invariant distribution is denoted $\mu(\varepsilon)$. The *limit invariant distribution* (as the rate of experimentation tends to zero) $\mu^* = \lim_{\varepsilon \to 0} \mu(\varepsilon)$ exists and is an invariant distribution of the unperturbed process P^0 (see e.g. [7], [12], or [5]).

The limit invariant distribution singles out a stable prediction of the unperturbed dynamics, in the sense that, for any $\varepsilon > 0$ small enough, the play approximates that described by μ^* in the long run. The states in the support of μ^*, i.e. $\{s \in S \mid \mu^*(s) > 0\}$ are called *stochastically stable states* or long-run equilibria. Clearly, the set of stochastically stable states is the union of some absorbing sets of the original, unperturbed chain ($\varepsilon = 0$).

In the sequel, whenever we say absorbing sets or states, we refer to the dynamics without experimentation. Since the perturbed dynamics is irreducible, no confusion should arise.

2.4. Radius and Coradius of a state. Given two absorbing sets X and Y, let $c(X,Y) > 0$ (referred to as the *transition cost* from X to Y) denote the minimal number of mistakes necessary for a direct transition from X to Y, i.e. a positive probability path starting in an element of X and leading to an element in Y, which does not go through any other absorbing set.

Transitions need not be direct, though. Define a path from X to Y as a finite sequence of absorbing sets $P = \{X = S_0, ..., S_K = Y\}$. Let $S(X,Y)$ be the set of paths from X to Y. Given a path P, define its length $l(P)$ as the number of elements of the sequence minus 1, so that $P = \{X = S_0, ..., S_{l(P)} = Y\}$. We extend the cost function to paths by $c(P) = \sum_{k=1}^{l(P)} c(S_{k-1}, S_k)$.

Now, define $C(X,Y) = \min_{P \in S(X,Y)} c(P)$ to be the minimal number of mistakes required for a (possibly indirect) transition from X to Y.

The *Radius* of an absorbing set X is defined as

$$R(X) = \min \{C(X,Y) \,|Y \text{ is an absorbing set, } Y \neq X\}$$

i.e. the minimal number of mistakes needed to leave X towards another absorbing set.

[5] We mean that a combination of institutions is picked up according to a pre-specified probability distribution having full support, for instance uniformly. The exact distribution does not affect the results, as long as it has full support, and does not depend on ε.

Intuitively, the radius measures how easy it is to destabilize an absorbing set. To obtain a measure for the accessibility of an absorbing set, we define the coradius of X as

$$CR(X) = \max\{C(Y,X)\,|Y \text{ is an absorbing set, } Y \neq X\}$$

Ellison [5] provides a powerful result which states that, if $R(X) > CR(X)$ for a given absorbing set X, then X is the unique stochastically stable set. The intuition is clear, for the inequality $R(X) > CR(X)$ simply expresses the idea that X is easier to reach than to leave. This result, though, is not enough for our purposes. We will make use of a finer result which makes use of the concept of modified coradius. This concept takes into account that paths from Y towards X which pass through other absorbing sets have an increased probability which can be measured exactly by the radius of the intermediate absorbing sets.

Given a path $P = \{S_0, ..., S_{l(P)}\}$, define the modified transition costs $c^*(P) = c(P) - \sum_{k=1}^{l(P)-1} R(S_k)$. Define accordingly $C^*(X,Y) = \min_{P\in S(X,Y)} c^*(P)$. Then, the modified coradius is given by

$$CR^*(X) = \max\{C^*(Y,X)\,|Y \text{ is an absorbing set, } Y \neq X\}$$

The following Lemma, taken from [5, Theorem 2], states that, whenever the radius exceeds the (modified) coradius, the absorbing set is easier to reach than to destabilize, and hence it is stochastically stable.

Lemma 3. *Let A be an absorbing set. Then:*

(i) If $R(A) \geq CR^*(A)$, the states in A are stochastically stable.

(ii) If $R(A) > CR^*(A)$, the only stochastically stable states are those in A.

(iii) If the states in an absorbing set B are stochastically stable and $R(A) = C^*(B,A)$, the states in A are also stochastically stable.

Proof. Part (ii) follows immediately from [5, Theorem 2]. Part (i) follows from part (iii) taking B to be any absorbing set containing stochastically stable states (which always exist). Part (iii) follows from [5, Theorem 3]. ■

3. Stochastic Stability

We now turn to the analysis of the stochastically stable states. We will show that, for an arbitrary number of traders, goods, and feasible institutions, full coordination on a Walrasian institution for each good is stochastically stable. It turns out, though, that full coordination on certain non-market clearing institutions is also stochastically stable.

3.1. Stability of Walrasian institutions. The Walrasian state W is the state where, for each good k, all traders coordinate in the corresponding fully competitive institution w_k. That is, $A(W) = \{w_k\}_{k=1}^K$.

Theorem 4. *Under A1, A2, and D, W is stochastically stable.*

Proof. We will show that, for any state s, there exists a chain of single mutations connecting s with the competitive state. Hence, the modified coradius of W is equal to 1.

Start at state s, and consider a good k such that there is some nonempty institution z for k other than w_k. With a single mutation, some agent will choose institution w_k. By Lemma 1 and Assumption D, with positive probability one agent who was previously trading good k at z will leave institution z. Repetition of this argument ends with a state where institution z becomes empty. Repetition of the argument for all other nonempty institutions for good k ends with a state s' where the only nonempty institution for good k is w_k Note that, by A2, none of the empty institutions can become nonempty in the process.

This new state s' might be absorbing or not. If it is not absorbing, there exists a positive probability path, involving no further mutations, connecting s' to a new, absorbing state s''. By A2, the only nonempty institution for good k in s'' is also w_k.

$>$From absorbing state s'', the same argument (involving a single mutation) allows to reach a new (absorbing) state where the only nonempty institution for another good k' is w_k. Repetition of this arguments leads to the state W.

Since the radius of all intermediate absorbing states is 1 and they are left with a single mutation, the modified coradius of W is 1. Since the radius of W is at least 1, it follows from Lemma 3 that W is stochastically stable. ■

3.2. Stability of other institutions.

Lemma 5. *Let the set of available institutions be given by $\{z_k(r_k), w_k\}_{k=1}^{K}$, where w_k is fully competitive for good k, and $z_k(r_k)$ is an alternative institution for good k with rationing parameter r_k. Let s be a fixed strategy profile. For each vector $r = (r_1, \ldots, r_K)$, let $p(r)$ the corresponding equilibrium price vector. Then, $p(r)$ is continuous in r.*

Proof. Straightforward. ■

Theorem 6. *Assume A1,A1*,A2, and D. For generic economies, there exist $\underline{r}_k < 1$ and $\overline{r}_k > 1$ for all k such that, if $z_k(r_k)$ is an institution for good k with rationing parameter $r_k \in]\underline{r}_k, \overline{r}_k[$, the state ω where all traders coordinate at the institutions $z_k(r_k)$ for all k is stochastically stable.*

Proof. Since W is stochastically stable, it suffices to show that there exists a positive probability transition from W to the new state which involves only chains of single mutations (hence the modified coradius of the new state is 1).

We start at state W and proceed iteratively. At each step, we will show that one mutation suffices for all traders to move away from w_k and fully coordinate at $z_k(r_k)$.

Take good 1. Let S_1 be the set of strategy profiles s such that $A(s) \cap Z_1 = \{w_1, z_1)\}$ and $A(s) \cap Z_k = \{w_k\}$ for all $k \neq 1$.

Given $s \in S_1$ and $z_1 = z_1(1)$, let $\varepsilon_s^1 = |p_s(z_1(1)) - p_s(w_1)|$ where $p_s(z_1(1))$ and $p_s(w_1)$ are the equilibrium prices at the two nonempty institutions for good 1 (which are actually both fully competitive). Let $0 < \delta < \frac{1}{2} \min \left\{ \varepsilon_s^1 \,\middle|\, s \in S, \, \varepsilon_s^1 > 0 \right\}$.

By continuity, there exist $\underline{r}_1' < 1$ and $\overline{r}_1' > 1$ such that, for every $\underline{r}_1' < r_1 < \overline{r}_1'$, $|p_s(z_1(r_1)) - p_s(z_1(1))| < \delta$ for all $s \in S_1$, where $p_s(z_1(1))$ is the equilibrium price when $z_1 = z_1(1)$ and $p_s(z_1(r_1))$ is the equilibrium price when $z_1 = z_1(r_1)$.

We want to show that, for any $s \in S_1$, and for r_1 close enough to 1, there always exists a trader at w_1 who wants to change to $z_1(r_1)$. Since there are no other nonempty institutions, by A2 it suffices to show that some trader wants to switch away from w_1.

Note that, if $s \in S_1$ is such that $p_s(z_1(1)) < p_s(w_1)$, then $p_s(z_1(1)) + \delta < p_s(w_1)$. By A1*(i), there exists $\underline{r}_1\,(p_s(z_1(1)) + \delta, p_s(w_1))$ such that any buyer at w_1 who observes a price lower than or equal to $p_s(z_1(1)) + \delta$ at institution $z_1(r_1)$ with $r_1 \geq \underline{r}_1\,(p_s(z_1(1)) + \delta, p_s(w_1))$ switches to $z_1(r_1)$.

Let $\underline{r}_1''$ be the maximum among all the $\underline{r}_1\,(p_s(z_1(1)) + \delta, p_s(w_1))$ for profiles $s \in S_1$ such that $p_s(z_1(1)) < p_s(w_1)$.

Analogously, if $s \in S_1$ is such that $p_s(z_1(1)) > p_s(w_1)$, then $p_s(z_1(1)) - \delta > p_s(w_1)$ and there exists $\overline{r}_1\,(p_s(z_1(1)) - \delta, p_s(w_1))$such that any seller at w_1 who observes a price higher than or equal to $p_s(z_1(1)) - \delta$ at institution $z_1(r_1)$ with $r_1 \leq \overline{r}_1\,(p_s(z_1(1)) - \delta, p_s(w_1))$ switches to $z_1(r_1)$. Let $\overline{r}_1''$ be the minimum among all the $\overline{r}_1\,(p_s(z_1(1)) - \delta, p_s(w_1))$ for profiles $s \in S_1$ such that $p_s(z_1(1)) > p_s(w_1)$.

Let $\underline{r}_1 = \max(\underline{r}_1', \underline{r}_1'') < 1$ and $\overline{r}_1 = \min(\overline{r}_1', \overline{r}_1'') > 1$ and fix now any $r_1 \in]\underline{r}_1, \overline{r}_1[$.

Let $s \in S_1$. Suppose first $r_1 < 1$. If $p_s(z_1(r_1)) \geq p_s(w_1)$, by A1(ii), (weak) sellers at w_1 want to switch to $z_1(r_1)$ with positive probability. Hence, we are left with the case $p_s(z_1(r_1)) < p_s(w_1)$.

Generically, $p_s(z_1(1)) \neq p_s(w_1)$. By construction, $p_s(z_1(r_1)) < p_s(w_1)$ implies that $p_s(z_1(1)) < p_s(w_1)$ and $p_s(z_1(r_1)) < p_s(z_1(1)) + \delta < p_s(w_1)$. Since, by hypothesis, $r_1 \geq \underline{r}_1\,(p_s(z_1(1)) + \delta, p_s(w_1))$, it follows that buyers switch from w_1 to $z_1(r_1)$.

The case $r_1 > 1$ is analogous. This shows that, as long as both w_1 and $z_1(r_1)$ are nonempty, under Assumption D traders will move from the former to the latter with positive probability.

Suppose now traders are coordinated at state W. After an initial mutation where one trader switches to $z_1(r_1)$, the argument above shows that there exists a positive probability path to the absorbing state where all traders are coordinated at $z_1(r_1)$ and w_k for $k \neq 1$.

Let now S_2 be the set of strategy profiles s such that $A(s) \cap Z_1 = \{z_1(r_1))\}$, $A(s) \cap Z_2 = \{w_2, z_2)\}$and $A(s) \cap Z_k = \{w_k\}$ for all $k \geq 3$. An analogous argument shows that there exist $\underline{r}_2 < 1$ and $\overline{r}_2 > 1$ such that, if $z_2(r_2)$ is an institution for good 2 with rationing parameter $r_2 \in]\underline{r}_2, \overline{r}_2[$, a single mutation from the previous absorbing state suffices to trigger a transition to the absorbing state where all traders are coordinated at $z_1(r_1)$, $z_2(r_2)$ and w_k for $k \geq 3$. The argument proceeds then iteratively. ■

4. EXAMPLE: A SIMPLE EXCHANGE ECONOMY

We now analyze a simple example in order to show the intuition for the general results. This will also serve the purpose to exemplify that our assumptions about the learning process do no contradict the standard framework, where the demand is derived from utility maximisation.

Take a very simple exchange economy with four agents, denoted by A, A', B and B'. There are two goods, 0 and 1. All traders are endowed with the same utility functions

$$U^i(q_0^i, q_1^i) = q_0^i q_1^i, \qquad i = A, B, A', B'$$

with q_0^i, q_1^i denoting the quantities i consumes of good 0 and 1. The initial endowments are given by $e^A = e^{A'} = (e_0^A, e_1^A) = (3, 1)$ and $e^B = e^{B'} = (e_0^B, e_1^B) = (1, 3)$.

Good 0 is traded at a fully competitive market institution without any rationing. It serves as numeraire. All traders have to choose simultaneously and independently the market institution at which each of them wants to trade good 1. A fully competitive institution w is feasible for good 1, but traders can also choose an alternative non-competitive institution z with $r_z = r > 1$.[6] Since traders might opt for different institutions, the price of good 1 need not be the same for all traders. Denote the price at which trader i trades good 1 by p^i.

Utility maximization gives the following excess demand functions:

$$\begin{aligned} \left(x_0^i, x_1^i\right) &= \left(\frac{p^i - 3}{2}, \frac{3 - p^i}{2p^i}\right) \qquad i = A, A' \\ \left(x_0^j, x_1^j\right) &= \left(\frac{3p^j - 1}{2}, \frac{1 - 3p^j}{2p^j}\right) \qquad j = B, B' \end{aligned}$$

We first analyze the outcome of the different possible distributions of traders over the institutions. A state is given by $s = (s_A, s_{A'}, s_B, s_{B'})$ where s_i is the institution for good 1 chosen by trader i. Taking into account the possibility of relabeling of the traders, we have to consider eight different states:

- $s_1 = (w, w, w, w)$. All traders coordinate on the market clearing institution. This implies that the price is the same for all traders, $p^A = p^{A'} = p^B = p^{B'} = p$. The equilibrium condition is given by

  $$2\frac{p - 3}{2p} + 2\frac{3p - 1}{2p} = 0$$

  This implies that the equilibrium price is one, that each traders consumes 2 units of each good, and that the achieved utility is four.

- $s_2 = (z, z, z, z)$. All traders coordinate on the non-market clearing institution. This implies that the price is the same for all traders, $p^A = p^{A'} = p^B = p^{B'} = p$. Due to the distribution of the initial endowments, in equilibrium traders B and B' have to be the sellers of good 1. Hence, the equilibrium condition is given by

  $$2\frac{1}{r}\frac{1 - 3p}{2p} + 2\frac{3 - p}{2p} = 0$$

  This leads to:

  $$p = \frac{1 + 3r}{3 + r}.$$

[6] By symmetry, similar results can be derived in the presence of a non-market clearing institution with $r < 1$.

Traders A and A' are not rationed in that case. Hence, their realized trades coincide with their excess demands.

$$(\tilde{x}_0^i, \tilde{x}_1^i) = \left(-\frac{4}{3+r}, \frac{4}{1+3r}\right) \qquad i = A, A'$$

leading to

$$\begin{aligned}(q_0^i, q_1^i) &= \left(\frac{5+3r}{3+r}, \frac{5+3r}{1+3r}\right) \qquad i = A, A' \\ U^i(r) &= \frac{(5+3r)^2}{(1+3r)(3+r)} \qquad i = A, A'\end{aligned}$$

The realized utilities of traders A and A' are strictly decreasing in r for all r larger than 1. $U^A(1) = 4$, and for r going to infinity, the utility converges to the no-trade utility of 3.

Traders B and B' are rationed in good 1. Their realized trades are given by:

$$\left(\tilde{x}_0^j, \tilde{x}_1^j\right) = \left(-p\tilde{x}_1^j, \frac{1}{r}x_1^j\right) = \left(\frac{4}{3+r}, -\frac{4}{1+3r}\right) \qquad j = B, B'$$

leading to

$$\begin{aligned}\left(\tilde{q}_0^j, \tilde{q}_1^j\right) &= \left(\frac{7+r}{3+r}, \frac{9r-1}{1+3r}\right) \qquad j = B, B' \\ U^j(r) &= \frac{(9r-1)(7+r)}{(1+3r)(3+r)} \qquad j = B, B'\end{aligned}$$

The realized utilities of traders B and B' are strictly increasing in r at $r = 1$. It reaches its maximum at $r \simeq 2.2$ and decreases for larger r. For r going to infinity, the utility converges to the no-trade utility of 3.

As it can be easily seen, if r converges to one, the realised trades converge to the quantities of state s_1. Hence, $U^A(1) = U^B(1) = 4$.

- $s_3 = (w, z, w, w)$. A, B, and B' opt for the market clearing institution, whereas A' chooses the non-market clearing institution. The analysis for this state is equivalent to that of state (z, w, w, w).

  The equilibrium price of good 1 at the non-market clearing institutions must be such that the individual excess demand of trader A' is zero. This implies $p^{A'} = 3$. Of course this price is only virtual, because no actual trade of good 1 is possible for trader A'. His utility level is given by $U^{A'} = 3$.

  The other traders coordinate on the market clearing institution. Denote the price for all these traders by p^A. The equilibrium condition is given by

$$2\frac{1-3p^A}{2p^A} + \frac{3-p^A}{2p^A} = 0.$$

  This implies $p^A = \frac{5}{7}$.

Traders are not rationed in that case. Hence, their realized trades coincides with their excess demands.

$$\begin{aligned}(\widetilde{x}_0^A, \widetilde{x}_1^A) &= \left(-\frac{8}{7}, \frac{8}{5}\right) \\ \left(\widetilde{x}_0^j, \widetilde{x}_1^j\right) &= \left(\frac{4}{7}, -\frac{4}{5}\right) \qquad j = B, B'\end{aligned}$$

leading to

$$\begin{aligned}(\widetilde{q}_0^A, \widetilde{q}_1^A) &= \left(\frac{13}{7}, \frac{13}{5}\right) \\ \left(\widetilde{q}_0^j, \widetilde{q}_1^j\right) &= \left(\frac{11}{7}, \frac{11}{5}\right) \qquad j = B, B'\end{aligned}$$

The realized utility of trader A is $\frac{169}{35}$, whereas the realized utility of traders B and B' is $\frac{121}{35}$.

- $s_4 = (w, w, w, z)$. A, A' and B opt for the market clearing institution, whereas B' chooses the non-market clearing institution. The equilibrium price of good 1 at the non-market clearing institutions must be such that the individual excess demand of trader B' is zero. This implies $p^{B'} = \frac{1}{3}$. Again this price is only virtual, because no actual trade of good 1 is possible for trader B'. His utility level is given by $U^{B'} = 3$.

  The other traders coordinate on the market clearing institution. Denote the price for all these traders by p^A. The equilibrium condition is given by

$$\frac{1 - 3p^A}{2p^A} + 2\frac{3 - p^A}{2p^A} = 0.$$

  This implies $p^A = \frac{7}{5}$.

  Traders are not rationed in that case. Hence, their realized trades coincides with their excess demands

$$(\widetilde{x}_0^i, \widetilde{x}_1^i) = \left(-\frac{4}{5}, \frac{4}{7}\right) \qquad i = A, A'$$
$$(\widetilde{x}_0^B, \widetilde{x}_1^B) = \left(\frac{8}{5}, -\frac{8}{7}\right)$$

  leading to

$$(\widetilde{q}_0^i, \widetilde{q}_1^i) = \left(\frac{11}{5}, \frac{11}{7}\right) \qquad i = A, A'$$
$$(\widetilde{q}_0^B, \widetilde{q}_1^B) = \left(\frac{13}{5}, \frac{13}{7}\right)$$

  The realized utility of trader B is $\frac{169}{35}$, whereas the realized utility of traders A and A' is $\frac{121}{35}$.

- $s_5 = (z, z, z, w)$. A, A', and B opt for the non-market clearing institution, whereas B' chooses the market clearing institution. The equilibrium price of good 1 at the market clearing institutions must be such that the individual excess demand of trader B' is zero. This implies a (virtual) price of $p^{B'} = \frac{1}{3}$. The utility level of B' is $U^{B'} = 3$.

  The other traders coordinate on the non-market clearing institution. Denote the price for all these traders by p^A. The equilibrium condition is given by

  $$\frac{1}{r}\frac{1-3p^A}{2p^A} + 2\frac{3-p^A}{2p^A} = 0.$$

  This implies

  $$p^A = \frac{1+6r}{3+2r}$$

  Traders A and A' are not rationed in that case. Hence, their realized trades coincides with their excess demands.

  $$(\widetilde{x}_0^i, \widetilde{x}_1^i) = \left(-\frac{4}{3+2r}, \frac{4}{1+6r}\right) \qquad i = A, A'$$

  leading to

  $$(\widetilde{q}_0^i, \widetilde{q}_1^i) = \left(\frac{5+6r}{3+2r}, \frac{5+6r}{1+6r}\right) \qquad i = A, A'$$

  The realized utility of traders A and A' are strictly decreasing in r. For $r = 1$, their utilities are $\frac{121}{35.}$, and for r going to infinity, the quantities and the utilities converge to the no-trade levels.

  Trader B is rationed in good 1. His realized trades are given by:

  $$(\widetilde{x}_0^B, \widetilde{x}_1^B) = \left(-p\widetilde{x}_1^B, \frac{1}{r}x_1^B\right) = \left(\frac{8}{3+2r}, -\frac{8}{1+6r}\right)$$

  leading to

  $$(\widetilde{q}_0^B, \widetilde{q}_1^B) = \left(\frac{11+2r}{3+2r}, \frac{-5+18r}{1+6r}\right)$$
  $$U^B(r) = \frac{(11+2r)(-5+18r)}{(1+6r)(3+2r)}.$$

  At $r = 1$, the realized utility of trader B is $\frac{169}{35}$, and strictly increasing in r. It reaches its maximum at $r \simeq 1.65$ with a utility of about 5.14. For higher r the utility is strictly decreasing. For r going to infinity, the quantities and the utility converge to the non-trade levels.

- $s_6 = (z, w, z, z)$. A, B, and B' opt for the non-market clearing institution, whereas A' chooses the market clearing institution. The equilibrium price of good 1 at the market clearing institutions must be such that the individual excess demand of trader A' is zero. This implies a (virtual) price of $p^{A'} = 3$. The utility level of A' is $U^{A'} = 3$.

The other traders coordinate on the non-market clearing institution. Denote the price for all these traders by p^A. The equilibrium condition is given by

$$2\frac{1}{r}\frac{1-3p^A}{2p^A}+\frac{3-p^A}{2p^A}=0.$$

This implies

$$p^A=\frac{2+3r}{6+r}.$$

Trader A is not rationed in that state. Hence, his realized trades coincide with his excess demands.

$$(\widetilde{x}_0^A,\widetilde{x}_1^A)=\left(-\frac{8}{6+r},\frac{8}{2+3r}\right)$$

leading to

$$(\widetilde{q}_0^A,\widetilde{q}_1^A)=\left(\frac{10+3r}{6+r},\frac{10+3r}{2+3r}\right)$$
$$U^B(r)=\frac{(10+3r)^2}{(6+r)(2+3r)}.$$

The realized utility of trader A is strictly decreasing in r. For $r=1$, his utility is $\frac{169}{35.}$, and for r going to infinity, the quantities and the utility converge to the non-trade levels.

Traders B and B' are rationed in good 1. Their realized trades are given by:

$$\left(\widetilde{x}_0^j,\widetilde{x}_1^j\right)=\left(-p\widetilde{x}_1^j,\frac{1}{r}x_1^j\right)=\left(\frac{4}{6+r},-\frac{4}{2+3r}\right)\qquad j=B,B'$$

leading to

$$\left(\widetilde{q}_0^j,\widetilde{q}_1^j\right)=\left(\frac{10+r}{6+r},\frac{2+9r}{2+3r}\right)\qquad j=B,B'$$
$$U^j(r)=\frac{(10+r)(2+9r)}{(6+r)(2+3r)}\qquad j=B,B'$$

At $r=1$, the realized utilities of traders B and B' are $\frac{121}{35}$, and strictly increasing in r. They reach their maximum at $r\simeq 3.25$ with utilities of about 3.8. For higher r the utilities are strictly decreasing. For r going to infinity, the quantities and the utilities converge to the non-trade levels.

- $s_7=(w,z,w,z)$. A and B opt for the market clearing institution, whereas A' and B' choose the non-market clearing institution. In this state, traders A and B realize the same prices, quantities and utilities as in state s_1.

$$p^A=p^B=1$$
$$U^A=U^B=4.$$

On the other hand, traders A' and B' realize the same prices, quantities and utilities as in state s_2.

$$p^{A'} = p^{B'} = \frac{1+3r}{3+r}$$
$$U^{A'}(r) = \frac{(5+3r)^2}{(1+3r)(3+r)}$$
$$U^{B'}(r) = \frac{(9r-1)(7+r)}{(1+3r)(3+r)}$$

The realized utility of trader A' is a strictly decreasing function in r for all r larger than 1. $U^A(1) = 4$, and for r going to infinity, the utility converges to the no-trade utility of 3. The realized utilities of trader B' is strictly increasing at $r = 1$. It reaches its maximum at $r \simeq 2.2$ and decreases for larger r. For r going to infinity, the utility converges to the no-trade utility of 3.

- $s_8 = (w, w, z, z)$. A and A' opt for the market clearing institution, whereas B and B' choose the non-market clearing institution. Thus, no trade is possible. This state leads to the same outcome as (z, z, w, w). All traders get the same no-trade utility of three. The virtual prices are given by

$$p^A = p^{A'} = 3$$
$$p^B = p^{B'} = \frac{1}{3}$$

We go on by characterizing the absorbing sets of the unperturbed learning processes.

Lemma 7. *Under A1 and A3, states s_1 and s_2 constitute the only absorbing sets of the unperturbed learning process.*

Proof. >From A3 it is immediately clear that states s_1 and s_2 are absorbing. In order to show that they are the only absorbing sets, we will show that for any other state s there exists a positive probability path from ω to either state 1 or state 2.

State 3: In state 3, trader A', who is at the non-market clearing institution, is exposed to a price of 3, whereas in the market clearing institution the price of good 1 is $\frac{5}{7}$. Hence, by A1(i) there is a positive probability that he switches to the market clearing institution, leading to state 1. Note in state 3, $U^{A'} = 3$, whereas the utility of all other traders is strictly above three. Hence, our assumptions about the learning process are compatible with an approach, where learning would be directly driven by the realized utility levels.

State 4: In state 4, trader B', who is at the non-market clearing institution, is exposed to a price of $\frac{1}{3}$, whereas in the market clearing institution the price of good 1 is $\frac{7}{5}$. Hence, by A1(ii) there is a positive probability that he switches to the market clearing institution, leading to state 1. Note again that in state 4, $U^{B'} = 3$, whereas the utility of all other traders is strictly above three.

State 5: In state 5, trader B', who is at the market clearing institution, is exposed to a price of $\frac{1}{3}$, whereas in the market clearing institution the price of good 1 is

$\frac{1+r}{3+2r} > \frac{1}{3}$. Hence, by A1(ii) there is a positive probability that he switches to the non-market clearing institution, leading to state 2. Note again that in state 5, $U^{B'} = 3$, whereas the utility of all other traders is strictly above three.

State 6: In state 6, trader A', who is at the market clearing institution, is exposed to a price of 3, whereas in the market clearing institution the price of good 1 is $\frac{2+3r}{6+r} < 3$. Hence, by A1(i) there is a positive probability that he switches to the non-market clearing institution, leading to state 2. Note again that in state 6, $U^{A'} = 3$, whereas the utility of all other traders is strictly above three.

State 7: In state 7, trader A', who is at the non-market clearing institution, is exposed to a price of $\frac{1+3r}{3+r} > 1$, whereas in the market clearing institution the price of good 1 is 1. Hence, by A1(i) there is a positive probability that he switches to the market clearing institution, leading to state 4. As we have already seen, in state 4 trader B' might switch to the market clearing institution, leading to state 1. Note again that in state 7, $U^{A} = 4$, whereas $U^{A'} < 4$.

State 8: In state 7, trader B, who is at the non-market clearing institution, is exposed to a price of $\frac{1}{3}$, whereas in the non-market clearing institution the price of good 1 is 3. Hence, by A1(ii) there is a positive probability that he switches to the market clearing institution, leading to state 5. As we have already seen, in state 5 trader B' might switch to the market clearing institution, leading to state 1. ■

Next we show that both states are stochastically stable:

Lemma 8. *Under A1* and A3, and if r is not too large,* $R(s_1) = R(s_2) = CR(s_1) = CR(s_2) = 1$.

Proof. Since there are only two absorbing sets, by definition of Radius and Coradius it holds that $R(s_1) = CR(s_2)$ and $R(s_1) = CR(s_2)$. We will now first analyze $R(s_1)$ and then $R(s_2)$.

i) $R(s_1) = 1$: Assume that trader A' makes a mistake and switches to the non-market clearing institution. Now we are in state s_3, where the price in the non-market clearing institution is 3, whereas the price in the market clearing institution is $\frac{5}{7}$. If r is not too large, $A1^*(ii)$ implies that B and B' switch with a strictly positive probability to the non-market clearing institution in the unperturbed learning process. This leads to state s_6. As we have already seen in the proof of the previous lemma, the unperturbed learning process implies a strictly positive probability to switch from state s_6 to state s_2.

i) $R(s_2) = 1$: Assume that trader B' makes a mistake and switches to the market clearing institution. Now we are in state 5, where the price in the non-market clearing institution is $\frac{1+6r}{3+2r}$, whereas the price in the market clearing institution is $\frac{1}{3}$. A1(i) implies that A and A' switch with a strictly positive probability to the market clearing institution in the unperturbed learning process. This leads to state s_4. As we have already seen in the proof of the previous lemma, the unperturbed learning process implies a strictly positive probability to switch from state s_4 to state s_1. ■

5. CONCLUSION

Our results can be interpreted in two ways. On the one hand, coordination on the market-clearing institutions is obtained independently of the characteristics of the alternative available trading institutions. This strong stability result shows that the market-clearing "assumption" is indeed, to a certain extent, justified.

On the other hand, some alternative non market-clearing institutions are also stochastically stable. Hence, nothing guarantees that the actually used trading institutions are efficient - some regulatory interventions might be necessary to improve the functioning of trading institutions. Furthermore, non-market clearing "stable" institutions can be deliberately designed, if it is in the interest of a market designer.

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