

International Integration, Common Exposure and Systemic Risk in the Banking Sector: An Empirical Investigation*

Nicole Allenspach[†] Pierre Monnin[‡]

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[†]Swiss National Bank. E-mail: nicole.allenspach@snb.ch.

[‡]Swiss National Bank and University of Zurich. E-mail: pierre.monnin@snb.ch.

Abstract

This paper analyzes the impact of ongoing financial integration and increase in cross-border activities on banks' common exposure to shocks and on banking sector systemic risk. For that, we study the evolution of correlations between large international banks' asset-to-debt ratios over 1993-2006 and compute a systemic risk index for the same period. We find that banks' common exposure to shocks has significantly decreased until 2000 and rapidly increased afterwards. Systemic risk follows a totally different pattern. No trend emerges and, instead, we observe two peaks: one in 1998 (LTCM and Russian crisis) and one in 2002-2003 (stock market downturn). These findings suggest that, contrary to a widespread belief, higher correlations between banks do not necessarily induce higher systemic risk. We then provide evidence that systemic risk is mainly driven by banks' individual risk-taking rather than by their common exposure to risks.

Keywords: Systemic risk, Co-movements, Banking sector, International integration.

JEL-Classifications: F36, G15, G21.

1 Introduction

Worldwide, the banking sector has gone through profound transformations in the last decades. Technical progresses in financial engineering and in communications technologies as well as global deregulation policies have significantly modified the international financial landscape. In Europe, the launch of the Euro has additionally contributed to accelerate these changes.¹ An obvious outcome of these developments is an acceleration of international financial markets integration. At the firm level, financial institutions—and large banks in particular—profit now from a much easier access to a wider range of markets and financial instruments. Yet, the consequences of this new environment on banks' common exposure to shocks and on systemic risk in the banking sector are ambiguous. With respect to banks' common exposure to shocks, a wider range of markets and financial tools offers banks an opportunity to differentiate themselves by implementing their own specific business strategy. Adopting different strategies reduces the common exposure of banks to shocks. At the same time, however, easy access and low entry costs are likely to increase the number of banks competing on a particular market, thereby inducing an increase in these banks' common exposure to shocks. Similar holds for systemic risk. Access to new markets and new financial techniques offers financial institutions—on the one hand—more possibilities to manage and diversify their risks, which is beneficial in terms of systemic risk. On the other hand, given that banks compete on the same markets, they are vulnerable to similar shocks. Furthermore, intensified cross-border linkages between banks increase the risk of contagion. Both effects are rather negative for systemic stability. To sum up: From a theoretical point of view, it is not evident which of these effects dominate.

In this paper, we try to clear up these ambiguities with empirical data by studying a panel of large international banks over a period running from 1993 through to 2006. Concretely, we intend to answer two questions: 1) what was the impact of interna-

¹See Baele *et al.* (2004).

tional integration on banks' common exposure to shocks between 1993 and 2006? And 2) what was the impact of international integration on systemic risk in the international banking sector between 1993 and 2006? Note that these questions are not identical. Monnin (2004) shows with a simple numerical example that a higher common exposure does not necessarily imply higher systemic risk. Indeed, an increase in common exposure can be compensated by a decrease in banks' total exposure, making the overall systemic risk to decrease. This situation can occur, for example, when two banks switch from independent risky strategies (no common exposure, very high individual risk) to a common safe strategy (complete common exposure, but to a very low risk). By answering the two questions mentioned above, we also aim to study in more detail the empirical link between common exposure to shocks and systemic risk during the period under consideration.

To estimate the impact of international integration on common exposure, we analyse the evolution of the co-movements – i.e. correlation – between banks' asset-to-debt ratios (AD ratios henceforth). As shown in Section 2.1, shocks to AD ratios sum up both shocks to bank's assets and liabilities as well as their effects on each other. AD ratios movements can thus be considered as a good summary of movements in bank's overall condition. A high (low) correlation between AD ratios suggests that both banks are similarly (differently) affected by shocks and thus that they have a high (low) common exposure to shocks.

To estimate the impact of international integration on systemic risk, we compute a systemic risk index based on Lehar (2005) and study its evolution during the period 1993-2006. Lehar's index measures the probability of observing a systemic crisis – defined as a given number of simultaneous bank defaults - in the banking sector at a given point in time. We finally try to assess if there is a link between the evolution of banks' common exposure (i.e. correlation between AD ratios) and systemic risk.

Several studies have analyzed co-movements in banks' characteristics. DeNicolò

and Kwast (2002) find a significant rise in stock return correlations between large US banking institutions during the 1990s. In a similar study for the European Union, Schröder and Schüller (2003) show that the correlations between 13 national bank stock indices have risen significantly in the last years. Brasili and Vulpes (2005) draw a similar conclusion when studying the correlations between European banks' distance-to-defaults. Hawkesby, Marsh and Stevens (2005) analyze the correlations between equity returns (and between credit default swaps premia) for a sample of European and US large and complex financial institutions (LCFI). They find a relatively high degree of co-movements in asset prices of LCFIs compared to a control group of non-financials. However, their results also show that a considerable degree of heterogeneity remains between different sub-groups of the sample: There exists, e.g., a divide between European and US banking institutions. The general conclusion from these studies is that co-movement between banks has increased in the last decade, which suggest that banks are becoming more similar with time and face more and more the same risks. Most of these studies conjectured that the observed increased in co-movements leads to a higher systemic risk.

Our paper differs from the previous studies in two ways: firstly, we use a new method based on Ledoit, Santa-Clara and Wolf (2003) to estimate the joint dynamic of the AD ratios as a whole (i.e. for all banks at the same time), whereas previous studies concentrated on the dynamic between pairs of banks. The resulting variance-covariance matrix can then be used directly in the computation of the systemic risk index. Secondly, and this is our main contribution, we study in detail the link between common exposure and systemic risk. We assess if a higher common exposure to shocks (i.e. higher correlation) is associated with higher or lower systemic risk or if it does not play any role for systemic risk. As mentioned before, other studies often claim that higher correlation yields to higher systemic risk, without formally checking this assumption.

The main results of our analysis are the following. Firstly, we find that the correlation between banks' AD ratios decreases in the first part of the sample period, and increases after 2000. This suggests that banks have adopted different business strategies, and thus reduced their common exposure to shocks, in response of changes in the banking sector environment until 2000. After this date, however, their strategies seem to become increasingly similar and their common exposure rises. This finding holds for the whole sample as well as for different sub-groups (namely North America and European Union). However, the degree of common exposure differs between these groups. Correlations between North-American banks tend to be higher than between their European counterparts. Co-movements between US and European banks are far less pronounced than within each regional sub-group, suggesting that these two groups are (at least partially) exposed to different shocks.

Secondly, in contrast to the correlation analysis, we do not find any significant trend in the systemic risk index. The latter is rather characterized by two peaks at the end of 1998 and at the end 2002 - beginning 2003. These two periods correspond to two well-known episodes of high level of stress on the banking sector: the LTCM and the Russian crisis at the end of 1998 and a persistent downturn on stock market in 2002-2003. Taking a closer look on different sub-groups, we find that the high level of systemic risk in 2002-2003 is mainly attributable to European banks suffering additionally from bad economic conditions in the European economy. In the US sample, the crisis period 2002-2003 is less pronounced but an additional (however less explicit) crisis period is detected around 1994-1995.

Thirdly, our results point out that correlation between banks is not a reliable measure of systemic risk. The link between correlation and our systemic risk index is weak and its direction can change depending on the period considered. On the contrary, the distance-to-default, which is a combination of AD ratio volatility and level, turns out to be a very reliable factor explaining the systemic risk index. In other words, systemic

risk seems to be the consequence of each banks' individual risk taking (i.e., its distance-to-default) rather than of the banks' common exposure to shocks (correlation). This finding warns against viewing systemic risk as a pure correlation phenomenon and highlights the danger of high and volatile leverage at the bank individual level. Note, however, that once the effects of the distance-to-default are taken into account, we find that correlation is positively associated with systemic risk. In other words, for a given level of individual risk and reserves, a higher common exposure implies a higher systemic risk.

The paper is structured as follows: Section 2 explains the methodology used to estimate the correlation dynamics and the systemic risk index. The data used is described in Section 3. Section 4 studies the correlation dynamics between large international banks' AD ratios. Section 5 presents the estimated systemic risk index and compares its evolution with the correlation dynamics. Section 6 concludes.

2 Methodology

As mentioned in the previous section, we need two ingredients to answer the questions at the centre of this paper: the evolution of the correlations between AD ratios and the one of the systemic risk index. To get them, we proceed in four steps: 1) we make some assumptions about bank asset and debt dynamics, 2) we use these assumptions to recover the AD ratio from equity and debt data using Merton's method (as the AD ratios are not directly observable), 3) we estimate the dynamic of the AD ratios, including the dynamic of their covariances, using a multivariate GARCH model and 4) we use the estimated dynamic to compute the systemic risk index. The next four sections describe these four steps in details. In addition, the next section shows why shocks to AD ratios sum up both shocks to bank assets and debts as well as their effects on each others.

2.1 Asset and debt dynamic

The AD ratio is defined as the ratio between the asset market value of a bank and its debt face value. Unfortunately, asset market values are not directly observable, but, following Merton (1974), they can be estimated by modelling bank's equity as a call option on bank's assets. However, to recover the AD ratios from observed equity prices with Merton's technique, we have to assume that asset market values and debt face values follow a multivariate Itô process such as

$$\begin{bmatrix} d\mathbf{A}_t \\ d\mathbf{D}_t \end{bmatrix} = \begin{bmatrix} \mathbf{A}_t & 0 \\ 0 & \mathbf{D}_t \end{bmatrix} \begin{bmatrix} \boldsymbol{\mu}_A \\ \boldsymbol{\mu}_D \end{bmatrix} dt + \begin{bmatrix} \mathbf{A}_t & 0 \\ 0 & \mathbf{D}_t \end{bmatrix} \begin{bmatrix} \boldsymbol{\Upsilon}_{AA} & \boldsymbol{\Upsilon}_{AD} \\ \boldsymbol{\Upsilon}_{DA} & \boldsymbol{\Upsilon}_{DD} \end{bmatrix} \begin{bmatrix} d\mathbf{w}_A \\ d\mathbf{w}_D \end{bmatrix} \quad (1)$$

We have that $d\mathbf{A}_t$ and $d\mathbf{D}_t$ are $(N \times 1)$ vectors containing the instantaneous change in the banks' assets A_t^i and debts D_t^i , respectively. N is the number of banks. \mathbf{A}_t and \mathbf{D}_t are $(N \times N)$ matrices containing the assets A_t^i and the debts D_t^i in their respective diagonals, all other elements being zeros. $\boldsymbol{\mu}_A$ and $\boldsymbol{\mu}_D$ are $(N \times 1)$ vectors regrouping the constant instantaneous growth rate of banks' assets and debts, respectively. $d\mathbf{w}_A$ and $d\mathbf{w}_D$ ($N \times 1$) are vectors of independent Wiener processes. They represent the individual shocks to assets and debts of each bank at time t . Finally $\boldsymbol{\Upsilon}_{AA}$, $\boldsymbol{\Upsilon}_{AB}$, $\boldsymbol{\Upsilon}_{BA}$ and $\boldsymbol{\Upsilon}_{BB}$ are $(N \times N)$ matrices which regroup the instantaneous responses of assets and debts to the different shocks. For example, v_{AD}^{ij} (the ij -th element of $\boldsymbol{\Upsilon}_{AD}$) is the instantaneous response of the i -th asset value to a shock in the j -th debt.

The diagonals of the matrices $\boldsymbol{\Upsilon}_{AA}$ and $\boldsymbol{\Upsilon}_{DD}$ are the direct responses of the bank's assets and debt to their own shocks. The diagonal of the matrix $\boldsymbol{\Upsilon}_{AD}$ ($\boldsymbol{\Upsilon}_{DA}$) is the direct response of a bank's assets (debt) to a shock affecting its own debt (assets). All other elements are the indirect responses of a bank's assets and debt to shocks affecting other banks' assets and debt. They represent the contagion effects between banks through interlinkages. Note that we do not assume any symmetrical response

between banks or between asset and debts. For example, the correlation between assets and debt can be different according to whether the shock affects the assets or the debt. Similarly, the response of bank i to a shock to bank j can be different from a bank j 's response to bank i 's shock.

Without loss of loss of generality (see Appendix A), we can rewrite equation (1) in the reduced form

$$dz_t = \boldsymbol{\mu}dt + \boldsymbol{\Upsilon}dw \quad (2)$$

where

$$\boldsymbol{\Upsilon}\boldsymbol{\Upsilon}' = \boldsymbol{\Omega}$$

\mathbf{z}_t is the $(n \times 1)$ vector regrouping the log AD ratios $z_t^i = \ln(A_t^i/D_t^i)$, $\boldsymbol{\mu}$ is a $(n \times 1)$ vector with the drift in log AD ratios and $\boldsymbol{\Omega}$ is the $(n \times n)$ variance-covariance matrix between instantaneous log AD ratio changes.

2.2 Recovering AD ratios from equity prices

Merton (1974) first suggested to model the bank's equity as a call option on the bank's assets to compute the bank's default probability estimated by market participants. This method can also be used to recover the bank's AD ratio from the equity price. Merton's method is based on the fact that if, at debt's maturity time $t + T$, the value of the bank's assets is smaller than its debt ($A_{t+T}^i < D_{t+T}^i$), then it is not rational for the shareholders to exercise the option, i.e., the bank defaults. If the bank defaults, the value of the equity is then zero. Merton's model states therefore that the value of bank i 's equity at time $t + T$ is:

$$E_{t+T}^i = \max(A_{t+T}^i - D_{t+T}^i, 0) \quad (3)$$

where E_t^i is the bank's stock price.

An equity with such payoffs is similar to an exchange option². If both the assets and the debt are log normally distributed, as stated in Equation (1), its initial value can easily be computed (Margrabe 1978) and is equal to:

$$E_t^i = A_t^i \Phi(d_1) - D_t^i \Phi(d_2) \quad (4)$$

with

$$d_1 = \frac{z_t^i + \left(\sigma_{z_t^i}^2/2\right) T}{\sigma_{z_t^i} \sqrt{T}}$$

$$d_2 = d_1 - \sigma_{z_t^i} \sqrt{T}$$

where $\Phi(\cdot)$ is the cumulative normal distribution and $\sigma_{z_t^i}^2$ is the conditional variance of the log AD ratio. Dividing both sides of Equation (4) by D_t^i yields

$$X_t^i = Z_t^i \Phi(d_1) - \Phi(d_2) \quad (5)$$

where X_t^i is the equity-to-debt ratio of bank i at time t and Z_t^i is its AD ratio at time t . Using Itô's lemma, we have that

$$\sigma_{x_t^i} X_t^i = \sigma_{z_t^i} Z_t^i \Phi(d_1) \quad (6)$$

where $\sigma_{x_t^i}^2$ is the conditional variance of the log equity-to-debt ratio.

The bank i 's debt D_t^i and equity price E_t^i are directly observable. We can form the equity-to-debt ratio X_t^i and compute its conditional variance $\sigma_{x_t^i}^2$. With X_t^i and $\sigma_{x_t^i}$ known, we are left with two unknown variables in Equation (5) and (6): the AD ratio Z_t^i and its conditional variance $\sigma_{z_t^i}$. The AD ratio Z_t^i can be recovered by simultaneously

²Exchange options are sometimes also referred to as options to exchange one asset for another.

solving Equation (5) and (6) using a numerical iterative process.^{3,4}

Note that this method gives the correct AD ratio only if the market participants correctly interpret the information they have about the banks. In particular, they should understand correctly the inter-dependence between banks and integrate it in their valuation. Note also that this method is not the only one that can be used to recover the AD ratio.⁵ However, Hovakimian and Kane (2000) show that the differences in the AD ratio valuations given by these different methods are small.

2.3 Estimation of the AD ratio joint dynamics

The next step in our procedure is to estimate the dynamic process in Equation (2) with empirical data. We choose to model the AD ratio dynamic with a multivariate GARCH model (M-GARCH). The main advantage of this kind of model is that it allows for *time-varying* variances and covariances. This is necessary in our context since we are interested in the evolution of the correlation through time.

The equivalent of Equation (2) in discrete time is:

$$\begin{aligned}\Delta \mathbf{z}_t &= \boldsymbol{\mu}_z + \mathbf{u}_t \\ \mathbf{u}_t &\sim N(0, \mathbf{H}_t)\end{aligned}\tag{7}$$

where $\Delta \mathbf{z}_t$ is a $(N \times 1)$ vector containing the variables Δz_t^i , $\boldsymbol{\mu}_z$ is a $(N \times 1)$ vector of constants, \mathbf{u}_t is a $(N \times 1)$ vector of white noise residuals and \mathbf{H}_t denotes the $(N \times N)$

³Note that only the "marginal" variance of the equity-to-debt ratios is necessary to recover the AD ratios. In particular, this technique does not require to know the correlations between the banks' equities (see Zhou (2001) for another illustration with a bivariate model). Therefore, this technique can be applied separately to each bank for a given T .

⁴Other techniques are available to recover the market asset value. Duan (1994) estimates it by maximizing a likelihood function. Vassalou and Xing (2004) use an iterative process that does not require the second equation.

⁵See for example, Bichsel and Blum (2004) for a model without stochastic debt, Ronn and Verma (1986) for a model with forbearance or Saunders and Wilson (1995) for a model with an infinite-maturity put.

conditional variance-covariance matrix of the residuals. We use the diagonal *Vech* model (Bollerslev, Engle and Wooldridge 1988) specification for the dynamic of \mathbf{H}_t . In this model, the conditional covariance h_t^{ij} between bank i and bank j depends only on the past covariance and the past residuals:

$$h_t^{ij} = \omega_{ij} + \alpha_{ij}u_{t-1}^i u_{t-1}^j + \beta_{ij}h_{t-1}^{ij} \quad (8)$$

The model has the property that when two banks experience trouble at the same time, their correlation increases and they will tend to move together in the future. This property is important as it captures the empirical finding that the correlation between banks seems to increase during bad periods. Yet, correlation will also increase when both banks are hit by a positive shock; i.e., the covariances do not behave asymmetrically.

In matrix form, Equation (8) has the following representation

$$\mathbf{H}_t = \mathbf{C} + \mathbf{A} \otimes \mathbf{u}_{t-1} \mathbf{u}_{t-1}' + \mathbf{B} \otimes \mathbf{H}_{t-1} \quad (9)$$

where the coefficient matrices \mathbf{C} , \mathbf{A} and \mathbf{B} are $(N \times N)$ matrices regrouping the parameters ω_{ij} , α_{ij} and β_{ij} , respectively. The symbol \otimes denotes the Hadamar product of two matrices.⁶ The covariance matrix \mathbf{H}_t has some distinctive characteristics to be respected by the estimation method. First, \mathbf{H}_t is symmetric implying that \mathbf{C} , \mathbf{A} and \mathbf{B} must also be symmetric. Secondly, \mathbf{H}_t is a positive semidefinite matrix and thus, any estimation of it must also be positive semidefinite.

A natural way to estimate \mathbf{H}_t and the coefficient matrices \mathbf{C} , \mathbf{A} and \mathbf{B} seems to use maximum likelihood estimates as it is usually done for univariate GARCH models. Unfortunately, this is not feasible because (i) the parameters are too numerous and so intricately linked that existing optimization algorithms do usually not converge, and (ii) maximum likelihood estimation does not necessarily give positive semidefinite

⁶The Hadamar product is the elementwise product of two matrices: $\mathbf{U} \otimes \mathbf{V} = (u_{ij}v_{ij})$.

covariance matrices. To cope with this second problem, econometricians usually impose additional conditions on the model coefficients to ensure that the matrix \mathbf{H}_t is positive semidefinite.⁷ In addition to the fact that such restrictions might not make sense from an economic point of view, Kroner and Ng (1998) have shown that M-GARCH results are very sensitive to different specifications.

We choose to follow a different approach to estimate the coefficient matrices \mathbf{C} , \mathbf{A} and \mathbf{B} : the Flexible M-GARCH method developed by Ledoit, Santa-Clara and Wolf (2003). This procedure has the advantage to solve both problems previously mentioned without imposing *a priori* restrictions on the coefficients. It is based on a *decentralized* estimation of the coefficients. Ledoit *et al.* propose to estimate the coefficient matrices \mathbf{C} , \mathbf{A} and \mathbf{B} in two steps. In the first step, each coefficient of the matrix is independently estimated with a univariate or bivariate GARCH model. Thus, the estimation of a large matrix is reduced to several univariate and bivariate problems for which conventional univariate and bivariate GARCH estimation techniques are easy to apply. As indicated above, the resulting estimated coefficient matrices $\hat{\mathbf{C}}$, $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ will obviously not necessarily ensure the variance-covariance matrix to be positive semidefinite. Thus, in a second step, Ledoit *et al.* apply a result from Ding and Engle (2001) stating that positive semidefinite coefficient matrices are a sufficient condition to yield (almost surely) a positive semidefinite variance-covariance matrix. The second part of the procedure is thus to find the positive semidefinite matrices $\tilde{\mathbf{C}}$, $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$ that are the closest to the $\hat{\mathbf{C}}$, $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ initial matrices.⁸ The coefficient matrices $\tilde{\mathbf{C}}$, $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$ given by the Flexible M-GARCH estimation are then used to compute the conditional variance-covariance dynamic with Equation (8) and, in the next step, to compute the systemic risk index.

⁷See Ding and Engle (2001) for a recent comparison of the restrictions used by different models.

⁸The positive semidefinite matrix \mathbf{X} , which is the closest from the initial matrix \mathbf{Y} , can be found by using a simple algorithm developed by Sharapov (1997).

2.4 Construction of the systemic risk index

Our systemic risk index follows Lehar (2005). The index is an estimation of the *probability of a systemic banking crisis* at time t . In this paper, a systemic crisis is defined as follows:

Definition 1 *A systemic crisis occurs when a percentage θ of the banking system becomes insolvent within the next k periods.*

Definition 1 requires another definition, which specifies under which conditions a bank defaults.

Definition 2 *The bank i defaults if the market value A_t^i of its assets falls below the face value D_t^i of its debt within the next k periods (i.e. $A_{t+j}^i < D_{t+j}^i$ for at least one $j \in [0, k]$ or equivalently $Z_{t+j}^i < 1$ for at least one $j \in [0, k]$).*

Given Definitions 1 and 2, the systemic risk index can be expressed as:

$$I_t(\theta) = \Pr \left[\sum_{i=1}^N \theta_t^i b_t^i > \theta \right] \quad (10)$$

where N is the number of banks, θ_t^i is the weight⁹ of bank i in the banking system at time t and

$$b_t^i = \begin{cases} 1 & \text{if } A_{t+j}^i < D_{t+j}^i \text{ for at least one } j \in [0, k] \\ 0 & \text{otherwise} \end{cases}$$

b_t^i is a dummy variable that takes the value 1 if bank i goes bankrupt in the next k periods. There are various ways to determine a bank's weight θ_t^i in the banking sector (e.g. equal weight, proportion of a bank's assets in the total banking sector's assets, proportion of a bank's interbank deposits in the total system, etc.). We chose to give an equal weight to each bank.

⁹The individual weights are normalized such that $\sum_{i=1}^N \theta_i = 1$.

Unfortunately, it is not possible to compute analytically the probability of a systemic crisis. To estimate it, we proceed with a Monte-Carlo simulation. An alternative would have been to use approximation techniques such as the one proposed by Carmona and Durrleman (2004). The Monte-Carlo simulation, yet, has the advantage of being easier and faster to implement and more flexible for further developments (e.g. modification of the dynamic or explicit modelling of contagion effects). A description of the simulation algorithm to estimate the systemic risk index is given in Appendix B.

3 Data

The raw data needed to compute the correlation between the AD ratios and to construct the systemic risk index consist of individual banks' balance sheet data (debt) and market information (equity prices). Data on debt is taken from Bloomberg while equity prices stem from Datastream. As data on debt is not available on a monthly basis, quarterly and - for some banks - yearly data have been transformed into monthly data by linear interpolation.

We constructed two different datasets. The first dataset comprises monthly data on 27 large international banks from November 1992 until June 2006 (long sample).¹⁰ The second dataset (short sample) comprises data on a total of 39 large international banks—including the 27 institutions already represented in the first dataset—from June 1997 until June 2006.¹¹

¹⁰The long sample consists of banks from: Germany (3), France (1), Italy (2), Netherlands (2), Spain (2), Sweden (2), Switzerland (1), UK (3), USA (2), Canada (5), Australia (4).

¹¹The short sample consists of banks from: Belgium (3), Germany (3), France (2), Italy (3), Netherlands (2), Spain (2), Sweden (2), Switzerland (1), UK (5), USA (7), Canada (5), Australia (4).

4 Common exposure to shocks

To get an idea of how banks' common exposure to shocks has evolved in time, we study the dynamic of the AD ratio correlations. More precisely, we try to identify a potential common trend in the correlations between each pairs of banks. A high AD ratio correlation indicates that two banks are both equally affected by a shock, i.e. that they have a high common exposure to shocks. As explained in the introduction, this is a sign that both banks have a similar business strategy or are competing on the same markets. Thus, if we observe a common upward trend in all correlations, we can conclude that, in the aggregate, banks' common exposure has increased and that banks have become more similar.

We can get a first idea on the trend in correlation by observing the evolution of the average correlation (cf Figure 1). In both samples, the average correlation decreases until about 2000 and then increases regularly. This pattern is also observed in the average correlation between pairs of North American banks, pairs of European Union banks and between pairs of banks from each sub-group. This is a first indication that, in the aggregate, banks' common exposure to shocks has decreased until about 2000 and then increased.

To study more precisely this hypothesis, we estimate the common trend to all correlations in a panel data analysis. We then test for a break in the slope of the common trend. Concretely, we estimated the following system of equations

$$y_t^{ij} = \gamma + \delta t + u^{ij} + \varepsilon_t^{ij} \quad (11)$$

where y_t^{ij} is a logit transformation¹² of the correlation between bank i and j at time t , γ is the average (logit) correlation, δ is the slope of the common trend, u^{ij} is a fixed

¹²More precisely, $y_t^{ij} = \ln\left(\frac{v_t^{ij}}{1 - v_t^{ij}}\right)$ where $v_t^{ij} = (cor(i, j) + 1)/2$. This transformation insure that y_t^{ij} is distributed over $]-\infty; +\infty[$ whereas the correlation is bounded between -1 and 1 .

FIGURE 1: CORRELATION BETWEEN AD RATIOS (LEFT: LONG SAMPLE, RIGHT: SHORT SAMPLE)

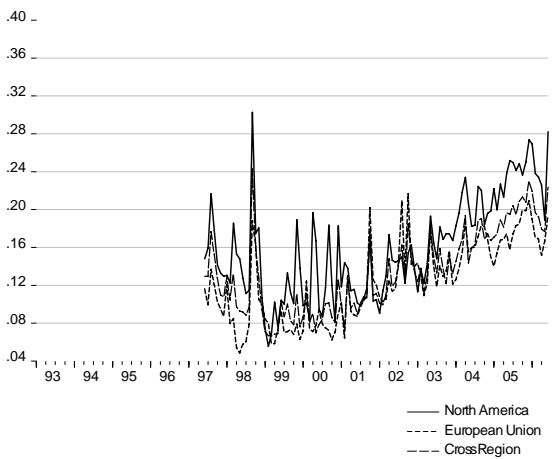
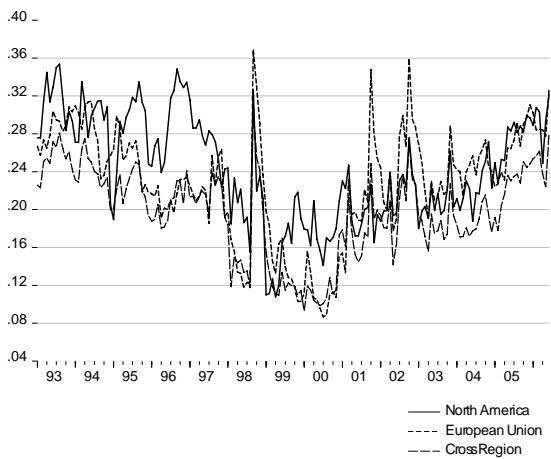
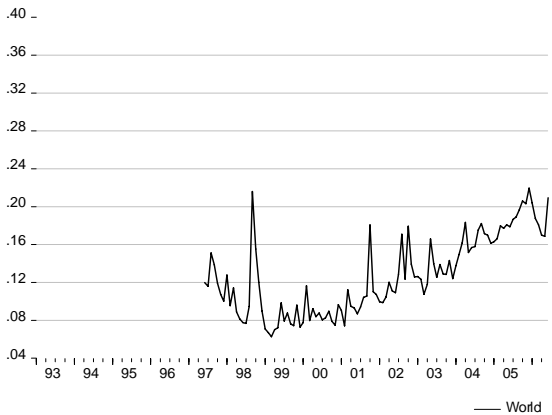
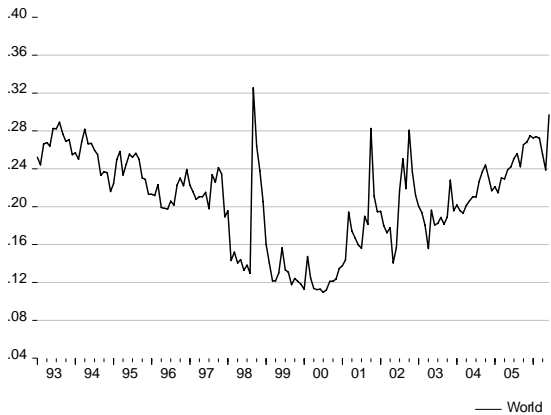


TABLE 1: TEST FOR A BREAK IN THE COMMON TREND AND ESTIMATED SLOPES OF THE TREND

| Sample | Region | sup F -stat | p -value | Break date | α | β before break | β after break |
|-----------|----------------|---------------|------------|------------|----------|----------------------|---------------------|
| 1993-2006 | World | 1604.05 | <1% | 2000.04 | 0.5918 | -0.0034 | 0.0033 |
| | European Union | 491.46 | <1% | 1999.12 | 0.6742 | -0.0043 | 0.0040 |
| | North America | 100.82 | <1% | 2000.06 | 0.6845 | -0.0031 | 0.0026 |
| | Cross | 362.65 | <1% | 2000.01 | 0.5593 | -0.0032 | 0.0028 |
| 1997-2006 | World | 644.99 | <1% | 1999.10 | 0.2488 | -0.0026 | 0.0027 |
| | European Union | 114.28 | <1% | 1999.08 | 0.2308 | -0.0025 | 0.0027 |
| | North America | 199.37 | <1% | 2001.11 | 0.3084 | -0.0011 | 0.0049 |
| | Cross | 276.64 | <1% | 1999.12 | 0.2698 | -0.0028 | 0.0030 |

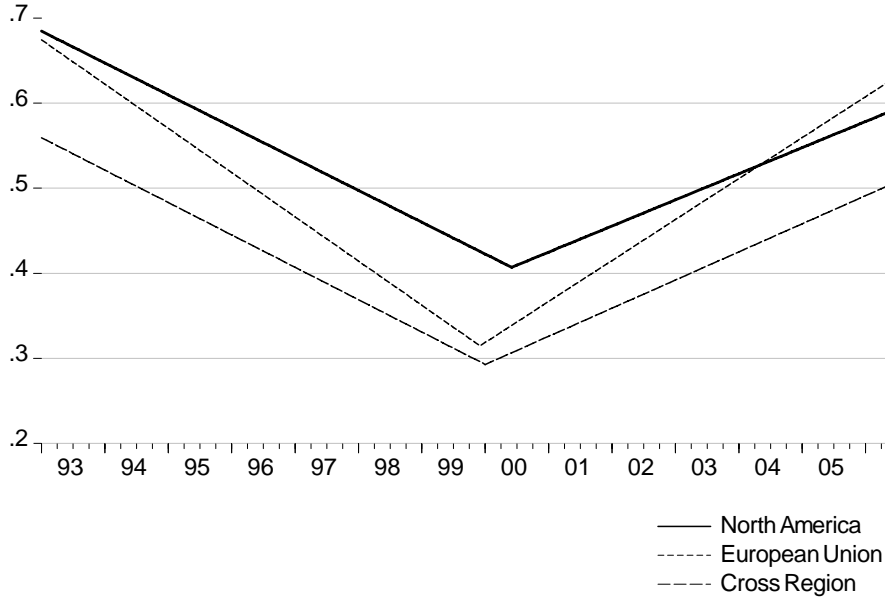
All coefficients are significant at a 1% confidence level. A sup F -statistics greater than 16.64 indicates that the null hypothesis of no break is rejected at a 1% confidence level.

effect particular to each pair and ε_t^{ij} is a independant heteroskedastic error term, which is normally distributed with variance σ_{ij}^2 . We then tested the hypothesis of a break at time t_0 in the slope δ of the trend. We used the test developed by Bai and Perron (1998 and 2003) which simultaneously estimates the most probable break date and then tests if the break is statistically significant. We do the estimation and the test for both sample and for the entire set of correlations, for correlations between North American banks, for correlations between EU banks and for correlations between North American and EU banks. The results are presented in Table 1.

The estimated break dates lie between August 1999 and June 2000 (with an exception for the North American banks in the short sample). It indicates that a change in banks' common exposure occurred around the beginning of this century. Before this date, the trends are negative, which implies that the common exposure to shocks had a tendency to decrease. This suggests that, during this period, banks were differentiating from each other, adopting different strategies or markets. After 2000, the trends reverse and common exposure to shocks increases, hinting at increasing similarities or interdependencies between banks. An increase in banks co-movements since 1999 is also documented by Brasili and Vulpes (2005).

This trend reverse is observed in both regions and between these regions. Figure 2

FIGURE 2: ESTIMATED COMMON TRENDS IN AD RATIOS CORRELATION (LONG SAMPLE, IN LOGIT TRANSFORMED UNITS)



displays the estimated trend for correlation between North American banks, between EU banks and between banks of each region.¹³ Except at the end of the sample, the correlation between North American banks is higher than between EU banks. North American banks seem to be more commonly exposed to shock, or more similar, than EU banks. The correlation between EU and North American banks is the lowest, indicating that banks from different regions are less commonly exposed to shocks or more differentiated. This result is in line with Hawkesby, Marsh and Stevens (2005), who find a high degree of heterogeneity between both sub-groups and a higher correlation between US banks. Note finally that the slope of the EU banks is steeper in both phases. EU banks have differentiated more strongly during the pre-2000 period and then have become more similar than North American banks, to the point that the correlation between EU banks seems higher now than between US banks.

¹³The trend for the short sample are not presented here but can be obtained by the authors. The conclusions are similar.

5 Evolution of systemic risk

Do the changes observed in the banking industry in the past years have any impact on the systemic risk? In particular, does the increase in common exposure to shocks observed since 2000 generate a higher systemic risk? To answer these questions, we constructed a set of indices of systemic risk. These indices are based on Lehar (2005). They reflect the probability to observe a systemic crisis in the banking sector. A systemic crisis is defined as the simultaneous default of a given proportion of the banking sector (cf. Section 2.4). We computed a global systemic risk index for all the banks in our sample and two regional sub-indices, one for North American banks and one for EU banks. For all three regions (World, North America, EU), we computed two indices: one for which a default of 10% of the banking sector triggers a crisis, one for which 20% of the banking sector must default. We computed these 6 indices for both samples (cf. Section 3), which makes a total of 12 systemic risk indices. These indices are presented in Figure 3.

The global index points out two periods of high systemic risk: at the end of 1998 and at end 2002 - beginning 2003. These two episodes correspond to the LTCM and Russian crisis in 1998 and to the stock market downturn in 2002-2003. The systemic risk during the rest of the sample is less acute. The 1998 peak is observed in both the US and the EU sub-indexes. The EU banks seem to have been more affected than the North American banks in 2002-2003, probably because they were also facing bad economic conditions at that time. In the US banking system, a period of higher systemic risk is also observed in 1994-1995, which translates by a slightly higher systemic risk in the global index.

5.1 Trends in the systemic risk index

A quick look at Figure 3 suggests that the path of the systemic risk index is very different from the evolution of banks' common exposure to shocks presented in Figure

FIGURE 3: SYSTEMIC RISK INDEX (LEFT: LONG SAMPLE, RIGHT: SHORT SAMPLE)

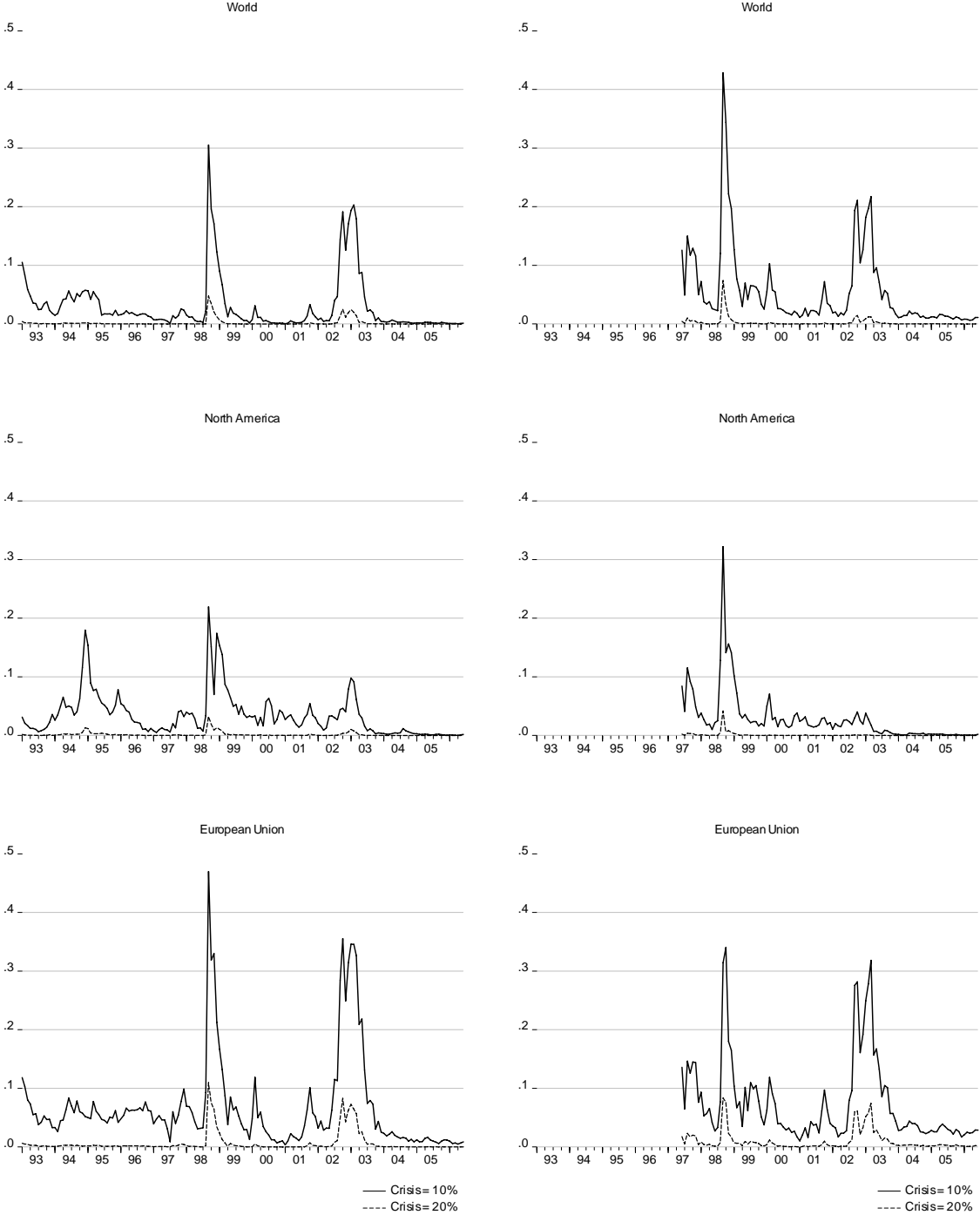


TABLE 2: TREND IN THE SYSTEMIC RISK INDEX

| Sample | Index | Trend before 2000 | Trend after 2000 |
|-----------|-------------------|-------------------|------------------|
| 1993-2006 | World 10% | -0.0196** | 0.0109 |
| | World 20% | -0.0233** | 0.0034 |
| | North America 10% | 0.0075 | -0.0110 |
| | North America 20% | 0.0018 | -0.0079 |
| | EU 10% | 0.0031 | 0.0019 |
| | EU 20% | 0.0022 | 0.0142 |
| 1997-2006 | World 10% | -0.0151 | -0.0076 |
| | World 20% | -0.0435 | 0.0012 |
| | North America 10% | -0.0223 | -0.0343** |
| | North America 20% | -0.0397 | -0.0118** |
| | EU 10% | -0.0120 | -0.0030 |
| | EU 20% | -0.0299 | -0.0004 |

* (**) indicates that the coefficient is significant at a 5% (1%) confidence level. The percentage associated to each index corresponds to the proportion of the banking system that need to default to trigger a crisis (e.g. World 10% corresponds to the systemic risk index for which a 10% of the banking system has to default to trigger a crisis).

1. The latter has a distinct V-shape, whereas the former is characterized by two peaks of higher systemic risk for the banking sector. This visual impression is confirmed when we try to fit a trend with a break to the systemic risk index at the beginning of 2000, as we did for correlations (see Equation 11). The results of this regression are presented in Table 2. Since the index is a probability bounded between 0 and 1, we estimate the coefficients with a logit regression using weighted least squares as suggested by Greene (2000).¹⁴

Most indices do not display any significant trend. Only the global and the North American systemic risk indices seem to have significantly decreased before 2000 in the long sample and after 2000 in the short sample, respectively. Furthermore, no significant break date is detected. This result contrasts with the unambiguous trends and breaks observed in the AD ratio correlation: While a clear V-shaped trend appears in the dynamic of banks' co-movements, no apparent trend is detected in the systemic risk index' pattern . Moreover, when a trend is detected in the systemic risk index, it is not always associated to the same direction in the co-movement's trends (i.e., two

¹⁴All regressions in this section are made using this method.

episodes of negative trends are detected for the systemic risk index, one in a period of declining correlation and one in a period of increasing correlation).

Many other studies record a similar rise in banks' co-movements and most of them conclude, without explicitly checking it, that this increase induces a higher systemic risk. Given our results, however, the direction of this link is questionable. Therefore, assessing the systemic risk by looking at the level of banks' common exposure to shocks seems to give an ill-defined picture of the situation.

5.2 Are common exposure and systemic risk related?

The results of the previous section raise questions about the link between banks' common exposures (i.e. AD ratio correlations) and systemic risk in the banking sector. Do common exposures really play a role for systemic risk? How can we interpret a change in common exposure in terms of systemic risk? From the construction of the systemic risk index (see Section 2.4), it is obvious that three elements determine its value: 1) the *correlation structure* between banks' AD ratios, 2) the *volatilities* of the AD ratios and 3) the *level* of the AD ratios. While the first component captures the systemic characteristics of a banking sector, the last two components are bank-specific. Combined in the *distance-to-default*¹⁵, they describe a bank's individual risk taking. The systemic risk index is a function of these systemic and bank-specific dimensions. Unfortunately, we do not know the exact form of this function. We can guess though that it is likely to be non linear.

To get an idea about each factor's influence on the systemic risk index, we compute the rank correlation between the systemic risk index and (i) the banks' AD ratio correlations and (ii) the banks' distance-to-default. The rank correlation statistics instead of the traditional (linear) correlation (Pearson coefficient) are chosen as they measure the link between two variables independently of the form taken by the function

¹⁵The distance-to-default is equal to the level of the AD ratio divided by its volatility.

TABLE 3: RANK CORRELATION BETWEEN THE SYSTEMIC RISK INDEX AND DIFFERENT FACTORS

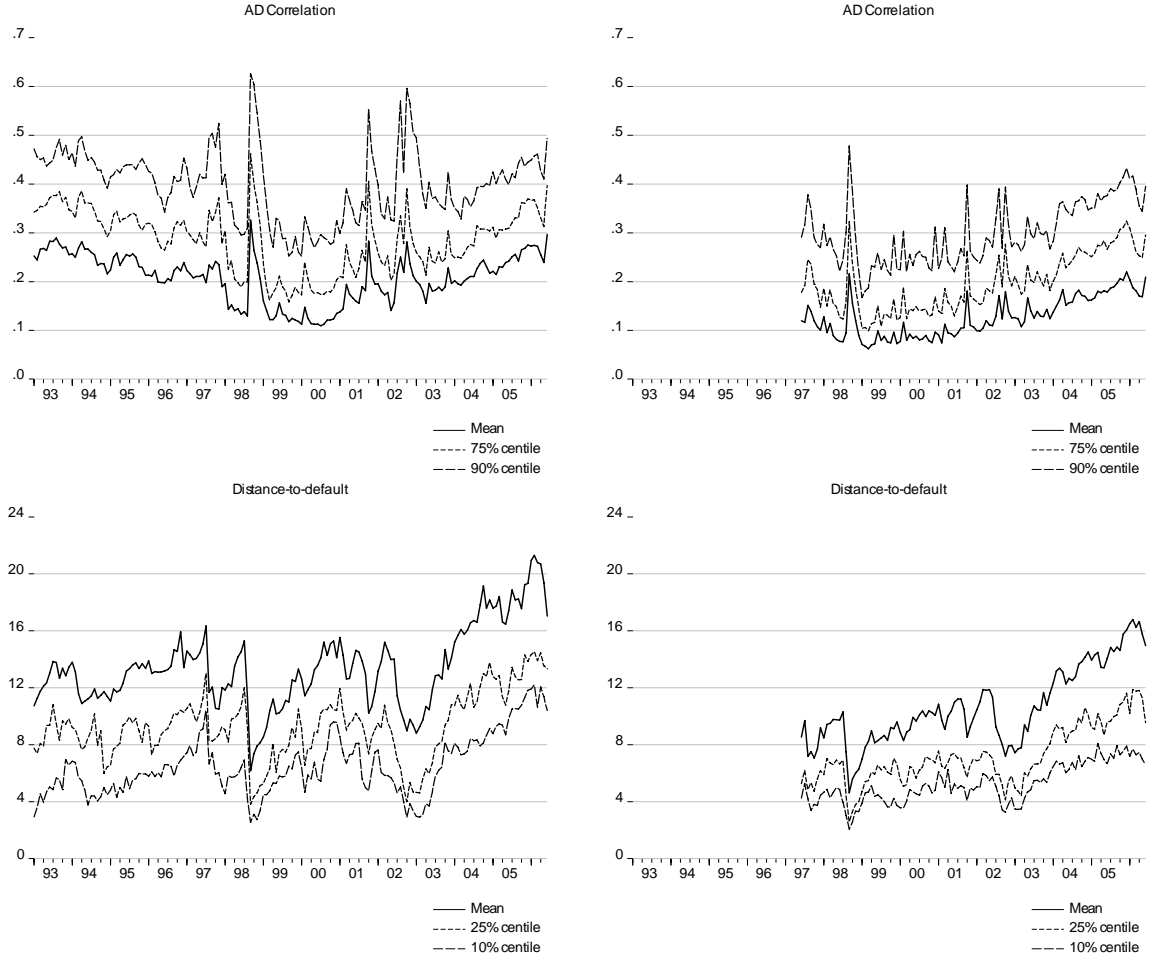
| Sample | Index | Factor | Correlation | | Distance-to-default | |
|-----------|-----------|----------------|----------------------|---------------|----------------------|---------------|
| | | | Spearman coefficient | Kendall's tau | Spearman coefficient | Kendall's tau |
| 1993-2006 | World 10% | Average | 0.2369 | 0.1620 | -0.8587 | -0.6833 |
| | | 75% percentile | 0.2874 | 0.1949 | -0.8549 | -0.6752 |
| | | 90% percentile | 0.4291 | 0.2880 | -0.9046 | -0.7429 |
| | World 20% | Average | 0.3041 | 0.2254 | -0.7880 | -0.6379 |
| | | 75% percentile | 0.3310 | 0.2464 | -0.7553 | -0.6093 |
| | | 90% percentile | 0.4731 | 0.3518 | -0.8008 | -0.6470 |
| 1997-2006 | World 10% | Average | -0.3510 | -0.2339 | -0.9348 | -0.7919 |
| | | 75% percentile | -0.3424 | -0.2319 | -0.9375 | -0.7976 |
| | | 90% percentile | -0.3041 | -0.2159 | -0.8984 | -0.7256 |
| | World 20% | Average | -0.0618 | -0.0472 | -0.7630 | -0.5939 |
| | | 75% percentile | -0.0579 | -0.0429 | -0.7730 | -0.6097 |
| | | 90% percentile | -0.0470 | -0.0401 | -0.6837 | -0.5238 |

The fourth (fifth) column gives the Spearman (Kendall) coefficient between different systemic risk indices and the mean, 75% centile and 90% centile of the correlation between banks' AD ratio. The sixth (seventh) column gives the Spearman (Kendall) coefficient between different systemic risk indices and the mean, 25% centile and 10% centile of the banks' distance to default.

linking them. We use the Spearman rank-order correlation coefficient and the Kendall measure of correlation to compute the rank correlation between the systemic risk index and the banks' average ratio correlation on the one hand and between the systemic risk index and the banks' average distance-to-default on the other hand. However, the average correlation (or average distance-to-default) is a single number and might not adequately reflect the complete structure of correlations (distance-to-defaults) in the banking sector. In particular, it is possible that the systemic risk index is mainly influenced by extreme values of correlations or distance-to-defaults (i.e., by banks that are extremely commonly exposed or extremely close to default). To check for that, we also use the 75% (25%) and the 90% (10%) percentiles of the correlations (distance-to-defaults). The evolution of the average and centiles of correlations and distance-to-defaults are displayed in Figure 4. The rank correlation between the systemic risk index and these different measures are presented in Table 3.¹⁶

¹⁶We present the results for the World index only. The results for the regional systemic risk indices can be obtained by the authors. Their conclusions do not differ from those presented here.

FIGURE 4: EVOLUTION OF THE AD CORRELATIONS AND THE DISTANCE-TO-DEFAULTS (LEFT: LONG SAMPLE, RIGHT: SHORT SAMPLE)



The results for the rank correlation show that the link between systemic risk and banks' common exposure (i.e., AD ratio correlation) is ambiguous. A positive relationship is identified in the long sample, while the same relationship appears to be negative or non-significant in the short sample. On the opposite, the link between systemic risk and distance-to-default is always negative. Moreover, the rank correlation between systemic risk and distance-to-default is always stronger than the one between systemic risk and common exposure. We thus draw the following main conclusion: Low distance-to-default is a much stronger and much more reliable sign of systemic risk than high correlation. The effect of banks' common exposure to shocks on systemic risk is weaker and can even change direction depending on the period.

The strong link between distance-to-default and systemic risk is illustrated by Figure 5, in which a logit transformation of the systemic risk index is plotted against AD ratio correlation (left) and distance-to-default (right), respectively. Clearly, the dispersion with the distance-to-default is smaller than with correlation. Interestingly, with this transformation, the link between the systemic risk and the distance-to-default seems to be relatively linear.

Table 4 presents the results of a linear regression of the (logit of the) systemic risk index on the AD ratio correlation and on the distance-to-default, respectively. The coefficient on the distance-to-default is significantly negative in all specifications. The degree of correspondance between the index and the estimated regression is striking. The results from the regression with correlation are less convincing: We have a poor coefficient of partial correlation and its sign is either non significant or negative. Slightly better results are obtained when using the 90% centile of correlation.

Not surprisingly, these regression results coincide with those obtained from the rank correlation analysis. The common exposure (i.e., AD ratio correlation) is a poor predictor of the systemic risk index and the direction of its relation changes depending on the period. The distance-to-default, by contrast, explains well the systemic risk

TABLE 4: REGRESSION OF THE SYSTEMIC RISK INDEX ON CORRELATION AND DISTANCE-TO-DEFAULT

| Sample | Regressor | World 10% | | World 20% | |
|-----------|-------------------------------------|-------------|--------|-------------|--------|
| | | Coefficient | R^2 | Coefficient | R^2 |
| 1993-2006 | Mean correlation | 7.6496* | 0.0396 | 17.4344** | 0.1416 |
| | 75% centile of correlations | 10.4503** | 0.1271 | 20.1928** | 0.3220 |
| | 90% centile of correlations | 15.8976** | 0.5599 | 25.3561** | 0.7423 |
| | Mean distance-to-default | -0.5073** | 0.7761 | -0.8087** | 0.8164 |
| | 25% centile of distance-to-defaults | -0.6618** | 0.8601 | -1.1114** | 0.8683 |
| | 10% centile of distance-to-defaults | -0.9528** | 0.8545 | -1.4776** | 0.7810 |
| 1997-2006 | Mean correlation | -4.1073 | 0.0234 | -0.4085 | 0.0001 |
| | 75% centile of correlations | -3.2653 | 0.0306 | -0.3175 | 0.0001 |
| | 90% centile of correlations | -3.6337* | 0.0473 | 0.0664 | 0.0000 |
| | Mean distance-to-default | -0.5672** | 0.8735 | -0.9852** | 0.8830 |
| | 25% centile of distance-to-defaults | -0.7476** | 0.8908 | -1.2656** | 0.8935 |
| | 10% centile of distance-to-defaults | -1.0445** | 0.7599 | -1.9254** | 0.7650 |

* (**) indicates that the coefficient is significant at a 5% (1%) level.

index. This difference in precision is illustrated by Figure 6. In each panel, the actual systemic risk index is compared with a proxy given by a linear function of one of the two factors (i.e., common exposure and distance-to-default, respectively). The match between the systemic risk index and its proxy based on the 10% percentile of distance-to-default is striking (right and lower panel of Figure 6). Taking distance-to-default as a proxy for measuring systemic risk seems to give a fairly good and easily obtainable estimation of it.

However, while the distance-to-default seems to be the main factor driving the systemic risk, the common exposure might account for the portion of the systemic risk index that is not explained by the distance-to-default. To check that, we compute the rank correlation between the common exposure and the residuals of a regression of the systemic risk index on the distance-to-default and the correlation. The idea is to check if a positive residual (i.e., an "excess" of systemic risk given what is estimated by distance-to-default) is associated with a high or a low common exposure. The rank correlations are presented in Table 5.

The results show that residuals are positively correlated with common exposure.

FIGURE 5: SYSTEMIC RISK INDEX VS. AD CORRELATIONS (LEFT) OR DISTANCE-TO-DEFAULT (RIGHT) (LONG SAMPLE)

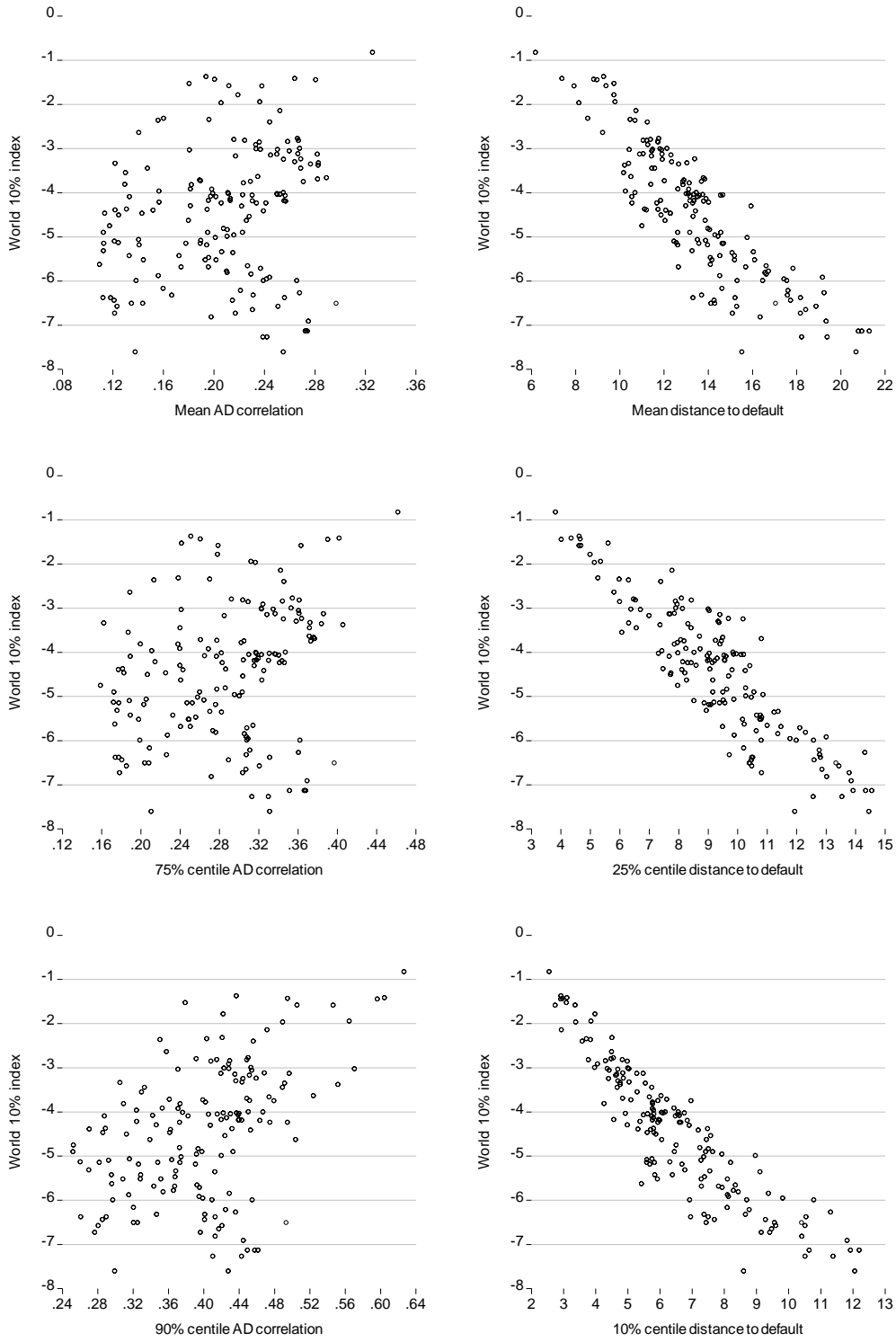


FIGURE 6: ESTIMATED SYSTEMIC RISK INDEX (LONG SAMPLE)

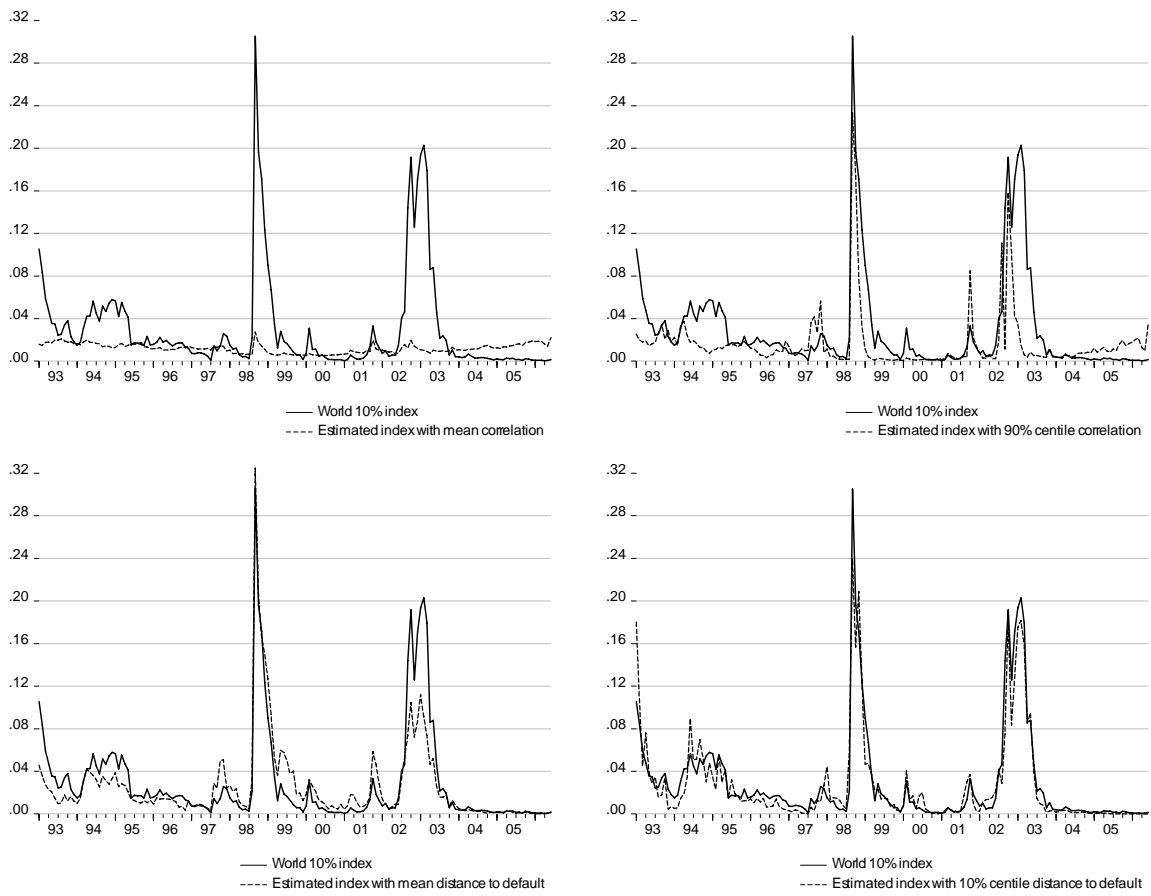


TABLE 5: RANK CORRELATION WITH RESIDUALS

| Sample | Residuals from equation using: | Regressor | Spearman coefficient | Kendall's tau |
|--------|--------------------------------|-----------------------|----------------------|---------------|
| 1993 | Mean distance-to-default | Mean correlation | 0.6468 | 0.4638 |
| - | | 75% perc. correlation | 0.5845 | 0.4142 |
| 2006 | | 90% perc. correlation | 0.5158 | 0.3677 |
| | 10% perc. distance-to-default | Mean correlation | 0.3944 | 0.2752 |
| | | 75% perc. correlation | 0.3476 | 0.2366 |
| | | 90% perc. correlation | 0.3079 | 0.2079 |
| 1997 | Mean distance-to-default | Mean correlation | 0.6509 | 0.4540 |
| - | | 75% perc. correlation | 0.6185 | 0.4166 |
| 2006 | | 90% perc. correlation | 0.5300 | 0.3683 |
| | 10% perc. distance-to-default | Mean correlation | 0.4914 | 0.3428 |
| | | 75% perc. correlation | 0.4630 | 0.3211 |
| | | 90% perc. correlation | 0.3757 | 0.2565 |

The degree of correlation is significantly higher than between common exposures and the systemic risk index (cf. Table 3), suggesting that the effect of common exposures on the unexplained part of systemic risk is greater than on the systemic risk itself. We also see that the rank correlations are all positive. This means that, *once the distance to default is taken into account*, a higher common exposure correlation always induces a higher systemic risk.

To conclude, we find that the systemic risk dynamic does not match the pattern of the banks' common exposure. This indicates that common exposure is probably not the main factor explaining systemic risk. Indeed, further analysis reveals the banks' distance-to-default as the main driving force. However, we find that common exposures explain relatively well the deviation of systemic risk from the value implied by distance-to-default. We also find that, once the distance to default is taken into account, higher common exposure induces higher systemic risk.

6 Conclusion

This paper answers two questions. The first one is: how has international banks' common exposure to shocks evolved in the last decade in response to the fundamental changes observed in the banking sector's environment? To answer this question, we estimate the correlations between large international banks' asset-to-debt (AD) ratios over 1993-2006 with the Flexible MGARCH approach developed by Ledoit, Santa-Clara and Wolf (2003). We find a decreasing trend until 2000 followed by an increasing trend since. This suggests that during the nineties, banks (or at least some of them) have taken advantage of the new technologies and markets available to them to pursue their own business strategy and to differentiate from each other, thus reducing their common exposure. Since 2000, however, the banks' common exposure to shocks increased rapidly, which could indicate that they adopt increasingly similar strategies. This finding is also robust for different sub-groups of the sample.

The paper's second question concerns the impact of these trends on systemic risk in the banking sector. From a theoretical point of view, ongoing financial market integration and rising cross-border activities may have both favourable and adverse effects on the stability of the banking system. To explore this question empirically, we construct a systemic risk index based on Lehar (2005) where systemic risk is defined as the probability of a joint failure of a critical number of banks. In contrast to the correlation analysis, no clear trend emerges. Instead, we observe two peaks in the end of 1998 (LTCM and Russian crisis) and in 2002-2003 (stock market downturn), the latter mainly hurting European banks.

The different patterns observed for banks' common exposure and for the systemic risk contrasts with the widespread view that systemic risk increases with banks' co-movement. Our further results confirm that correlation between AD ratios is not a reliable measure for systemic risk. Instead, we find that the distance- to-default—a measure that combines the volatility and the level of the AD ratios—is the main driver

of the systemic risk index. Once this distance to default is taken into account, however, correlation is positively associated with systemic risk.

These findings have two direct consequences for supervisory authorities: first, they show that systemic risk cannot be viewed as a pure correlation phenomenon. Instead, they highlight the danger of high and volatile leverage. According to our results, the main driver of systemic risk is the size of the risks taken by each bank individually (distance-to-default) rather by their common exposure to shocks (AD ratio correlation). Thus, supervisors concerned by systemic stability should first concentrate on making sure that banks are not taking disproportionate risks. Reducing inter-linkage or enforcing diversification in the banking sector seems to be a second best policy. Second, from the monitoring point of view, co-movements between banks seem to be a spurious measure of systemic risk. Taken individually, it gives, in the best case, a weak signal about systemic risk or, in the worst case, a signal of wrong direction. To be useful and unambiguous about the evolution of systemic risk, co-movement must be interpreted in combination with distance-to-default.

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A Reduced form asset un debt dynamic

Developping each equation of the system

$$\begin{bmatrix} d\mathbf{A}_t \\ d\mathbf{D}_t \end{bmatrix} = \begin{bmatrix} \mathbf{A}_t & 0 \\ 0 & \mathbf{D}_t \end{bmatrix} \begin{bmatrix} \boldsymbol{\mu}_A \\ \boldsymbol{\mu}_D \end{bmatrix} dt + \begin{bmatrix} \mathbf{A}_t & 0 \\ 0 & \mathbf{D}_t \end{bmatrix} \begin{bmatrix} \boldsymbol{\Upsilon}_{AA} & \boldsymbol{\Upsilon}_{AD} \\ \boldsymbol{\Upsilon}_{DA} & \boldsymbol{\Upsilon}_{DD} \end{bmatrix} \begin{bmatrix} d\mathbf{w}_A \\ d\mathbf{w}_D \end{bmatrix}$$

yields

$$\begin{aligned} dA_t^i &= A_t^i \mu_A^i dt + A_t^i \left(\sum_{j=1}^n v_{AA}^{ij} dw_A^j + \sum_{j=1}^n v_{AD}^{ij} dw_D^j \right) = A_t^i \mu_A^i dt + A_t^i \sigma_{A^i} dw_{A^i} \\ dD_t^i &= D_t^i \mu_D^i dt + D_t^i \left(\sum_{j=1}^n v_{DA}^{ij} dw_A^j + \sum_{j=1}^n v_{DD}^{ij} dw_D^j \right) = D_t^i \mu_D^i dt + D_t^i \sigma_{D^i} dw_{D^i} \end{aligned}$$

which are both Itô processes where $\sigma_{A^i} = \left(\sum_{j=1}^n (v_{AA}^{ij})^2 + \sum_{j=1}^n (v_{AD}^{ij})^2 \right)^{1/2}$ and $\sigma_{D^i} = \left(\sum_{j=1}^n (v_{DA}^{ij})^2 + \sum_{j=1}^n (v_{DD}^{ij})^2 \right)^{1/2}$. Using Itô's formula, we get that

$$\begin{aligned} d \ln A_t^i &= \left(\mu_A^i - \frac{1}{2} \sigma_{A^i}^2 \right) dt + \sigma_{A^i} dw_{A^i} \\ d \ln D_t^i &= \left(\mu_D^i - \frac{1}{2} \sigma_{D^i}^2 \right) dt + \sigma_{D^i} dw_{D^i} \end{aligned}$$

To get the dynamic of dz_t^i , note that $z_t^i = \ln A_t^i - \ln D_t^i$, which implies that $dz_t^i = d \ln A_t^i - d \ln D_t^i$. Using the two previous equations in this expression gives

$$dz_t^i = \mu_z^i dt + \sigma_{z^i} dw_{z^i}$$

where

$$\begin{aligned}\mu_z^i &= \mu_A^i - \mu_D^i - \frac{1}{2}(\sigma_{A^i}^2 - \sigma_{D^i}^2) \\ \sigma_{z^i} &= \left(\sum_{j=1}^n (v_{AA}^{ij})^2 + \sum_{j=1}^n (v_{AD}^{ij})^2 + \sum_{j=1}^n (v_{DA}^{ij})^2 + \sum_{j=1}^n (v_{DD}^{ij})^2 \right)^{1/2}\end{aligned}$$

We can see that the dynamic of the AD ratio sums up the instantaneous growth rate of the bank's asset and debts as well as all interactions with shocks to its own and other banks' assets and debts.

Regrouping this equation for all the banks in one single system yields

$$dz_t = \boldsymbol{\mu}dt + \boldsymbol{\Upsilon}d\mathbf{w}$$

B Simulation algorithm

To compute the systemic crisis index $I_t(\theta)$, we start the simulation at time t with the vector of AD ratio \mathbf{z}_t and the estimated covariance matrix \mathbf{H}_t . The simulation uses the following algorithm:

1. Generate a vector \mathbf{e}_t containing N independent standard white noises. \mathbf{e}_t simulates the independent shocks to the AD ratios.
2. Generate the vector \mathbf{u}_t from the vector \mathbf{e}_t with a Choleski decomposition of the variance-covariance matrix \mathbf{H}_t . \mathbf{u}_t is the total effect of the different individual shocks \mathbf{e}_t on each AD ratio.
3. Generate the new vector $\mathbf{z}_{t+1} = \mathbf{z}_t + \Delta\mathbf{z}_{t+1}$ with Equation (7).
4. For each bank, check if $z_{t+1}^i < 0$ (insolvency condition). If one bank becomes insolvent, set $b_t^i = 1$.

5. Check if there is a systemic crisis, i.e. if $\sum_{i=1}^N \theta_t^i b_t^i > \alpha$. Stop the algorithm if there is a systemic crisis.
6. If there is no systemic crisis, compute the new variance-covariance matrix \mathbf{H}_{t+1} with Equation (9), the vector \mathbf{u}_t generated in Step 2 and the estimated coefficient matrices $\tilde{\mathbf{C}}$, $\tilde{\mathbf{A}}$ and $\tilde{\mathbf{B}}$.
7. Repeat Steps 1 to 6 k times.

For each period, repeat this algorithm M times. The probability of a systemic crisis is then estimated by the number of times that the algorithm has stopped over M .